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An Investigation into the relative nature of the production of selected private and government services

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AN INVESTIGATION INTO THE RELATIVE NATURE OF THE PRODUCTION OF SELECTED PRIVATE AND GOVERNMENT SERVICES

By

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INTRODUCTION

The service sector of the economy has been growing in its importance since the early 1900's. It has grown both absolutely and relatively to the two other major subgroups of the economy, agriculture and industry. Since 1929 the service sector real money share of the economy's output has been growing only slightly faster than the other two sectors, but its share of total employment has grown from about 40 percent to over 55 percent in 1967. Considering this great increase, there has been surprisingly little research done about the service sector. There are, of course, some reasons for this lack of study--primarily, the unavailability of usable data.

Within the service sector, the government sector has had even less attention given to the nature and the production of its services. There have been several studies evaluating the absolute cost of government services, but these have generally been descriptive in nature.

Neoclassical economic theory states that the price mechanism will tend to push the market towards the efficient allocation of resources in the absence of constraints. The government sector does not have this mechanism working for it directly. Therefore, it can be hypothesized that there may be a difference in the production of goods and services between the private and government sectors of the economy.

In order to evaluate the nature of the production of government services, it should be desirable to find similar private services to compare with the government production of services.

It is the purpose of this paper to investigate the relative nature of the production of selected private and government services, using statistically derived parameters for two types of production functions. These are the Cobb-Douglas and the constant elasticity of substitution (CES) production functions. The parameters to be derived are the substitution, returns to scale, and technological change parameters. By comparing the values of the parameters for the private and government sectors, differences and similarities will be evaluated. It is hoped that by comparing selected services some insights into the relative nature of the production of government services can be found. Also, this study should indicate whether future research is needed and identify a few problems associated with it.

This study was limited to an initial investigation for several reasons. First, the data available were not complete--only a few services were investigated. Second, the data, although it should be adequate, could be improved, only with great additional time and money. Third, the demand for services was assumed to be given. Finally, assumptions of the supply of factors, technology, and the operations of the firms were by necessity heroic.

Three agencies within the government sector and five private services were used in a comparison of the government and private sector. The government service agencies investigated were the Post Office Department; the Division of Disbursement or the Treasury Department; and the Department of Insurance or the Veterans Administration. The private services for which data were obtained were business services; finance, insurance, and real estate; insurance carriers, all services; and real estate. Data for the government services were taken entirely from the Bureau of the Budget Study, Measuring Productivity of Federal Government Organizations. Data for private services were taken from The National Income and Product Accounts, 1929-1965, Office of Business Economics; and Production and Productivity in the Service Industries, National Bureau of Economic Research. The time covered for both private and government services was 1949 to 1962.

Following this introduction, Chapter II gives a general description of the service industry noting features common to most government and private services and why these two sectors should be compared. Chapter III is a general development of some of the theoretical aspects of the nature of production, including both a graphical and mathematical development. Chapter IV gives a general mathematical development of the CES production function. A description of the data used with the CES production function is contained in Chapter V. The regression model used, an analysis of the results, and a discussion of the implication of the results, are found in Chapter VI. The last section, Chapter VII, contains the summary and conclusions.

CHAPTER II

THE SERVICE SECTOR

The purpose of this chapter is to provide a general description of the service sector, some characteristics common to most services, and to show why the private service sector should provide a good control group for an evaluation of the government sector.

From prior studies, one common problem has arisen: Just what is the service sector? Most economists have been in general agreement as to which major industries should be included within this sector but there is no general agreement as to how it should be defined. George Stigler recognized this problem in 1950.[1] Since then the definition of the service sector has varied from writer to writer but some common characteristics can be noted: an intangible product, the relationship of the product to the consumer, and the idea of a residual sector.[2] An intangible product did allow an industry to be placed within the service sector, but it was not a necessary condition for an industry to be placed in the service sector. Personal services were often produced and consumed simultaneously, but a wide variety

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[1] George J. Stigler, Trends in Employment in the Service Industries, p. 47.

[2] Victor R. Fuchs, The Service Economy (New York: NBER, 1968), pp. 15 and 16.

of business services were removed from the individual consumer. The final characteristic of the service sector, the residual sector--or all types of private businesses not classified as industry or agriculture, did range over the entire sector but said little about it. Obviously, agriculture, manufacturing, and mining should not be included but within each of these sectors, services were produced. The residual sector was just a convenient way to define all other businesses.

For the purpose of this paper, the service sector was defined to include wholesale and retail trade, finance, insurance, real estate, general government, and those industries commonly designated as personal and business services in the standard industrial classification system of the Bureau of the Budget.[3] This definition did not have any advantage over any other definition, but it was general enough to include a wide-range of services upon which the following discussion can be based.

The service sector had been characterized by a rapid growth in employment since 1920. There were primarily three reasons for this growth: "a more rapid growth of final demand (than industry or agriculture) for services, a relative increase (to industry and

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[3]U.S., Executive Office of the President, Bureau of the Budget, Standard Industrial Classification Manual (Washington, D.C.: Government Printing Office, 1947), pp. 147-211.

agriculture) in intermediate demand for services, and a relatively slow increase (to industry and agriculture) in output per man in services."[4] The income elasticity of the demand for services had been hypothesized as a major reason for the growth of the service sector. This had not been the case.[5] Engle's Law had been found to hold for agricultural goods, but this did not imply that services would be substituted for food. As an individual's income grew, there was a wide variety of goods and methods of savings to substitute for food. The fact that the service sector's share of national income grew slowly from 1929 to 1967, whether measured in constant or current dollars, supported this conclusion. It was estimated that the growth of the intermediate demand for services had contributed only about ten percent to the growth of service employment since 1929, within the service sector as a whole. In the last decade, business services grew at a greater rate than the service sector--which indicated an increasing demand for intermediate services. The main factor in the growth of intermediate demand for services was specialization. The growth trend in the use of specialized factors of production appeared to arise from the desire to lower costs and increase efficiency by the use of these specialized factors for a relatively short period of time. Another major factor in the growth of intermediate demand was the growth in population. This growth, especially when it

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[4]Fuchs, The Service Economy, p. 3.

[5]Fuchs, The Service Economy, p. 4.

brings greater urbanization, was found to be a major explanatory variable in the growth of services. The problems associated with urbanization and a growing awareness of these problems led to an expansion of services, especially within the government sector. Finally, the relatively slow growth in output per worker in the service sector appeared to be the major factor in the growth of employment. The service sector had the slowest growth in output per worker compared with the agricultural and industrial sectors. This slow growth seemed to be a function of a relatively slow growth in the quality of labor, a decrease in hours worked, a relatively slow growth of technology,[6] and, possibly, because of the increased size of the service sector.

The nature of the service sector's product and the growth of employment within this sector led to the hypothesis that the production of services may be characterized by a production process that differed from the other two sectors of the economy. If this was true, then there was reason to compare the government and private sectors.

There was little change in the service sector's share of national income, whether measured in real or money terms. There were several hypotheses for this lack of growth. The primary and most obvious was the rise in the price level and corresponding reduction in quantity of services demanded. Since there were no good measures for changes in the quality of a service, or most other goods, it was

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[6]Fuchs, The Service Economy, p. 62.

typically assumed there were little or no quality changes in either goods or services. Therefore, by assuming a price elasticity of demand of one, and since it appeared that the income elasticity was approximately one, then the share of services of the national income would be expected to have little or no change through time, ceterus paribus. That is, the negative substitution effect from an increase in the price of services was almost balanced by the positive increase in income through time.

There were several reasons why similar sectors should be investigated. If there were dissimilarities between the goods and service sectors in the characteristic of production, then they could not be compared adequately. The goods or industrial sector appeared to be operating under a very different type of production function than the service sector. The industrial sector seemed to have a much higher capital-labor ratio, and it was hypothesized that it may have had a greater elasticity of substitution between inputs. The higher capital-labor ratio had tended to raise the marginal product in labor in the industry sector faster than in the service sector. The rise in the marginal product of labor in industry was a function of an increase in the quality of human capital and an increase in units of capital in the goods sector. Since the growth of capital in the industry sector had grown at a faster rate, it appears as if the industry sector was able to substitute capital for labor with greater ease. This implied a greater elasticity of substitution in the industry sector, at least during the period of substitution.[7] (Since the elasticity of substitution of labor for capital and the elasticity of capital for labor were identical, hereafter the way in which inputs are substituted will be disregarded.)

Since it appeared that the elasticity of substitution was greater in the industry sector, this implied that the two sectors were operating under two different production functions. Stated more explicitly, if the production functions for the goods and service sector were given by

$$X_I = f(x_1, x_2, \ldots x_n, t_I) \text{ and } X_s = f(x_1, x_2, \ldots x_n, t_s)$$

respectively, X_I and X_s are the output of the goods and service sector, respectively, the x_i's, $i = 1, \ldots n$, are the inputs of both sectors, and t is an index of technology. If both sectors were perfectly competitive and all inputs are homogeneous, then a difference in the marginal products of the inputs implied a different technology between the two sectors. For example, if the marginal product of an input in the service sector was less than that of an input in the goods sector,

$$1 = \frac{\partial X_s}{\partial x_i} < \frac{\partial X_I}{\partial x_i} = 2$$

then

[7]Fuchs, The Service Economy, p. 62.

$$Q_s = \sum_{i=1}^{n} \frac{\partial X_s}{\partial x_i}(x_i) < \sum_{i=1}^{n} \frac{\partial X_r}{x_i}(x_i) = Q_I$$

If the output, Q, of the two sectors differ with the same input ratios, then there was either a difference in the technological relationship between inputs or a difference in efficiency. Therefore, the null hypothesis that these two sectors were similar did not appear to be the case.

Since the goods sector of the economy did not appear to be a good control group for measuring relative differences between the government and private sector, the service sector appeared to be a good alternative. Both of these sectors produced services and both appeared to produce their output with a relatively labor intensive production process. While this approach was an improvement, it could be refined. The government sector was primarily engaged in providing services to groups rather than to separate individuals--military defense, business statistics, police and fire protection and mail services. These services generally fell into the category of business rather than personal services. The ideal method of comparison would be to compare identical services in both sectors. This cannot be done in all cases for some services were produced only by the government. For example, postal services are primarily the responsibility of the federal government. This did not preclude the private sector from producing similar services, but it was unlikely that the private sector would produce exactly the same service and compete directly with the government sector.

The next question to ask was why compare these sectors and would any dissimilarities be found. The private sector used the price mechanism to allocate resources among firms. The price mechanism was imperfect; but in the long run, resources should tend to be allocated efficiently. In the government sector, the price mechanism did not operate directly. The legislative branch, in effect, determined the demand for a service and allocated the funds to provide for it. In producing a service, a governmental department could be influenced by factors that outweigh the pressures to produce the services as efficiently as possible. Some of the factors that could influence the production of services were empire building and inherent inefficiencies.

These factors were probably present in the private sector; but instead of being the primary factors in the allocation of resources, they should have been secondary. Stated somewhat differently, the null hypothesis was that similar government and private services were produced with the same technology. If the null hypothesis was not accepted, then some of the above reasons could contribute to the rejection of it.

To my knowledge, no one had investigated the null hypothesis stated above although there had been several studies measuring the productivity of government services.[8] These studies had primarily measured changes in output, labor, and capital. Data on these variables were then put into index number form and compared. Measures of the average product of labor and capital were then computed. This method did give some indication of the changes in the absolute magnitudes of these variables, but it went no further. Comparisons of index numbers for labor, capital, and outputs between sectors and among sectors had been tried but the usefulness of this procedure was doubtful, except for descriptive and gross predictors. These studies generally tried to measure whether changes in the average productivity of labor in one sector were greater than in another without considering the capital stock.

One study using production functions found that the changes in the combined productivity of labor and capital were not statistically different between the manufacturing and service sectors.[9] However, this result was obtained by using a linearly homogeneous Cobb-Douglas production function with implied similar production technologies. Also, data for the service sector was not deflated to reflect

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[8]U.S., Executive Office of the President, Bureau of the Budget, Measuring Productivity of Federal Government Organizations (Washington, D.C.: Government Printing Office, 1964), p. 10; Henry D. Lytton, "Public Sector Productivity," The Review of Economics and Statistics, Vol. 53, No. 2 (May, 1961), pp. 182-184.

