Luc Tartar

An Introduction to Sobolev Spaces and Interpolation Spaces

Springer

UMI

Contents

1 Historical Background 1

2 The Lebesgue Measure, Convolution 9

3 Smoothing by Convolution 15

4 Truncation; Radon Measures; Distributions 17

5 Sobolev Spaces; Multiplication by Smooth Functions 21

6 Density of Tensor Products; Consequences 27

7 Extending the Notion of Support 33

8 Sobolev's Embedding Theorem, $1 \leq p < N$ 37

9 Sobolev's Embedding Theorem, $N \leq p \leq \infty$ 43

10 Poincaré's Inequality 49

11 The Equivalence Lemma; Compact Embeddings 53

12 Regularity of the Boundary; Consequences 59

13 Traces on the Boundary 65

14 Green's Formula 69

15 The Fourier Transform 73

16 Traces of $H^s(R^N)$ 81

17 Proving that a Point is too Small 85

18 Compact Embeddings 89

19 Lax–Milgram Lemma 93

20 The Space $H(div; \Omega)$ 99

21 Background on Interpolation; the Complex Method 103

22 Real Interpolation; K-Method 109

23 Interpolation of L^2 Spaces with Weights 115

24 Real Interpolation; J-Method 119

25 Interpolation Inequalities, the Spaces $(E_0, E_1)_{\theta,1}$ 123

26 The Lions–Peetre Reiteration Theorem 127

27 Maximal Functions 131

28 Bilinear and Nonlinear Interpolation 137

29 Obtaining L^p by Interpolation, with the Exact Norm 141

30 My Approach to Sobolev's Embedding Theorem 145

31 My Generalization of Sobolev's Embedding Theorem 149

32 Sobolev's Embedding Theorem for Besov Spaces 155

33 The Lions–Magenes Space $H_{00}^{1/2}(\Omega)$ 159

34 Defining Sobolev Spaces and Besov Spaces for Ω 163

35 Characterization of $W^{s,p}(R^N)$ 165

36 Characterization of $W^{s,p}(\Omega)$ 169

37 Variants with BV Spaces 173

38 Replacing BV by Interpolation Spaces 177

39 Shocks for Quasi-Linear Hyperbolic Systems 183

40 Interpolation Spaces as Trace Spaces 191

41 Duality and Compactness for Interpolation Spaces 195

42 Miscellaneous Questions 199

43 Biographical Information 205

44 Abbreviations and Mathematical Notation 209

References 213

Index 215