Geophys. J. R. astr. Soc. (1984) **76**, 17–28

An introduction to wave propagation in anisotropic media

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Received 1983 May 1

Summary. Wave motion in an anisotropic solid is fundamentally different from motion in an isotropic solid, although the effects are often subtle and difficult to recognize. There are such a wide range of three-dimensional variations possible in anisotropic media that it is difficult to understand the behaviour of wave motion without experimentation. Laboratory experiments are very difficult to construct and extensive numerical experiments have now given many theoretical insights so that the behaviour of waves in anisotropic media is now comparatively well understood. This introduction summarizes some of the relationships and insights required for this understanding.

1 Introduction

Any homogeneous uniform material whose properties vary with direction is anisotropic, and its elastic behaviour with respect to appropriate seismic wavelengths can be described by effective elastic constants in one of a range of anisotropic symmetry systems. Seismic waves penetrating such anisotropic material display a number of characteristic and diagnostic effects, which are subtly different from those of waves propagating in isotropic solids.

The general theory of wave motion in anisotropic elastic solids is well known (Love 1944), with further notable contributions being made by Duff (1960), Lighthill (1960), Kraut (1963) and others. Uniform homogeneous elastic solids may be divided into eight anisotropic symmetry systems with distinct and individual properties. These eight systems include, as extrema, the isotropic system with maximum symmetry where every plane is a symmetry plane, and the triclinic system with minimum symmetry where there are no symmetry planes. The effects of propagation in any particular symmetry system cannot be directly investigated by means of general expressions, and it is only since numerical experiments have been made with digital computers that we have gained any real understanding of wave propagation in specific anisotropic symmetry systems. Such numerical applications require rather different theoretical developments from the previous general applications. The theory for these numerical applications has been reviewed by Crampin (1981). In this paper, we describe the physical behaviour of wave motion required to understand and interpret observations of wave propagation in anisotropic solids.

2 Basic assumptions

The theoretical developments reviewed in Crampin (1981) can be derived from five basic relationships, which may be found in any textbook of theoretical elasticity (for example, Love 1944), and they will not be proven here. However, we list them here, although they are not necessary for understanding this introduction, as they do provide the essential mathematical background on which this understanding is based.

(1) The equations of motion for waves propagating with infinitesimal displacements in a purely anisotropic medium in equilibrium are:

$$\rho\partial^2 u_j/\partial t^2 = c_{jkmn}\, u_{m,nk}; \tag{1}$$

where ρ is the density; u_j is the component of displacement in the jth direction; $\{c_{jkmn}\}$ is the fourth-order tensor of elastic constants; and $u_{m,nk} = \partial^2 u_m/\partial x_n \partial x_k$. All suffixes take the values 1, 2 and 3 unless otherwise specified, and the suffix *summation convention* is understood throughout, whereby if any suffix occurs twice it is put equal to 1, 2 and 3 in turn and the results summed.

(2) The three-dimensional generalization of Hooke's stress-strain relationship is:

$$\sigma_{jk} = c_{jkmn} u_{m,n}; \tag{2}$$

where $\{\sigma_{jk}\}$ is the second-order stress tensor; and $u_{m,n} = \partial u_m/\partial x_n$. Note that the stress tensor is necessarily a symmetric matrix with three mutually orthogonal principal directions of stress.

(3) The fourth-order tensor of elastic constants necessarily transforms by the tensor transformation law:

$$c'_{jkmn} = x'_{j,p}\, x'_{k,q}\, x'_{m,r}\, x'_{n,s}\, c_{pqrs}; \tag{3}$$

where $x'_{j,p} = \partial x'_j/\partial x_p$ are the direction cosines.

(4) The elastic tensor has the following symmetries:

$$c_{jkmn} = c_{kjmn} = c_{mnjk}. \tag{4}$$

We are immediately able to demonstrate from these basic equations an important property of the elastic tensor:

(5) *Theorem*: the plane $x_p = 0$ is a plane of (mirror) symmetry if and only if $c_{jkmn} = 0$ whenever one or three of j, k, m, n, is equal to p. A plane in an anisotropic elastic solid will possess mirror symmetry if the elastic constants are unchanged by reflection in the plane. The direction cosines representing a reflection in the $x_p = 0$ plane are:

$$x'_{j,k} = 0 \text{ for } j \neq k; \qquad x'_{j,k} = 1 \text{ for } j = k \neq p; \qquad \text{and} \qquad x'_{j,k} = -1 \text{ for } j = k = p.$$

