Peter W. Christensen · Anders Klarbring

An Introduction to Structural Optimization

Springer

Contents

1 Introduction 1
1.1 The Basic Idea 1
1.2 The Design Process 1
1.3 General Mathematical Form of a Structural Optimization Problem . 3
1.4 Three Types of Structural Optimization Problems 5
1.5 Discrete and Distributed Parameter Systems 7

2 Examples of Optimization of Discrete Parameter Systems 9
2.1 Weight Minimization of a Two-Bar Truss Subject to Stress Constraints 9
2.2 Weight Minimization of a Two-Bar Truss Subject to Stress and Instability Constraints 12
2.3 Weight Minimization of a Two-Bar Truss Subject to Stress and Displacement Constraints 14
2.4 Weight Minimization of a Two-Beam Cantilever Subject to a Displacement Constraint 18
2.5 Weight Minimization of a Three-Bar Truss Subject to Stress Constraints 21
2.6 Weight Minimization of a Three-Bar Truss Subject to a Stiffness Constraint 31
2.7 Exercises 33

3 Basics of Convex Programming 35
3.1 Local and Global Optima 35
3.2 Convexity 37
3.3 KKT Conditions 41
3.4 Lagrangian Duality 46
3.4.1 Lagrangian Duality for Convex and Separable Problems . . 47
3.5 Exercises 52

4 Sequential Explicit, Convex Approximations 57
4.1 General Solution Procedure for Nested Problems 57

4.2 Sequential Linear Programming (SLP) . . . 58
4.3 Sequential Quadratic Programming (SQP) . . . 59
4.4 Convex Linearization (CONLIN) . . . 59
4.5 The Method of Moving Asymptotes (MMA) . . . 66
4.6 Exercises . . . 72

5 Sizing Stiffness Optimization of a Truss . . . 77
5.1 The Simultaneous Formulation of the Problem . . . 77
5.2 The Nested Formulation and Some of Its Properties . . . 84
5.2.1 Convexity of the Nested Problem . . . 85
5.2.2 Fully Stressed Designs . . . 87
5.2.3 Minimization of the Volume Under a Compliance Constraint 88
5.3 Numerical Solution of the Nested Problem Using MMA . . . 91

6 Sensitivity Analysis . . . 97
6.1 Numerical Methods . . . 97
6.2 Analytical Methods . . . 98
6.2.1 Direct Analytical Method . . . 98
6.2.2 Adjoint Analytical Method . . . 99
6.3 Analytical Calculation of Pseudo-loads . . . 100
6.3.1 Bars . . . 101
6.3.2 Plane Sheets . . . 104
6.4 Exercises . . . 112

7 Two-Dimensional Shape Optimization . . . 117
7.1 Shape Representation . . . 117
7.1.1 Bézier Splines . . . 118
7.1.2 B-Splines . . . 120
7.2 Treatment of Geometrical Design Constraints . . . 127
7.2.1 C^1 Continuity Between Bézier Splines . . . 128
7.2.2 C^1 Continuity at a Point on a Line of Symmetry . . . 129
7.2.3 A Composite Circular Arc . . . 131
7.3 Mesh Generation and Calculation of Nodal Sensitivities . . . 132
7.3.1 B-Spline Surface Meshes . . . 133
7.3.2 Coons Surface Meshes . . . 134
7.3.3 Unstructured Meshes . . . 137
7.4 Summary of Sensitivity Analysis for Two-Dimensional Shape Optimization . . . 139
7.5 Exercises . . . 143

8 Stiffness Optimization of Distributed Parameter Systems . . . 147
8.1 Calculus of Variations . . . 147
8.1.1 Optimality Conditions and Gateaux Derivatives . . . 149
8.1.2 Handling a Constraint . . . 153
8.2 Equilibrium Principles for Distributed Parameter Systems . . . 156
8.2.1 One-Dimensional Elasticity . . . 156

8.2.2 Beam Problem . . . 158
8.2.3 Two-Dimensional Elasticity . . . 159
8.2.4 Abstract Equilibrium Principles . . . 162
8.3 The Design Problem . . . 163
8.3.1 Optimality Conditions . . . 166
8.3.2 The Stiffest Rod . . . 168
8.3.3 Beam Stiffness Optimization . . . 170
8.4 Exercises . . . 174

9 Topology Optimization of Distributed Parameter Systems . . . 179
9.1 The Variable Thickness Sheet Problem . . . 179
9.1.1 Problem Statement and FE-Discretization . . . 179
9.1.2 The Optimality Criteria (OC) Method . . . 182
9.2 Penalization of Intermediate Thickness Values . . . 188
9.2.1 Solid Isotropic Material with Penalization (SIMP) . . . 188
9.2.2 Other Penalizations . . . 190
9.3 Well-Posedness and Potential Numerical Problems . . . 190
9.3.1 The Archetype Problem and an Analogy . . . 190
9.3.2 Numerical Instabilities . . . 191
9.4 Restriction of the Archetype Problem . . . 193
9.4.1 Bounds on the Design Gradient . . . 194
9.4.2 Filters . . . 195
9.5 Relaxation of the Archetype Problem . . . 198
9.6 Exercises . . . 200

Answers to Selected Exercises . . . 203

References . . . 207

Index . . . 209