AN INVESTIGATION OF STUDENT CONJECTURES IN STATIC AND DYNAMIC GEOMETRY ENVIRONMENTS

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AN INVESTIGATION OF STUDENT CONJECTURES IN STATIC AND DYNAMIC

GEOMETRY ENVIRONMENTS

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A Dissertation

Submitted to

the Graduate Faculty of

Auburn University

In Partial Fulfillment of the

Requirements for the

Degree of

Doctor of Philosophy

Auburn, Alabama
May 13, 2005

DISSERTATION ABSTRACT

AN INVESTIGATION OF STUDENT CONJECTURES IN STATIC AND DYNAMIC GEOMETRY ENVIRONMENTS

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171 Typed Pages

Directed by W. Gary Martin

This study was designed to investigate the mathematical conjectures formed by high school geometry students when given identical geometric figures in two different types of geometric environments. Student conjectures formed in a static geometry environment were compared with those formed in a dynamic geometry environment generated by dynamic geometry software. These conjectures and the environments in which they were formed were examined both quantitatively and qualitatively.

Results indicate that students who used dynamic geometry software made more relevant conjectures, fewer false conjectures, and the conviction in the correctness of their conjectures was higher when compared to students working in a static geometry environment. These differences were found to be statistically significant using linear regression analysis.

Qualitative data was collected by means of participant observations, a survey instrument, selected participant interviews, and a qualitative analysis of the conjectures made by the students in each environment. Qualitative analysis focused on the following themes: Student concepts of conjecture and proof, student preferences concerning each environment, the kind of language used in the conjectures formed in each environment, the ability to find counterexamples using dynamic geometry software, the "dragging" techniques used by the participants using dynamic geometry software, and the students conviction in the output generated by dynamic geometry software.

Results indicated a strong preference for the dynamic environment and a high conviction in the output generated by dynamic geometry software. The language used in forming conjectures in the dynamic environment was noticeably different and reflected the environment itself. The participants' concept of proof included both inductive and deductive frames when dynamic geometry software was available, and many of the students had difficulty with forming and finding of counterexamples using dynamic geometry software when confronted with a false conjecture.

ACKNOWLEDGEMENTS

The author would like to thank the participants of this study, the administers of the school used in this study, the teachers and educators that served as outside graders, the advisory committee and chair for their support, and my wife and daughter for their patience and understanding.

Style Manual Used:

Publication Manual of the American Psychological Association, Fifth Edition

Computer Software Used:

Microsoft Word 2002

Geometers' Sketchpad version 4.02

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CHAPTER I: INTRODUCTION

Reform measures in mathematics education have called for an increased emphasis on the inductive processes of exploration and conjecture. Mathematical conjectures are formed by observing data, recognizing patterns and making generalizations. These generalizations are unproven statements based on inductive reasoning (Serra, 1997). The use of conjecture as a means of instruction contrasts with the traditional pedagogy of memorizing and proving already known geometric theorems. The National Council of Teachers of Mathematics (NCTM) recommends that the practice of conjecture be an integral part of instruction regarding mathematical reasoning and proof across the grades. In particular, the subject of geometry is particularly well suited for exploration and conjecture (NCTM, 2000).

Over the last decade, a number of dynamic geometry software packages have been developed that allow students to construct, measure, distort, and explore geometric figures. Two of the most popular geometry software packages used today are Cabri Geometre (Laborde, 1990) which is marketed by Texas Instruments, and Geometer's Sketchpad (Jackiw, 1991) which is marketed by Key Curriculum Press. Geometer's Sketchpad version 4.02 (Jackiw, 2001) was used to create the dynamic geometry environment used in all of the activities and instruments in this study.

Dynamic geometry software enables students to examine many cases without having to reconstruct the figure. With dynamic geometry software students are able to select any vertex, segment, or any other part of a geometric figure and move it using the computer's mouse to "drag" that part causing a distortion of the figure. Figures can be constructed so that the defining characteristics of a specific type of geometric figure are maintained. For example, a constructed parallelogram in a dynamic geometry environment will always keep the opposite sides of the quadrilateral parallel. Therefore, the students are able to distort a parallelogram into another parallelogram by the "dragging" process. Any measured sides or angles will automatically change accordingly. By continuing this process the user essentially has a continuum of parallelograms to investigate rather than one specific static figure. Figure 1 illustrates dragging a constructed parallelogram in a dynamic geometry environment. Dynamic geometry software was used to construct this figure.

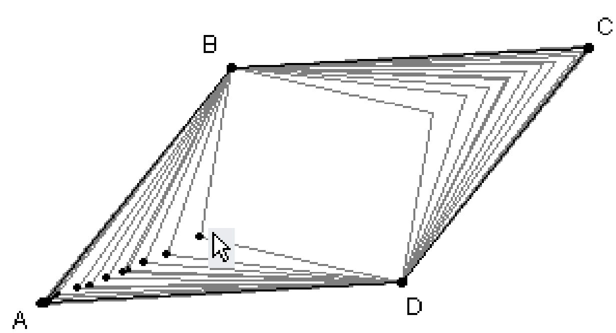

Figure 1. Dragging point A to form a continuum of parallelograms in a dynamic geometry environment.

Deforming the figure by dragging allows students to directly observe how various components of geometric figures and their measures are affected by dynamic changes. By generalizing the patterns that emerge during these explorations and observing changes in the figures and their measures, students may be able to form their own mathematical conjectures (Glass, Deckert, Edwards, & Graham, 2001). The use of dynamic software enables students to examine many cases, thus extending their ability to formulate and explore conjectures.

The challenge for teachers of geometry is to integrate dynamic geometry environments in their teaching as a way of encouraging students to explore ideas and develop conjectures while continuing to help them understand the need for proofs or counterexamples of conjectures (NCTM, 2000). The classical approach to proof can be enriched by the advent of dynamic geometry software. Haddas and Hershkowitz (1999) claimed, “A main pedagogical feature of many dynamic geometry based learning environments is that the discovery and conviction of geometrical facts is greatly enhanced by means of dynamic processes” (p. 25). With the use of dynamic geometry software, students are no longer solely reliant on stated theorems and formal proofs of those theorems to verify geometric principles. They now have a tool that will facilitate conjecturing and aid in exploration of geometric principles.

Purpose and Objective

If developing conjectures is to be an integral part of secondary geometry courses, then the tools and environments presented during instruction should enhance the students' ability to form conjectures. This study will investigate differences in students’ geometric conjectures in both static and dynamic geometry environments. This study will explore whether the use of a dynamic geometry environment significantly increases the students' ability to form worthwhile conjectures.

In order to address this question, high school geometry students were allowed to independently conjecture in both static and dynamic geometry environments. The static environment did not allow students to distort the figures by dragging. In the dynamic environment, students were able to drag the figures and observe changes in the figures and the measurements provided. Note that the figures presented in both environments were identical with the exception of the dragging capabilities.

Research Questions

This study addressed the following questions:

1. Does the number of relevant conjectures formed by students significantly differ in static and dynamic geometry environments?
2. Does the number of false conjectures formed by students significantly differ in static and dynamic geometry environments?
3. Does students' conviction in their conjectures significantly differ in these environments?
4. How do the variables of gender, mathematics achievement, and geometric reasoning level relate with the students' ability to conjecture in these different environments?

It was hypothesized that in the dynamic geometry environment, students will produce significantly more relevant conjectures, significantly fewer false conjectures, and that student’s conviction in all conjectures will be significantly greater when compared to the conjectures formed in the static environment. The variables of gender, mathematics achievement, and geometric reasoning level were used as covariates and as exploratory variables.

This study also included several qualitative themes addressing other aspects of the students’ experience during the course of the study. Four *a priori* themes were developed as a result of the initial literature review, prior to the collection of data. The four a priori themes were: Student preferences concerning each environment, the students’ concepts of conjecture and proof, the students’ ability to form and find counterexamples in the dynamic environment, and the students’ conviction in the output generated by dynamic geometry software. Specific instruments designed to address these themes will be discussed in Chapter III. During the analysis of data, two more emergent themes were developed. These emergent themes were: The use of dynamic language in the conjectures formed in the dynamic environment and a description of different dragging techniques used by the students. The selection and analysis of all six of these themes will be discussed further in Chapters III and V.

Qualitative methods and studies of mixed design are increasingly common in the field of mathematics education (Schoenfeld, 2000). By including both quantitative methods to answer specific research questions and qualitative methods to explore the above themes, this study was designed, not only to compare, but also to describe student conjecturing in two different geometric environments.

Summary

This study investigated students' ability to form geometric conjectures in both static and dynamic geometry environments. All participants were exposed to both environments and participated in up to eight lab activities that allowed them to conjecture independently in each of the geometric environments. These activities were aligned to the curriculum of a secondary geometry course throughout the semester and were used as part of the instruction during regular class hours.

In order to lay the foundation for this study, the next chapter will review literature concerning traditional geometric proof instruction, different levels of geometric reasoning, the role of student conjecturing in mathematics instruction, and the uses of dynamic geometry software. Chapter III discusses the methodology used to collect and analyze both the quantitative and qualitative data obtained in this study. Chapter IV reports all of the quantitative results while Chapter V reports the results of the qualitative themes addressed in this study. The final chapter will discuss the research questions and the qualitative themes in terms of implications and suggestions for further research.