These direction cosines impose the following conditions on the reflected elastic constants c'_{jkmn} by the tensor transformation (3);

$$c'_{jkmp} = -c_{jkmp}; \;\; c'_{jkpp} = c_{jkpp}; \;\; c'_{jpmp} = c_{jpmp}; \;\; c'_{jppp} = -c_{jppp}; \;\; \text{and} \;\; c'_{pppp} = c_{pppp}; \tag{5}$$

and all their equivalents by the symmetry relationships (4). Thus, the constants are invariant under these conditions if and only if $c_{jkmn} = 0$ whenever one or three of i, k, m, n are equal to p.

Note that repeated application of this theorem demonstrates that if the elastic tensor possesses two orthogonal symmetry planes ($x_1 = 0$ and $x_2 = 0$, say) then the third mutually orthogonal plane ($x_3 = 0$) is also a plane of mirror symmetry.

3 Body-wave phase velocities

The velocities of plane body-waves in anisotropic media are obtained by substituting the expressions for plane waves into the equations of motion (1) to yield three simultaneous equations in ρc^2, where c is the phase velocity. There are several ways to do this. The conventional technique is to express the equations for the velocity in a particular direction in terms of direction cosines with respect to a coordinate system fixed in the anisotropic solid. These lead to three simultaneous Kelvin-Christoffel equations which can be solved comparatively easily for three distinct body-waves, which generally have three distinct velocities. Although the Kelvin-Christoffel equations are convenient for calculating velocities in homogeneous material, they lead to unwieldy expression for calculations in multilayered models.

The preferred technique, proposed by Crampin (1970), is to rotate the elastic tensor so that all problems are reduced to propagation with apparent velocity c in the x_1-direction. This has the advantage that all analytical expressions and computer programs can be written in concise general forms by making use of the summation convention for repeated suffices, which are well adapted for computer manipulation. Thus at the expense of an initial rotation of the elastic tensor, calculations in all symmetry systems can be calculated by the same computer program. This technique is one of the key features permitting the numerical developments (Crampin 1981). We shall demonstrated the use of the technique in the calculation of plane body-waves.

A plane-wave propagating in the x_1-direction, with phase velocity c, can be written:

$$u_j = a_j \exp[i\omega(t - x_1/c)] \quad \text{for} \quad j = 1, 2, 3. \tag{6}$$

Substituting (5) into the equation of motion (1), we have three equations:

$$\begin{aligned} \rho c^2 a_1 &= c_{1111} a_1 + c_{1121} a_2 + c_{1131} a_3; \\ \rho c^2 a_2 &= c_{2111} a_1 + c_{2121} a_2 + c_{2131} a_3; \\ \rho c^2 a_3 &= c_{3111} a_1 + c_{3121} a_2 + c_{3131} a_3; \end{aligned} \tag{7}$$

where we have omitted the common multiplier $(-i\omega)^2 \exp[i\omega(t - x_1/c)]$.

These three simultaneous equations may be solved in a variety of ways. However, the preferred technique for numerical solution is to write the equations as linear eigenvalue problems, which are particularly well suited to solution by computer. We have:

$$(\boldsymbol{T} - \rho c^2 \boldsymbol{I})\, \boldsymbol{a} = 0; \tag{8}$$

where $\boldsymbol{T}$ is the 3×3 matrix:

$$\boldsymbol{T} = \begin{pmatrix} c_{1111} & c_{1121} & c_{1131} \\ c_{2111} & c_{2121} & c_{2131} \\ c_{3111} & c_{3121} & c_{3131} \end{pmatrix}; \tag{9}$$

$\boldsymbol{I}$ is the 3×3 identity matrix; and $\boldsymbol{a}$, with elements a_j, is the amplitude vector of the displacements.

The matrix $\boldsymbol{T}$ is a principal minor of the real symmetric positive-definite matrix of elastic constants, and is also a real symmetric positive-definite matrix. Consequently, the eigenvalue problem (8) has three real positive roots for ρc^2 with orthogonal eigenvectors.