[9]Phoebus J. Dhrymes, "A Comparison of Productivity Behavior in Manufacturing and Service Industries," The Review of Economics and Statistics, Vol. 45 (February, 1963), pp. 64-68.

increases in the price of services. The hypothesis that dissimilar outputs could be produced under different technological conditions did not appear to be seriously challenged.

CHAPTER III

A DEVELOPMENT OF THE NEOCLASSICAL THEORY OF PRODUCTION

This chapter is intended to introduce and develop that part of the neoclassical theory of production needed for the development and understanding of the CES production function. This primarily involved obtaining a definition of an isoquant and the elasticity of substitution. The chapter serves as an introduction to the basic concepts and tools used to analyze the nature of production.

Throughout this chapter there were several simplifying assumptions. They were: (1) two homogeneous non-negative inputs, labor and capital, with a fixed quality; (2) a fixed technology; (3) a continuous production function with first and second derivations; and (4) all inputs infinitely divisible. To make the discussion conform to economic reality, it must also be assumed that the production function equation does not contain a constant term. That is, if the quantity of either labor or capital was zero, there would be no output. The symbols used were as follows: L = labor, K = capital, X = output.

In developing a theory of production, two cases will be presented: (1) production with one fixed and one variable input and (2) both inputs variable. Only the case of two inputs and one output was considered for this was the only case directly applicable to this study. The theory could be easily expanded to the n factor case for more generality.

When dealing with one variable input, the idea of a fixed input was meant to mean the fixed input was given as a constant. A fixed input, by definition, placed the discussion in the short run which, in turn, implies that technology was also given and fixed.

When discussing production theory, the idea of a production function was defined as follows:

> A production function shows the maximum output attainable from any specified set of inputs. . . The production function is a single-valued mapping from input space into output space in as much as the maximum attainable output for any stipulated set of inputs is unique.[1]

It should be emphasized that the production function shows the "maximum attainable output." Without this constraint, the production function would imply an output plane with an infinite number of possible outputs, that is, it would give an indeterminate solution.

The production function gave the technical relationship between inputs and outputs. It described the condition under which a given level of output could be produced. That is, a production function was a set of technological conditions under which a firm was forced to produce, subject to certain constraints. (The cost constraints placed

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[1]C. E. Ferguson, The Neoclassical Theory of Production and Distribution (Cambridge, England: Cambridge University Press, 1969), p. 7.

upon a firm or industry were embodied in the supply curves of factors of production. The limits placed upon production were given by the demand curves for the firm's output.)

The general functional relationship between output and inputs for the one variable case was

$$X = f(L,\overline{K}) \qquad 3.1a$$

where $\overline{K}$ means that K was fixed. This implies the function acts as if output was solely a function of labor:

$$X = f(L|K = K_1) \qquad 3.1b$$

where $K = K_1$ means given some amount of capital, K, is equal to some specified amount, K_1. The marginal product of labor (MP_L) is the change in output caused by a change in the labor input. The MP_L is derived by taking the first partial derivative of output with respect to labor as in equation 3.1b.

$$MP_L = \frac{\partial X}{\partial L} = f_L = \frac{\partial f(L|K = K_1)}{\partial L} \qquad 3.2$$

Equation 3.2 emphasized that the MP_L is also a function of K. The marginal product of capital ($MP_K = f_K$) was zero. The MP_K had to be zero since the first partial derivative of a function with respect to a non-existent variable was zero. The average product of labor (AP_L) is equal to the output divided by total labor inputs.

$$AP_L = \frac{X}{L} = \frac{f(L|K = K_1)}{L} \qquad 3.3$$

As with MP_L, the AP_L is a function of all inputs.

Several relationships between the total product, marginal production, and average product should be noted. Consider Figure 1 where

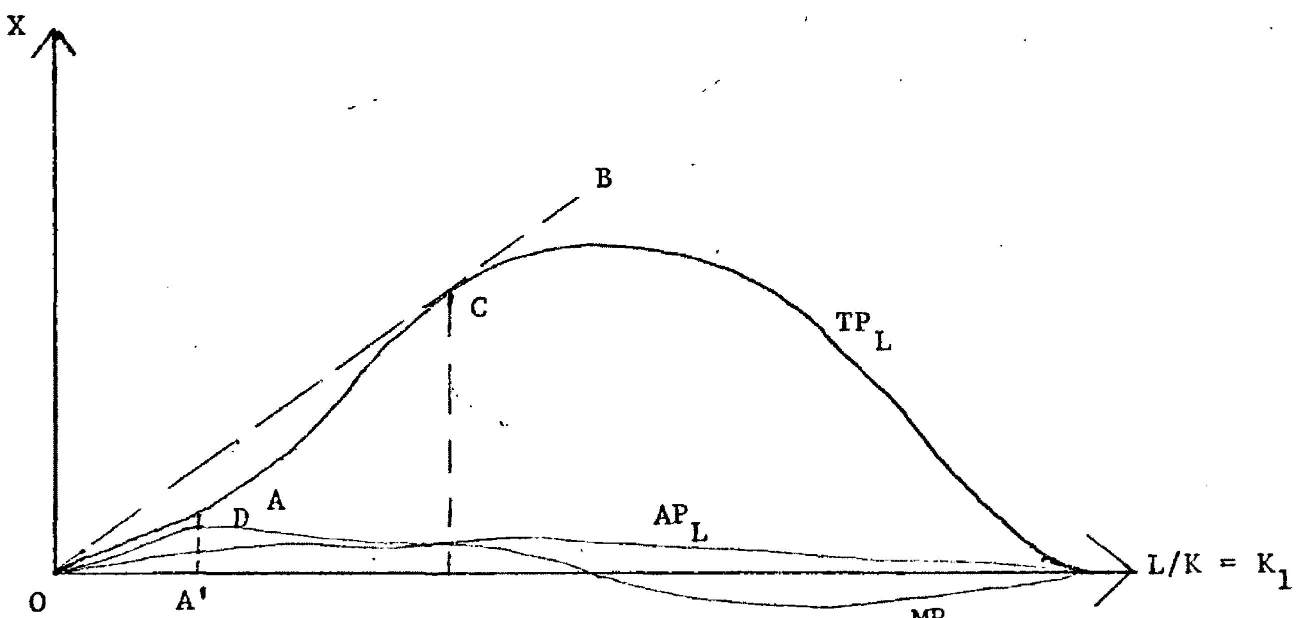

Figure 1

in this drawing, the total product curve (TP) started from the origin, increased and then decreased, as did the average and marginal product curves. The total product curve was drawn in this fashion to clearly present the law of variable proportions.

> With a given state of technology, if the quantity of one productive service is increased by equal increments, the quantity of the other productive services remaining fixed, the resulting increment of product will decrease after a point.[2]

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[2]Ferguson, The Neoclassical Theory, p. 69.

The law of variable proportions referred specifically to a production function having a fixed factor. The law of diminishing returns (marginal returns) was similar, but referred to the limits of production, that is, $\lim_{x_i \to \infty} X$ was less than the maximum value of X where $x_i \to \infty$ means the input, x_i, was increased without bound. The marginal product curve, with one variable input, was the slope of the total product curve. The marginal product curve reached a maximum when

$$\frac{\partial X}{\partial L} = 0 \text{ and } \frac{\partial^2 X}{\partial L^2} = f_{LL} < 0 \qquad 3.4$$

at point A. The marginal product curve was increasing if $f_{LL} > 0$, the second partial derivative of output with respect to labor, and decreasing marginal returns if $f_{LL} < 0$. Graphically, the average product curve was the slope of a straight line passing through the origin and intersecting the total product curve at a point. The average product curve reached a maximum where this line becomes tangent to the total product curve at C on line OB. The average product curve was increasing when $\frac{\partial (X/L)}{\partial L} > 0$ and average product is decreasing when $\frac{\partial (X/L)}{\partial L} < 0$. The average product curve reached a maximum when

$$\frac{\partial (X/L)}{\partial L} = 0 \text{ and } \frac{\partial^2 (X/L)}{\partial L^2} < 0 \qquad 3.5$$

The average product curve also reached its maximum value when the marginal product equaled the average product within the relevant range.

This can be seen from

$$\frac{\partial(X/L)}{\partial L} = \frac{1}{L} f_L - \frac{X}{L} = \frac{1}{L} MP_L - AP_L \qquad 3.6$$

The final point of interest for the single input variation case was the stages of production. The stages of production generally referred to the short run and to the nature of the average product of the variable factor. Stage one was generally the area of increasing average returns. Stage two included the area of decreasing average returns and positive marginal returns. Stage three occurred at the point of zero marginal returns to the variable factor.

It must be emphasized that the delineation of the stages of production as described above only referred to a linearly homogeneous production function. A homogeneous production function of degree V referred to a specific type of functional relationship between the proportionate increases in input usage and the changes in output. Given a production function

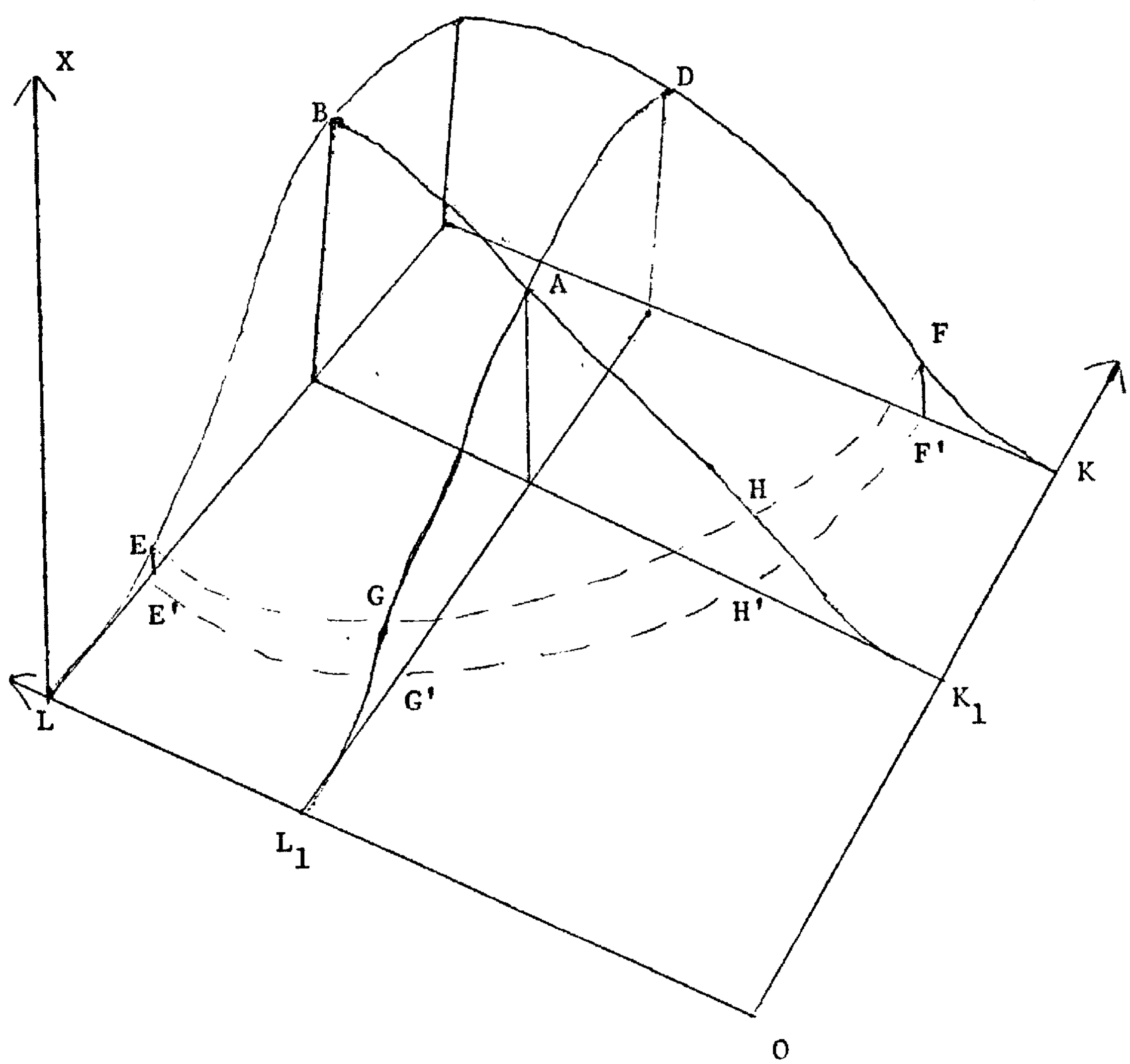
$$X = f(x_1, x_2, \ldots, x_n) \qquad 3.7$$

and increasing all factors proportionately by a factor, a, then,

$$X = f(ax_1, ax_2, \ldots, ax_n) = a^V f(x_1, x_2, \ldots, x_n) \qquad 3.8$$

where V referred to the degree of homogeneity. The degree of homogeneity also implies the returns to scale for a production function.