CHAPTER II: LITERATURE REVIEW

This chapter outlines research in several areas that have important consequences for understanding the role of dynamic geometry environments on student conjecturing. Since a primary motivation for introducing student conjecturing in the mathematics classroom is to enhance the instruction of proof, literature concerning traditional proof instruction in geometry is discussed. The van Hiele theory of geometric reasoning also plays a significant role in understanding the deductive process of proof; therefore, literature concerning the van Hiele theory of geometric reasoning will also be reviewed in this chapter.

Studies that focus on the role of conjecture in the classroom are included in this chapter as well as literature that links the process of conjecturing with the use of dynamic software and to the instruction of proof. Several recent studies have investigated the uses of dynamic geometry software and its applications in the geometry classroom. The debate over the role of dynamic geometry software with regards to the instruction of proof is a common theme in recent studies and several of those studies will be discussed in this chapter as well. Although most of the studies describe research conducted in the secondary classroom, some studies using preservice teachers and practicing teachers as participants are included in this chapter. These studies provide insight on these participants' attitude toward proof and the use of dynamic geometry software in the classroom.

Traditional Proof Instruction

The traditional axiomatic approach to proof gives little or no place for conjecturing. Theorems are simply given to the students by the teacher or by the textbook. These theorems are assumed true, and students are asked to verify them by using deductive reasoning, often in a formal two-column proof format. This traditional approach to proof has, in the past, left many students disenchanted with the proof process (Battista & Clements, 1995). Senk (1989) has reported that geometric proof was among the most difficult and disliked mathematical topics for college-bound students in the United States. Senk (1985) also stated the typical high school mathematics program provides virtually no opportunity for students to practice proof writing outside the context of geometry class. Difficulty with proof is not merely a problem for students in the United States. Data from the Third International Mathematics and Science Study (TIMSS) indicate that, in general, students worldwide have particular difficulties organizing arguments (National Center for Education Statistics, 1998).

Findings suggest that the transition to proof is too abrupt in the traditional mathematics curriculum and that this transition is often difficult for even those who have done superior work in preceding courses. Within this traditional environment, tasks concerning proofs are presented in the form "prove that …", where the statement to be proved is already provided to students (Furinghetti, Olivero, & Paola, 2001). Battista and Clements (1995) claimed that most mathematics instruction and textbooks lead us to believe that mathematicians make use only of formal proof and deductive reasoning based on axioms. However, in reality, mathematicians pose problems, analyze examples, make conjectures, look for counter examples, and revise conjectures all as part of creating mathematics. Deductive proof is seen as the final step of this creative process.

In a study consisting of 2699 students in 99 geometry classes from five states, Senk (1985) reported that only about 30% of the students in a full year geometry course that taught proof were able to master proofs that were similar to those presented in standard secondary geometry textbooks. In this study, a wide variety of schools and students were used to achieve a realistic cross section of students with regards to achievement and socioeconomic conditions.

Senk (1985) found three specific instructional issues that should be addressed as a result of this study. First, many students could not get started after listing the given statements, suggesting that teachers need to pay special attention to helping students begin a chain of deductive reasoning. Second, many students cited the theorem to be proved within the proof itself, suggesting that teachers should place greater emphasis on the meaning of proof. Finally, many students had difficulty with embedded figures and auxiliary lines showing the need to instruct students how, why, and when they can transform a figure in a proof.

Mingus and Grassl (1999) described the "attitude barrier" concerning formal proof, which prevents students from taking the risk of justifying or explaining their reasoning to others in the classroom. This attitude is realized by frustration and a disdain for proofs causing students to quit before even attempting to write a proof. This attitude toward and inability to do proof is a serious deficiency in a student's mathematical training.

Mingus and Grassl (1999) reported on two different studies concerning proof frames and abilities. In a study of elementary and secondary preservice teachers, Mingus and Grassl (1999) examined the participants' experience with proof and their beliefs concerning proof. The participants included 30 preservice elementary teachers enrolled in a mathematics content course and 21 preservice secondary mathematics teachers enrolled in an abstract algebra course. The study found that most of the elementary preservice teachers (80%) and most of the secondary math preservice teachers (55%) had either no proof instruction in secondary school or just the traditional axiomatic proof instruction in a high school geometry class. The majority of both elementary and secondary preservice teachers felt uncomfortable with proof and felt unprepared to tackle formal proofs in college level mathematics. They felt that their secondary mathematics curriculum had not prepared them for this task, and most of the participants felt that some kind of proof instruction should be incorporated into the earlier grades to help prepare and nurture students toward formal proof.

Mingus and Grassl (1999) also reported on a study using 215 middle and high school students that attempted to judge students' ability to produce a convincing mathematical argument. The students were asked to show that there are just as many even numbers as there are odd numbers. The problem was presented in written form during school hours, and 170 of the students provided written justification of their reasoning. Responses included elegant arguments involving one-to-one correspondence, the role of digits in the unit's position, and proofs by contradiction. The study found that most of the students were able to provide a proof at some level and, surprisingly, the younger students in the sixth to eighth grades often showed the most creativity in their justifications.

The implications of these two studies led to the conclusion that proofs should be encountered as early as possible in the curriculum and that a broad view of proof should be adopted to encourage a wide variety of mathematical ideas. More precision and breadth should be expected as students advance through the grades. Teachers should also demonstrate appropriate proofs as they reinforce the students' attempts at proof at all levels (Mingus & Grassl, 1999).

The way in which students perceive the role of figures used in geometric proofs was the subject of a study by Martin and Harel (1989a). The study asked 410 university students enrolled in a lower division mathematics course to judge the correctness of the following geometric statement: "A segment connecting the midpoints of two sides of a triangle is one half the length of the third side" (p. 266). This statement is sometimes referred to as the midsegment theorem.

Three different instruments were used in this study. The first two instruments used general proofs of the statement accompanied by combinations of general figures and specific "non-generic" figures of triangles. For each figure, participants were asked if the given proof was valid for that figure. The third instrument used an argument tailored to a specific triangle. It was also accompanied with both general and particular figures of triangles. Like the other two instruments, the participants were asked to judge the validity of the given argument for each of the figures provided. The results of this study provided two interesting findings concerning the students' concepts of proof with regards to geometric figures. First, the use of particular "non-generic" figures did not appear to influence the students' judgment about the correctness of a proof. Second, students indicated that new proofs would have to be done if the figures were changed. In other words, students conceived that the proof was only valid for the figure that was provided and not necessarily valid for all figures of that "type". This inability to generalize to other figures is indeed a problem for students' concept of proof, and the authors suggested further research on student interpretations of figures in geometric proof.

Martin and Harel (1989b) also reported on a study of proof frames of preservice elementary teachers. In this study 101 preservice elementary teachers enrolled in a sophomore level mathematics course were given a proof instrument in which both a familiar generalization and an unfamiliar generalization were given. Along with these generalizations, the instrument provided inductive examples and patterns as well as a general proof, a false proof, and a particular proof.

The results of this study showed that students accepted both inductive and deductive arguments as proofs. The authors postulated that inductive and deductive arguments represent two different proof frames constructed by students as the result of experiences outside and inside the mathematics classroom. Martin and Harel (1989b) concluded that the inductive frame, which is formed earlier, is not deleted when students acquire the deductive frame. This finding is relevant to reform recommendations concerning the instruction of proof and to this study. Having students form conjectures in geometry class may indeed help them develop their ability to use inductive reasoning, although it must be emphasized that inductive arguments are not valid mathematical "proofs". Teachers must recognize the students' "inductive frame" and how to incorporate this frame into formal proof instruction.

All of these studies address different aspects of proof instruction and offer suggestions to improve the instruction of proof. Students should be exposed to methods of justification in both inductive and deductive frames at an earlier age. The act of conjecturing and justifying using both inductive and deductive methods may indeed better prepare students for formal proof instruction in secondary school mathematics. The role of figure in geometric proofs and how students perceive the figures is an important element in the students' concept of proof. Teachers in elementary and middle grades should be trained to recognize and encourage creative ways for students to justify their conclusions. The National Council of Teachers of Mathematics recommends that instructional programs from kindergarten through secondary school incorporate investigation and conjecturing. Such activities should help students develop and evaluate mathematical argument and proof, select and use various types of arguments, and recognize proof as a fundamental aspect of mathematics in general (NCTM, 2000).

The van Hiele theory of Geometric Reasoning

The van Hiele theory of geometric reasoning originated in the respective dissertations of Dina van Hiele-Geldof and her husband Pierre van Hiele in 1957 (Fuys, Geddes, & Tischler, R., 1988). The van Hieles described stages of geometric reasoning that students pass through as they acquire geometric knowledge and cognitive sophistication with regards to geometric reasoning. The "four level" van Hiele model of geometric reasoning is summarized as follows (de Villiers, 1996; Gutierrez & Jaime, 1998):

- Level 1 - Recognition: Students recognize figures by appearance and recognize squares, rectangles, triangles, etc. by their shape. They do not explicitly identify the properties of these figures.
- Level 2 – Analysis: Students begin analyzing the properties of figures and use proper terminology to define and describe figures. They do not yet inter-relate the figures and their properties.
- Level 3 – Ordering: Students are able to classify and inter-relate figures by their properties. Students can order properties of figures by short chains of deductions.
- Level 4 – Deduction: Students start developing longer sequences of statements and begin to understand the concept of deduction, the role of axioms, theorems, and proof.