These equations immediately demonstrate some of the fundamental features of body-wave propagation in anisotropic media. The three real roots of (8) show that there are three body-waves in every direction of phase propagation with orthogonal particle motion and with velocities which, in general, are different and vary with direction. These waves correspond to a quasi P-wave, qP, with approximately longitudinal particle polarization, and two quasi shear-waves, $qS1$ and $qS2$, with approximately transverse particle motion. The particle polarizations of these body-waves are fixed in the material along any direction of phase propagation, and, in general, will not be parallel to the displacements of P-, SV-, and SH-waves in isotropic horizontally layered solids. These properties have distinctive effects, particularly on shear-wave propagation. A shear wave entering a region of anisotropy necessarily has to split into the two or more fixed polarizations appropriate for that particular direction of energy propagation. (There will generally be a qP-wave component as well, but this is usually small, and can be neglected.) These fixed quasi shear-waves travel at difference velocities and separate in time, so that on re-entry into an isotropic region the original pulse cannot be reconstructed. Thus the passage through a region of anisotropy writes a characteristic signature into the polarization of the shear wavetrains, which, because shear waves have a unique velocity in isotropic media, is preserved for the remaining isotropic sections of the path. The phenomenon is illustrated schematically in fig. 4 of Crampin, Evans & Atkinson (1984b).

The Earth has a complicated velocity structure, and the small velocity variation with direction expected in anisotropy is unlikely to be resolved except when examining velocity variations in many directions *in one plane*. Almost the only situation where we can do this in the Earth is when examining the variation with azimuth of P_n-wave velocities. It is significant that the earliest, and most widely recognized, anisotropy in the Earth is the velocity anisotropy of P_n-waves beneath the thin homogeneous oceanic crust (Hess 1964; Raitt *et al.* 1969; and many others).

4 Anisotropic symmetry systems

Almost all we need to know about anisotropic symmetry systems and most of the analysis in Crampin (1981) can be derived from equations (1) to (5). Equations (1) and (2) show that there are $3^4 = 81$ elastic constants, which the symmetry conditions (4) reduce to 21 independent constants. These are usually written in the form of the symmetric matrix in Fig. 1(a).

Now it might be though that media with up to 21 independent elastic constants would have far too many possible variations to be easily classified. In fact, anisotropic symmetry systems are very readily classified by the relative arrangement of planes of (mirror) symmetry into eight distinct symmetry systems. All the more common symmetry systems have one or more symmetry planes with the spatial orientations shown in Fig. 2, and the corresponding matrices of elastic constants in Fig. 1(b). These symmetry systems have distinct and characteristic properties, which in conventional crystallography are discussed in terms of point groups, space lattices, and other fundamental geometrical considerations. However, in wave-propagation analysis the most distinctive features of the different symmetry systems are the arrangements of symmetry planes.

The relationship between the orientation of the symmetry planes in Fig. 2 and the particular matrix of elastic constants in Fig. 1(b) is straightforward. The monoclinic system, for example, has one plane of mirror symmetry, $x_3 = 0$, say. Consequently, we know from equation (5) that the monoclinic elastic constants c_{jkmn} equal zero whenever one of three of $j, k, m, n = 3$. Replacing such constants by zero in Fig. 1(a), we obtain the arrangement of elastic constants in the monoclinic system in Fig. 1(b). Similarly, the orthohombic system has three mutually perpendicular symmetry planes, say $x_1 = 0$, $x_2 = 0$ and $x_3 = 0$. Replacing the elements in Fig. 1(a) which have one or three of their suffixes equal to 1, 2 and 3, taken separately, we have the arrangement of constants in the orthorhombic system in Fig. 1(b). The relationships can also be demonstrated for the other symmetry systems, although the procedure is more complicated because the tensor of elastic constants must be rotated to get the particular symmetry plane normal to one of the coordinate directions so that we can apply equation (5).

