AN INVESTIGATION OF BUBBLE PLUME MIXING BY COMPARISON WITH LIQUID JET MIXING

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Mixing of liquids in tanks by means of a non-reacting gas-bubble plume was studied by using a tracer response method. The circulation time was obtained from the period of a damping oscillation of the response curve. The mixing time was defined as the time required to reduce the concentration variation to within 1% of the mixed mean value and was measured by an impulse response.

The results indicate that the induced liquid flow-rate varies as the first power of the submergence depth of the nozzle and as the 1/3 power of both the gas flow rate and the horizontal cross-section of the tank. When the circulating flow in the tank dominates the mixing, the ratio of mixing time to circulation time is nearly equal to that of the liquid jet mixing, and increases with decreasing distance between nozzle and side wall of the tank. When gas is injected in a central region of the tank, the mixing time becomes larger with decreasing depth of liquid, due to an effect of relative stagnation. Rapid mixing is achieved by injecting gas at a mid-radius of the tank, where the effects of both tank wall and relative stagnation can be excluded.

Introduction

Mixing of liquids in open-top tanks by means of a gas-bubble plume finds broad application in many industrial mixing processes including metals processing and biological (and chemical) water treatment processes.

On application to prevention of ice formation in lakes by raising warm bottom water to the surface, Konovalov[7] showed an approximate analysis of the liquid flow induced by a two-dimensional air-bubble plume from a line gas source submerged in water. Hussain and Siegel[3] proposed a two-phase axisymmetric jet model in infinite surrounding liquid region. They compared the analytical liquid flow with experiments reported by Kobus,[6] Baines[2] and Konovalov.[7] Recent studies[1,12] on batch mixing by means of gas showed that rapid mixing with multiple nozzle injections was accomplished by a certain asymmetric arrangement of nozzles and that rapid mixing with a single nozzle injection could be realized by situating the nozzle near the side wall of the tank.

In this study, mixing characteristics were measured by the impulse response method described in a previous paper.[10] The mean circulation time was obtained from the mean period of a damping oscillation of the response curve. The mixing time was defined as the time when the concentration dropped to within 1% of the mixed mean value. For comparison of the results with those of liquid jet mixing, characteristics of mixing by use of a vertical liquid jet were also measured since the reported data of liquid jet mixing[8,13] were confined to the case of a jet introduced axisymmetrically.

Prior to discussing bubble-plume mixing, some characteristics of vertical liquid-jet mixing will be described briefly. For bubble-plume mixing, an empirical correlation for induced liquid flow will be proposed on the basis of the measured circulation time. The mixing time will be discussed through comparison of the results with those of vertical liquid-jet mixing and through application of the two-parameter model for relative stagnation.

1. Experimental

Impulse response experiments were conducted to measure the mean circulation time and mixing time. The apparatus used for bubble-plume mixing studies is shown diagramatically in **Fig. 1**. Four cylindrical tanks, 30, 56, 80 and 104 cm in inside diameter, were used. Gas was taken from a compressed air main. The volumetric flow rate of air was varied from 1 to 10^4 cm^3/s. It was measured with an orifice meter and expressed as the flow rate at atmospheric pressure. A plastic pipe of 2.54-cm inner diameter was inserted through the side wall of the tank near the bottom. This horizontal piping allows a continuous change in radial position of the injection nozzle. Three nozzles were provided: two air orifices of 1 and 2.5 cm internal

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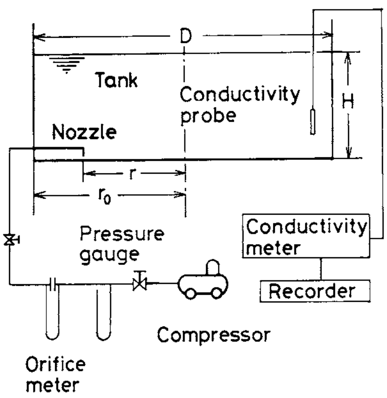


Fig. 1. Flow sheet of experimental apparatus for bubble-plume mixing.

diameter and a 3-cm ID sintered porous-plate sparger with a mean pore diameter of 100 μm. The opening of the orifice faces towards the bottom of the tank so that gas leaves the orifice steadily with negligible upward momentum. Tap water at room temperature (10–28°C) was used as a test medium and concentrated (15 wt%) NaCl solution as a tracer. The tracer volume was 50–150 cm^3.