To further examine the effect of the van Hiele levels on proof writing ability, Senk (1989) used a sample of 241 secondary school students enrolled in full year geometry classes. She found that proof writing ability correlated significantly with van Hiele level even when entering knowledge of geometry and geometry achievement throughout the course were used as covariates. She concluded that a student's entering van Hiele level could serve as a strong predictor of the students' ability to master proof in a secondary geometry course.

In order to measure the students' proof writing achievement, a six item test was given to each student with a 35 minute time limit. This test contained two short answer items and four full proofs. This test was given to the students during the last month of the school year and contained proof items similar to those found in typical secondary geometry texts. A Cronbach's alpha reliability coefficient of .85 was reported for the students taking the test. Students who were able to correctly prove at least three out of the four given proofs were considered to have "mastered" proof writing.

This study also supported the notion that formal deductive proof in geometry requires at least thinking at level three in the van Hiele hierarchy. Senk (1989) used a "five level" van Hiele scale in which level five represents the ability to perform rigorous mathematical proofs. At the onset of the study only 15 of the 241 participants were at level three and none at levels four and five. By the end of the study 68 students had reached level three, 13 had reached level four, and four had reached level five. Only 22% of the students who reached level two were able to master proof whereas 57%, 85%, and 100% of the students who reached levels three, four, and five respectively were able to master proof. It should be noted, however, that the majority of the students (65%) were still at level two or below at the end of the course.

Students must pass through lower levels of geometric thought before they can attain higher levels, and this process does take a considerable amount of time. The van Hiele theory suggests that instruction should gradually progress through lower levels of geometric thought before students begin a proof-oriented study of geometry. Because students cannot bypass levels and achieve understanding, prematurely dealing with formal proof can cause students to rely on memorization without understanding. The traditional axiomatic approach to formal proof then is unlikely to be productive for the vast majority of students in high school geometry (Battista & Clements, 1995).

The Role of Conjecture

An alternate approach to formal proof instruction in geometry would involve including attention to the inductive step of conjecture as a prelude to formal deductive proof. The discovery of geometric facts by conjecturing may lead to formal and informal justifications of why the conjectures are true, thus laying the groundwork for deductive proof. Students can be given an environment in which to explore geometric situations and derive their own conjectures rather than relying on the teacher or the textbook to tell them the mathematical truths that can be found in any particular geometric situation.

This kind of instruction is consistent with recommendations from the National Council of Teachers of Mathematics (NCTM). The NCTM supports instruction in which students are able to learn with understanding rather than memorize mathematical facts and procedures. Learning with understanding includes proposing ideas and conjectures as well as evaluating those ideas and conjectures (NCTM, 2000).

The NCTM's "Reasoning and Proof" standard for all students from pre-kindergarten through high school includes the following four elements:

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof. (NCTM, 2000, p. 56)

As seen from this list, conjecturing and various types of reasoning are coupled with formal proof. As students move through the grades, conjecturing and informal justifications should prepare students for the more rigorous act of deductive proof in secondary school mathematics, but they are certainly never abandoned.

The mathematical environment provided for students when they form conjectures may indeed determine the quality of their conjectures and the conviction in the conjectures that they form. de Villiers (1992) reported on a study concerning students' conviction on given mathematical conjectures. The study hypothesized that the majority of children would base their conviction of the truth of the given statements on the authority of the teacher and/or textbook rather than on personal conviction. de Villiers (1992) also hypothesized that the majority of the children would not easily distinguish false statements on their own, but would be dependent on the authority of the teacher and/or textbook for this distinction.

To test his hypothesis, de Villiers (1992) studied 40 grade 10 students from two different schools and 99 grade 11 students from five different schools in order to determine what mathematical statements the students were convinced or doubtful about and the reasons for their conviction. All students were given a series of 15 geometric statements in 35 minutes and asked to respond with one of four different codes. The codes are as follows:

Code 1: Believe it is true from own conviction.

Code 2: Believe it is true because it appears in the textbook or because the teacher said so.

Code 3: Do not know whether it is true or not.

Code 4: Do not think it is true.

It was found that the majority of pupils based their conviction on authoritarian grounds rather than personal conviction, as hypothesized. The students in this study found formal proof to be less convincing than verification from an authority and very few students (2%) could accurately identify false statements. This study implies that students may not be adequately prepared to conjecture on their own when simply handed a paper of mathematical statements (de Villiers, 1992). Students may indeed need a proper environment that promotes the act of conjecture through experimentation and manipulation.

Furinghetti, Olivero, and Paola (2001) supported presenting students with tasks that use dynamic exploration to help foster the ability to conjecture. Such explorations and the resulting conjectures might support students in the process towards proof and make the way of doing mathematics in the classroom closer to the way of mathematicians. Furinghetti, Olivero, and Paola (2001) used the exploration of short, easily-understood geometric statements that foster discovery, conjecture, and do not suggest a method of proof. They reported on a classroom experiment that used the following geometric statement given to a small group of adolescent geometry students:

> You are given a right-angled triangle ABC, AB being the hypotenuse.
> Take a point P on AB. Draw the parallel lines to AC and BC through P.
> Name H and K, the points of intersection with AC and BC respectively.
> For which position of P does the line HK have minimum length?
> (Furinghetti, Olivero, & Paola, 2001, p. 324)

The students were grouped in threes and required to form conjectures, which then needed to be justified or proved to the entire class. The use of an open problem helped students produce strategies and because of the need for communicating them to their colleagues in the classroom, these strategies were explicit.

Furinghetti, Olivero, and Paola (2001) reported that almost all students were actively engaged in the activity, and a variety of different strategies were explored and discussed among the groups. The use of videotape allowed the students, teacher, and researchers to capture important learning moments and statements that would normally be overlooked or forgotten by regular observation. Although the use of dynamic geometry software was not part of the original classroom experiment, a follow up activity did use dynamic geometry software for demonstration purposes.

Boero, Garuti, and Mariotti (1996) reported on a teaching experiment that used conjecture as a stepping-stone toward proof. Once again the setting of an open-ended problem set the stage for the experiment:

> In the past years we observed that the shadows of two vertical sticks on the horizontal ground are always parallel. What can be said of the parallelism of shadows in the case of a vertical stick and an oblique stick? Can shadows be parallel? At times? When? Always? Never? (Boero, Garuti, & Mariotti, 1996, p. 3)

This problem was posed to thirty-six eighth grade Italian students in two separate mathematics classes. Many of the students started working with thin sticks or pencils. The absences of sunlight caused students to use dynamic mental processes to formulate conjectures. The conjectures were discussed, with the help of the teacher, until two collective statements resulted:

1. If sun rays belong to the vertical plane of the oblique stick, shadows are parallel. Shadows are parallel only if sun rays belong to the vertical plane of the oblique stick.

2. If the oblique stick is on a vertical plane containing sun rays, shadows are parallel. Shadows are parallel only if the oblique stick is on a vertical plane containing sun rays. (Boero, Garuti, & Mariotti, 1996, p. 5)

Although these statements could be combined into one conjecture in a compact "if and only if" form, they were left as is for purposes of proof. The proof exercises for this activity involved both working in pairs and teacher guided discussions. Activities focused on what it means to prove something in mathematics and the concept of "necessary and sufficient" conditions.

In this study, all of the participants actively took part in developing the initial conjecture and 29 of the 36 were able to complete all of the follow-up proof exercises in a productive way. Videotape analysis of the classroom activities showed more than half of the students exploring the problem in various dynamic ways by using props or hands to simulate movement. During these explorations conjectures were often abandoned or modified. The authors of this report claimed that because of the relative success of the use of "dynamic" mental processes educators should seek to find ways to incorporate dynamic environments into traditionally "static" mathematical situations (Boero, Garuti, & Mariotti, 1996).

The Role of Dynamic Geometry Software

Throughout the last two decades a number of dynamic geometry software packages have been marketed that allow users to construct, measure, distort, and explore geometric figures (Battista & Clements, 1995; de Villiers, 1996). Because of the dynamic nature of the environment created by these packages, students are able to explore multiple figures without having to reconstruct them. The Geometric Supposer software series (Schwartz & Yerushalmy, 1986) was a precursor to modern dynamic geometry software packages. It allowed students to choose a primitive shape, such as a triangle or a quadrilateral, and perform measurements and constructions on it. The program would then repeat the same operations on other shapes allowing students to explore the generality of the consequences of their constructions (Battista & Clements, 1995).

One of the first true "dynamic" geometry programs produced was Cabri-Geometre (Laborde, 1990), a French program that was introduced to the international mathematics education community at a conference in Budapest in 1988 (de Villiers, 1996). Unlike the Geometric Supposer, this "state of the art" package had dragging capabilities that gave the user instant control over the dynamic processes. Other packages soon followed, including the Geometer's Sketchpad (Jackiw, 1991) which is widely used in the United States as the premier dynamic geometry software package (Battista & Clements, 1995).