There are four arrangements of planes not shown in Fig. 2. These are the isotropic system with two elastic constants where all planes are symmetry planes, and three rather uncommon symmetry systems or subsystems. The uncommon systems are the triclinic system with up to 21 independent elastic constants whose only symmetry is reflection in the origin, and the two subsystems marked (2)* in Fig. 1(b). The trigonal system (2)* has no planes of symmetry, but the x_3-axis is a three-fold rotation axis, so that the system is

(a)

c_{1111}	c_{1122}	c_{1133}	c_{1123}	c_{1131}	c_{1112}
c_{2211}	c_{2222}	c_{2233}	c_{2223}	c_{2231}	c_{2212}
c_{3311}	c_{3322}	c_{3333}	c_{3323}	c_{3331}	c_{3312}
c_{2311}	c_{2322}	c_{2333}	c_{2323}	c_{2331}	c_{2312}
c_{3111}	c_{3122}	c_{3133}	c_{3123}	c_{3131}	c_{3112}
c_{1211}	c_{1222}	c_{1233}	c_{1223}	c_{1231}	c_{1212}

(b)

MONOCLINIC

a	b	c	.	.	d
b	e	f	.	.	g
c	f	h	.	.	i
.	.	.	j	k	.
.	.	.	k	m	.
d	g	i	.	.	n

ORTHORHOMBIC

a	b	c	.	.	.
b	d	e	.	.	.
c	e	f	.	.	.
.	.	.	g	.	.
.	.	.	.	h	.
.	.	.	.	.	i

TRIGONAL (1)

a	b	c	d	.	.
b	a	c	-d	.	.
c	c	e	.	.	.
d	-d	.	f	.	.
.	.	.	.	f	d
.	.	.	.	d	x

where x = (a-b)/2

TRIGONAL (2)*

a	b	c	d	g	.
b	a	c	-d	-g	.
c	c	e	.	.	.
d	-d	.	f	.	-g
g	-g	.	.	f	d
.	.	.	-g	d	x

where x = (a-b)/2

TETRAGONAL (1)

a	b	c	.	.	.
b	a	c	.	.	.
c	c	d	.	.	.
.	.	.	e	.	.
.	.	.	.	e	.
.	.	.	.	.	f

TETRAGONAL (2)*

a	b	c	.	.	g
b	a	c	.	.	-g
c	c	d	.	.	.
.	.	.	e	.	.
.	.	.	.	e	.
g	-g	.	.	.	f

HEXAGONAL

a	b	c	.	.	.
b	a	c	.	.	.
c	c	d	.	.	.
.	.	.	e	.	.
.	.	.	.	e	.
.	.	.	.	.	x

where x = (a-b)/2

CUBIC

a	b	b	.	.	.
b	a	b	.	.	.
b	b	a	.	.	.
.	.	.	c	.	.
.	.	.	.	c	.
.	.	.	.	.	c

ISOTROPIC

a	b	b	.	.	.
b	a	b	.	.	.
b	b	a	.	.	.
.	.	.	x	.	.
.	.	.	.	x	.
.	.	.	.	.	x

where x = (a-b)/2

Figure 1. (a) Usual representation of the symmetric fourth-order tensor of elastic constants. If x, y, z are principal axes and the c_{ijkm} are all different, this has the form of a triclinic symmetry system. (b) The range of possible symmetry systems (excluding triclinic) with x, y, z, are principal axes.