In the experiments, the tracer was instantaneously added to the system, by dumping a cup of tracer on the liquid surface or by puncturing a tracer-containing small balloon with a pin at various depths below the surface. The change in tracer concentration with time was measured by using an electric conductivity meter (M & S Instrument CD-35 MII). The conductivity probe was made from a platinum wire 1 mm diameter.

In experiments for vertical liquid-jet mixing the same tanks and nozzles as those for bubble-plume mixing were used with the exception that the nozzle opening faces towards the surface of the liquid. A complete description of the experimental method has been given in a previous paper.[10]

2. Results and Discussion

2.1 Mixing by use of vertical liquid jet

Figure 2 shows an example of the impulse response. An oscillation appears which damps and disappears within a short time compared to the mean residence time. The mean circulation time was obtained from the mean period of the oscillation. The results at $r/r_0 \geq 0.7$ showed that the mean circulation time was independent of the radial position of injection. The dimensionless mean circulation time $(t_C/t_R)/(d/L)$ is listed in **Table 1**. For the configuration of this study, the jet axis-length L is equal to the submergence depth of nozzle, $(H-h_i)$. The dimensionless mean circulation time ranges from 0.8 to 1.8, in accordance with that for horizontal or inclined jet mixing.[10] This fact indicates that as a representative length scale of tank

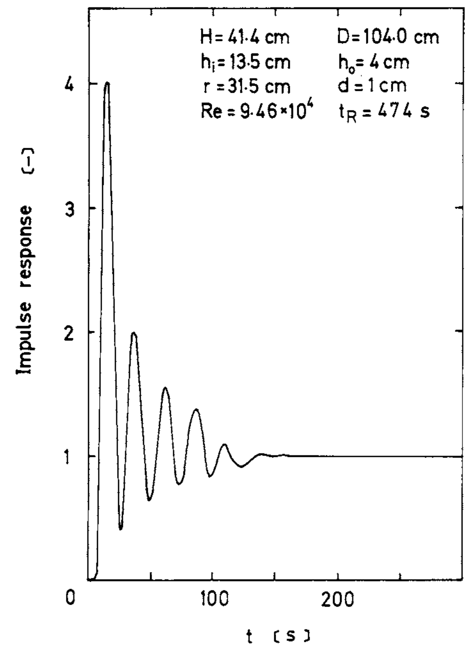


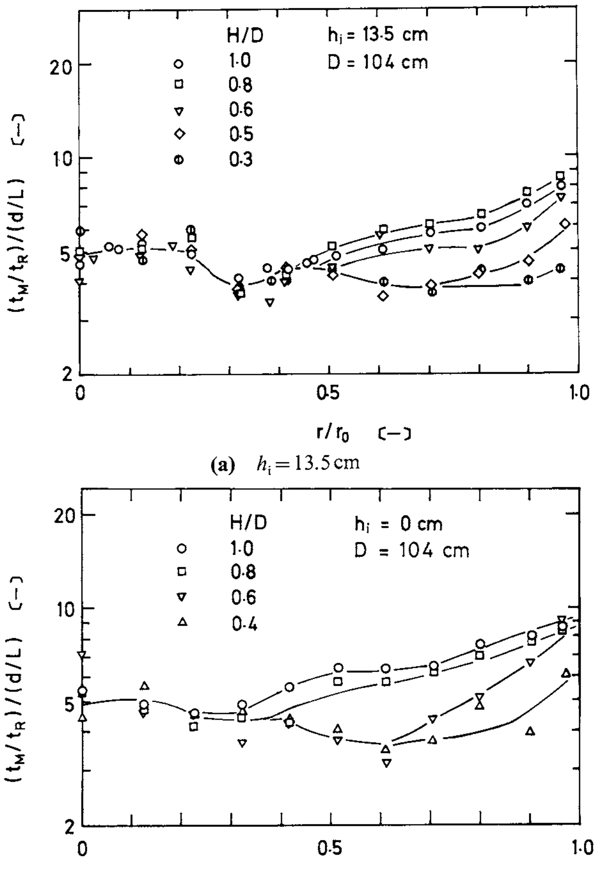
Fig. 2. Typical example of impulse response for liquid-jet mixing.