As stated in de Villiers (1996):

> The development of dynamic geometry software in recent years is certainly the most exciting development in geometry since Euclid. Besides rekindling interest in some basic research in geometry, it has revitalized the teaching of geometry in many countries where Euclidean geometry was in danger of being thrown into the trashcan of history (p. 25).

The Role of Conjecture using Dynamic Geometry Software

de Villiers (1996) provided a framework in which dynamic software can be used to verify true conjectures and construct counterexamples for false conjectures. He suggested that teachers and educators in mathematics should focus more on teaching and developing the process aspects of mathematics. Teachers should allow students to actively construct their knowledge in the class rather than being presented with preplanned content. The following figure shows de Villiers' (1996) illustration of the role of conjecture and counterexample in the proof process.

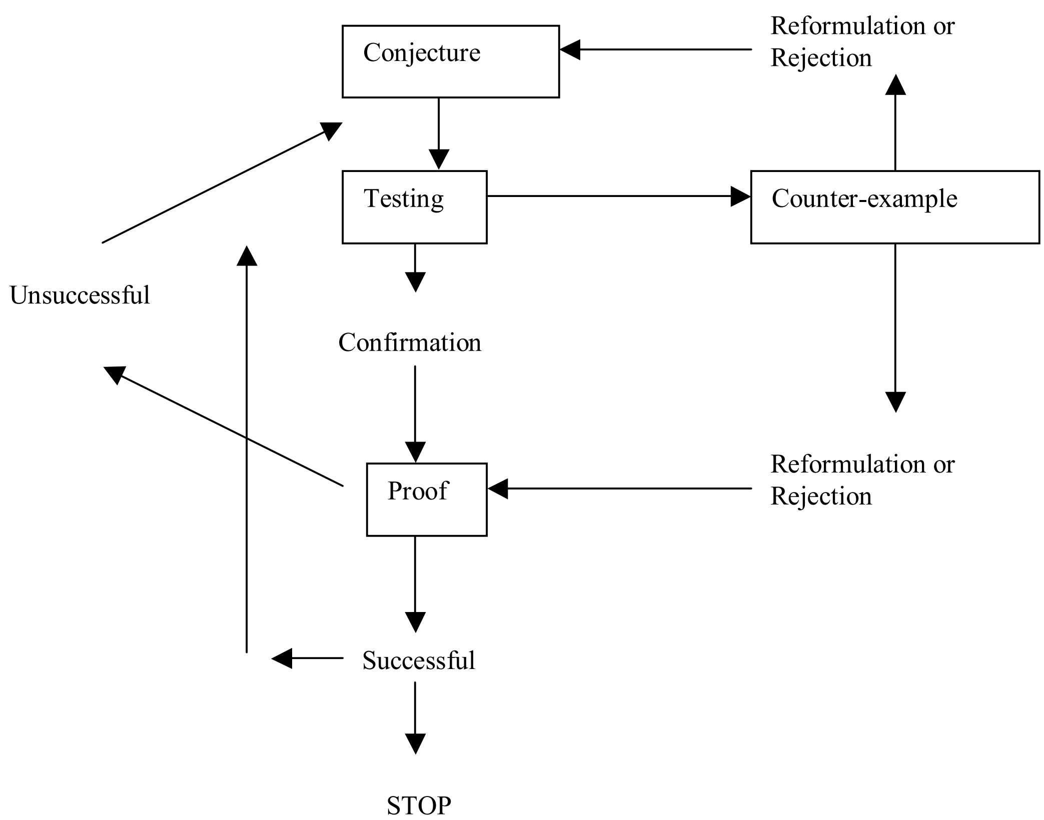

Figure 2. A flow chart of student research in geometry showing the role of conjecture and counterexample in the proof process (de Villiers, 1996. p. 27).

In figure 2, the step referred to as "testing" would use some kind of dynamic geometry software to test the students' conjectures. Students can test to see if their conjectures hold true under the dragging process. By examining a continuum of figures in the dynamic environment, students can confirm or reject their conjectures. If students discover counterexamples to their conjectures during this dynamic process, they can adapt their conjecture to accommodate the counterexample or start afresh with a new conjecture. Eventually this process of exploration, discovery, and testing leads to an attempt to prove the conjecture deductively. An unsuccessful attempt at the proof leads the student back to the conjecturing phase where as a successful attempt will end the process unless the student wishes to make further conjectures.

Implications of Dynamic Geometry Software

Arcavi and Hadas (1996) outlined some of the educational implications brought about by the use of dynamic geometry technologies:

- In the dynamic geometry environment, students are led to explore and to play with many particular cases. As a result these observations may add insight and provide a basis for proving and further exploration.
- Students can conduct explorations and conjecture independently without the need for a teacher to confirm or judge the outcome. The role of the teacher can then be that of a guide that forces students to take a stance on conjectures and asks the all-important question 'why?'
- Making sense of a situation while playing with the situation itself first enhances both the understanding of the situation and the representations used to analyze the situation such as measures, graphs, and symbols.
- Traditional boundaries between mathematical subdisciplines are blurred and connections are enhanced. One subdiscipline may serve as the model for another enhancing the student's sense of consistency among various branches of mathematics.

Arcavi and Hadas (1996) described several important components of the dynamic geometry environment that contribute to its success, including the attributes of visualization, experimentation, surprise, feedback, and need for proof. Dynamic geometry environments allow students to construct visual images with certain properties and then transform them continuously in real time thus adding to the visual experience of the user. Dynamic environments enhance the student's ability to experiment, to look for extreme cases and negative examples. This experimentation is the basis of stating generalizations and forming and dismissing conjectures (Arcavi & Hadas, 1996). When the element of surprise is incorporated into dynamic geometry activities, students may become more thoughtfully engaged in the problem at hand. The impact of puzzlement or curiosity is not a negative or judgmental result but rather causes students to question themselves and enhances meaningful learning. This element of surprise may also motivate the students' need for proof. In this way, the dynamic environment supports deductive proof and helps the students "close the circle" by engaging in proof (Arcavi & Hadas, 1996). By allowing multiple representations such as Cartesian graphs, measurements, and animations, mathematical connections are enhanced. While seeing objects in a dynamic state, students can observe the act of variation that is betrayed in static representations. Dynamic geometry environments support the notion of increasing and decreasing measures as well as the concept of variation and extrema (Arcavi & Hadas, 1996).

Goldenburg and Cuoco (1998) raised issues concerning student perceptions of the dynamic geometry environment. Exactly what do students perceive on the computer screen using dynamic software? Do they see a continuum of figures? Or do they perceive several discrete cases? Do students have to re-examine their existing definitions to suit the dynamic environment?

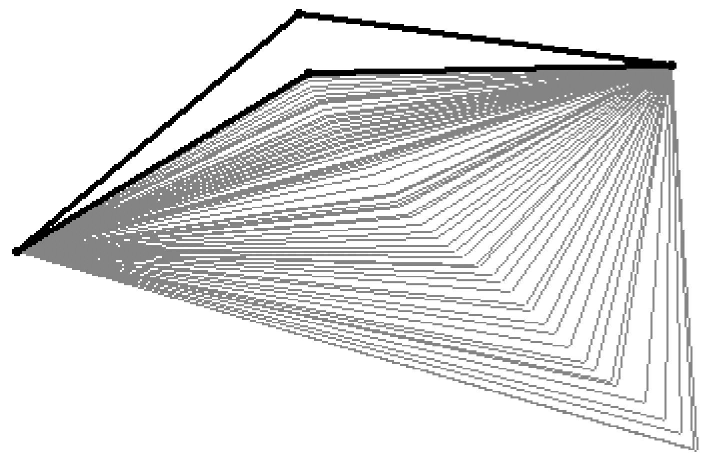

Figure 3. A continuum of quadrilaterals under the drag mode using Geometers Sketchpad (Jackiw, 1991).

To illustrate this last question, consider students investigating a construction in which the midpoints of the four sides of a quadrilateral are connected in order. The resulting figure referred to as the “midsegment quadrilateral” appears to be a parallelogram. The students may wish to explore and conjecture that this will be true for all quadrilaterals. While exploring by “dragging”, students are likely to create figures that they do not consider to be quadrilaterals, such as the degenerate triangle that is formed as the quadrilateral transforms from convex to concave or the “crossed bowtie” figure that is not even considered to be a polygon. Notice that the conjecture, however, still holds for these “monster” polygons. How do students resolve these cases? Do they ignore them, redefine their existing notion of quadrilateral, or treat them as separate cases (Goldenberg & Cuoco, 1998)?

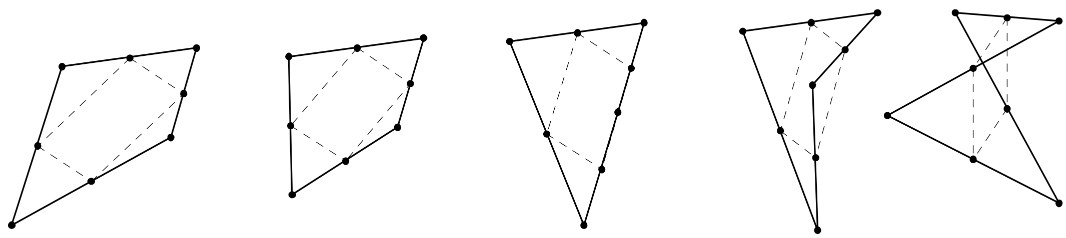

Figure 4. The exploration of the midsegment quadrilateral.

