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An inventory model with ramp type demand, time varying holding cost and price discount on backorders

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C H R O N I C L E

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A B S T R A C T

In this paper, an inventory model is studied with ramp type demand rate where holding cost is expressed as linearly increasing function of time. The study includes some features that are likely to be associated with certain types of inventory, like inventory of seasonal fruits and vegetables, newly launched fashion items, etc. When stock on hand is zero, the inventory manager offers a price discount to customers who are willing to backorder their demand. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval.

1. Introduction

In classical inventory models, the demand rate is assumed to be constant while in reality, time plays an important role in the inventory system. In the case of ramp type demand rate, the demand increases with time up to certain limit and then ultimately stabilizes and becomes constant. This type of demand rate is very commonly seen when some new brand of consumer goods comes to the market, for example, some kind of newly launched mobile phones, fashion goods, garments, automobiles, cosmetics, etc. Hill (1995) first considered the inventory models for increasing demand followed by a constant demand. Mandal and Pal (1998) extended the Hill's inventory model for deteriorating items and allowing shortage. Wu et al. (1999) constructed the inventory model with ramp type demand rate such that the inventory period is longer than the linear increasing period of the demand. Wu (2001) investigated the inventory model with ramp type demand rate and Weibull deterioration rate. Giri et al. (2003) developed an inventory model with ramp type demand and more generalized Weibull deterioration distribution. Panda et al. (2008) gave an optimal replenishment policy for the perishable seasonal product in a season with ramp type demand rate. Sharma et al. (2009) developed an economic order quantity (EOQ) model for variable rate of deterioration having a ramp type demand rate. Mandal

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(2010) derived an EOQ model for deteriorating items, assuming a ramp type demand rate. Pradhan and Tripathy (2012) developed an interactive inventory model between producer and buyer in the presence of an extraordinary purchase and the credit period with Weibull deterioration for items and ramp type demand. In most models, holding cost is known and constant. But holding cost may not always be constant. In case of seasonal fruits and vegetables, the longer these items are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost. The variability in the holding cost was first introduced by Muhlemann and Valtis-Spanopoulous (1980). They developed an EOQ model with a constant demand rate where holding cost is expressed as a percentage of the average value of capital invested in the stock. Weiss (1982) discussed the same model considering holding cost per unit as a nonlinear function of the length of time for which the item was held in stock. Goh (1994) considered two types of holding cost variation: (a) a nonlinear function of storage time and (b) a nonlinear function of storage level. Giri et al. (1996) developed a generalized EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. Alfares (2007) presented an inventory system with stock dependent demand, in which the holding cost is a step function of storage time. He considered two types of holding cost variation in terms of storage time: retroactive increase, and incremental increase. Roy (2008) developed an inventory model for deteriorating items by considering demand rate and holding cost as linear functions of time.

In classical inventory models with shortages, it is generally assumed that the unmet demand is either completely lost or completely backlogged. However, it is quite possible that while some customers leave, others are willing to wait till the fulfilment of their demand. In some situations, the inventory manager may offer a discount on backorders and/or reduction in waiting time to tempt customers to wait. Pan and Hsiao (2001) proposed a continuous review inventory model by considering the order quantity and with negotiable backorders as decision variables. Ouyang et al. (2003) developed a periodic review inventory model with backorder discounts to accommodate more practical features of the real inventory systems. Chuang et al. (2004) discussed a distribution free procedure for mixed inventory model with backorder discount and variable lead time. Uthayakumar and Parvati (2008) considered a model with only first two moments of the lead time demand known, and obtained the optimum backorder price discount and order quantity in that situation. Pal and Chandra (2012) studied a deterministic inventory model with shortages. They considered only a fraction of the unmet demand is backlogged, and the inventory manager offers a discount on it. Pal and Chandra (2014) developed a periodic review inventory model with stock dependent demand, permissible delay in payment and price discount on backorders.

In this paper, an inventory model is developed where holding cost is expressed as linearly increasing function of time and demand rate is a ramp type function of time. The manager offers his customer a discount in case he is willing to backorder his demand when there is a stock-out. The paper is organized as follows. Assumptions and notations are presented in Section 2. In Section 3, the model is formulated and the optimal order quantity and backorder price discount determined. In Section 4, numerical examples are cited to illustrate the policy and to analyze the sensitivity of the model with respect to the model parameters. Concluding remarks are given in Section 5.

2. Notations and Assumptions

To develop the model, we use the following notations and assumptions.