Table 1. Mean circulation time

H/D	h_i [cm]	r/r_0	t_C [s]	$(t_C/t_R)/(d/L)$
1.0	13.5	0.96	16.7	1.28
1.0	13.5	0.90	17.2	1.32
1:0	13.5	0.84	18.3	1.40
1.0	13.5	0.81	15.1	1.16
0.8	13.5	0.96	19.7	1.43
0.8	13.5	0.90	18.1	1.31
0.6	13.5	0.96	11.9	0.81
1.0	0	0.96	16.9	1.50
1.0	0	0.90	15.4	1.38
1.0	0	0.80	15.8	1.41
0.8	0	0.96	14.6	1.26
0.8	0	0.90	15.3	1.32
0.8	0	0.80	16.3	1.41
0.8	0	0.70	18.3	1.58
0.6	0	0.96	14.3	1.24
0.6	0	0.90	14.8	1.29
0.6	0	0.80	14.3	1.24
0.4	0	0.96	21.5	1.78

the jet axis length L is more suitable than the diameter and depth of the tank.

To compare the mixing characteristics for various conditions, the mixing time was defined as the time from the start of mixing to the time when the variation of measured concentration dropped to within 1% of the mixed mean concentration. Because of a scatter of data (maximum difference 20%) for each set of system parameters, the mixing time was measured more than three times. An average of the measurements was recorded as the final result. All the dimensionless mixing time $(t_M/t_R)/(d/L)$ in the circulation flow regime $(Re \geq 3 \times 10^4)$[10] was independent



(a) $h_i = 13.5$ cm

(b) $h_i = 0$ cm

Fig. 3. Dimensionless mixing time as a function of radial position of nozzle and liquid depth.

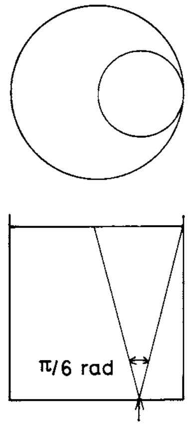


Fig. 4. Cone of vertical angle $\pi/6$ rad in tank.

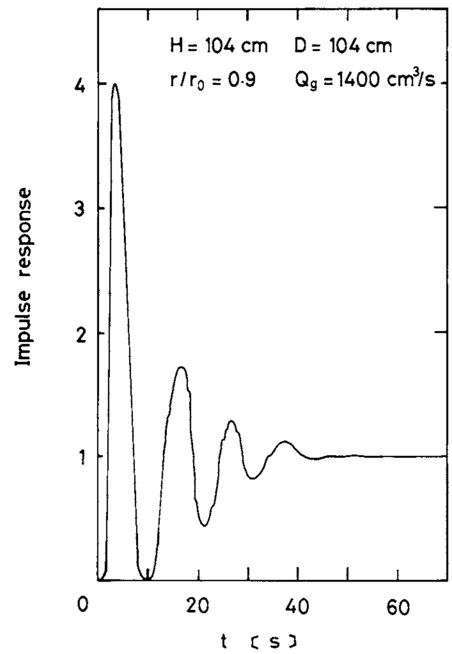


Fig. 5. Typical example of impulse response for bubble-plume mixing.

of *Re* and the inner diameters of both nozzle and tank. **Figure 3** shows the dimensionless mixing time plotted against dimensionless radial position of nozzle, r/r_0. The dimensionless mixing time is nearly constant, with a value of 3–6 at $r/r_0 \lesssim 0.5$, increasing with increasing r/r_0 at $r/r_0 \gtrsim 0.5$. The increase can be interpreted by the wall effect described in a previous paper.[10] The extent of the wall effect is expressed in a unified manner by a cone in a tank as shown in **Fig. 4**; the cone of vertical angle $\pi/6$ rad is concentric to the jet. When the cone contacts the side wall of the tank before the termination of the jet, the circular jet evolves into a wall jet along the wall. Since the spreading rate of the wall jet is 8.5 times greater parallel to the wall than normal to the wall, the wall jet spreads widely along the wall, inducing circulations of small variance of circulation time.[10] This is reflected by the large mixing time and by the precise damping oscillation on the response curve.