If V > 1, V < 1, or V = 1, then there were increasing, decreasing, or constant returns to scale respectively. Of course, returns to scale only referred to the long run when all factors were variable. While returns to scale only referred to the long run, the degree of homogeneity imposed definite restrictions on the relevant stage of production in the short run. Consider a production surface showing all combination of inputs and output. The short run case was equivalent to passing a place (EFG) through the production surface perpendicular to the capital axis as in Figure 2.



A portion of the total product curve of labor is K_1AB. It shows the level of output for each quantity of labor, given that the capital input was equal to K_1. On the same production surface, the quantity of labor could be fixed and the quantity of capital be allowed to vary or L_1AD. The production surface represents a given technology, the marginal and average products of capital could be computed and superimposed on a graph similar to Figure 1. This was done in Figure 3. In Figure 3 the ratio of L to K, K fixed, was increased for movements to the right. Conversely, moving from right to left decreased the L/K ratio, L fixed. The marginal, average, and total product curves for labor should be read from right to left. The marginal, average, and total product curves for capital should be read

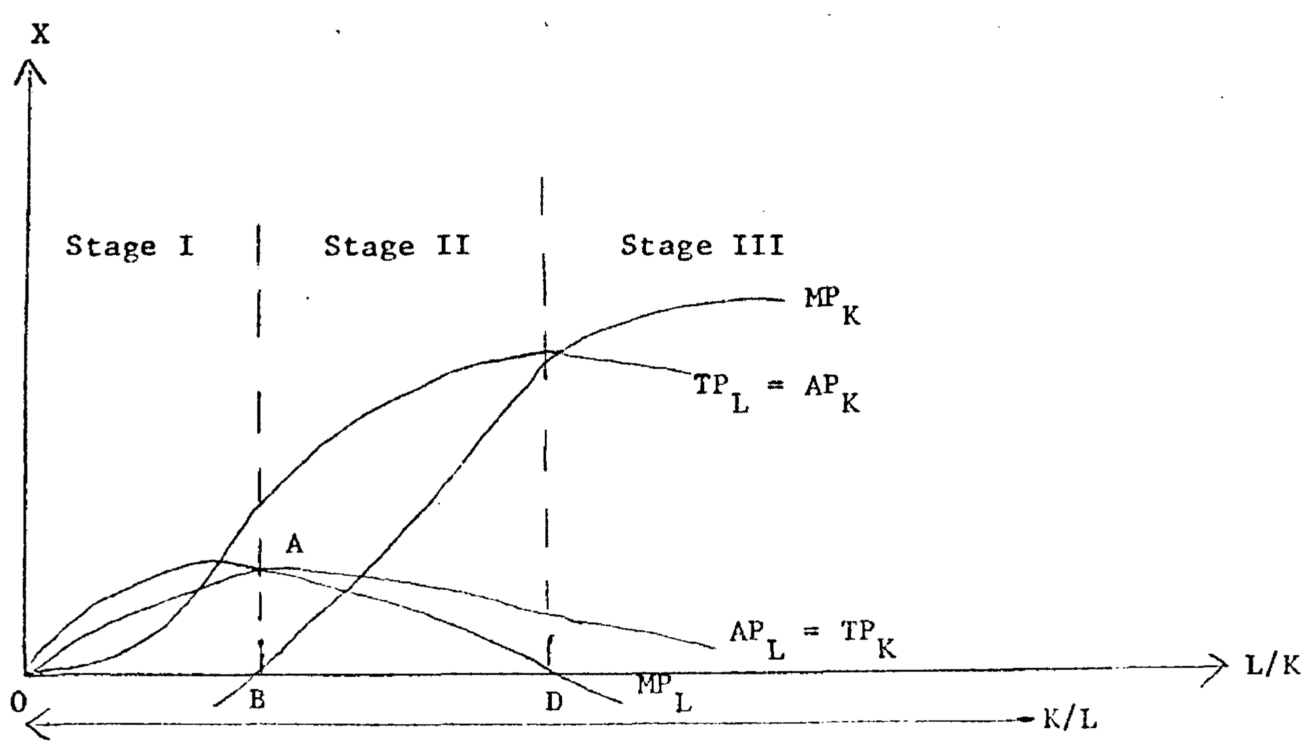


Figure 3.

from left to right. It was important to remember that increasing L/K from left to right was the same as increasing K/L from right to left in Figure 3. The total product curve for labor (TP_L) is determined by adding more units of labor to <u>one</u> fixed unit of capital, L/K increases. The total product of labor derived by adding more units of L to <u>one</u> fixed unit of capital was the output per (one) unit of capital (AP_K). Similarly, if labor was fixed at one unit, the total product of capital was the total product per (one) unit of labor (AP_L). It was seen that if the fixed factor was not assumed to be equal to one unit then the equality between the total product curve for labor and average product of capital would not have held.[3]

If a production function was homogeneous of degree one, the relevant stage of production is where the marginal products of both inputs are positive and decreasing. As was seen in the last figure, this was the area of decreasing average returns to labor and a positive but a decreasing marginal product of labor. This could be shown using Euler's relationships which in economic literature states that the sum of marginal products times their rate of use exhausts the total product, if the function was linearly homogeneous, and was of the form

$$X = L \cdot MP_L + K \cdot MP_K \qquad 3.9$$

[3]Richard A. Bilas, *Microeconomic Theory: A Graphical Analysis* (New York: McGraw-Hill, Inc., 1967), p. 119.

First, place $MP_K = 0$ in equation 3.9, then $X = L \cdot MP_L$ or $\frac{X}{L} = MP_L$. The average product of labor equals the marginal product of labor, point A, Figure 3. Now place $MP_L = 0$ so $X = K \cdot MP_K$ or $\frac{X}{K} = MP_K$, point C. Between points b and d, both the marginal product of labor and capital were positive. This was the stage of rational production.

Now, if the degree of homogeneity (V) was greater than one, increasing returns to scale, then equation 3.9 must be written as

$$V \cdot X = L \cdot MP_L + K \cdot MP_K \qquad 3.10$$

where V indicates the degree of homogeneity. As was seen, the sum of marginal products times their rate of use did not necessarily exhaust the total product. Using Euler's relationship (3.10) and rearranging terms

$$MP_L + \frac{K}{L} MP_K = V \frac{X}{L} \qquad 3.11a$$

and

$$V \cdot AP_L - MP_L = \frac{K}{L} MP_K \qquad 3.11b$$

the relevant stage of production was found. If $V > 1$ and the marginal product of capital was equal to zero, this implies AP_L was less than MP_L for all positive input ratios. Therefore, the relevant stage of production was extended to include a portion of area under the total product curve where the average product of labor was increasing for both marginal product curves were greater than zero. If $0 < V < 1$, the relevant stage of production was the area where the marginal product curves had a positive value, but the stage of rational production would be reduced to that area where $AP_L > MP_L$ by a factor V. If $V < 0$, this implied absolute diminishing returns to scale and the marginal product of one factor must always be negative when the marginal product of the other factor was positive. There would only be stage I and III; stage II cannot occur.

The general functional notation for simultaneous input variation production function is

$$X = f(K,L) \tag{3.12}$$

The same relationships hold for the marginal and average product of labor as in the discussion above. The concepts could now be applied to capital. In long run, inputs were allowed to be substituted for one another in response to changes in technology and price ratios.

The long run factor substitutions were usually represented graphically by isoquant curves. "An isoquant is a locus of input combinations each of which is capable of producing the same level of output."[4] These curves also showed the response of output to changed levels of input usage. The isoquant map represented a fixed technology. An isoquant curve could be graphically derived from Figure 2 by passing a plane through the production surface parallel to the KL plane. Then by projecting the intersection of this plane and the production surface onto the KL plane results in an isoquant. By

[4]Ferguson, The Neoclassical Theory, p. 64.

repeating the above procedure, a number of isoquants could be formed. This was equivalent to passing the plane EFG through the production surface and projecting the line EGHF onto the KL surface to form E'G'H'F' (see Figure 2). A portion of the final isoquant map was represented by Figure 4. Output increased from level I to level IV by increasing the use of one or both inputs. As the input combination changed from point A through point C on isoquant II, for instance; output remains the same. The ratio of marginal products of labor and capital was given by the negative of the slope of an isoquant at a point. This ratio is called, in general, the marginal rate of technical substitution of capital for labor ($MRTS_{K/L}$) and was derived by taking the total differential of the equation of the isoquant.

$$dX = f_L \cdot dL + f_K \cdot dK = 0 \qquad 3.13a$$

then rearranging

$$-\frac{dK}{dL} - \frac{f_L}{f_K} = MRTS_{K/L} \qquad 3.13b$$

The $MRTS_{K/L}$ showed the rate at which one input could be substituted for another and retain the same level of output. There are several other interesting characteristics of isoquants. The $MRTS_{K/L}$ is completely described on any one isoquant. Since the $MRTS_{K/L}$ is a ratio, it does not depend on the magnitudes of the inputs, if the production function is homogeneous of degree one. Also, the $MRTS_{K/L}$ usually is defined for positive values of the marginal products. Given an isoquant map as in Figure 4, it was seen that the slope of an isoquant continuously decreases from left to right between points A' and B' on isoquant IV, Figure 4. Outside this range, the slope assumed a positive value. At point A', the marginal product of capital was zero which implied a $MRTS_{K/L}$ approaching infinity.

$$MRTS_{K/L} = \frac{MP_L}{MP_K} = \frac{MP_L}{0} \cong \infty \qquad 3.14$$

Similarly, at point B', the marginal product of labor was equal to zero. Since both the marginal product of capital and labor were positive between point A' and B', this was the relevant stage of production. Above point A', the marginal product of capital was negative and that of labor was positive, implying stage I for labor and stage III for capital. Beyond point B', the converse was true. Therefore, outside of the A'B' range, it would pay the entrepreneur to throw away some of the input with the negative marginal product. The relevant stage of production was bordered by isoclines OC and OD in Figure 5. An isocline is defined as "a locus of points along which the marginal rate of technical substitution is constant."[5] This was the same as saying the slope of all isoquants were equal at the point of inter-

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[5]Ferguson, The Neoclassical Theory, p. 86.