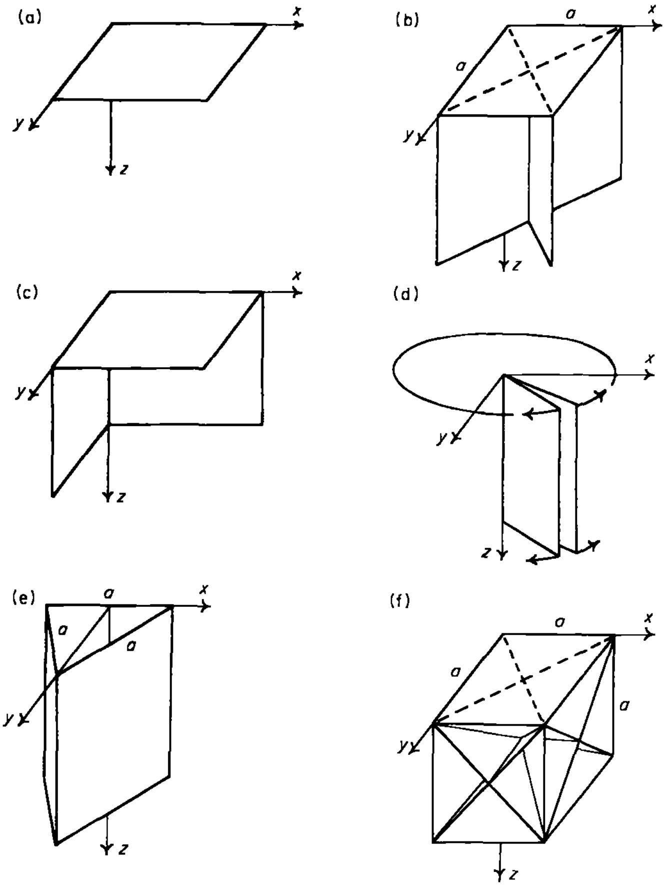

Figure 2. Orientation of symmetry planes of the more common symmetry systems: (a) monoclinic; (b) tetragonal; (c) orthorhombic; (d) hexagonal; (e) trigonal; and (f) cubic.

unchanged by $2\pi/3$ rotations about this axis. Similarly, the tetragonal system marked (2)* has one plane of symmetry, $x_3 = 0$, and the x_3-axis is a four-fold rotation axis, so that the system is unchanged by $2\pi/4$ rotations. Note that these last three systems are theoretically possible, but they do not commonly occur in any mineral or inclusion assemblages and they have not been used in any numerical calculations.

Table 1 lists the number of independent elastic constants and the arrangement of symmetry planes for the various anisotropic symmetry systems. It is worth noting that the symmetry systems cannot be arranged in any unique order of increasing complexity or increasing symmetry. Each system has unique and individual properties unlike any other symmetry system. However, these properties refer to the overall behaviour of the symmetry system. Wave motion in any particular symmetry plane is very similar to motion in any other symmetry plane, and wave motion in any off-symmetry plane has generalized behaviour which is not dependent on the particular symmetry system.

5 Symmetry planes

The orientation of the anisotropic symmetry planes to the free surface and to the direction of propagation has major effects on seismic phenomena. In many circumstances, the

Table 1. Symmetry planes and shear-wave singularities in anisotropic symmetry systems.

Symmetry system	Number of independent *elastic* constants	Number and orientation of symmetry planes (referred *to principal axes)*	Shear-wave singularities Kiss	Inter-*section*	Point
Cubic	3	three identical: *x*-, *y*- and *z*-cuts; six identical: planes joining opposite sides of cube	6	0	8
Hexagonal	5	one *z*-cut; ∞ planes through axis of symmetry (*z*-axis)	2	{2, 0}	0
Trigonal[b]	6 (7)[a]	three identical: sides of triangular prism	0	0	{$2^{c}+6$; $2^{c}+18^{d}$; etc.[d]}
Tetragonal[b]	6 (7)[a]	two identical: *x*- and *y*-cuts; one *z*-cut; two identical: planes joining opposite sides of prism	2	0	8
Orthorhombic	9	three distinct: *x*-, *y*- and *z*-cuts	0	0	{4; 12[d]; etc.[d]}
Monoclinic	13	one *z*-cut	0	0	{8; etc.[d]}

[a] Possible but rarely occurring configurations.
[b] The names of these systems refer to two possible elastic tensors (see text): the systems with fewer constants occur most commonly.
[c] Point singularities on axes.
[d] Systems with more complicated patterns of singularities (usually of less common occurrence).

presence or absence of symmetry planes in a few critical orientations with respect to the direction of phase propagation has more fundamental effects on the behaviour of the seismic wave than the particular anisotropic symmetry system. Symmetry planes have two major effects on body waves. *P* and *SV* motion is decoupled from *SH* motion: (1) when the propagation direction lies in a vertical symmetry plane; and (2) when the propagation direction lies in a symmetry plane, and the polarization of the *P*-wave and one of the *S*-waves is parallel and the polarization of the other *S*-wave is at right angles to the symmetry plane. The effect of symmetry-plane orientations on surface waves is discussed in Crampin (1981).