Notations

$I(t)$ = inventory level at time point t

b = fraction of the demand backordered during stock out

b_0 = upper bound of backorder ratio

K = ordering cost per order

P = purchase cost per unit

s_1 = backorder cost per unit backordered per unit time
s_2 = cost of a lost sale
π = price discount on unit backorder offered
π_0 = marginal profit per unit
T = length of a replenishment cycle
T_1 = time taken for stock on hand to be exhausted, $0 < T_1 < T$
S = maximum stock height in a replenishment cycle
s = shortage at the end of a replenishment cycle

Assumptions

1. The model considers only one item in inventory.
2. Replenishment of inventory occurs instantaneously on ordering, that is, lead time is zero.
3. Shortages are allowed, and a fraction b of unmet demands during stock-out is backlogged.
4. Holding cost $h(t)$ per item per unit time is assumed to be time dependent
   $h(t) = h + \alpha t$ where $\alpha>0$, $h>0$
5. The demand rate $R(t)$ is assumed to be a ramp type function of time t
   $R(t) = D_0[t-(t-\mu)H(t-\mu)]$
   where D_0 and μ are positive constants and $H(t-\mu)$ is the Heaviside's function defined as follows: $H(t-\mu)=\begin{cases}1 & for\ t \geq \mu \\ 0 & for\ t < \mu\end{cases}$

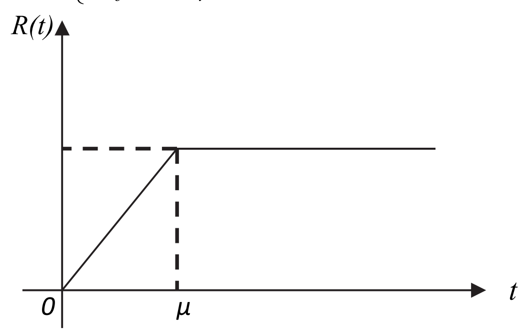

Fig. 1. The ramp type demand rate

6. The time taken for stock on hand to be exhausted (T_1) is greater than μ.
7. During the stock-out period, the backorder fraction b is directly proportional to the price discount π offered by the inventory manager. Thus,
   $b = \frac{b_0}{\pi_0}\pi$, where $0 \leq b_0 \leq 1,\ 0 \leq \pi \leq \pi_0$

3. Model Formulation

The planning period is divided into reorder intervals, each of length T units. Orders are placed at time points 0, T, $2T$, $3T$, …, the order quantity being just sufficient to bring the stock height to a certain maximum level S. Depletion of inventory occurs due to demand during the period $(0, T_1)$, $T_1 < T$, and in the interval (T_1, T) shortage occurs, of which a fraction b is backlogged. Hence, the variation in inventory level with respect to time is given by

$$\frac{d}{dt}I(t) = \begin{cases} -D_0 t, & \text{if } 0 < t < \mu \\ -D_0\mu, & \text{if } \mu < t < T_1 \\ -bD_0\mu, & \text{if } T_1 < t < T \end{cases}$$

Since $I(0)=S$ and $I(T_1)=0$, we get

$$I(t)=\begin{cases} -\dfrac{D_0 t^2}{2}+S, & \text{if } 0<t<\mu \\ D_0\,\mu(T_1-t), & \text{if } \mu<t<T_1 \\ bD_0\mu(T_1-t), & \text{if } T_1<t<T \end{cases}$$

Hence,

$$S=\frac{D_0\mu}{2}(2T_1-\mu) \text{ and } s=bD_0\mu(T-T_1)$$

Then,

Ordering cost during a cycle (OC) = K

Holding cost of inventories during a cycle (HC)

$$=\int_0^{T_1} h(t)I(t)dt=\int_0^{\mu} h(t)I(t)dt+\int_{\mu}^{T_1} h(t)I(t)dt=\frac{1}{6}D_0\mu\alpha T_1^3+\frac{1}{2}D_0\mu h T_1^2-\frac{1}{6}D_0\mu^3 h-\frac{1}{24}D_0\mu^4\alpha$$

Backorder cost during a cycle (BC)

$$=-s_1\int_{T_1}^{T} I(t)dt=\frac{s_1 bD_0\mu}{2}\left(T-T_1\right)^2$$

Lost sales cost during a cycle (LC) $=\left(1-b\right)D_0\mu(T-T_1)$

Purchase cost of inventory during a cycle is (PC)

$$=P\left(\frac{D_0\mu}{2}\left(2T_1-\mu\right)+bD_0\mu\left(T-T_1\right)\right)$$