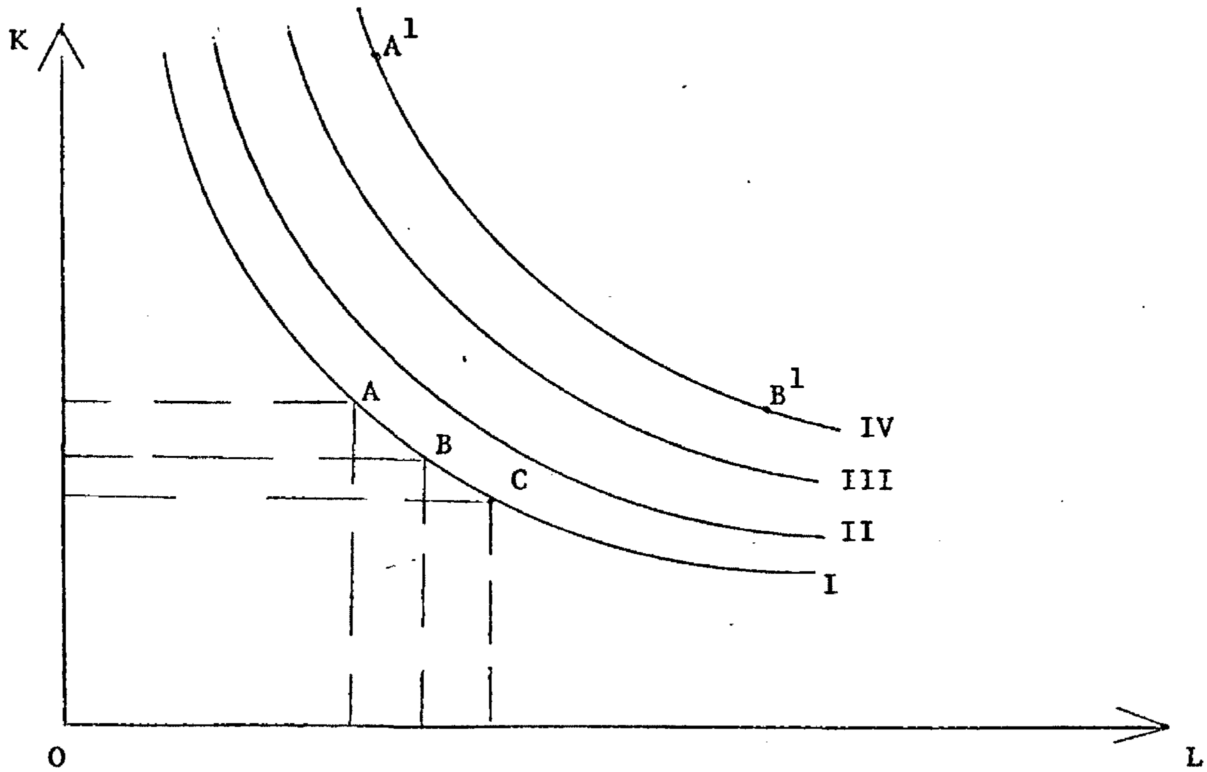

Figure 4

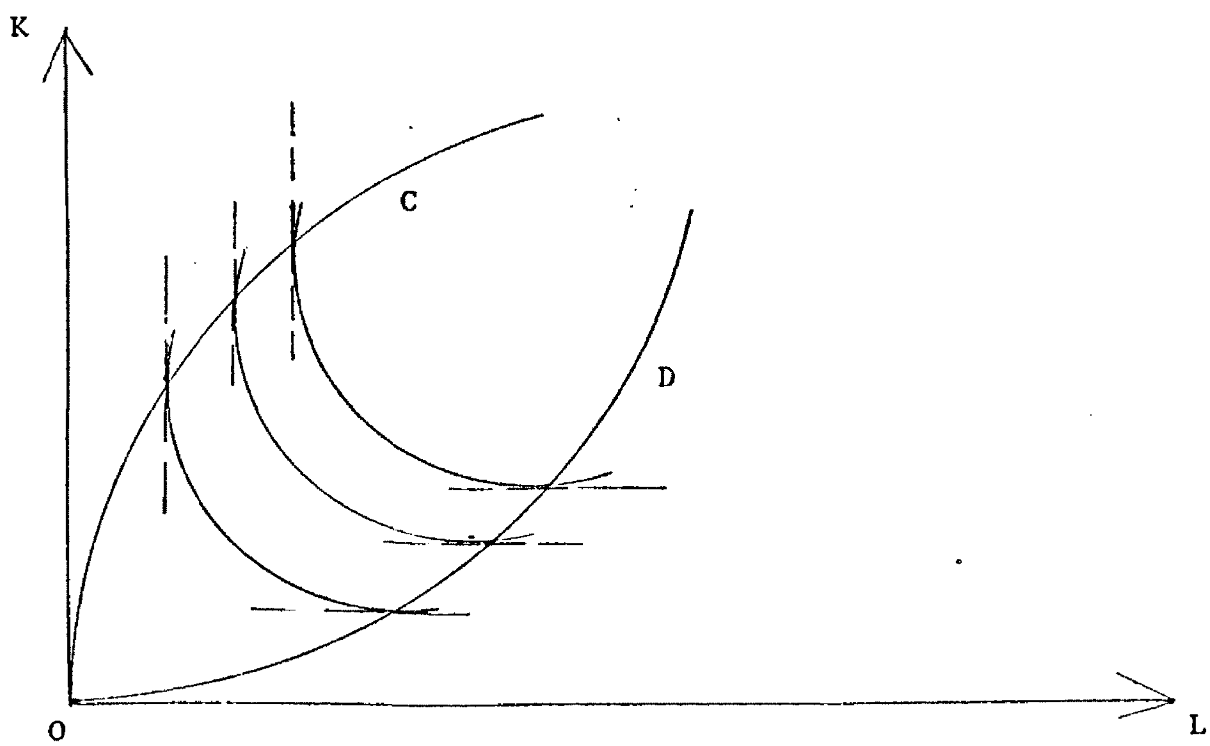

Figure 5

section with the isoclines. In Figure 5, along ob, the $MRTS_{K/L}$ was equal to zero.

From the fact that the MRTS decreases along an isoquant for changing input ratios, a substitution curve can be derived. This curve shows the $MRTS_{K/L}$ for every input ratio. The elasticity of this curve is called the elasticity of substitution. It is a measure of the relative responsiveness of the marginal products of the inputs to changes in the capital-labor ratio. It is sometimes called the elasticity of technical substitution. The elasticity of substitution, as implied by the substitution curve, holds only for movements along an isoquant, not for movements between them. The formula for the elasticity of substitution (E) is given by

$$E = -\frac{\Delta\frac{K}{L}}{\frac{K}{L}} \cdot \frac{\frac{MP_L}{MP_K}}{\Delta\frac{MP_L}{MP_K}} \qquad 3.15$$

where Δ means a small change. The general formula for E was derived in terms of the production function. Let $y = \frac{K}{L}$ and $S = MRTS_{K/L}$ so that equation 3.15 could be rewritten as

$$E = \frac{dy}{y} \cdot \frac{S}{dS} \qquad 3.16$$

Now take the total differential of y

$$dy = \frac{\partial \frac{K}{L}}{\partial L} dL + \frac{\partial \frac{K}{L}}{\partial K} dK \qquad 3.17a$$

and rearrange terms so that

$$dy = \frac{L(dK) - K(dL)}{L^2} \qquad 3.17a$$

or

$$dy = -\frac{S \cdot L + K}{L^2} dL \qquad 3.18$$

The total differential for S is given by the general formula

$$dS = \frac{\partial S}{\partial L}dL + \frac{\partial S}{\partial K}dK \qquad 3.19$$

and again rearranging terms

$$dS = -S\frac{\partial S}{\partial K} - \frac{\partial S}{\partial L}\, dL. \qquad 3.20$$

Also

$$\frac{\partial S}{\partial K} = \frac{\partial \frac{f_L}{f_K}}{\partial K} = \frac{f_K \cdot f_{LK} - f_L \cdot f_{KK}}{f_K^2} \qquad 3.21$$

and

$$\frac{\partial S}{\partial L} = \frac{\partial \frac{f_L}{f_K}}{\partial L} = \frac{f_K \cdot f_{LL} - f_L f_{LK}}{f_K^2} \qquad 3.22$$

Now substitute equations 3.18, 3.20, 3.21, and 3.22 into equation 3.16 and the formula for E based upon the general production function was derived:

$$E = -\frac{f_L \cdot f_K(L \cdot f_L + K \cdot f_K)}{L \cdot K(f_{LL} \cdot f_K^2 - 2 \cdot f_{KL}\ f_K \cdot f_L + f_{KK} \cdot f_L} \qquad 3.23$$

This expression also showed that the elasticity of substitution of labor for capital is the same as the elasticity of capital for labor. This result was obtained because the general formula for E is a symmetrical.

Within the relevant stage of production, the limiting values of E are zero and infinity. To see this equation 3.18 and 3.19 should be substituted into 3.16. This resulted in

$$E = \frac{S}{L \cdot K} \frac{L \cdot S + K}{(\partial S/\partial K^2) - (\partial S/\partial L)} \qquad 3.24a$$

where

$$S \cdot \frac{\partial S}{\partial K} - \frac{\partial S}{\partial L} = - \frac{dS}{dL} = \frac{d^2K}{dL^2} \qquad 3.24b$$

which was the slope of an isoquant, so that

$$E = \frac{S}{L \cdot K} \quad \frac{L \cdot S + K}{(d^2K/dL^2)} \qquad 3.25$$

If two inputs were perfect substitutes, the isoquant would have been a straight line; and E would have gone to infinity. If two inputs were perfect compliments, an isoquant would have been shaped with a right angle and the slope of the isoquant approaches infinity; therefore, E approached zero.[6]

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[6]Ferguson, The Neoclassical Theory, p. 92.

$$E = -\frac{\Delta\frac{K}{L}}{\frac{K}{L}} \cdot \frac{\frac{MP_L}{MP_K}}{\Delta\frac{MP_L}{MP_K}} \qquad 3.15$$

Remember the $MRTS_{K/L}$ was given by the first derivative of the formula for an isoquant. If the isoquant was a straight line, the value of the first derivative is a constant. The term $\Delta MRTS_{K/L}$ is equivalent to taking the second derivative of the formula of an isoquant. Any derivative of a constant is zero. Now if $\Delta MRTS_{K/L}$ was equal to zero, the isoquant is a straight line; the denominator if equation 3.15 would be equal to zero; and the value of E would approach infinity. Also, if ΔMRTS approached infinity, the numerator goes to infinity and E would approach zero.

Whenever production functions were examined, the concept of technological change naturally arises. The two most widely discussed models were those initially presented by Hicks and Harrod. Hicks' definition of technological progress is primarily a short run concept; the supply of inputs is relatively fixed. Hicks defined technological progress or inventions "according as their initial effects are to increase, leave unchanged, or diminish the ratio of the marginal product of capital to that of labor. We call these inventions labor-saving, neutral, and capital-saving respectively."[7] Given the $MRTS_{K/L}$, then technological progress is neutral if--for a given input

[7] J. R. Hicks, The Theory of Wages (London: MacMillan and Company, Limited, 1933), pp. 121-122.

price ratio--the $MRTS_{K/L}$ remains unchanged from one period of time to another. If the $MRTS_{K/L}$ was decreased from one period to another, then an invention would be capital using. Finally, if an invention would be labor using if the $MRTS_{K/L}$ was increased.

Harrod's technological change is defined as neutral, capital using or labor using if "for a given interest rate the capital-output ratio remains unchanged, increases, or decreases."[8] This was the same as saying the relative share of capital remains unchanged, increases, or decreases, for a given capital-output ratio. Since the capital-output ratio is just the inverse of the average product of capital, a Harrod technological change was easily graphed. In Figure 6, competitive equilibrium was assumed so that the marginal product of capital equaled the rate of interest. For a given interest rate, B, a neutral technological change just rotated the average product curve to the right, from point C to D; the average product remained the same for a given capital-output ratio. If a technological change was capital using, then at a given interest rate, B, the average product of capital would be increased from C to some point above D. For a labor-using invention, the average product of capital fell to a point below C. Harrod's technology implied a long run concept since the interest rate must remain invariant to changes in technology and the capital-output ratio.

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[8]R. F. Harrod, Toward a Dynamic Economics (London: MacMillan and Company, Limited, 1949), pp. 22-23.

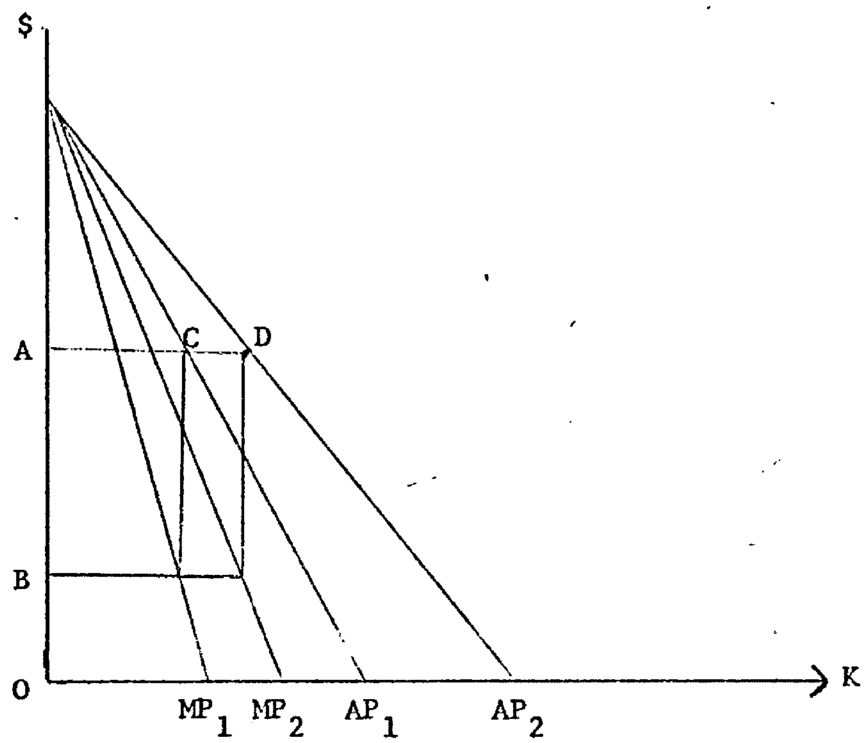

Figure 6

Hicks and Harrod neutrality were the same "if, and only if, the production function is characterized by constant output elasticities at all points"[9] (on the production surface). The output elasticity of an input is just the ratio of the marginal product of an input to the average product. For example, the output elasticity of labor, E_L, is

$$E_L = \frac{\partial X}{X} \div \frac{\partial L}{L} = \frac{\partial X}{\partial L} \cdot \frac{L}{X} = \frac{MP_L}{AP_L} \qquad 3.26$$

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[9] Ferguson, The Neoclassical Theory, p. 221.