The velocities of body-waves propagating in symmetry planes have particularly simple relationships with the elastic constants for weakly anisotropic solids. The three body-wave phase velocities in the plane $x_3 = 0$ area:

$$\rho V_P^2 = A + B_c \cos 2\theta + B_s \sin 2\theta + C_c \cos 4\theta + C_s \sin 4\theta;$$
$$\rho V_{SP}^2 = D + E_c \cos 4\theta + E_s \sin 4\theta; \qquad (10)$$
$$\rho V_{SR}^2 = F + G_c \cos 2\theta + G_s \sin 2\theta;$$

where

$$A = [3(c_{1111} + c_{2222}) + 2\,(c_{1122} + 2c_{1212})]/8;$$

$$B_c = (c_{1111} - c_{2222})/2;$$

$$B_s = (c_{2111} + c_{1222});$$

$C_c = [c_{1111} + c_{2222} - 2\,(c_{1122} + 2\,c_{1212})]/8;$

$C_s = (c_{2111} - c_{1222})/2;$

$D = [c_{1111} + c_{2222} - 2\,(c_{1122} - 2c_{1212})]/8;$

$E_c = -C_c;$

$E_s = -C_s;$

$F = (c_{1313} + c_{2323})/2;$

$G_c = (c_{1313} - c_{2323})/2;$

$G_s = c_{2313};$

ρ is the density; θ is the azimuth measured from the x_1-direction (x_1 and x_3 are not necessarily principal axes); and *SP* and *SR* are shear waves polarized parallel and at right angles, respectively, to the symmetry plane. The *P*-wave equation in (10) was first derived by Backus (1965) and the shear-wave equations by Crampin (1977). The equations are correct to the first order in the difference between the anisotropic and isotropic elastic constants. Note that the equations take a particularly simple form when θ is measured from a direction of sagittal symmetry: using the appropriate conditions for $x_2 = 0$ to be a plane of symmetry in the theorem in Section 2, the coefficients of the sine term in equations (10) vanish and leave the reduced equations with cosine terms.

Both the full and reduced equations are of great value in modelling studies, especially modelling studies of cracked solids (Crampin 1978; Crampin, McGonigle & Bamford 1980). The equations are simple relationships between velocities and elastic constants, which are linear in the elastic constants and valid for all symmetry planes. The restriction to the symmetry plane is severe. In other planes, the variations for *P*-waves may depend on second-order differences between the elastic constants so that the variations have significantly 6θ and 8θ terms (Crampin 1982), and the shear-wave variations are usually completely inappropriate. The shear-wave equations break down because the shear-wave phase velocities form two distinct sheets which touch only in a number of singular points, which for most symmetry systems are confined to symmetry planes (Crampin & Yedlin 1981; Crampin 1981). Table 1 lists the number of shear-wave singularities in the various symmetry systems. There are three types of singularity: kiss singularities, where two phase-velocity sheets come into tangential contact at a point; intersection singularities possible only in hexagonal systems, where the two phase-velocity sheets come into contact along a (circular) line and, in some senses, may be thought of as two sheets intersecting; and point singularities where the two phase-velocity sheets come into contact at the vertices of conical projections from each surface. These singularities are a very common phenomena, and may cause anomalies in the propagation and energy transmission of rays of shear waves propagating near the direction of singularities (Crampin 1981).

6 Energy transport

A consequence of the phase velocity being a vector and varying with direction in anisotropic media is that the wavenumber, κ, the number of wavelengths in unit distance, is also a vector and varies with direction. This means that the expression for the group velocity in isotropic media, $U = \partial\omega/\partial\kappa$, must also be written as a vector:

$$U = (\partial\omega/\partial\kappa_1,\ \partial\omega/\partial\kappa_2,\ \partial\omega/\partial\kappa_3). \tag{11}$$

Thus the energy transport of seismic waves of all types propagating in uniform purely