Hence, the cost per unit length of a replenishment cycle is given by

$$C(T_1,T,b)=\frac{1}{T}[\text{OC+HC+BC+LC+PC}]$$

$$=\frac{1}{T}\begin{pmatrix} K+\dfrac{1}{6}D_0\mu T_1^2\left(\alpha T_1-3h\right)-\dfrac{1}{24}D_0\mu^3\left(\alpha\mu+4h\right) \\ +\dfrac{1}{2}D_0\mu\left(T-T_1\right)\left(s_1 b\left(T-T_1\right)+2b\left(P-s_2\right)+2s_2\right)+\dfrac{1}{2}D_0\mu P\left(2T_1-\mu\right) \end{pmatrix}=\frac{N(T_1,T,b)}{T}$$

The optimal values of T_1, T and b, which minimize $C(T_1,T,b)$, must satisfy the following equations:

$$\left(\frac{1}{2}\alpha T_1+h+s_1 b\right)T_1=s_1 bT+\left(1-b\right)\left(s_2-P\right) \tag{1}$$

$$D_0\mu\left(s_1 b\left(T-T_1\right)+b\left(P-s_2\right)+s_2\right)=C(T_1,T,b) \tag{2}$$

$$T-T_1=\frac{2(s_2-P)}{s_1} \tag{3}$$

Result 3.1: For given T and b, $C(T_1,T,b)$ is a convex function of T_1.

Proof: We have

$$\frac{\partial^2 C(T_1,T,b)}{\partial T_1^2}=\frac{D_0\mu}{T}\left(\alpha T_1+h+s_1 b\right)>0$$

Hence, the result.

Result 3.2: For given T_1 and b, $C(T_1,T,b)$ is convex in T.

Proof:

$$\frac{\partial}{\partial T}C(T)=0 \text{ gives } -\frac{N}{T^2}+\frac{1}{T}\frac{\partial N}{\partial T}=0 \Rightarrow N=T\frac{\partial N}{\partial T}, \text{ where N is the numerator of } C(T)$$

So, for any T satisfying $\frac{\partial}{\partial T}C(T)=0$,

$$\frac{\partial^2}{\partial T^2}C(T)=\frac{2N}{T^3}-\frac{1}{T^2}\frac{\partial N}{\partial T}-\frac{1}{T^2}\frac{\partial N}{\partial T}+\frac{1}{T}\frac{\partial^2 N}{\partial T^2}=\frac{1}{T}\frac{\partial^2 N}{\partial T^2}=\frac{s_1 b D_0 \mu}{T}>0$$

This is possible only if $C(T)$ is convex in *T*.

Hence the result.

4. Numerical Illustration and Sensitivity Analysis

Since it is difficult to find closed form solutions to the sets of Eqs. (1-3), we numerically find solutions to the equations for given sets of costs using the statistical software MATLAB. The following tables show the change in optimal inventory policy with change in a model parameter, when the other parameters remain fixed.

Table 1

The optimal inventory policy for different values of *h*, when K = 500, D_0 = 100, α = 0.6, P = 5, μ = 0.25, s_1 = 6 and s_2 = 7

h	T_1	T	b	$C(T_1,T,b)$
3	2.63	3.30	0.2992	374.28
5	2.09	2.76	0.3775	419.64
7	1.75	2.42	0.7080	454.89
8	1.63	2.29	0.7354	469.90
9	1.52	2.18	0.7267	483.56

Table 2

The optimal inventory policy for different values of s_1, when K = 500, D_0 = 100, α = 0.6, P = 5, μ = 0.25, h = 4 and s_2 = 7

s_1	T_1	T	b	$C(T_1,T,b)$
4	2.16	3.16	0.7148	375.57
8	2.42	2.92	0.3451	411.50
10	2.48	2.88	0.3767	419.78
14	2.56	2.84	0.2573	429.75
15	2.57	2.84	0.2354	431.46

Table 3

The optimal inventory policy for different values of s_2, when K = 500, D_0 = 100, α = 0.6, P = 5, μ = 0.25, h = 4 and s_1 = 6

s_2	T_1	T	b	$C(T_1,T,b)$
8	2.22	3.22	0.3307	383.41
13	2.26	4.92	0.3340	389.05
14	2.33	5.33	0.3333	399.18
16	2.51	6.18	0.6875	423.90
20	2.94	7.94	0.7202	484.45