Possible negative consequences

The use of dynamic geometry software as an instructional tool is not without controversy. Jones (1999) reported on some data from a longitudinal study designed to examine how using the dynamic geometry package Cabri-Geometre (Laborde, 1990) mediates the learning of certain geometrical concepts. He conducted case studies of five pairs of 12 year old pupils working through a series of specially designed tasks that involved the construction of various quadrilaterals. Jones (1999) concluded that children working with computers might focus on the screen product at the expense of reflection upon its construction. Students modified the figure "to make it look right" rather than debugging any problems in the construction process. Students did not necessarily appreciate how the computer tools they used constrained their behavior. After making inductive generalizations, students frequently failed to apply them to a new situation (Jones, 1999).

Lange (2002) described a classroom episode in which students used dynamic geometry software to explore perpendicular bisectors. The instructor was expecting the students to discover that any point on the perpendicular bisectors is equidistant to the endpoints of the segment being bisected; this was the "targeted" conjecture of the exercise. The author described how many students failed to observe this property and instead focused on non-conjectures that were already given or were irrelevant. The instructor's goal in this classroom exercise was to link three main components of proof writing: observation, conjecture, and then deduction. Students struggled with what observations should be made in the dynamic geometry environment which made the next step of conjecture difficult.

Both Jones (1999) and Lange (2002) pointed out difficulties that students have adapting to the dynamic geometry environment. It may indeed take some students a considerable amount of time and exposure to realize the potential of the dynamic geometry environment in terms of proper construction techniques and conjecturing in geometry.

Dynamic Language

Mariotti (2001) analyzed a long-term teaching experiment carried out in the ninth and tenth grades aimed at introducing pupils to theoretical thinking by using dynamic geometry software. The analysis of the teaching experiment was aimed at discussing the specific role played by dynamic geometry software. It was found that the use of dynamic geometry software in the generation of a conjecture is based on the internalization of the dragging function as a logic control, which is able to transform perceptual data into a conditional relationship between hypothesis and thesis (if…then). The use of dynamic geometry software on the computer created a channel of communication between the teacher and the pupil based on a shared language (Mariotti, 2001).

Jones (2000) also looked at students' interaction with the dynamic geometry environment and how it affected the students reasoning and language used in their explanations. Like Mariotti (2001), students developed a language that reflected the dynamic environment and used that language to communicate their mathematical explanations. Jones (2000) also reported on a longitudinal study of 12-year old students who completed a 30-week unit of geometry that used dynamic geometry software as a means of delivery. Students passed through three phases during this unit that focused primarily on the properties and classification of quadrilaterals.

In the first stage students relied on description rather than actual explanation. Mathematical language and reasoning were not yet present. In the second stage, explanations became more mathematically precise and were influenced by the dynamic environment itself. For example, words like "dragging" were used in the explanations. In the third stage of the teaching unit students were providing explanations that were entirely in the context of the dynamic geometry environment.

Both of these studies show how the dynamic geometry environment becomes part of the actual mathematics being addressed over time. Language that reflects the environment is used in explanations and as a means of communication between pupil and teacher. The environment promotes action leading to the if-then form of a conjecture and this action becomes part of the reasoning behind the explanations (Jones, 2000).

Dynamic Software and Proof

Dynamic software has the potential to encourage both explanation and proof, because it makes it so easy to pose and test conjectures. "Unfortunately, the successful use of this software in exploration has lent support to a view among many educators that deductive proof in geometry should be downplayed or abandoned in favor of an entirely experimental approach to mathematical justification" (Hanna, 2001, p. 13).

Pandiscio (2002) reported on a study that examined preservice teachers' conception of proof and the use of dynamic software. In this case study, four preservice teachers who were enrolled in a semester long course centered on effective secondary mathematics pedagogy were used as the participants. These participants were given two geometric situations to explore using a dynamic software package name Geometers' Sketchpad. Pandiscio (2002) used the following problems:

- *Are the six small triangles that are formed by the intersection of the medians of a given triangle congruent? Do they have equal area? Investigate and then prove.*
- *If two secant segments are drawn from a point outside a circle, then the product of the measures of one secant segment and its external part is equal to the product of the measures of the other secant segment and its external part. Explore these relationships, and then prove the theorem* (p. 215).

Surveys, observations, and interviews of the participants revealed that after using dynamic software, preservice mathematics teachers expressed the concern that high school students will believe proofs are unnecessary. They still believed that a formal proof is different from "proof by many examples," but after repeated use of dynamic software, they questioned the value of the formal proof for high school students.

Through observation it was noted that the participants explored the problems more deeply with the software than without. The participants claimed that the greatest value of dynamic software is in helping students understand key relationships that are embedded in the figures being explored rather than the proofs of these relationships. This study suggests that the use of dynamic geometry software does not strengthen the perception of preservice teachers that proof is critical to high school geometry. In fact, this study suggested that such a powerful inductive tool may actually decrease the perceived need for students to write deductive formal proofs (Pandiscio, 2002).

Laborde (2001) reported on a case study of four secondary geometry teachers' attitudes related to the use of dynamic geometry software in their classes. The study examined two veteran teachers with experience in dynamic geometry software, a novice teacher who was experienced in computer science and finally a veteran teacher who had little familiarity in using technology in mathematics teaching. Specifically, the study focused on the tasks that were explored by these teachers' students using dynamic geometry software.

For the novice teacher, technology was used to show and confirm that certain geometric theorems and postulates were indeed true and that the figures did behave as expected. The veteran teacher who had little experience with dynamic geometry used the technology mainly for observation and for construction, but pencil and paper environments were provided in addition to the dynamic environment. The two teachers who were experienced in the uses of dynamic geometry software tended to have more open explorations to make use of the drag mode to enhance student conjecture and to pose problems for further exploration and discussion at the end of the activities. This case study illustrated how different types of teachers may use dynamic geometry software. Teachers that are familiar and comfortable with the technology may tend to use tasks that lead to student exploration and discovery. They may allow students to work independently using the technology to form conjectures that would perhaps lead toward deductive proof or counter-examples (Laborde, 2001).

A study by Marrades and Gutierrez (2000) specifically examined the use of dynamic geometry software to facilitate students' transition to deductive proof in geometry. These authors considered two case studies of pairs of students working in a unit that used Cabri-Geometre (Laborde, 1990). Each pair of students was part of a class of sixteen secondary students of age fifteen or sixteen. All students in this class used dynamic geometry software twice a week throughout the unit of 30 activities. The typical activity in this unit would first ask the students to construct geometric figures and then explore them using the Cabri software. Students were then asked to make conjectures about the figures, and finally they were asked to justify their conjectures.

Marrades and Gutierrez (2000) classified the justifications as empirical or deductive. Empirical justifications were inductive in nature and relied on observation of the figures. These observations were classified as "naïve empiricism", "crucial experiment", or "generic example". Naïve empiricism referred to justifying the conjecture by showing that it is true in one or more examples, usually selected without a specific criterion. Crucial experiment justifications used a specific, carefully selected example. Students were aware of the need for generalization so they would choose an example that was as non-particular as possible although it was not considered as a representative of any other example. Students assumed that the conjecture was always true if it was true in this example. The final type of empirical justification, generic example, was based on a specific example seen as a characteristic representative of its class. This kind of justification also used abstract reasons for the truth of the conjecture by means of operations or transformations on the chosen example.

Two types of deductive justifications were also classified in this study. “Thought experiment” referred to the use of a specific example to help the student organize their justifications; however, the use of axioms, definitions, and accepted theorems was used to form deductions. “Formal deduction” referred to the use mental operations without the help of specific examples. In this kind of justification only generic aspects of the problem are discussed. This kind of justification is akin to formal proof.

The researchers collected data by collecting handed in worksheets, downloaded files of the sketches constructed by the participants, and informal videotaped interviews. The results showed that even though the students did not reach the stage of formal deduction, the use of the dynamic geometry software increased the quality of students’ conjectures over time and justifications moved toward deduction by reaching the empirical stage of generic example. Therefore, the students had looked beyond specific shapes and moved toward making conjectures for general figures.

Marrades and Gutierrez (2001) reached several conclusions based on the results of this study:

1) Dynamic geometry software may well help secondary school students understand the need for abstract justifications and formal proofs.

2) The types of justifications and the phases in the process of producing justifications are complementary elements and allow us to make a detailed analysis of the solutions of proof problems.

3) By stating carefully organized sequences of problems, and giving students free time to explore these problems, it is possible to have students progress toward more elaborated types of justifications.

4) Students need a considerable amount of time devoted to experimentation using dynamic geometry software before they become confident with formal deduction.

This study is an example of how dynamic geometry software can prepare students to tackle the task of deductive proof. The authors claim that "we need to know students' conception of mathematical proof in order to understand their attempts to solve proof problems, that is, what kinds of arguments convince students that a statement is true?" (Marrades & Gutierrez, 2001, p. 88).

