

An Introduction to Soil Dynamics

Theory and Applications of Transport in Porous Media

Series Editor:

Jacob Bear, *Department of Civil and Environmental Engineering,
Technion – Israel Institute of Technology, Haifa, Israel*

Volume 24

For further volumes: <http://www.springer.com/series/6612>.

An Introduction to Soil Dynamics

Arnold Verruijt

Delft University of Technology, Delft,
The Netherlands

 Springer

Arnold Verruijt
Delft University of Technology
2628 CN Delft
Netherlands
a.verruijt@verruijt.net

A CD-ROM accompanies this book containing programs for waves in piles, propagation of earthquakes in soils, waves in a half space generated by a line load, a point load, a strip load, or a moving load, and the propagation of a shock wave in a saturated elastic porous material.

Computer programs are also available from the website <http://geo.verruijt.net>

ISBN 978-90-481-3440-3

e-ISBN 978-90-481-3441-0

DOI 10.1007/978-90-481-3441-0

Springer Dordrecht Heidelberg London New York

Library of Congress Control Number: 2009940507

© Springer Science+Business Media B.V. 2010

No part of this work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission from the Publisher, with the exception of any material supplied specifically for the purpose of being entered and executed on a computer system, for exclusive use by the purchaser of the work.

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Preface

This book gives the material for an introductory course on Soil Dynamics, as given for about 10 years at the Delft University of Technology for students of civil engineering, and updated continuously since 1994.

The book presents the basic principles of elastodynamics and the major solutions of problems of interest for geotechnical engineering. For most problems the full analytical derivation of the solution is given, mainly using integral transform methods. These methods are presented briefly in Appendix A. The elastostatic solutions of many problems are also given, as an introduction to the elastodynamic solutions, and as possible limiting states of the corresponding dynamic problems. For a number of problems of elastodynamics of a half space exact solutions are given, in closed form, using methods developed by Pekeris and De Hoop. Some of these basic solutions are derived in full detail, to assist in understanding the beautiful techniques used in deriving them. For many problems the main functions for a computer program to produce numerical data and graphs are given, in C. Some approximations in which the horizontal displacements are disregarded, an approximation suggested by Westergaard and Barends, are also given, because they are much easier to derive, may give a first insight in the response of a foundation, and may be a stepping stone to solving the more difficult complete elastodynamic problems.

The book is directed towards students of engineering, and may be giving more details of the derivations of the solutions than strictly necessary, or than most other books on elastodynamics give, but this may be excused by my own difficulties in studying the subject, and by helping students with similar difficulties.

The book starts with a chapter on the behaviour of the simplest elementary system, a system consisting of a mass, supported by a linear spring and a linear damper. The main purpose of this chapter is to define the basic properties of dynamical systems, for future reference. In this chapter the major forms of damping of importance for soil dynamics problems, viscous damping and hysteretic damping, are defined and their properties are investigated.

Chapters 2 and 3 are devoted to one dimensional problems: wave propagation in piles, and wave propagation in layers due to earthquakes in the underlying layers, as first developed in the 1970s at the University of California, Berkeley. In these chapters the mathematical methods of Laplace and Fourier transforms, characteristics,

and separation of variables, are used and compared. Some simple numerical models are also presented.

The next two chapters (Chaps. 4 and 5) deal with the important effect that soils are usually composed of two constituents: solid particles and a fluid, usually water, but perhaps oil, or a mixture of a liquid and gas. Chapter 4 presents the classical theory, due to Terzaghi, of semi-static consolidation, and some elementary solutions. In Chap. 5 the extension to the dynamical case is presented, mainly for the one dimensional case, as first presented by De Josselin de Jong and Biot, in 1956. The solution for the propagation of waves in a one dimensional column is presented, leading to the important conclusion that for most problems a practically saturated soil can be considered as a medium in which the solid particles and the fluid move and deform together, which in soil mechanics is usually denoted as a state of undrained deformations. For an elastic solid skeleton this means that the soil behaves as an elastic material with Poisson's ratio close to 0.5.

Chapters 6 and 7 deal with the solution of problems of cylindrical and spherical symmetry. In the chapter on cylindrically symmetric problems the propagation of waves in an infinite medium introduces Rayleigh's important principle of the radiation condition, which expresses that in an infinite medium no waves can be expected to travel from infinity towards the interior of the body.