Table 4

The optimal inventory policy for different values of *P*, when K = 500, D_0 = 100, h = 4, α = 0.6, μ = 0.25, s_1 = 6 and s_2 = 7

P	T_1	T	b	$C(T_1,T,b)$
1	2.18	4.18	0.3327	278.43
2	2.16	3.82	0.4141	300.74
3	2.17	3.50	0.4955	327.30
4	2.22	3.22	0.3333	359.39
6	2.50	2.83	0.2139	446.50

Table 5
The optimal inventory policy for different values of α, when $K = 500$, $D_0 = 100$, $h = 4$, $P = 5$, $\mu = 0.25$, $s_1 = 6$ and $s_2 = 7$

α	T_1	T	b	$C(T_1,T,b)$
0.1	2.56	3.22	0.3060	388.84
0.5	2.37	3.03	0.3137	396.76
1.1	2.17	2.84	0.3460	406.64
1.9	1.99	2.65	0.4133	417.48
2.3	1.92	2.58	0.6061	422.21

The above tables show that, for other parameters remaining constant,

(a) both T_1 and T are decreasing in h and α but increase as s_2 increase;
(b) b, and hence π, decreases with increase in s_1, but increases with h, s_2 and α;
(c) T_1 is increasing in s_1, s_2 while T is decreasing in h, P and s_1.

The above observations also indicate that, with the aim to minimizing total cost, the policy should be to maintain high inventory level for low holding cost and purchase cost but high lost sales cost. Also, higher the backorder cost, lower should be the price discount offered and for higher lost sales cost, higher price discount should be offered.

Table 6 gives the percentage change in the total cost over an inventory cycle with change in the model parameters.

Let us consider the following model parameters: $K = 500$, $D_0 = 100$, $\alpha = 0.6$, $P = 5$, $\mu = 0.25$, $h = 4$, $s_1 = 6$ and $s_2 = 7$. Table 6 gives the percentage change in the total cost over an inventory cycle with change in the model parameters.

Table 6
Percentage change in total cost with change in the model parameters

Parameter		% change in total cost	Parameter		% change in total cost
Name	Value		Name	Value	
	3	-6.09		4	-5.77
	5	5.29		8	3.25
h	7	14.14	s_1	10	5.33
	8	17.90		14	7.83
	9	21.33		15	8.26
	1	-30.14		8	-3.80
	2	-24.54		13	-2.38
P	3	-17.88	s_2	14	0.16
	4	-9.83		16	6.36
	6	12.03		20	21.55
	0.1	-2.43			
	0.5	-0.45			
α	1.1	2.03			
	1.9	4.75			
	2.3	5.94			

From the results of Table 6, it is quite evident that the model is highly sensitive to changes in the holding cost, purchase cost and lost sales cost compare to other model parameters.

Sensitivity analysis is performed by changing (increasing or decreasing) the parameters by 5% and 10% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are shown in Table 7 as follows.

Table 7
The results of sensitivity analysis

Parameter	% change	% change in T_1	% change in T	% change in b
h	-10	4.78	3.72	58.75
	-5	2.33	1.81	59.10
	5	-2.21	-1.72	28.36
	10	-4.32	-3.36	60.30
s_1	-10	-1.73	1.13	20.45
	-5	-0.83	0.53	28.18
	5	0.76	-0.47	59.86
	10	1.46	-0.89	58.88
s_2	-10	5.00	-3.90	37.70
	-5	2.32	-2.09	22.86
	5	-1.99	2.35	83.10
	10	-3.65	4.95	42.62
P	-10	-2.58	3.56	145.58
	-5	-1.37	1.72	12.08
	5	1.54	-1.59	25.27
	10	3.26	-3.03	21.45
α	-10	0.98	0.76	30.50
	-5	0.48	0.38	30.39
	5	-0.47	-0.37	30.20
	10	-0.94	-0.73	30.86

5. Conclusion

The paper studies an inventory model with ramp type demand allowing shortages. It is also considered that holding cost is linearly increasing function of time. The study includes some features that are likely to be associated with certain types of inventory in real life, like inventory of seasonal fruits and vegetables, newly launched fashion items, electronic goods, etc. A fraction of the demand is backlogged, and the inventory manager offers a discount to each customer who is ready to wait till fulfilment of his demand. The optimum ordering policy and the optimum discount offered for each backorder are determined by minimizing the total cost in a replenishment interval. Through numerical study, it is observed that for low backorder cost, it is beneficial to the inventory manager to offer the customers high discount on backorders.

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