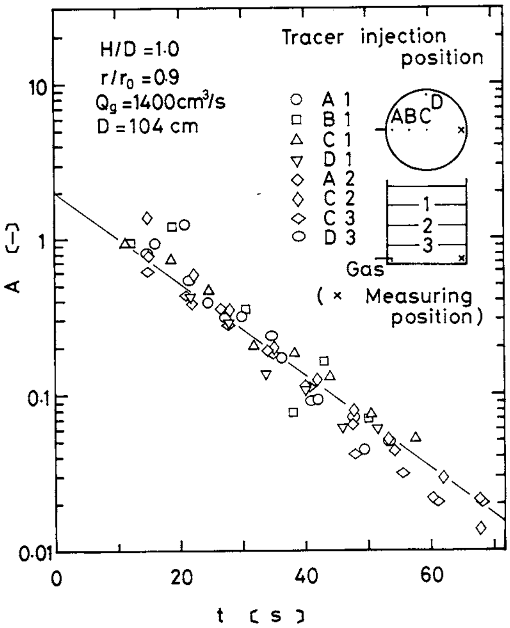
A nozzle position r/r_0 between 0 and 0.5 is recommended for rapid mixing irrespective of the level in the tank. When liquid depth is small, horizontal or inclined jet mixing[10] is preferable to vertical jet mixing because the dimensionless mixing time is nearly equal to one another and consequently the mixing time is inversely proportional to the jet axis length L at the same values of d and t_R.

2.2 Response curve for bubble-plume mixing

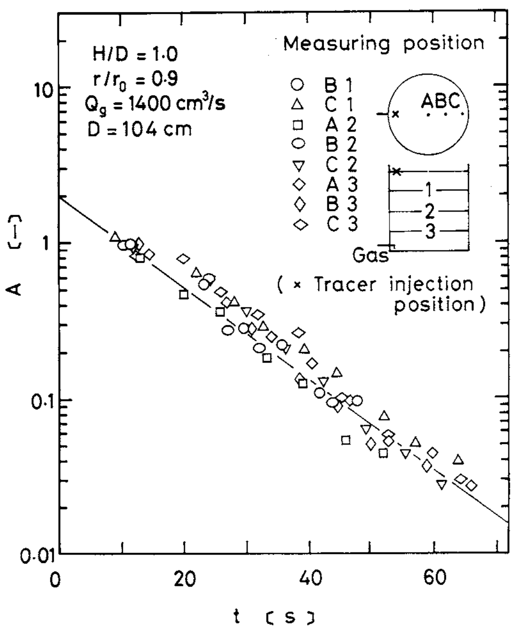
Figure 5 shows an example of the impulse response. Similarly to liquid-jet mixing, damping oscillation appears when gas is injected near the side wall of the tank. **Figure 6** shows typical series of the decaying amplitude as a function of time. The decay of amplitude is independent of tracer-injection position and measuring position, and can be correlated by an exponential function. The solid line shows the expression of the recycle model[5]:

$$A = 2\exp(-2\pi^2\sigma_C^2 t/t_C) \qquad (1)$$

with the dimensionless variance of the circulation time, σ_C^2, as a parameter.