Chapters 8 and 9 give the basic theory of the theory of elasticity for static and dynamic problems. Chapter 8 also gives the solution for some of the more difficult problems, involving mixed boundary value conditions. The corresponding dynamic problems still await solution, at least in analytic form. Chapter 9 presents the basics of dynamic problems in elastic continua, including the general properties of the most important types of waves: compression waves, shear waves, Rayleigh waves and Love waves, which appear in other chapters.

Chapter 10, on confined elastodynamics, presents an approximate theory of elastodynamics, in which the horizontal deformations are artificially assumed to vanish, an approximation due to Westergaard and generalized by Barends. This makes it possible to solve a variety of problems by simple means, and resulting in relatively simple solutions. It should be remembered that these are approximate solutions only, and that important features of the complete solutions, such as the generation of Rayleigh waves, are excluded. These approximate solutions are included in the present book because they are so much simpler to derive and to analyze than the full elastodynamic solutions. The full elastodynamic solutions of the problems considered in this chapter are given in Chaps. 11–13.

In soil mechanics the elastostatic solutions for a line load or a distributed load on a half plane are of great importance because they provide basic solutions for the stress distribution in soils due to loads on the surface. In Chaps. 11 and 12 the solution for two corresponding elastodynamic problems, a line load on a half plane and a strip load on a half plane, are derived. These chapters rely heavily on the theory developed by Cagniard and De Hoop. The solutions for impulse loads, which can be found in many publications, are first given, and then these are used as the basics for the solutions for the stresses in case of a line load constant in time. These solutions should tend towards the well known elastostatic limits, as they indeed do.

An important aspect of these solutions is that for large values of time the Rayleigh wave is clearly observed, in agreement with the general wave theory for a half plane. Approximate solutions valid for large values of time, including the Rayleigh waves, are derived for the line load and the strip load. These approximate solutions may be useful as the basis for the analysis of problems with a more general type of loading.

Chapter 13 presents the solution for a point load on an elastic half space, a problem first solved analytically by Pekeris. The solution is derived using integral transforms and an elegant transformation theorem due to Bateman and Pekeris. In this chapter numerical values are obtained using numerical integration of the final integrals.

In Chap. 14 some problems of moving loads are considered. Closed form solutions appear to be possible for a moving wave load, and for a moving strip load, assuming that the material possesses some hysteretic damping.

Chapter 15, finally, presents some practical considerations on foundation vibrations. On the basis of solutions derived in earlier chapters approximate solutions are expressed in the form of equivalent springs and dampings.

The text has been prepared using the L^AT_EX version (Lamport, 1994) of the program T_EX (Knuth, 1986). The P_CT_EX macros (Wichura, 1987) have been used to prepare the figures. Modern software provides a major impetus to the production of books and papers in facilitating the illustration of complex solutions by numerical and graphical examples. In this book many solutions are accompanied by parts of computer programs that have been used to produce the figures, so that readers can compose their own programs. It is all the more appropriate to acknowledge the effort that must have been made by earlier authors and their associates in producing their publications. A case in point is the paper by Lamb, more than a century ago, with many illustrative figures, for which the computations were made by Mr. Woodall.

The programs used to produce many of the illustrations in the book can be downloaded from the website <http://geo.verruijt.net>. Updates of these programs will be published on this website. Early versions of the book have been published on this website, leading to helpful comments by readers from all over the world.

Many thanks are due to Professor A.T. de Hoop for his many helpful and constructive ideas and comments, and to Dr. C. Cornejo Córdova for several years of joint research. Further comments will be greatly appreciated.

Contents

1	Vibrating Systems	1
1.1	Single Mass System	1
1.2	Characterization of Viscosity	2
1.3	Free Vibrations	3
	Small Damping	4
	Critical Damping	5
	Large Damping	6
1.4	Forced Vibrations	6
	Dissipation of Work	9
1.5	Equivalent Spring and Damping	10
1.6	Solution by Laplace Transform Method	11
1.7	Hysteretic Damping	13
2	Waves in Piles	17
2.1	One-Dimensional Wave Equation	17
2.2	Solution by Laplace Transform Method	18
	2.2.1 Pile of Infinite Length	18
	2.2.2 Pile of Finite Length	20
2.3	Separation of Variables	21
	2.3.1 Shock Wave in Finite Pile	22
	2.3.2 Periodic Load	24
2.4	Solution by Characteristics	26
2.5	Reflection and Transmission of Waves	29
2.6	The Influence of Friction	34
	Infinitely Long Pile	37
2.7	Numerical Solution	38
	Example	39
2.8	A Simple Model for a Pile with Friction	42
	Problems	43