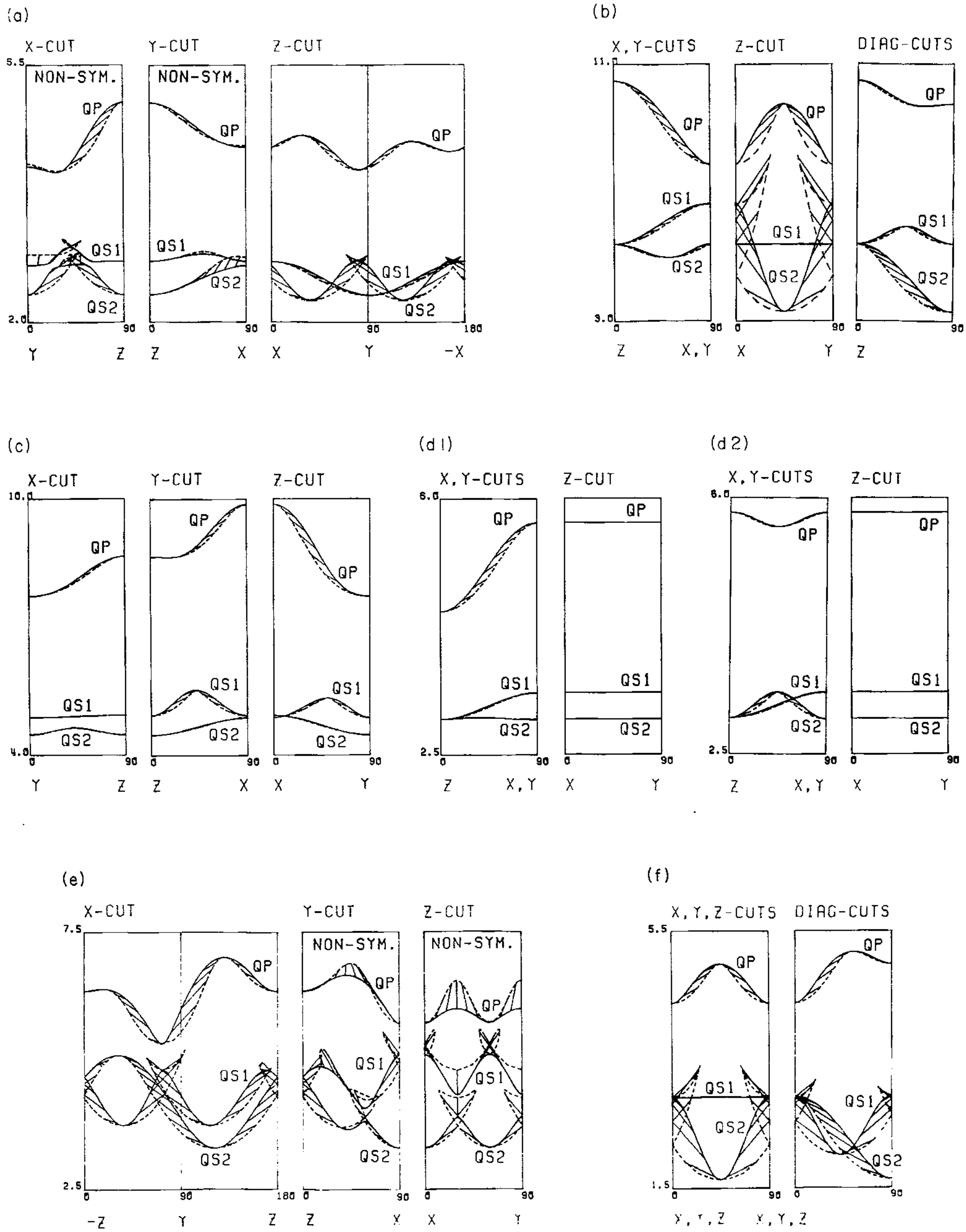

Figure 3. Examples of velocity variations of the three body-waves over planes in six anisotropic symmetry systems. The variations are shown over quadrants in the x-, y- and z-cuts (these are symmetry planes unless otherwise indicated), and those symmetry planes not included in this corner. The principal axes are indicated below the variations. Solid lines are exact phase velocities; dashed lines are group velocities (in non-symmetry planes the group velocities are projected on to the plane of phase variation). Group and phase velocities are joined every 10° of phase velocity direction. (a) *monoclinic* BIPHPQ: a biplanar cracked structure, where two quadrants of the z-cut variations are shown; (b) *tetragonal* rutile; (c) *orthorhombic* olivine; (d1) *hexagonal* – dry parallel cracks, HCD1 from Crampin (1984); (d2) *hexagonal* – liquid-filled parallel cracks, HCS1 from Crampin (1984); (e) *trigonal* α-*quartz*: the two other sides of the triangular prism are symmetry planes with the same variations as the x-cut, where two quadrants of the variations are shown; (f) *cubic* silicon.

elastic anisotropic media is never normal to the plane of constant phase, except in a few very restrictive cases such as body-waves propagating perpendicular to the symmetry axis of a solid with hexagonal anisotropic-symmetry.

The plane of constant phase of a plane wave propagating in the x_1-direction in a uniform anisotropic solid travels at a constant phase velocity $\partial\psi/\partial\kappa_1 = c$, so that from (12) the group velocity becomes:

$$U = (c, \partial\omega/\partial\kappa_2, \partial\omega/\partial\kappa_3). \tag{12}$$

We see that the energy of a plane wave travels at the phase velocity perpendicular to the plane, but also has a component of motion parallel to the plane.