(a) Fixed measuring position



(b) Fixed tracer injection position

Fig. 6. Amplitude A as a function of time

2.3 Induced liquid circulation for bubble-plume mixing

Figure 7 shows that the mean circulation time at $r/r_0 \gtrsim 0.6$ is independent of the radial position of the nozzle. In **Fig. 8**, the mean circulation time is plotted against the volumetric flow rate of gas, Q_g. It is independent of liquid depth and nozzle diameter, and is correlated well with a solid line for each tank diameter. The variation of t_C with Q_g is essentially a power function:

$$t_C \propto Q_g^{-1/3} \tag{2}$$

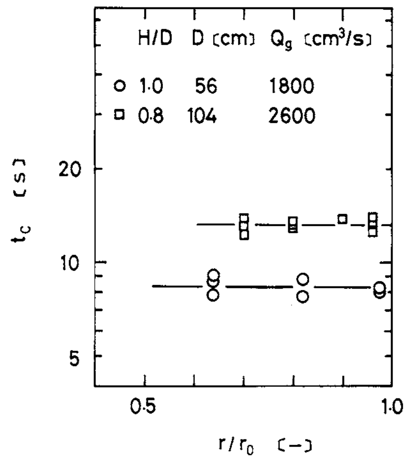
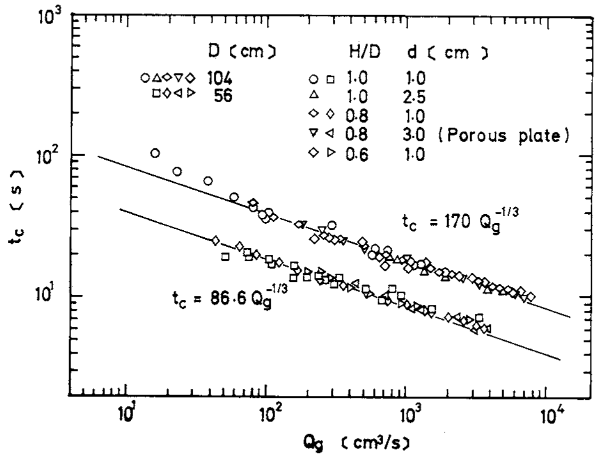
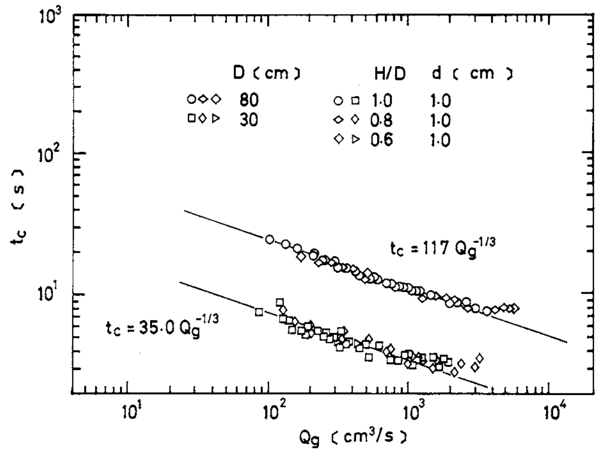


Fig. 7. Mean circulation time as a function of radial position of nozzle.



(a) D=56 and 104 cm



(b) D=30 and 80 cm

Fig. 8. Mean circulation time as a function of flow rate of gas.

In general, the flow rate of circulating liquid in tank is expressed as[10]:

$$Q_l = V/t_C \tag{3}$$

Substituting Eq. (2) into Eq. (3) gives a bubble-driven liquid circulation. **Figure 9** shows the circu-

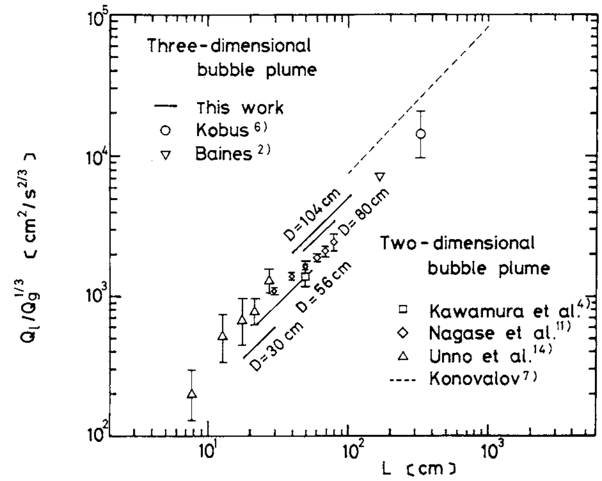


Fig. 9. Volumetric flow rate of liquid as a function of submergence depth of nozzle.