Mudaly and de Villiers (1999) investigated students' use of dynamic software in order to explore student conviction in discovered conjectures and their need for further explanation concerning those conjectures. The study asked the following questions: Are students convinced about the truth of their discovered geometric conjecture and what is their level of conviction? Do they require further conviction? Do they exhibit a desire for an explanation for why the result is true? Can they construct a logical explanation for themselves with guidance and do they find it meaningful? The following figure illustrates a question that was used to prompt investigation.

Sarah, a shipwreck survivor manages to swim to a desert island. As it happens, the island closely approximates the shape of an equilateral triangle. She soon discovers that the surfing is outstanding on all three of the island's coasts and crafts a surfboard from a fallen tree and surfs everyday. Where should Sarah build her house so that the total sum of the distances from the house to all three beaches is a minimum?

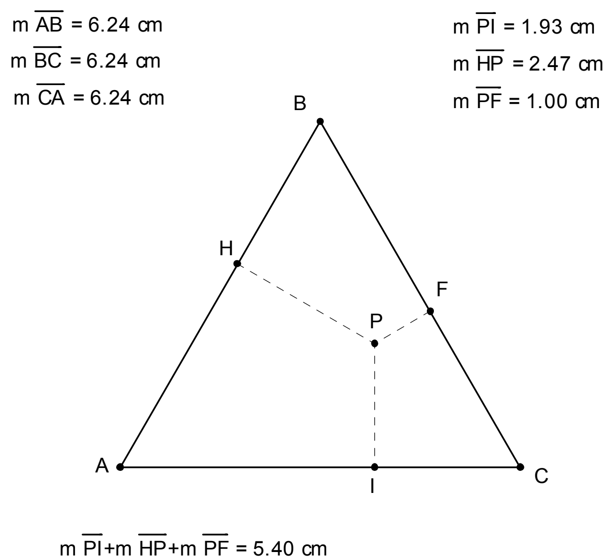

Figure 5. Shipwreck problem used by Mudaly and de Villiers (1999, p. 2).

Students were able to use this dynamic figure to discover that the sum of the lengths of the three segments PI, HP, and PF remains constant while dragging point P within the confines of the equilateral triangle.

The equilateral triangle figure, already constructed in a dynamic geometry environment, was presented to a sample class of fourteen year old students. After exploring the problem using the dynamic features of the software, most of the students were completely convinced that, indeed, the sum of the lengths of the three segments PI, HP, and PF remains constant while dragging point P, and most of them wanted to know why. The research indicated that the learners displayed a need for further explanation for a result, independent of their need for conviction. Given such high levels of conviction one might expect that it should have made no difference to the students whether there was some logical explanation for the result. Yet they found the result surprising and expressed a strong desire for an explanation that was effectively utilized to introduce them to proof as a means of explanation rather than verification (Mudaly & de Villiers, 1999).

Hadas, Hershkowitz, and Schwarz (2000) found similar results in their study that introduced the elements of contradiction and uncertainty with the aid of dynamic geometry software. Two activities were developed for a sample of eighth grade students and designed to cause contradiction or uncertainty in the students' initial intuitive conjectures. The first activity was concerned with the interior angle sum and the exterior angle sum of convex polygons. Students initially conjectured about the sum of the interior angles of various convex polygons with the aid of dynamic geometry software. Many students conjectured that indeed the sum of the interior angles increased by 180 degrees with each additional side or angle. Many of them even expressed the relationship algebraically. This result may have caused the students to then assume that the sum of the exterior angles would also increase as the number of sides increased. In fact 37 out of 49 responses indicated just that. The fact that the sum of the exterior angles is always 360 degrees for convex polygons was a surprising result that contradicted the students' conjectures. This result was also discovered by means of dynamic geometry software;

however, the explanation of why this sum is constant motivated students to go further into interesting inductive, deductive, and visual arguments (Hadas, Hershkowitz, & Schwarz, 2000).

In the next activity, students used dynamic geometry to explore the conditions for congruent triangles, to investigate if and when two triangles having several congruent parts are congruent. In one of the tasks, the students were asked if it is possible to construct a triangle with one side and two angles congruent to another triangle but that is not congruent to that triangle. This task was designed to cause uncertainty among the students. Some of the students were able to make the construction using dynamic geometry software while others claimed that the triangles must be congruent. Those students who claimed that the triangles must stay congruent had constructions in which the corresponding sides were the included sides of the two congruent angles in both of the triangles. Those who assigned the congruent side as the included side in one triangle and a non-included side in the other triangle realized through the dragging process that these triangles were not congruent. In fact in this case the triangles are similar (Hadas, Hershkowitz, & Schwarz, 2000). Figure 6 illustrates these two possibilities.

Once again students were driven toward deductive explanations to resolve this uncertainty. Results of this study show that most of the students resolved this problem using deductive means and discovered the triangle congruence propositions in the process (Hadas, Hershkowitz, & Schwarz, 2000).

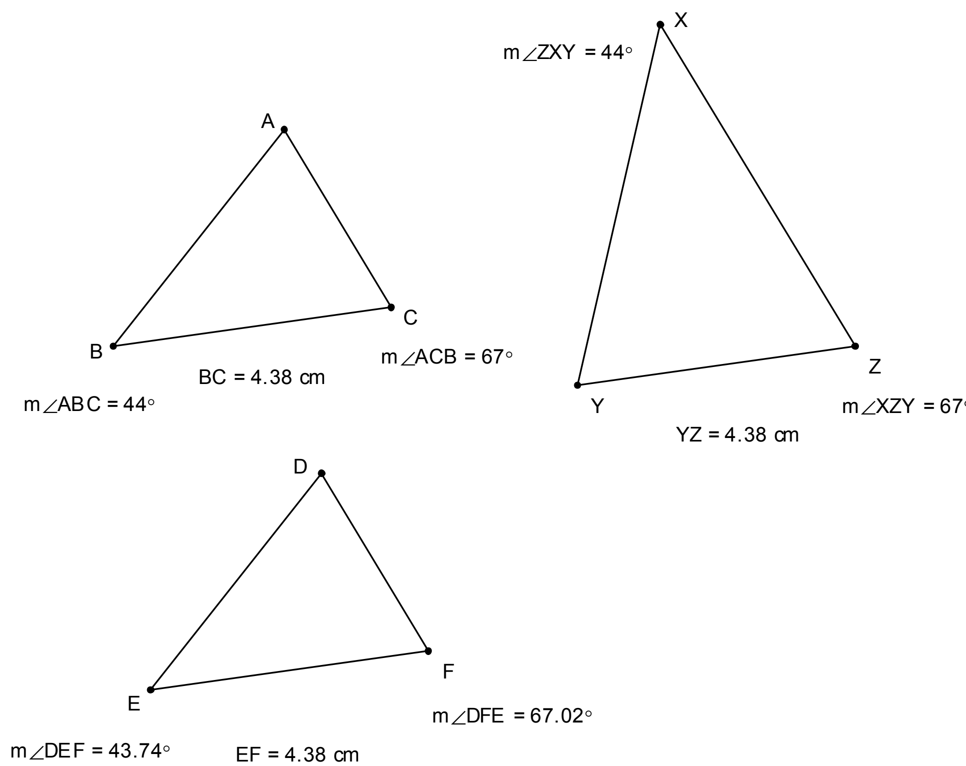

Figure 6. Constructions of triangles with one congruent side and two congruent angles, showing that the positioning of the congruent parts determines the congruence of the triangles.

These last two studies contrast with the findings in Pandiscio (2002) which suggested that preservice teachers may find the use of dynamic software a hindrance to proof instruction and that their students would not be motivated to use deductive proof after using dynamic software. With proper guidance students were motivated and were able to construct deductive arguments after exploring and conjecturing in the dynamic geometry environment.

These dynamic geometry studies explore a variety of issues concerning the use of the dynamic geometry environment in the classroom. Topics include student perceptions and misconceptions, student conjectures and conviction, student need and ability to use dynamic geometry software to communicate and foster proof, and teacher as well as pre-service teachers' attitude and uses of dynamic geometry software. The recurring themes of conjecture and proof in all of these studies make it evident that these topics are of utmost importance in studies concerning the use of dynamic geometry software.

Summary

This chapter has reviewed literature concerning student's abilities and attitudes concerning proof. A brief discussion of van Hiele theory was provided to show the relation to proof and possible causes for the failure of traditional proof instruction. Literature concerning the role of conjecture in the proof process was then followed by a discussion of the dynamic geometry environment created by computer software packages and the role of dynamic geometry software in terms of conjecture and proof. This chapter serves to lay the groundwork for this study. The following chapter details the methodology used to collect both quantitative and qualitative data, a description of the context of the research, a detailed explanation of the instruments that were used, the variables that were measured, and the methods used to analyze the data.

CHAPTER III: METHODOLOGY

This chapter provides a detailed outline of the methods employed to answer the quantitative research questions and to address the qualitative themes regarding student conjecture in geometry and the use of dynamic geometry software. A discussion of the participants used in the study and the instruments used on those participants will be followed by the procedures for collecting data and analyzing that data. Note that the researcher of this study was also the instructor of the students used in this study and is a full-time employee of the school and the district in which this study took place.