That is, the output elasticity is the proportional change in output caused by a change in one input times its relative rate of use. The output elasticity of an input was a function of all inputs. The sum of all output elasticities for a production function are equal to the function coefficient. The function coefficient is the "elasticity of output with respect to an equiproportionate variation of all inputs."[10]

One point should be noted concerning technological change within a firm and for an economy. Within a firm, technological change would most likely be an erratic process because of fixed resources and because once a technological advance was incorporated into the production process, it was quite possible that there would be a significant increase in productivity. In the aggregate, technological change tends to be a smooth-flowing process. The erratic process of technological change within firms tends to average out when summed over the economy.

The following chapter is a mathematical development of the CES production function equation. Its derivation was based upon the concepts developed in this chapter.

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[10]Ferguson, The Neoclassical Theory, p. 76.

CHAPTER IV

DEVELOPMENT OF THE MODEL

In order to measure the relative nature of the production function equations for selected private and government services, it was necessary to have a model which would (1) be economically meaningful, (2) be feasible, and (3) answer the question of interest. The basic model used in this paper was the constant elasticity of substitution (CES) production function. This model was derived directly from the general definition of the elasticity of substitution. The CES production function can be derived in several ways.[1] The following derivation was taken largely from Brown.[2] The reason for choosing Brown's method was that it allowed the returns to scale to vary.

As stated above, the derivation of the CES production function stems directly from the definition of the elasticity of substitution. The elasticity of substitution can be described from

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[1]C. E. Ferguson, "Substitution, Technical Progress, and Returns to Scale," American Economic Review, Papers and Proceedings, Vol. 55, No. 2 (1965), pp. 298-299; C. E. Ferguson, The Neoclassical Theory of Production and Distribution (Cambridge: Cambridge University Press, 1969), pp. 101-103; K. Arrow, H. Chenery, B. Minas, and R. Solow, "Capital-Labor Substitution and Economic Efficiency," The Review of Economics and Statistics, Vol. 42 (August, 1962), pp. 230.

[2]M. Brown and J. S. DeCani, "Technological Change and the Distribution of Income," International Economic Review, Vol. 43, No. 3 (September, 1963), pp. 305-309.

In Chapter III, an isoquant was shown to represent a constant level of output for changing input ratios. Since, for every point on an isoquant, there is a unique input ratio; either input is a function of the other input,

$$K = f(L) = f \qquad \text{or} \qquad L = f(K) = g \qquad 4.1$$

given X equal to some fixed output. F and g are just shorthand expressions for the functional relationships. To find the explicit equation for 4.1 in terms of the elasticity of substitution, some manipulation was done.

The $MRTS_{K/L}$ is defined along an isoquant as

$$MRTS_{K/L} = \frac{dK}{dL} = f' \qquad 4.2$$

The elasticity of substitution then becomes

$$E = \frac{d\frac{K}{L}}{dS} \cdot \frac{S}{\frac{K}{L}} = \frac{d\frac{K}{L}}{df'} \cdot \frac{f'}{\frac{K}{L}} \qquad 4.3$$

also

$$\frac{d\frac{K}{L}}{df'} = \frac{L\frac{dK}{df'} - K\frac{dK}{df'}}{L^2} = \frac{1}{f''}\frac{Lf - f}{-L^2} \qquad 4.4$$

$$\frac{df^1}{dL} = f'' \text{ or } \frac{dL}{df^1} = \frac{1}{f''} \text{ and } \frac{df}{df''} = \frac{df}{dL} \cdot \frac{dL}{df^1} = \frac{f'}{f''} \qquad 4.5$$

Now 4.4 was substituted into 4.3 to obtain a nonlinear second order differential equation for E

$$E = \frac{Lf'^2 - ff'}{Lff''} \qquad 4.6$$

By reducing 4.6 to a first order linear differential an explicit formula for K = f(L) was found. First set 4.6 equal to zero and let E be some constant not equal to zero so that

$$f'' + \frac{1}{EL} \cdot f' = \frac{1}{Ef}(f')^2 = 0 \qquad 4.7$$

Start the reduction process by substituting

$$f(L) = \exp[h(L)] = -\exp(h) \qquad 4.8a$$

(Exp. is just the exponential e; e = 2.718...)

where

$$f'(L) = h'(L)\exp[h(L)] = h' \cdot \exp(h) \qquad 4.8b$$

and $$f''(L) = h''(L)\exp[h(L)] + [h'(L)]^2\exp[h(L)] \qquad 4.8c$$

into equation 4.7 to obtain

$$h'' + \frac{1}{EL} h' + \frac{E - 1}{E} h'^2 = 0 \qquad 4.9$$

Continue the reduction process by placing

$$h' = Q(L) = Q \qquad 4.10$$

and substitute again

$$\frac{Q^1}{Q^2} + \frac{1}{EL}\cdot\frac{1}{G} + \frac{E-1}{E} = 0 \qquad 4.11$$

to yield a nonlinear second order differential equation. The final step in the reduction process was completed by letting $Q(L) = \frac{1}{R(L)}$ and

$$R'(L) = -\frac{Q^1(L)}{(Q(L))^2} \qquad 4.12$$

to yield a first order linear differential equation

$$R^1 = \frac{1}{EL} R = \frac{E-1}{E} \qquad 4.13$$

To obtain a solution for 4.13, it was placed in the form of an exact equation, letting the constant factor of integration be

$$\exp -\frac{1}{E} \quad \frac{1}{L}dL = \exp -\frac{1}{E} \log L = L^{-\frac{1}{E}} \qquad 4.14$$

and multiplying equation 4.13 by this factor. This resulted in

$$L^{-\frac{1}{E}}\frac{dR}{dL} - \frac{L^{-\frac{1}{E}}}{EL}R = \frac{E-1}{E}L^{-\frac{1}{E}} \qquad 4.15a$$

or

$$d\ R\cdot L^{-\frac{1}{E}} = \frac{E-1}{E}\cdot L^{-\frac{1}{E}}\cdot dx \qquad 4.15b$$

Now integrate both sides to obtain

$$RL^{-\frac{1}{E}} = L^{-\frac{1+E}{E}} + C \qquad 4.16$$

where C is an arbitrary constant of integration. Rearrange terms to get

$$R = L\ 1 + CL^{\frac{1-E}{E}} \qquad 4.17$$

which is the general formula for an isoquant.

The general formula for the CES production function with returns to scale parameter was found from Euler's Relationships and equation 4.17. Now, from above

$$R(L) = \frac{1}{Q(L)} = \frac{1}{h'(L)} = \frac{1}{\frac{d(\text{Log } f(L))}{dL}} = \frac{1}{\frac{f'(L)}{f(L)}} = \frac{f'(L)}{f'(L)} = \frac{dL\ f(L)}{df} \qquad 4.18$$

and the initial function relationships let K = F(L) so that df = dK. Now in equation 4.17 place $R = \frac{f(L)}{f'(L)}$, from equation 4.18 so that

$$\frac{dL\ f(L)}{df} = L + CL^{\frac{1}{E}} \qquad 4.19a$$

or rearranging

$$\frac{dL}{L + CL^{\frac{1}{E}}} = \frac{dK}{K} \qquad 4.19b$$

Rearrange equation 4.19 for easier integration by setting

$$\frac{dK}{K} = \frac{L^{\frac{1}{E}}}{C + L^{\frac{E-1}{E}}} dL \qquad 4.20$$

and equation 4.20 yields, after a little manipulation,

$$L^{1-\frac{1}{E}} - ak^{1-\frac{1}{E}} = C \qquad 4.21$$

where a is the result of the constant factor of integration. Equation 4.21 is the equation of an isoquant with the constant and arbitrary value of C. The slope of equation 4.21 must be negative so replace a by -J so that 4.21 now becomes

$$L^{1-\frac{1}{E}} + JK^{1-\frac{1}{E}} = C \qquad 4.21a$$

Since output is a function of the production surface, it is also a function of isoquants which make up the entire production surface. Let output be a function of equation 4.21.

$$X = f\ L^{1-\frac{1}{E}} + JK^{1-\frac{1}{E}} = f(h) \qquad 4.22$$

Using equation 4.22 and Euler's Relationship, the explicity equation for the CES production function was found. The general equation for Euler's Relationship for a homogeneous function of degree υ is

$$XV = K\frac{\partial X}{\partial L} + L\frac{\partial X}{\partial L} \qquad 4.23$$

Since $\frac{\partial X}{\partial K} = J\ 1-\frac{1}{E}\ K^{-\frac{1}{E}}$ from equation 4.22, 4.23 can be rewritten as

$$VX = L^{1-\frac{1}{E}}\ L^{-\frac{1}{E}}\ \frac{dX}{dL} + JK^{1-\frac{1}{E}} = 1-\frac{1}{E}\ L^{1-\frac{1}{E}} + JK^{1-\frac{1}{E}}\ \frac{dX}{dL}$$

$$= 1-\frac{1}{E}\ h\frac{dX}{dL} \text{ or } \frac{dX}{X} = \frac{V}{1-\frac{1}{E}}\ \frac{dh}{h} \qquad 4.24$$

Integrate both sides and substitute the value for h (equation 4.22) into the resulting equation to find

$$\text{Log } X = \text{Log } L^{1-\frac{1}{E}} + JK^{1-\frac{1}{E}}\ \frac{V}{1} - \frac{1}{E} + \text{Log } Ag\ \frac{V}{1} - \frac{1}{E} \qquad 4.25$$

By letting $J = \frac{g}{1-g}$ and $\frac{1}{E} - 1 = e$ and again rearranging the final form of the production function was found.

$$X = A(gK^{-e} + (1-g)L^{-e})^{\frac{\upsilon}{e}} \qquad 4.26$$

By taking the limit of e ($\lim_{e\to\infty} e = 1$) or as E goes to one reduces the CES to the Cobb-Douglas production function.

Several restrictions were placed on equation 4.26. The values of the parameters, to conform to what economic reality should be, are as follows: 1. $A > 0$, 2. $0 < g < 1$, 3. $E > 0$, and $V > 0$. If A was less than zero, it implies that output would be negative with positive values of inputs. When g is less than zero, capital always has a negative contribution to output. For $g > 1$, labor's contribution is negative. If the elasticity of substitution was less than zero, this implies the marginal product of one input is negative. Production is taking place in either stage I or stage III. If $V < 0$, then there is no relevant stage of production; and there are absolute diminishing returns to scale. Of course, positive levels of inputs must be used.

The parameters A, g, e, and v are called the efficiency, input intensity, substitution, and the returns to scale or homogeneity parameters respectively. The efficiency parameter, A, could be explained either in terms of another production function or in terms of shifts of a particular production function through time. In relation to another production function, A describes the relative efficiency of the production process given the input ratio, g, e, and v. That is, if the efficiency parameters--A_1 and A_2--for two different firms were not equal, given $g_1=g_2$, $e_1=e_2$, $v_1=v_2$, and $\frac{K_1}{L_1}=\frac{K_2}{L_2}$, then X_1 must be greater than X_2, implying greater efficiency in the use of inputs if A_1 was greater than A_2. The difference in efficiencies between firms or changes in efficiency through time was graphed. In Figure 7, the difference in efficiency between two firms was represented by the distance between f_1 and f_2.[3]

When dealing with one firm, a change in efficiency was represented by upward shift in the production function, from f_2 to f_1 by a multiple of the change in A at each point along the production

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[3]R. M. Solow, "Technological Change and the Aggregate Production Function," The Review of Economics and Statistics, Vol. 39 (August, 1957), p. 313.