3	Earthquakes in Soft Layers	45
3.1	Earthquake Parameters	46
3.2	Horizontal Vibrations	47
3.2.1	Unloaded Soil Layer	48
3.2.2	Soil Layer with Surface Load	50
3.3	Shear Waves in a Gibson Material	52
3.4	Hysteretic Damping	54
3.4.1	Basic Equations	54
3.4.2	Unloaded Soil Layer	56
3.4.3	Soil Layer with Surface Load	58
3.5	Numerical Solution	60
	Basic Equations	60
	Problems	63
4	Theory of Consolidation	65
4.1	Consolidation	65
4.1.1	Undrained Compression of a Porous Medium	66
4.1.2	The Principle of Effective Stress	67
4.2	Conservation of Mass	68
4.3	Darcy's Law	71
4.4	Equilibrium Equations	72
4.5	Drained Deformations	75
4.6	Undrained Deformations	76
4.7	Cryer's Problem	78
	Basic Equations	78
	Boundary Conditions	79
	Initial Response	80
	Solution of the Problem	80
	Inverse Laplace Transformation	81
	Results	81
4.8	Uncoupled Consolidation	82
	Constant Isotropic Total Stress	83
	Horizontally Confined Deformations	84
4.9	Terzaghi's Problem	85
	Solution	86
	Settlement	87
	Problems	90
5	Dynamics of Porous Media	91
5.1	Basic Differential Equations	91
5.1.1	Conservation of Mass	91
5.1.2	Conservation of Momentum	93
5.1.3	Constitutive Equations	94
5.2	Propagation of Plane Waves	95
5.3	Special Cases	96
5.3.1	Undrained Waves	96

- 5.3.2 Rigid Solid Matrix 97
- 5.4 Analytical Solution 99
 - 5.4.1 Periodic Solution 99
 - 5.4.2 Response to a Sinusoidal Load 103
 - 5.4.3 Approximation of the Solution 104
 - 5.4.4 Numerical Verification 107
 - 5.4.5 Response to a Block Wave 108
- 5.5 Numerical Solution 109
- 5.6 Conclusion 111
- 6 Cylindrical Waves 113**
 - 6.1 Static Problems 113
 - 6.1.1 Basic Equations 113
 - 6.1.2 General Solution 115
 - 6.1.3 Examples 115
 - 6.2 Dynamic Problems 120
 - 6.2.1 Sinusoidal Vibrations at the Cavity Boundary 120
 - 6.2.2 Equivalent Spring and Damping 125
 - 6.3 Propagation of a Shock Wave 127
 - 6.3.1 Solution by Laplace Transform 127
 - 6.3.2 Solution by Fourier Series 129
 - 6.4 Radial Propagation of Shear Waves 131
 - 6.4.1 Sinusoidal Vibrations at the Cavity Boundary 132
 - Problems 134
- 7 Spherical Waves 137**
 - 7.1 Static Problems 137
 - 7.1.1 Basic Equations 137
 - 7.1.2 General Solution 139
 - 7.1.3 Examples 139
 - 7.2 Dynamic Problems 143
 - 7.2.1 Propagation of Waves 144
 - 7.2.2 Sinusoidal Vibrations at the Cavity Boundary 145
 - 7.2.3 Propagation of a Shock Wave 149
 - Problems 154
- 8 Elastostatics of a Half Space 157**
 - 8.1 Basic Equations of Elastostatics 158
 - 8.2 Boussinesq Problems 160
 - 8.2.1 Concentrated Force 162
 - 8.2.2 Uniform Load on a Circular Area 163
 - 8.3 Fourier Transforms 164
 - 8.3.1 Line Load 165
 - 8.3.2 Strip Load 168
 - 8.4 Axially Symmetric Problems 169
 - 8.4.1 Uniform Load on a Circular Area 170