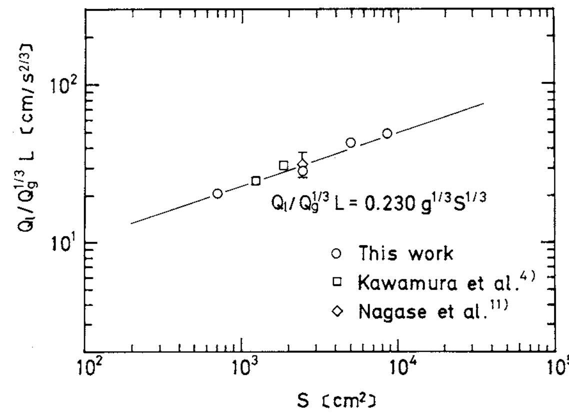


Fig. 10. Volumetric flow rate of liquid as a function of cross-sectional area of tank.

lation flow rate of liquid plotted as $Q_l/Q_g^{1/3}$ against the plume-axis length L. (For the configuration of this study the length L is equal to the liquid depth H.) A power function:

$$Q_l \propto L Q_g^{1/3} \tag{4}$$

correlates the results of this study and other investigators.[2,4,6,7,11,14] (The other investigators who used quite different measuring techniques have shown clearly that the liquid flow rate Q_l is independent of r/r_0 at $0 \leq r/r_0 \leq 1$.) It is worth noting that Eq. (4) is consistent with the results for two-dimensional bubble plume from a line source of gas in a rectangular tank. However, a proportional constant differs in each correlation, indicating a dependence of Q_l on tank diameter. **Figure 10** shows $Q_l/Q_g^{1/3}L$ plotted against the horizontal cross-sectional area of the tank, S, which is a representative horizontal dimension of both circular and rectangular tanks. The results are well correlated by:

$$Q_l = 0.230 L (g S Q_g)^{1/3}\,; \qquad 30\,\text{cm} \leq D \leq 104\,\text{cm}\,, \quad 0 \leq r/r_0 \leq 1 \tag{5}$$

where g is gravitational acceleration. Figure 10 correlates also the results of the other investigators[4,11] which point out a dependence of the liquid flow rate on the cross-sectional area of the tank.

2.4 Mixing time for bubble-plume mixing

When the gas injection is made in a central region of the tank, no damping oscillation appears in the response curve. Depending on tracer-injection position and measuring position, the concentration decreases (after an overshoot) or increases monotonically to the mixed mean value. This means that the circulation flow is not dominant in the mixing and that a relatively stagnant region occurs in the tank. To compare the mixing characteristics for all injection positions, the mixing time defined in section 2.1 was utilized. For the system showing a relatively stagnant region, the mixing time was measured more than five times because of the large scatter of data (maximum difference 30%). An average of measurements was recorded as the final result.

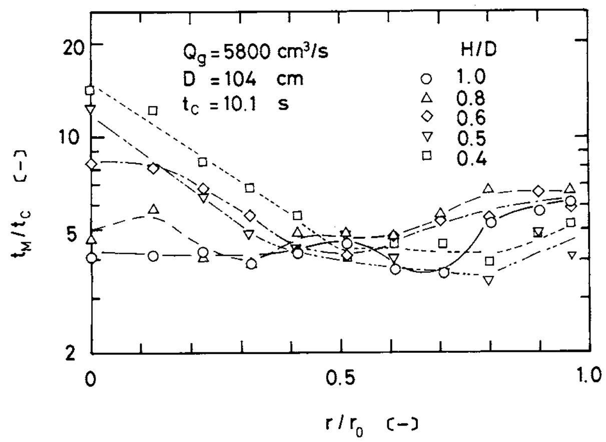
The measured mixing time showed the same dependence on Q_g as the circulation time at $r/r_0 > 0.5$, so that the mixing time was normalized by dividing by the circulation time which was calculated from Eqs. (3) and (5). **Figure 11** shows the normalized mixing time plotted against the gas-injection position for various values of Q_g and D. Similarly to that for liquid-jet mixing (see Fig. 3), the results at $r/r_0 > 0.5$ indicate a relatively large value, i.e., the retardation of mixing due to the wall effect. However, the variation of mixing time is not monotonic to r/r_0 and H/D, and consequently the extent of the wall effect cannot be expressed solely by the cone shown in Fig. 4. Thus there exists a difference in the wall effect between the buoyant two-phase plume and the momentum jet.