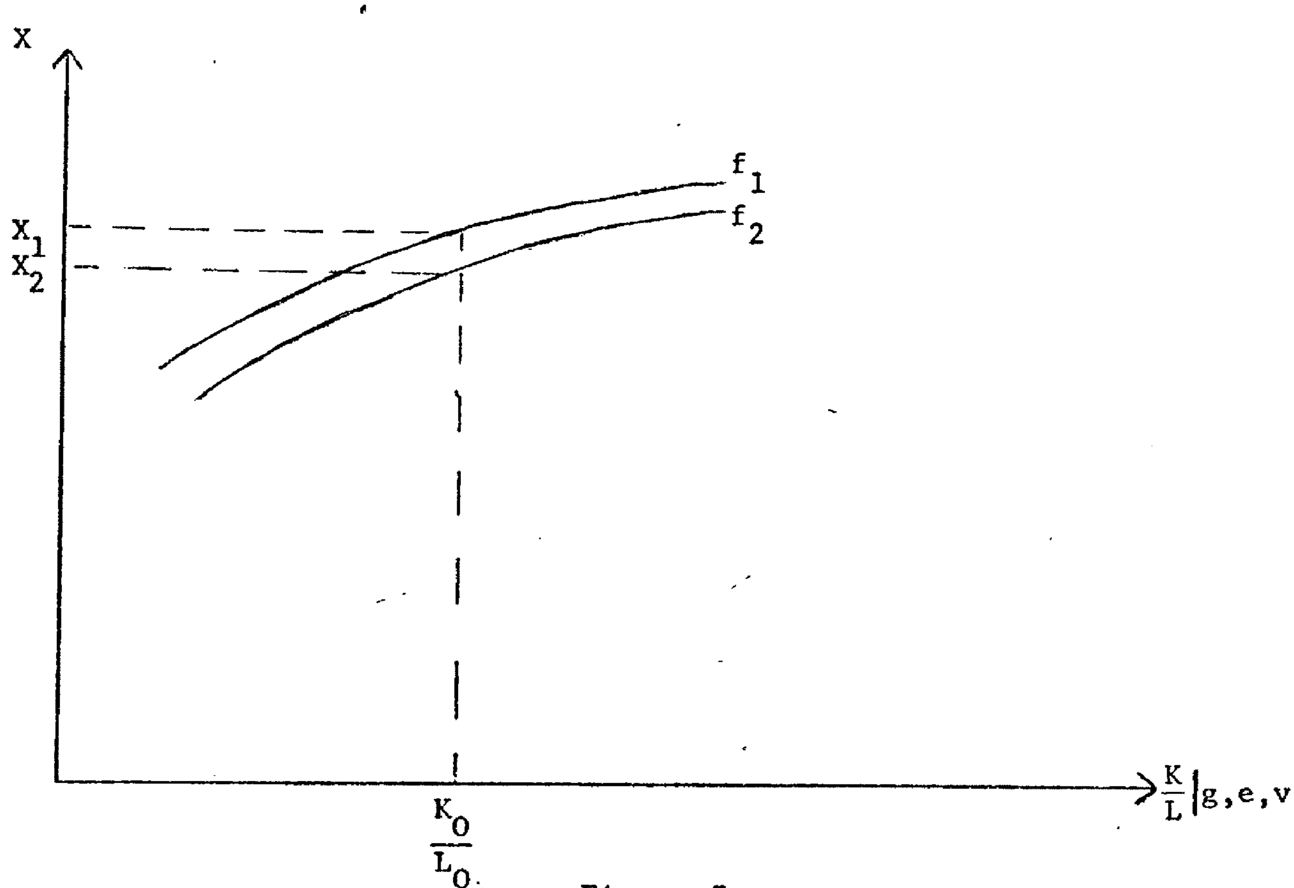

Figure 7

function. The graphical representation of changes in efficiency shown in Figure 7 was, in the strict sense, only representative of a linear homogeneous production function because, in this case, output was solely a function of the input ratio. To represent a function homogeneous of a degree > 1, it would be necessary to graph the production function in three dimensions. A change in A in the CES production function acts as a Hicks neutral technological change. Hicks' neutrality was defined as an invariant relationship between the capital-labor ratio and the marginal rate of technical substitution of capital for labor. The relationship between the efficiency parameter and Hicks' neutrality can easily be seen. The marginal rate of technical substitution of capital for labor was given by

$$MRTS_{K/L} = \frac{MP_L}{MP_K} = \frac{g}{1-g}\,\frac{L}{K}^{\frac{1}{E}} \qquad 4.27$$

Since the $MRTS_{K/L}$ did not contain A, it remained invariant to changes in A, that is, $\frac{\partial MRTS_{K/L}}{\partial A} = 0$. Therefore, any change in A had no effect on the relationship between inputs and the $MRTS_{K/L}$ remained the same for all previous values of the capital-labor ratio. Also, there was no problem with returns to scale since the v parameter cancels itself out in the process of taking the partial derivatives of the CES equation.

The input intensity parameter, g, derived its name from its effect on the labor-capital ratio, given the marginal rate of technical substitution. From equation 4.27, it could be seen that, given the value of E and $MRTS_{K/L}$, the greater the value of g, the smaller the labor-capital ratio. So, for each labor-capital ratio, given E, g determined the rate at which capital was substituted for labor.

The substitution parameter, e, was just the elasticity of substitution plus a constant as could be seen from the definition of e,

$$e = \frac{1}{E} - 1 \qquad 4.28$$

The returns to scale or homogeneity parameter, e, was also straightforward in its interpretation. The linearly homogeneous form of equation 4.26 is

$$X = A(gK^{-e} + (1-g)L^{-e})^{-\frac{1}{e}} \qquad 4.29$$

This was seen from the fact the K^{-e} and L^{-e} taken to the $-\frac{1}{e}$ power reduces equation 4.27 to the first power. By taking equation 4.29 to the v^{th} power, it is automatically made a homogeneous equation of degree v. The definition of a homogeneous equation of degree v was given as,

$$X = vf(ZK,ZL) = Z^{v}f(K,L) \qquad 4.30$$

where Z referred to a proportion increase in all inputs. It could be seen that if v was greater than one, then output would increase more than proportionate to the increase in the inputs, Z. The converse is true for decreasing returns to scale, v > 1.

The marginal products of both inputs were positive and decreasing if v was either greater than zero or not too large. To see this, the first derivative of output with respect to the input K was found:

$$\frac{\partial X}{\partial K} = vA(gK^{-e} + (1-g)L^{-e})^{-\frac{v}{e}-1}(gK^{-e-1}) \qquad 4.31$$

Stated another way, the marginal products of both inputs were positive and decreasing as additional units of inputs were added. This implies that the production function describes the relevant stage of production if v > 0.

The limits of the marginal products depend upon the value of the elasticity of substitution. If E < 1, then the limit of both MP_K

and MP_L approached zero as either K or L approached infinity. When $E > 1$, the limiting value for the marginal products was a constant. When the value for E was greater than one, the two inputs were quite similar so one can be substituted for the other without bounds. The limit of output as one or the other inputs was expanded also depends on the elasticity of substitution. The limits for X, given E, were consistent with the above results. If $E < 1$, then the limit of X approached a constant; and if $E > 1$, then the limit of X approached infinity.[4]

The Cobb-Douglas production function is a special case of the CES production function. The elasticity of substitution was, like the CES, constant, but given equal to one. The substitution parameter, e, went to zero and was dropped from the equation resulting in the general Cobb-Douglas form

$$X = AK^c L^b \qquad 4.32$$

The efficiency parameter, A, acted in an identical manner as in the CES production function. The two parameters--c and b--were the output elasticity of each input. The output elasticity is defined as the "proportional change in output induced by a change in an input relative to the proportional change in this input."[5] The sum of c and

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[4]M. Brown, On the Theory and Measurement of Technological Change (London: Cambridge University Press, 1966), p. 50.

[5]Ferguson, The Neoclassical Theory, p. 76.

b is called the function coefficient where the function coefficient is defined as "the elasticity of output with respect to an equiproportional variation of all inputs."[6] If $c + b > 1$, then there were increasing returns to scale. If $c + b < 1$, then the function was subject to decreasing returns to scale. For a value of $c + b = 1$, then there were constant returns to scale. The limits of output, and the marginal products of the inputs were the same as in the CES function when the elasticity of substitution was equal to one.

To use the CES production function with time series data, an additional term was added to the derived function. This term was a trend term which allows for technological change. In this paper, this trend term was noted by $\exp(mt) = e^{mt}$. The e is just the exponential e and t is a trend term denoting the time period, $t = 1, \ldots, n$. The constant m was to be estimated along with the values for A, e, g, and v. The derived value for m indicates the type of technological progress taking place through time. The general way to introduce $\exp^{mt}$ into the CES production is

$$X = A\ gK^{-e} + [1-g][\exp(mt)L]^{-e\ -\frac{v}{e}} \qquad 4.33$$

This form represented Harrod's neutral technological change; Hicks' neutral technological change was represented in the CES by, n:

[6]Ferguson, The Neoclassical Theory, p. 79.

$$X = A \exp[mt][gK^{-e} + (1-g)L^{-e}]^{-\frac{v}{e}} \quad 4.34$$ [7]

A caveat must be stated with regard to measuring technological change for a firm or sector and also about the returns to scale of a production function. First, as noted above, technological change usually cannot be represented adequately by a smooth exponential time trend for a firm or sector. Within a firm, there were usually certain fixed assets that could not be altered during a short period of time. When a technological change occurred within a firm, there was most likely a sudden shift in the production function. The extent of this shift depended upon the nature of the technological change. The same ideas could apply to a sector of the economy. If one firm gained a comparative advantage over the other firms in a sector because of technological change, then, barring constraints, the other firms could, in a relatively short period of time, introduce the innovation into their productive process. If this type of reaction would occur among firms, then the production function for the sector would shift. This shift would be less abrupt than that for a firm but would most likely not be a smooth, continuous change.

The simplest way to circumvent this problem would be to use cross-section data and find the trend in technological change in one time period. Unfortunately, this could not be done in this study

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[7]Ferguson, "Substitution, Technical Progress, and Returns to Scale," p. 299.

because of the lack of data. When the disaggregate time service data was regressed, there were three alternatives. The first would be to build a model allowing for discontinuous shifts in the production. Second, the time trend would be left out but no technological change seemed to be a very heroic assumption. Finally, the exponential trend would be included as an approximation for technological change. The last method was used for the data in this study and appeared to fairly closely resemble a smooth continuous production function. Technological change appeared to act fairly slowly and continuously over time. This result was partially caused by the lack of any great technological change. Probably the most significant innovation was the introduction of computers, although their introduction to the service sector appeared to have been spread over time.

A second deviation from standard neoclassical theory was assuming factors were paid their marginal products while allowing returns to scale to vary. There is no problem if returns to scale were constant. From Euler's relationships, it was seen that if there were increasing or decreasing returns to scale then the sum of the inputs times their rate of use consumes less or more than the total product. The possibility of one or more factors receiving more or less than their marginal product is not too serious as long as the departure from constant returns to scale was not large.[8] If the

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[8]Marvin Frankel, Discussion, American Economic Review, Papers and Proceedings, Vol. 55 (1965), pp. 307-309. This problem has only been mentioned.

deviation of the returns to scale was large, then there was no way to reconcile the conflict between returns to scale and factors being paid their marginal products. This conflict is usually ignored and for the lack of anything better it was disregarded in this paper.[9]

The CES production function was just one of many models that could be used to analyze the relative nature of the production of government and private services. Since Douglas' initial studies with the use of his famous production function, there have been many alternative models proposed. The following will be a brief review of a few of the models.