Participants

Participants were recruited from two secondary school geometry classes taught by the researcher. These two classes were designated as class A and class B for the purposes of this study. All students in these classes were asked, but not required, to participate in the study. Parental permission was obtained for each participant before the study began. All students who volunteered and returned a written permission form signed by a parent or legal guardian were included in the study. Forty-two out of a possible fifty-five students returned written permission forms. The remaining students still participated in all of the conjecturing lab activities; however, their collected data was not analyzed as part of this study.

The participants were enrolled in a public high school in a southern city of the United States. The total school population for grades 9-12 is approximately 1100. Approximately 70% of the school population is African American, 20% White, 7% Latino, and 3% Asian. The geometry classes in which the subjects were enrolled were neither remedial or honors classes and reflected the general school population in terms of race, ethnicity, and achievement. Most of the students in these geometry classes were sophomores or juniors with one freshman and four seniors included.

A wide range of mathematical abilities was represented; however, data obtained on mathematics achievement indicated that most of the participants were in the lower percentiles in mathematics achievement when compared with national data. The measure of mathematics achievement used in this study was the students' most recent score on the mathematics section of the Preliminary Scholastic Aptitude Test (PSAT) already on file prior to the study. Figure 7 shows the distribution of student scores on the 2003 PSAT math section. The national mean for this test was 44.5 for tenth grade students and 48.8 for eleventh grade students for the year 2003 (College Entrance Examination Board, 2003).

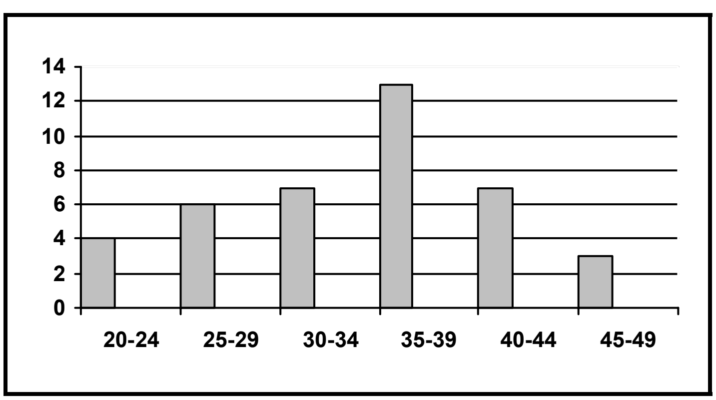

Figure 7. Participant distribution of 2003 PSAT mathematics scores

Forty-two students began the study, and one student was dropped because of lack of attendance. All of the students participating in the study were taking the course for the first time. The following table provides demographic data for the participants used in both classes. Note that the racial percentages are relatively close to the school averages and there is a balance of gender for the overall sample of students.

Table 1

Participants' gender and racial demographics

	Class A		Class B		Total
Race	Male	Female	Male	Female	
African American	6	6	6	11	29
White	3	2	0	2	7
Latino	3	0	1	0	4
Asian	0	0	0	1	1
Total	12	8	7	14	41

Instruments

Lab activities were developed to measure the students' conjecturing abilities in each environment, as well as their conviction in the correctness of their conjectures. In addition, two instruments were used to measure statistical covariates of the study. The mathematics section of the Preliminary Scholastic Aptitude Test (PSAT) was used to measure mathematics achievement and a Van Hiele geometric reasoning instrument was used to measure the students' geometric reasoning level. Two qualitative instruments were developed to explore qualitative themes: a student survey with open-ended questions and an interview protocol. Each of the instruments used are discussed in turn.

Conjecturing Lab Instruments

Eight different interactive lab instruments were developed with parallel versions for both the static and the dynamic geometry environments. Each of the lab instruments contained geometric figure(s) with appropriate measurements given. These figures were constructed using Geometer's Sketchpad (GSP) version 4.02 (Jackiw, 2001), a dynamic geometry software package. In the dynamic environment, students were able to deform the figures by using a dragging feature. This dragging utility gave the students the ability to observe how the components of the figure and the given measurements shown reflected the deformations. The static environment used the exact same figures and measurements; however, the dragging capabilities of the software were disabled. Therefore, the only notable difference between the two environments on any particular lab activity was the ability to drag. On each instrument, students were asked to state, in their own words, as many conjectures as they could about the geometric situations the figures represent. Students were then asked to rank their personal conviction of the truth of their conjectures on a scale of 1 to 10 for each conjecture, with 10 being the highest confidence in the truth of their conjectures.

The labs were designed to serve as introductory lessons before the actual theorems and postulates were presented to the class. For example, the parallelogram lab was completed prior to or on the day in which parallelograms were introduced in the course curriculum. This gave the students an opportunity to explore and conjecture about the properties of this figure before being presented with the theorems concerning parallelograms. Topics were selected to align with the course curriculum. The lab topics included:

1. Parallel Lines and Transversals
2. Angles of Triangles
3. Triangle Midsegments
4. Parallelograms
5. Diagonals of Rectangles and Squares
6. Diagonals of Rhombi and Kites
7. Trapezoids
8. Inscribed Angles and Circles

These instruments are presented in Appendix A.

The quality of the conjectures formed by the students was assessed using the following general rubric following Lange (2002):

- An **R** was assigned to a conjecture that was true and relevant to the lab activity. Relevant conjectures include the lab activities' "targeted" conjectures and other true conjectures that pertain to the figure(s) and had not yet been introduced in the course curriculum at the time of the activity.
- An **I** was assigned to conjectures that were true but irrelevant to the lab activity. These conjectures had either been introduced earlier in the course curriculum or they pertain to a more general figure than the figure(s) involved in the lab activity. For example, consider a student making the

following conjecture about a figure showing a parallelogram: “The sum of the interior angle measures of a parallelogram is 360 degrees.” This conjecture is true but irrelevant, since it is true for all quadrilaterals and was discussed earlier in the course.

- An **A** was assigned to a conjecture that was ambiguous and could not be reasonably interpreted as true or false. Conjectures of this kind were often poorly worded, and it was difficult to determine what the student is trying to communicate mathematically.
- An **F** was assigned to conjectures that were false. These conjectures may have been true for some cases but if a counterexample exists, mathematically speaking, the conjecture is false. For example, consider the following false conjecture: “A parallelogram has two acute angles and two obtuse angles.” Although this statement is true for many parallelograms, it is false if the parallelogram is a rectangle or a square.

For each lab instrument, item-specific rubrics that specify what kinds of conjectures are considered relevant, irrelevant, ambiguous, or false were developed; see appendix A. The validity of the instruments and their rubrics was validated by a mathematician and by a mathematics educator, both employed by a local university. Their suggestions were used in modifying the instruments to their final form.

All responses to the lab instruments were assessed by the researcher as well as an outside grader experienced in teaching secondary geometry. While different outside graders participated in the validation, only a single outside grader was used to assess each individual lab activity. For example, one outside grader was used to assess lab activity one where a different outside grader was used for lab activity two. Outside graders used the general rubric and item-specific rubrics to assign one of the four letters mentioned above to each of the subjects' conjectures. Interrater reliability percentage of agreement data is presented in chapter four of this report.

Geometric Reasoning Instrument

Participants completed an instrument to assess their entering level of geometric reasoning following the work of van Hiele (Gutierrez & Jaime, 1998) during the second week of class instruction. The instrument consisted of six open ended items with multiple parts. It was patterned after instruments suggested by Gutierrez and Jaime (1998) who have done extensive work with van Hiele levels of geometric reasoning. Gutierrez and Jaime (1998) state two sources of validity for the content used in these items. The first source of validity was a series of pilot studies conducted by the authors and the second source of validity was based on analysis made by several researchers with expertise in van Hiele theory (Gutierrez & Jaime, 1998). Gutierrez and Jaime (1998) also reported Guttman Coefficients ranging from .98 to 1.00. No additional information concerning validity and reliability is provided by Gutierrez and Jaime (1998). This instrument was graded by the researcher and an outside grader with knowledge of van Hiele theory using a general rubric suggested by the authors. This instrument ranked the students' geometric reasoning level (1 – 4) and is presented along with the rubric used in appendix B.

Mathematics Achievement Instrument

Information about mathematics achievement was obtained from student records already on file in the school's guidance department. Thirty-nine of the forty-one subjects had taken the Preliminary Scholastic Aptitude Test (PSAT) and the results were part of their school file. The score from the mathematics sections of this assessment was used to measure the students' mathematics achievement. The mathematics portion of this test consists of two timed 25 minute mathematics sections containing arithmetic, algebra, and geometry items. Comprehensive reviews and analyses are conducted to ensure that questions are a valid measure of mathematics problem solving ability and are fair for different groups of students (College Entrance Examination Board, 2003). Item types range from multiple choice, quantitative comparison, and free response. Reported scores for the PSAT mathematics sections may range from 20 to 80 points, with one point awarded for each correct response, 0 points for no answer, and a ¼ point deduction for each wrong answer on a multiple choice item. Percentile scores were also reported which compared the students nationally. A reliability coefficient of .87 was reported for the 2003 version of the mathematics portion of the PSAT (College Entrance Examination Board, 2003).