On the other hand, the occurrence of a relative stagnation is reflected by the large mixing time at $r/r_0 < 0.5$. To interpret the results, the two-parameter model[9] was applied as follows. If the tank is assumed to be divided by an imaginary plane into two compartments of an ideal stirred tank and an exchange rate of liquid between the two compartments, v, is finite, the dimensionless concentrations of tracer in the two compartments resulting from a pulse input in compartment 1 (volume fraction f) are expressed as:

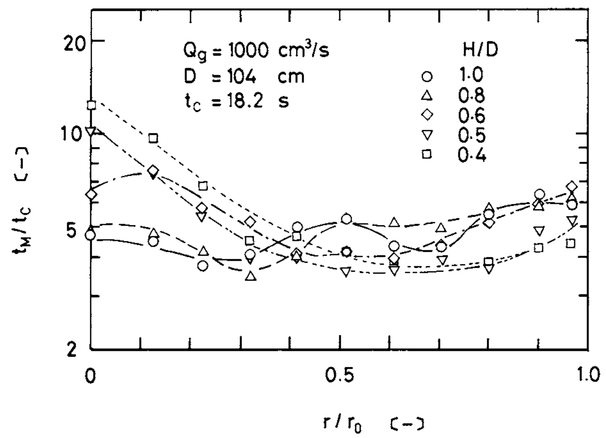
$$C_1 = 1 + \frac{1-f}{f} \exp\left[-\frac{t}{f(1-f)(V/v)}\right] \tag{6}$$

$$C_2 = 1 - \exp\left[-\frac{t}{f(1-f)(V/v)}\right] \tag{7}$$

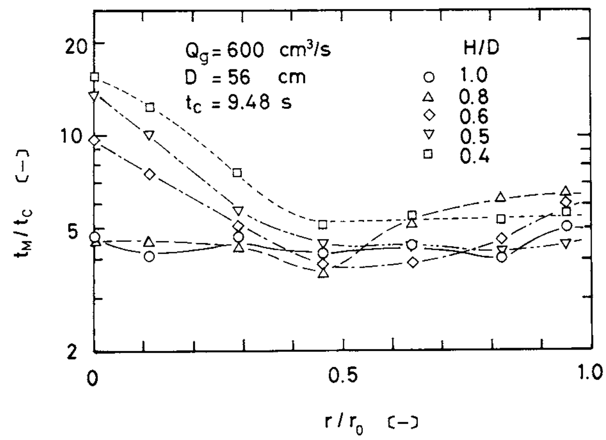
Thus the temporal increase or decrease in concentration depends on the positions of tracer injection and measurements. This fact is in accordance with



(a) $Q_g = 5800\,\mathrm{cm^3/s}$



(b) $Q_g = 1000\,\mathrm{cm^3/s}$



(c) $Q_g = 600\,\mathrm{cm^3/s}$

Fig. 11. Ratio of mixing time to circulation time as a function of radial position of nozzle and liquid depth.

experimental observation. The average mixing time is obtained from:

$$t_M = f t_{M1} + (1-f) t_{M2} \tag{8}$$

where t_{M1} and t_{M2} are the mixing times for compartments 1 and 2, respectively. Obtaining these values from Eqs. (6) and (7), and substituting them into Eq. (8) yield:

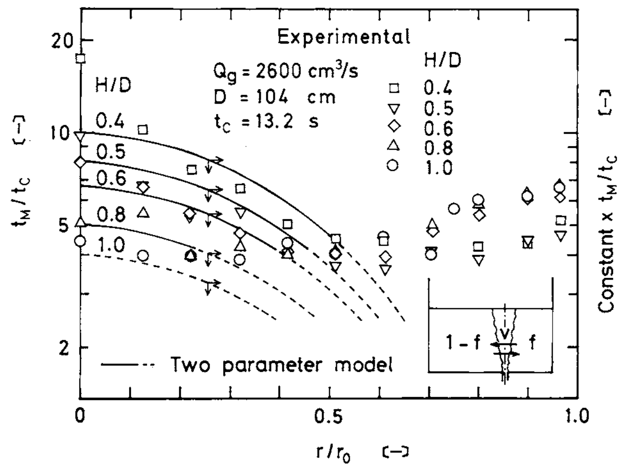


Fig. 12. Predicted and experimental ratios of mixing time to circulation time.

$$(t_M/t_C)/(Q_l/v) = f(1-f)\left(4.605 + f \ln \frac{1-f}{f}\right) \tag{9}$$

Hence, the normalized mixing time increases with decreasing exchange rate v and decreasing difference in volume fraction of the two compartments. **Figure 12** shows the mixing time calculated from Eq. (9) with assumptions: that v is proportional to Q_l and H/D; and that the imaginary plane including the plume axis is situated at a distance r from the axis of the tank. As can be seen, the calculation qualitatively shows retardation of mixing. These results strongly suggest that liquid exchange rate in the horizontal direction across the plume axis is small compared to that of the vertical liquid jet at the same value of Q_l or t_C. This is presumably due to the bubble-rich plume-core region which makes the velocity fluctuations in the transverse direction smaller than that of the single-phase momentum jets.

For rapid mixing, the above-mentioned two effects must be minimized. It is recommended that gas be injected at $r/r_0 = 0.5$, where the mixing time shows the minimum value irrespective of the liquid depth in the tank.

Conclusion

The circulation flow rate induced by a gas-bubble plume in a cylindrical tank is experimentally obtained by a tracer response method. An empirical equation correlates the circulation flow rates of two- and three-dimensional bubble plume in a unified manner. When gas is injected near the side wall of the tank, the mixing time normalized with the circulation time is nearly equal to that for vertical liquid-jet mixing and shows a retardation of mixing due to the wall effect. When gas is injected in the central region of the tank, the mixing time increases with decreasing radial position of the injection and decreasing depth of the contents. The increase in mixing time can be qualitatively explained by the two-parameter model for the system with a relative stagnation. For rapid mixing, it is recommended that gas be injected at mid-radius of the tank.

Nomenclature

A	= normalized amplitude of response signal	[—]
C_1, C_2	= concentration of tracer in compartments 1 and 2	[—]
D	= internal diameter of tank	[cm]
d	= internal diameter of nozzle	[cm]
f	= volume fraction of compartment	[—]
g	= acceleration of gravity	[cm/s^2]
H	= liquid depth in tank	[cm]
h_i	= height of nozzle from bottom of tank	[cm]
h_o	= height of suction pipe from bottom of tank	[cm]
L	= jet- and plume-axis length in tank	[cm]
Q_g	= volumetric flow rate of gas	[cm^3/s]
Q_l	= volumetric flow rate of circulating liquid	[cm^3/s]
q	= Volumetric flow rate of liquid through nozzle	[cm^3/s]
Re	= Reynolds number of liquid jet $(=(4/\pi)(qd/\nu))$	[—]
r	= radial position of nozzle	[cm]
r_0	= internal radius of tank	[cm]
S	= horizontal cross-sectional area of tank	[cm^2]
t	= time from start of mixing	[s]
t_C	= mean circulation time	[s]
t_M	= mixing time	[s]
t_R	= mean residence time	[s]
V	= volume of liquid in tank	[cm^3]
v	= volumetric flow rate of liquid exchanging between compartments	[cm^3/s]
ν	= kinematic viscosity of liquid	[cm^2/s]
$\sigma_C{}^2$	= dimensionless variance of circulation time	[—]

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