Uzawa introduced a generalization of the CES production function. This model is of the form

$$X = A(a_1 x_1^{-e} + \ldots + a_n x_n^{-e})^{-\frac{1}{e}} \qquad 4.35$$

where x_1, x_2, ... x_n refer to different factors of production. The primary characteristic of this model was that all partial elasticities of substitution are equal. This function could be generalized to allow subsets of variables, say, two types of labor and capital inputs, with equal partial elasticities of substitution while the partial elasticities between sets did not necessarily have to be equal. This type of function was of the form

[9]Frankel, Discussion, pp. 307-309. Frankel only mentioned the problem. He only seemed to feel that this is a major problem if the returns to scale parameter is greater than one.

$$X = A(a_1x_1^{-e_{12}} + a_2x_2^{-e_{13}})^{\frac{V}{e_{12}}}(a_3x_3^{-e_{34}} + a_4x_4^{-e_{35}})^{\frac{V_2}{-e_{34}}} \qquad 4.36$$

where $v_1 + v_2 = 1$, and $E_{12} = E_{13}$ and $E_{34} = E_{35} = 1$, but E_{12} may or may not equal E_{34}. This model, with appropriate data, would allow inputs to more closely meet the assumption of homogeneity.[10] With appropriate data, equation 4.36 would allow a finer comparison of the substitutability of factor inputs. A drawback associated with this model was the availability of data. This model, with the present sources of data, was not feasible for use with the service sector.

A wide range of models dealing with technological change were summarized by Bechmann and Sato.[11] They identified five main types of technological change and certain invariant relationships to test the type of technological change. Their regression results showed Hicks' and Harrod's neutrality generally provided a very good fit. This did not imply that some of the other models would not work

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[10] H. Uzawa, "Production Functions with Constant Elasticities of Substitution," Review of Economic Studies, Vol. 29, No. 81 (October, 1962), p. 291.

[11] M. J. Beckmann and R. Sato, "Aggregate Production Functions and Types of Technical Progress," American Economic Review, Vol. 50, No. 1 (March, 1969), pp. 91-95.

better for the service sector. Most of the models were of the Cobb-Douglas or CES type and generally only needed basic input and output data.

Another class of models has been presented by several persons.[12] These models took into account the possibilities of imperfect competition in the product and input markets. These models removed the restrictive assumptions of equality between the marginal products and rate of return to factors and the assumptions of perfectly elastic demand for the output. These models needed additional data on factor and product prices which put the problem of finding appropriate data out of reach.

A model, not generally used, should be mentioned. This is the variable elasticity of substitution model.[13] It allowed the elasticity of substitution to vary in response to changes in the input ratios and was probably a more accurate description of economic reality than assuming the elasticity of substitution constant. This was especially true when time series data was used where the inputs were not necessarily homogeneous through time. This model, however, introduced additional mathematical and statistical problems that this researcher was not capable of handling.

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[12] Brown, On the Theory and Measurement of Technological Change, p. 38.

[13] Ferguson, The Neoclassical Theory, p. 11.

The final type of model to be mentioned is the so-called "vintage model and the learning by doing models"[14] Both of these models allowed for variation in the productivity of inputs. The vintage model assumed that the additions to the capital stock of a firm would be more productive than the older capital stock. The learning by doing model allowed labor inputs to become more efficient with the passage of time. Both of these models allowed a quality change in the inputs with the passage of time. Unfortunately, they both needed data on capital inputs which were not available at this time for the service sector.

The CES production function has two main advantages over these other models. It is widely used and understood within the field of economics, and only data for output and for labor inputs were needed to determine the parameters of the production function. However, there was no economic basis for choosing the CES over the other models. It may be found that for a particular segment of the economy, one of the other models would be more appropriate when appropriate data becomes available.

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[14] Ferguson, The Neoclassical Theory, p. 305.

CHAPTER V

DESCRIPTION OF THE DATA

This chapter contains a description of the outputs provided by the service industries studied. Also, some of the problems associated with the data on the outputs and inputs of these service industries are examined. The basic data for this study were gathered from three sources.[1]

The governmental services investigated were produced by the Division of Disbursement within the Treasury Department, the Post Office Department, and the Veterans Department. The choice of these three services was dictated solely by the availability of data. Two other agencies were included in the Bureau of the Budget's study. They were the Bureau of Land Management and the Federal Aviation Agency. They were not included because the measures of output for these two services were tentative and extremely erratic.

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[1]U.S., Bureau of the Budget, Measuring Productivity of Federal Government Organizations (Washington, D.C.: Government Printing Office, 1964); U.S., Department of Commerce, Office of Business Economics, The National Income and Product Accounts of the U.S.: 1929-1964 (Washington, D.C.: Government Printing Office, 1964), pp. 20-21, 92-93; V. R. Fuchs, Editor, Production and Productivity in the Service Industries (New York: Studies in Income and Wealth, National Bureau of Economic Research, 1969), p. 22.

The Bureau of the Budget's study was designed to measure changes in productivity through time for selected governmental services. Changes in productivity were recorded by the use of index numbers. An intermediate output of this study was a measure of final output for each agency investigated. Final output included only those services produced for an outside agency or individual. All administrative functions and intra-agency outputs were not included in this measure. In effect, only those services produced which would have been sold if the agency was a private firm were included. This procedure allowed changes in the final output of an agency to be compared with that of private firms. The labor data for all three government services were payroll costs including overtime pay plus employee benefits, paid vacations, paid insurance, etc., and man-hours worked.

Data for all agencies were listed on a yearly basis, however the time periods covered for all agencies varied and each year was considered an observation. There were eight observations for the Division of Disbursements from 1955 through 1968, the Department of Insurance covered fourteen years from 1949 through 1962, and the Post Office Department observations covered ten years from 1953 to 1962.

Three distinct types of output were produced by the three agencies studied. The Division of Disbursement's primary output was payroll checks to government employees. This department experienced a very large growth in output from 1949 to 1962 while experiencing a decrease in labor inputs. The primary reason for the reduction in labor inputs was the introduction of electronic data processing equipment. This equipment allowed the Department to substitute capital for labor during this period. With such a large substitution of capital for labor, it was doubtful whether the inputs did remain homogeneous throughout this period. However, in the absence of a measure of quality change, it was assumed that the quality of inputs did remain fixed.

The Department of Insurance was engaged in providing life insurance services to veterans. Like the Division of Disbursement, it also incorporated new data processing equipment into their operations, during the period for which data were available--1955-1962. The same problems of changing quality of inputs is present. This agency's primary function was to process applications for veterans' insurance. This includes both disability and life insurance. The output of the agency was somewhat cyclical, depending upon the state of economy. During slowdowns in the economy, payments for life insurance tended to fall off. This agency provides a service quite similar to private insurance companies.

The Post Office Department provided a well-known service. The output of this department was primarily private mail service for private users. A few additional outputs were included such as the issuance of money orders, postal savings, and special mail services. These functions were only a small part of their total output and did not significantly affect the measure of total output. Data for this department were given from 1953 to 1962. Unfortunately, there was no private service of comparable size with which to compare it; however, the regression parameters will be listed to compare it with the other services in general. This department's outputs and inputs remained fixed in quality during the time period studied. The average hourly wage per employee was almost fixed in the time period study. There was also virtually no change in the type of capital assets used. This department was characterized by a very labor-intensive operation. Both output and inputs experienced about the same rate of growth.

No attempt was made to change the data derived by the Bureau of the Budget. All data were taken directly from the study, placed into index number form and used in the regression model. The index numbers for both the private and government sectors were computed by dividing each observation by sector for each input and output by the base year inputs and output figure for each respective sector. The base year was defined as the first year for which data were available for each agency.

All data on the private sector were taken from the Office of Business Economics' statistical tables in The National Income and Product Accounts of the United States, 1929-1964, and the National Bureau of Economic Research (NBER) Studies in Income and Wealth's publication, Production and Productivity in the Service Industries.[2] The National Income and Product Accounts provided data on gross product originating, the total number of full-time equivalent employees, and average annual earnings per full-time employee for all sectors of the economy. Gross product originating is the current dollar contribution of each sector to the gross national product. The Office of Business Economics' yearly deflators for their data in The National Income and Product Accounts was found in the NBER publication listed above. Output was deflated to arrive at a constant dollar measure of output. This deflator probably underestimated the true increase of output in the service sector.[3] The estimates of real output used in the following regressions were probably as good an estimate as possible given the present sources of data.[4]

The services included in this paper were the services provided in the following business categories: finance, insurance, and real estate; and personal and business services. These two main categories were further broken down so that regressions could also be run for a more homogeneous output. These services were used to provide a fair sampling with which to compare the government sector. The ideal situation would have similar services being examined. This unfortu-

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[2] U.S., Department of Commerce, Office of Business Economics, The National Income and Product Accounts, pp. 20-21, 92-93; V. R. Fuchs, Editor, Production and Productivity, p. 22.

[3] Fuchs, Editor, Production and Productivity, pp. 15-16.

[4] Fuchs, Editor, Production and Productivity, p. 18.

nately could not be done because of the unavailability of data. Miscellaneous business services included services provided by advertising agencies, research, mail and duplicating companies, and testing laboratories. Insurance carriers provided all types of insurance services, such as, health, life and property insurance, and insurance-related services. Real estate operations primarily included buying, selling, leasing, and developing land and property. Finance, insurance and real estate services were a combination of insurance companies, real estate operation and banking, credit agencies, security and commodity brokers. It was evident that these services produce a heterogeneous output. It was assumed that by using the deflated selling price of these services that they were reduced to equivalent units.

Data for all private services were collected from 1949 through 1962; there were fourteen observations for output, labor, and the wage rate for each private service industry. From the data collected, index numbers were formed in the same manner as for the government sector. In addition to the index numbers constructed with the fourteen observations, index numbers with base years of 1955 were constructed for data on the finance, insurance, and real estate, and miscellaneous business services industries.

A comparison of government and private data showed that the measurement of the labor inputs and wages was approximately the same. There was a problem of comparing the measures of output for the private and government sectors. While this appeared to be a large obstacle, it was not. The changes in government output were based on a physical measure of one unit of output in the base year. In subsequent years, output was still measured in base year units. This method did not include any arbitrary measures for a change in quality. Within the private sector, the total deflated output of the base year was assumed to be equal to one unit. The following year's output was based solely on the base year as in the measure of the government output. Since the data for each sector were in index number form they could be compared.

The data for both the government and private sectors were very limited. This was especially true of capital data. If data on capital inputs were available, it would allow several of the alternative models to be used.[5] Also, if a greater listing of data for private and government services were available, it would allow a closer comparison of the two sectors. Finally, if cross-section data on government services were available, this would reduce the problem of changing quality of inputs and outputs.

This was not to say that there was not other information available on the service sector. There was a great deal of data for different segments of the private service sector. However, this data

[5]Data on capital inputs were not required in the regression model used in Chapter VI.

was generally limited to one or two types of services. Also, the data from one source was not generally compatible with another source.

In summary, the limited data available precluded a detailed comparison of the government and private sectors. Also, the data available were somewhat questionable because only time series data were used in the regressions, and the problem of non-homogeneous units of input and output was always present. Even considering the problems associated with the data, it would be worthwhile to see if some results were obtainable, to see if the method used was workable before doing more sophisticated studies in this area.

CHAPTER VI

EMPIRICAL RESEARCH

The purpose of this chapter was to try to determine whether there were empirically identifiable differences in the nature of the production of similar private and governmental services. The method used was to estimate the substitution (e), returns to scale (v), and the technological change (m) parameters for the CES production function,

$$X = A(gK^{-e} + [1-g][\exp(mt)L]^{-e})^{-\frac{\upsilon}{e}} \qquad 6.1$$

in its Harrod neutral form. The efficiency (A) and distribution (g) parameters could not be estimated because of the unavailability of data on capital inputs. The t introduced in equation 6.1 and used in the regression model denotes the time period for each observation, t = 1, ..., n. The inclusion of this term implies that the quality of labor inputs increased from one observation to the next. This term has been found to be necessary when using time series regressions.[1]

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[1] C. E. Ferguson, "Substitution, Technical Progress, and Returns to Scale," American Economic Review, Paper and Proceedings, Vol. 53 (1965), p. 229.

