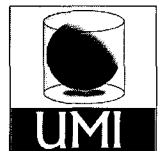


Luc Tartar

# An Introduction to Sobolev Spaces and Interpolation Spaces

 Springer



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