- 8.5 Mixed Boundary Value Problems 172
 - 8.5.1 Rigid Circular Plate 173
 - 8.5.2 Penny Shaped Crack 177
- 8.6 Confined Elastostatics 180
 - 8.6.1 An Axially Symmetric Problem 181
 - 8.6.2 A Plane Strain Problem 184
- 9 Elastodynamics of a Half Space 187**
 - 9.1 Basic Equations of Elastodynamics 188
 - 9.2 Compression Waves 189
 - 9.3 Shear Waves 189
 - 9.4 Rayleigh Waves 190
 - 9.5 Love Waves 194
 - 9.5.1 A Practical Implication 198
- 10 Confined Elastodynamics 201**
 - 10.1 Line Load on Half Space 202
 - 10.2 Line Pulse on Half Space 206
 - 10.3 Strip Load on Half Space 207
 - 10.3.1 Strip Pulse 207
 - 10.3.2 Inversion by De Hoop’s Method 208
 - 10.3.3 Constant Strip Load 212
 - 10.4 Point Load on Half Space 217
 - 10.5 Periodic Load on a Half Space 219
 - Displacement of the Origin 222
 - Vibrating Point Load 224
 - Problems 224
- 11 Line Load on Elastic Half Space 225**
 - 11.1 Line Pulse 225
 - 11.1.1 Description of the Problem 225
 - 11.1.2 Solution by Integral Transform Method 227
 - 11.1.3 The Vertical Displacement 230
 - 11.1.4 The Vertical Displacement of the Surface 244
 - 11.1.5 The Horizontal Displacement 249
 - 11.1.6 The Horizontal Displacement of the Surface 255
 - 11.2 Constant Line Load 260
 - 11.2.1 Isotropic Stress 262
 - 11.2.2 The Vertical Normal Stress 271
 - 11.2.3 The Horizontal Normal Stress 278
 - 11.2.4 The Shear Stress 284
 - Conclusion 289
 - Problems 290
- 12 Strip Load on Elastic Half Space 291**
 - 12.1 Strip Pulse 292

- 12.1.1 The Isotropic Stress 293
- 12.1.2 The Vertical Normal Stress 299
- 12.1.3 The Horizontal Normal Stress 305
- 12.1.4 The Shear Stress 308
- 12.2 Strip Load 312
 - 12.2.1 The Isotropic Stress 313
 - 12.2.2 The Vertical Normal Stress 320
 - 12.2.3 The Horizontal Normal Stress 326
 - 12.2.4 The Shear Stress 335
- Conclusion 342
- Problems 342

- 13 Point Load on an Elastic Half Space 345**
 - 13.1 Problem 345
 - 13.1.1 Basic Equations 345
 - 13.2 Solution 347
 - 13.2.1 Vertical Displacement of the Surface 349
 - 13.2.2 The Pekeris Procedure 350
 - 13.2.3 Numerical Evaluation of the Integrals 355
 - 13.2.4 Computer Program POINTLOAD 358
 - 13.2.5 Results 359

- 14 Moving Loads on an Elastic Half Plane 361**
 - 14.1 Moving Wave 361
 - 14.1.1 Basic Equations 361
 - 14.1.2 Solutions 363
 - 14.1.3 Solution 1 365
 - 14.1.4 Solution 2 367
 - 14.1.5 Completion of the Solution 369
 - 14.1.6 First Boundary Condition 370
 - 14.1.7 Second Boundary Condition 370
 - 14.1.8 The Two Constants 372
 - 14.1.9 Final Solution 372
 - 14.1.10 The Displacement of the Origin 373
 - 14.2 Moving Strip Load 374
 - 14.2.1 Vertical Displacement of the Surface 375
 - 14.2.2 Vertical Normal Stress 376
 - 14.2.3 Isotropic Stress 380
 - 14.2.4 Horizontal Normal Stress 381
 - 14.2.5 Shear Stress 382

- 15 Foundation Vibrations 385**
 - 15.1 Foundation Response 385
 - Circular Footing 386
 - 15.2 Equivalent Spring and Damping 389
 - 15.3 Soil Properties 391

15.4 Propagation of Vibrations 392

15.5 Design Criteria 392

Appendix A Integral Transforms 395

 A.1 Laplace Transforms 395

 A.1.1 Definitions 395

 A.1.2 Example 396

 A.1.3 Heaviside’s Expansion Theorem 397

 A.2 Fourier Transforms 398

 A.2.1 Fourier Series 398

 A.2.2 From Fourier Series to Fourier Integral 400

 A.2.3 Application 401

 A.2.4 List of Fourier Transforms 403

 A.3 Hankel Transforms 406

 A.3.1 Definitions 406

 A.3.2 List of Hankel Transforms 407

 A.4 De Hoop’s Inversion Method 409

 A.4.1 Introduction 409

 A.4.2 Example 410

Appendix B Dual Integral Equations 417

Appendix C Bateman-Pekeris Theorem 419

References 423

Author Index 427

Index 431

CD-ROM

A CD-ROM accompanies this book containing programs for waves in piles, propagation of earthquakes in soils, waves in a half space generated by a line load, a point load, a strip load, or a moving load, and the propagation of a shock wave in a saturated elastic porous material.