There were several ways by which the parameters for equation 6.1 can be estimated. Two of the methods required data on capital inputs and the rate of return for capital. The first method used a process of iteration to find the values of the parameters. With this process, the initial values for m and g must be placed into the regression model to compute the remaining values.[2] This process would work well as long as the initial values were not too far from the correct values. A second method would use the Taylor binomial expansion of 6.1 and assume that the remainder term was near zero with only two expansions.[3] This method also had one drawback. If the value for the elasticity of substitution departed from one by very much, the remainder term from Taylor expansion would become very large. This, in turn, would introduce a large error term into the estimate of the parameters. The third method would use a side condition to first estimate e, v, and m by least squares regression. The primary advantage of this method would be that only data on the wage rate (w) and labor inputs would be needed to estimate the values for these parameters. This would circumvent the problem of collecting

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[2]M. Brown, On the Theory and Measurement of Technological Change (London: Cambridge University Press, 1966), p. 103.

[3]J. Kementa, "On Estimation of the CES Production Function," International Economic Review, Vol. 8, No. 2 (June, 1967), p. 180.

and manipulating capital data.[4] The third method was used in this chapter.

The model used for the regressions was

$$\text{Log } X = a_0 + a_1 t + a_2 \text{ Log } w + a_3 \text{ Log } L \qquad 6.2$$

In this form

$$E = \frac{a_2}{a_3}, \; v = \frac{a_3 - a_2}{1 - a_2}, \text{ and } m = \frac{a_1}{a_3 - a_2} \qquad 6.3$$

This regression model had an additional favorable characteristic. The Hicks' neutral form of the CES production function

$$X = A[\exp(nt)](gK^{-e} + [1-g]L^{-e})^{-\frac{v}{e}} \qquad 6.4$$

was obtained by letting

$$n = \frac{a_1}{1 - a_3} \qquad 6.5$$

in the regression model given in equation 6.2.

Since there was no statistical reason for choosing either Hicks' neutral or Harrod's neutral technological progress, it was advantageous to be able to look at both forms.[5]

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[4]Brown, <u>On the Theory and Measurement</u>, p. 97.

[5]Ferguson, "Substitution, Technical Progress, and Returns to Scale," pp. 228-229.

If the Harrod neutral form was assumed, then Hicks' technological change could be investigated. Hicks' technological change is labor using or capital using as $-em \gtrless 0$. If Hicks' neutrality was assumed, then Harrod's technological change can be investigated. In this case, Harrod's technological change is capital or labor using as $E \gtrless 1$.[6]

The empirical results using equation 6.3 are shown in Table 6.1.

TABLE 6.1

	E	v	n	m
Miscellaneous Business Services	.688	.159	-.912	-.072
Insurance Carriers	.533	- .099	.049	-.030
Real Estate	6.095	-1.182	.667	-.250
Finance	.163	.087	.078	.011
All Services	.504	1.034	.810	.028
Division of Disbursements	-15.383	.820	.095	.020
Post Office Department	6.960	.634	.020	.011
Veterans Administration	- 2.025	.309	.074	.016

All regression coefficients for each regression were tested to determine if the coefficients were statistically different from zero at the ninety percent confidence level using an analysis of variance test. In all cases at least one parameter was not statistically different from zero. That is, it was found that the reduction of the sum of squares of deviation was not significant for at least one variable in each regression. This implied that the model developed in

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[6] C. E. Ferguson, The Neoclassical Theory of Production and Distribution (London: Cambridge University Press, 1969), p. 223.

Chapter IV possibly was not the appropriate one to use for the service sector. The CES production function was developed with the implicit assumption that all parameters were statistically significant. If one parameter was not significant then the entire model breaks down. For example, if the regression coefficients used to determine the value for the elasticity of substitution were not statistically significant, this implies that there was no value for the elasticity of substitution which was significant for the service sector.

For all but one regression, the multiple correlation coefficient of determination (R^2) was .983 or greater. The Veterans Administration had the lowest R^2 of .952. The high values for the R^2's indicated that the independent variable chosen may have been highly related. As a rough check for the above mentioned possibility of multicollinearity the time variable was regressed against the log of the wage rate index for the government service agencies. It was found that the time variable and percentage changes in the wage rate were highly correlated for the Division of Disbursements and the Department of Insurance. For the Post Office Department t and the logs of the labor input index were regressed against each other. These two independent variables were also found to be highly correlated.

The high correlation between t and percentage changes in the wage rate could be partially explained by the pricing policy for labor inputs by the government in which the wage rate increases were institutionalized in the legislative branch of the federal government.

Wage rate increases often came at specified intervals and by a given percent of existing wage rates. These contractural relationships between the wage rate and time did not necessarily allow the wage rate to change in response to changes in the marginal product of labor; therefore, the wage rate was not a good proxy for the capital stock. These contractural arrangements could not be handled by the CES production function. As mentioned in Chapter IV, the production function used in this paper was derived directly from the definition of the marginal rate of technical substitution and the elasticity of substitution. Both of these concepts imply that the inputs vary in response to changes in the marginal products of capital and labor.

A slightly different relationship from the one above held in the Post Office Department. This agency's production of services was characterized by a highly labor intensive production process. In order for output to increase through time the labor inputs must have increased if there was no substitution of capital for labor. In other words, the production function for the Post Office possibly was characterized by fixed-proportions production function. This was exactly what appears to have been the case in this agency.

Whenever there was a high correlation between independent variables, there was no reason to use both of the independent variables in the regression. "If both independent variables are used in the same regression it may not be possible to find the values of the individual regression coefficients with sufficient accuracy."[7] This again implied that the CES production function with the given data was not the appropriate model to use to analyze the service sector.

In addition to the above reasons for the poor results obtained in the regression, there were the problems associated with the data used in the regressions. As can be noted in table 6.1, there were several discrepancies from relevant economic theory. First, two of the estimated values for the elasticity of substitution were negative. This implied the marginal product of one factor was negative. Second, two of the returns to scale parameters were negative. This infers absolute decreasing returns to scale. Finally, four of the technological change parameters were negative. This implies technological retrogression.

Inaccurate measurements in obtaining the data used in the analysis was another cause of these discrepancies. The apparently poor data lent doubt to the following conclusions, including the lack of statistical significance of the regression parameters. The values for the CES production function parameters were tested separately from the regression coefficients. These parameters were also found to not be statistically different from zero at the ninety percent confidence level. Notwithstanding this doubt, the following paragraphs consider in more detail the values obtained for some of these parameters.

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[7]Michael J. Brennan, Jr., Preface to Econometrics, (Chicago: South-Western Publishing Company, 1960), p. 341.

The elasticity of substitution tended to be low in all sectors. This finding tended to confirm the belief that the service sector was labor intensive and indicated that factors were not readily substituted for one another. This result was consistent with the findings of Fuchs that the growth of employment in the service sector was greater than in the manufacturing and agricultural sector.[8]

The two negative values for the elasticity of substitution may have indicated that the government was either not able to substitute factor inputs for one another because of institutional arrangements or that there was a significant change in the quality of inputs through time. It may be recalled that the marginal product of labor is a function of the capital inputs, among other things. The change in the quality of capital inputs allowed a reduction in the number of employees within the Division of Disbursement and the Veterans Administration. If the introduction of electronic data processing equipment allowed a reduction in labor inputs and simultaneously increased the marginal product of capital, relative to that of labor, this would result in the elasticity of substitution obtaining a negative value. However, this negative value for E actually implied a shift in the shape of the production surface. That is, for any given input ratio, the value of the marginal rate of technical substitution probably changed because of changes in the functional relationship between inputs.

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[8]V. Fuchs, The Service Economy, (New York: National Bureau of Economic Research, 1968), p. 18.

The values for the return to scale parameter were all less than one for all but services taken together. However, they were all found not to be statistically different from zero. Except for the use of computers within some sectors, there were little or no technological advances.[9] The low values for the returns to scale parameters were a likely result of the fact that most service establishments were relatively small, compared to the industry sector. The private service sector has not appeared to have had a technological reason to increase the size of individual firms. Furthermore, as long as the service industries were relatively labor intensive, it was possible to establish a service establishment with little trouble. The values of the returns to scale parameter for government were generally larger than those of the private sector. Since the values were all less than one, however, little could be said other than that the degree of diminishing returns to scale may be smaller in the private sector. It should be remembered that the returns to scale parameters were not statistically different from zero. This implies that any proportionate increase in inputs would not increase output. This result was highly unlikely in any sector of the economy. This, along with the above mentioned doubts about the data, tended to support the idea that the CES production function was not the appropriate model to use.

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[9]When the CES production function is run with a returns to scale parameter, it tends to incorporate the effects of technological change parameter into the value of v.

Technological change within the private service sector was a mixture of labor using and capital using technological change. Of the four services produced within the private sector with economically meaningful parameters, three had Hicks' capital using technological change. This type of change was not unreasonable. With a relatively labor intensive production process and with the greater use of data processing equipment, it seemed possible that the marginal product of capital would be increasing relative to that of labor. All the technological change parameters showed the government experienced labor using technological change. A possible explanation was that combined with the introduction of electronic data processing equipment, there was a reduction in the size of the labor inputs. This reduction of labor inputs combined with improved capital equipment would have increased the marginal product of labor to that of capital if the reduction of labor inputs was large. It should be remembered that the technological change parameters also were not statistically different from zero. This implied that no technological advances took place. The possibility of no technological advance in the service sector was likely if the effects of electronic data processing were not significant considering the institutional pricing arrangements for labor.

In summary, the results found in this chapter cannot confirm any similarities or dissimilarities between the government and private service sectors. However, if data on government services were available for state or regional offices, cross-sectional analysis would be very beneficial in determining differences between sectors. With this approach, the labor and capital inputs would be relatively homogeneous as the model implied they should. It may be recalled that data encompassing nonhomogeneous inputs were one probable cause of the theoretically invalid parameters found in the empirical analysis. At the present time, the Office of Business Economics of the Department of Census is trying to develop a better measure for service output.[10] This measure, if developed, would allow further study with much greater accuracy.

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[10] V. Fuchs, Editor, Productivity and Production in the Service Industries (New York: Studies in Income and Wealth, National Bureau of Economic Research, 1969), p. 18.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The aim of this paper was to outline a method by which the relative nature of the production of government and private services could be compared. The method used was statistically to derive values for the parameters of a CES production function.

This method showed there were no statistical differences between the two sectors since at least one coefficient in each of the regressions was not significant. In addition to this, the values for some of the parameters derived were not economically relevant. A likely cause of the meaningless parameters was the lack of appropriate data. Also, institutional factors within the government sector may have made the CES production function inappropriate to use because wages may have not responded to changes in the marginal product of labor.

It was this researcher's opinion that if better data were available for both the government and private sectors, further research of this type could be conducted. If better data were available and if statistical differences between the two sectors were found then several observations and conclusions could be made. First, assume the returns to scale parameters for the private service sector were greater than those of the government sector. This might indicate that the government sector should decentralize the production of its services. Such a finding would be indicative of governmental inefficiency and a reason for more rapid increase in the cost of governmental services than in other services. If the elasticity of substitution was found to be larger in the government sector, this might indicate that the degree of specialization was relatively low, indicating the government sector may not have taken advantage of some of the inventions incorporated into the private service sector's production functions. A difference in the direction of technological change between the sectors would suggest further research. The types of technological progress in each sector could be evaluated to determine if the inventions used in one sector would be appropriate for use in the other sector.

Better data appeared to be a necessity. Because the wage rate and time were highly correlated, the wage rate could not be used as a proxy for capital equipment. As long as inputs were assumed to vary in response to changes in their marginal products, contractural wage policy invalidated this assumption. Further study in the area of estimation of the capital stock is necessary if meaningful results are to be obtained. Without capital data, it appeared as if the CES production function was not appropriate. If capital data were available, this would allow several alternative models to be run with a better chance of obtaining significant results and additional insights into the relative nature of the production of services.

Also, should no statistical differences be noted between sectors with the model then only one conclusion could be drawn: there are no inherent economic differences in the production functions of the two sectors. Any hypothesized inefficiencies with the goverment sector are probably present in the private sector. Attention is often given to the more rapid increase in the cost of governmental services than, for instance, the consumer price index. In the absence of statistical differences, these differential cost increases would be the likely result of the service intensive nature of governmental output.

As mentioned in Chapter V the Office of Business Economics is trying to develop a better measure of the output of the service sector. If a better measure of output is developed, this may allow a more detailed study of the service sector. It is hoped that the method now being developed will also be applicable to the government sector.

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