These properties have a number of consequences for the propagation of seismic rays. In general, a ray of seismic body waves is not perpendicular to the instantaneous plane of constant phase. The ray (energy) travels in a straight line between a source and a receiver in homogeneous media so that the travel time measured at a single point gives the group velocity. The phase velocity can only be determined by analysis of records from an array of receivers. Similarly, the place where a particular energy group strikes an interface is determined by the ray and group velocity, but the behaviour at the interface is determined by a generalized Snell's law applied to the propagation direction and phase velocity. This means that incident, reflected and refracted *propagation vectors* are coplanar at a plane interface, the incident, reflected and refracted *rays* are, in general, not coplanar.

Fig. 3 shows the relationship between variations of phase and group velocity for examples of different symmetry systems. Two examples of hexagonal systems are shown to indicate the wide range of angular patterns possible in most systems. In symmetry planes, the phase and group velocities are coplanar, and in non-symmetry planes the group velocities in Fig. 3 are projected on to the plane of phase-velocity variation. Lines at every 10° join velocities and directions along the ray (group-velocity curve) to the direction and velocity of the plane of constant phase (phase-velocity curve) for that ray.

There are cusps on the group velocity surfaces associated with areas of high curvature on the quasi shear-wave phase-velocity surfaces. Such cusps can cause marked variations in body-wave amplitudes and directions of ray paths for waves with curved wavefronts but cause remarkably little disturbance to plane wavefronts.

7 Discussion

Despite the fundamental differences in the equations for propagation in anisotropic media, seismograms through such media possess very few distinguishing features that enable the anisotropy to be diagnosed or estimated. The most distinctive feature of propagation in uniform anisotropic solids appears to be the variation of properties with direction (Fig. 3). However, the Earth has a very complicated structure with many lateral and vertical inhomogeneities with varying velocities and transmission coefficients. This means that conventional measurements of both body waves (*P*-wave amplitudes and arrival times) and surface waves (Rayleigh-wave amplitudes and dispersion curves) only yield reliable measurements of velocity anisotropy in exceptional circumstances, such as observations of P_n-waves propagating in a horizontal Moho beneath the uniform oceanic crust. It is particularly difficult to diagnose anisotropy from *P*-wave arrivals as there are no anisotropic effects immediately recognizable on the seismogram. The possibly large deviation of the group velocity and direction from the phase vector in strongly anisotropic media is not apparent at individual three-component seismograms because the *P*-wave polarization follows the ray very closely (Crampin, Stephen & McGonigle 1982), so that the observed behaviour appears to be very similar to that in isotropic media. Anisotropy, or transverse isotropy, is sometimes deduced from surface-wave dispersion observations when compatible isotropic inversions of Raleigh- and Love-waves cannot be found. However, failure to model any of the wide range of possible isotropic variations in the Earth will invalidate such deductions, as is now demonstrated by Mitchell (1984) for some of the original classic claims for transverse isotropy.

Although shear-wave splitting is the most distinctive feature of body-wave propagation in anisotropic solids, analysis of body-wave particle motion, in a particular shear-wave polarization, even in isotropic media has received very little attention in the past. The numerical experiments suggest that information about the differential shear-wave velocity anisotropy and the orientation of *in situ* anisotropy along the ray path can be extracted from polarization diagrams of the shear wavetrain at or near the surface (Crampin 1981). This is complicated by the behaviour of shear-waves incident at a free surface, where shear-waves suffer mode conversion, and phase changes generate phases before and after the direct shear-wave arrival (Booth & Crampin 1984), which may be mistaken for shear-wave splitting. However, subsurface recordings, as in vertical seismic profiles (Gal'perin 1971), will be free of the complications on recordings made on the free surface.

Exact inversion of this information for structure and anisotropic parameters will be difficult without prohibitive amounts of data. However, since anisotropy alignments are in almost all cases due to the former or current stress-fields and the degree of anisotropy is inherent to the constituents of the material or the current state of the stress, these anisotropy techniques open up ways of examining new parameters for describing the interior of the Earth, with a wide range of applications (Crampin, Chesnokov & Hipkin 1984a).

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