Survey Instrument

Student surveys were given to all participants after all lab activities had been concluded to assess their reactions to the experience. Questions on this instrument were open ended. They addressed the students' concept of conjecture, environment preferences, and the purpose of dragging in the dynamic environment. The purpose of this instrument was to address several of the *a priori* qualitative themes of this study:

Which environment do the students prefer and why? What is the students' concept of conjecture? How much do students use the dragging utility while in the dynamic environment? This survey instrument is presented in appendix C.

Interview Protocol

Interviews took place with ten selected participants representing a cross section of students by level of mathematics achievement. These interviews took place after all lab activities and the survey instrument were completed. Individual interviews were conducted with the students, who were seated at a computer station for the interview. Students were presented with geometric figures in the dynamic software environment, and conjectures associated with them. The students were asked to evaluate the truth of these given conjectures. Students were allowed, and sometimes encouraged, to drag the figures while the interview was taking place. The questions focused on the concept of proof versus conjecture, finding counterexamples to disprove false conjectures, and student conviction in the computer output of the dynamic environment. These questions coincided with three of the a priori qualitative themes addressed in this study. Particular attention was placed on the students' dragging technique during the interviews and the manner in which the students' used the dragging utility was noted in the researcher's journal. All interviews were audiotaped and transcribed. The interview protocol and accompanying figures are presented in appendix D.

Procedures

An informed consent form was distributed to all students of both classes during the first week of instruction. All of the students were invited to participate in the study, and those that returned the form with a parental signature were included in the study. No additional recruiting was necessary, and all students participated in the lab activities as a requirement of the course regardless of their participation in the study.

The van Hiele geometric reasoning instrument was administered during the second week of instruction. The PSAT scores for each participant were also obtained from the guidance department of the high school during the second week of instruction.

The eight conjecturing labs, both static and dynamic, consisted of 45-minute increments of time at the beginning of a 90-minute block of instruction. These labs were spread over eight weeks of instruction beginning on the fourth week. All lab activities took place in one of the school's computer labs. Students were seated at a computer station in which the file containing the lab activity was already open. The class that was assigned the static environment had their dragging tool hidden and only their text tool was functioning to allow them to write their conjectures. The class that was assigned the dynamic environment also had the dragging tool operational.

All students were shown how to use the dragging tool but no other training on the capabilities of the dynamic geometry software package was presented to the students until the conclusion of the study, since all of the figures presented in the activities were constructed by the researcher. Thus, the students did not have to construct, hide, or measure anything on the screen in order to make relevant conjectures. In the directions, students were asked to state any "new" conjectures not already discussed in the classroom concerning the figures in the lab activities. Students were also asked to rate their conviction in the truth of their conjectures on a 1-10 scale for each of their conjectures with 10 being the most confident.

The researcher's task during the lab activities was to ensure that the students were on task and were properly using the computer software and hardware. At no time did the researcher confirm the validity of any conjecture or help any student with their conjectures. All students received full credit for participating in each lab activity regardless of the quality of their conjectures or their participation in the study. Students saved their final output on a floppy disc provided by the researcher. The researcher collected each disc at the end of the conjecturing lab and reissued them before the next lab. When each lab activity was concluded, all students met in the classroom where the researcher led a discussion on the students' findings.

Class B participated in the static environment on lab activities 1, 3, 5, and 7 while class A participated in the dynamic environment during these lab activities. During activities 2, 4, 6, and 8 these assignments were reversed. The purpose of this assignment design was to ensure that all students were equally exposed to both environments so as to not to deprive any student from any educational benefits offered by the static or dynamic environments. Because of this design, it was necessary to run separate data analyses on the two different assignment cases (odd and even).

The qualitative data collection occurred following the completion of the final lab activity. The survey instrument was administered one week after the completion of the final lab activity. Students were selected for interviews based on their reported mathematics achievement scores to ensure that subjects represented various levels of mathematical achievement. Each interview lasted approximately fifteen minutes and was conducted privately with the researcher during the school day.

In addition, the researcher kept a journal throughout the course of the study to incorporate field notes recorded during the research activities. Field notes were taken after the completion of each lab activity and after each participant interview. These notes were immediately entered into an electronic journal in chronological order. Entries following the lab activities typically were a paragraph in length and included general observations of the classes, their use of the software, how quickly they finished the activity, and how well they participated during the discussion following the lab. Entries following the interviews made note of the participants' dragging technique during the interview as well as their general behavior during the interview process. The interviews themselves were audio taped and later transcribed.

Data Analysis

This study analyzed both quantitative and qualitative data. Both of these analyses are described in the following sections.

Quantitative Analysis

Descriptive statistics, correlations, and sequential linear regression analysis were used to answer the quantitative research questions. The following independent variables were collected for each participant: Gender (GENDER), mathematics achievement (ACHIEVEMENT), and van Hiele level (VH). The independent variable for environment (ENVIRONMENT) was recorded for each activity and for each student.

The following dependent variables were collected from each participant in each environment: mean score for the number of relevant conjectures (RELEVANT), mean score for the number of false conjectures (FALSE), and the mean conviction score for each conjecture (CONVICTION). Therefore, each student had a score for the number of relevant conjectures per lab activity and for the number of false conjectures per lab activity. These means were calculated separately for each environment since all participants conjectured in both environments.

To compute RELEVANT, the total number of relevant conjectures was summed across the activities in a particular environment for each participant and then divided by the number of labs completed by that participant in that environment. This yielded a mean score for the variable RELEVANT for each participant in each environment. False conjectures were treated in a similar manner to calculate the variable FALSE for each participant in each environment. The conviction score for each participant was obtained by calculating the mean of all convictions for each conjecture in a particular environment.

When differences between the researcher's rating and the outside grader's rating were present within a lab activity, the mean of those ratings was used. For example, if the researcher had found three relevant conjectures for a student on a particular lab activity, and the outside grader had found only two relevant conjectures for that same student on that same lab activity, a score of 2.5 was used for that lab activity.

A correlation matrix was created using all variables to determine zero order correlation coefficients. Three different sequential linear regressions were performed on the three dependent variables for each of the two assignment cases. The independent variables, ACHIEVEMENT, GENDER, and VH, were used as covariates in the sequential linear regression. Therefore, this analysis encompassed six sequential linear regressions with ENVIRONMENT being the independent variable of interest. Sequential regression analysis was used to test the following null hypotheses:

- The opportunity to alter a figure in the dynamic geometry environment by dragging does not significantly increase a student's ability to make relevant conjectures.
- The opportunity to alter a figure in the dynamic geometry environment by dragging does not significantly decrease a student's likelihood of making false conjectures.
- The opportunity to alter a figure in the dynamic geometry environment by dragging does not significantly increase a student's conviction in the conjectures they make.

Statistical analyses were also used to explore what role gender, mathematics achievement, and van Hiele level play in the conjecture process. All statistical analyses were conducted using the Statistical Package for the Social Sciences (SPSS) (Shannon and Davenport, 2001).

Qualitative Analysis

The qualitative analysis of this study used four sources of data: The survey instrument, the participant interviews, the researcher's journal, and the conjecture labs themselves. The survey instrument and the interview protocol were constructed to address the four *a priori* qualitative themes. These themes were developed prior to the collection of data, motivated by the literature reviewed for this study. They included: Student preferences concerning the conjecturing environments, students' concepts of conjecture and proof, students' ability to form and find counterexamples using dynamic geometry software, and students' conviction in the output generated by dynamic geometry software. The specific literature sources used to develop these themes are discussed in greater detail in Chapter V.

The qualitative analysis concentrated on coded key words and phrases found in all of the mentioned sources of data. The researcher's journal as well as all surveys, interviews, and conjectures were read thoroughly. On a second reading, key words and phrases were identified and placed on index cards along with their location on the instrument. These key words and phrases were assigned as "codes" that helped organize the data into several core categories. All codes and text associated with those codes were then transferred from index cards to a computer data base for easy access.

Bogdan and Biklen (1998) refer to codes as "words, phrases, patterns of behavior, subjects' ways of thinking, and events that repeat or stand out (p. 171)." The act of coding involves several steps: Searching your data for patterns and topics of interest, identifying words and phrases that identify those patterns and topics, placing these words and phrases into core categories, and using these categories to sort the data (Bogden & Biklen, 1998).

After the coding process, many of the core categories applied directly to the four *a priori* themes developed for this study. However, several other categories gleaned from the conjecturing labs and the researcher's journal suggested the development of two additional qualitative themes. Thus, emergent themes concerning dynamic language and dragging tendencies were added to the analysis.

CHAPTER IV: QUANTITATIVE RESULTS

This chapter reports the results of the quantitative data analysis conducted in order to answer the quantitative questions of this study. The descriptive statistics of each of the variables are reported first, followed by the interrater reliability percentage of agreement concerning the scores obtained from the instruments designed for this study. The results of zero-order correlations of all of the variables are reported followed by the results of the sequential regression analysis. This chapter concludes by summarizing how these results address the research questions as well as the exploratory findings.

Descriptive Statistics

The following tables provide descriptive statistics for the three dependent variables: RELEVANT, FALSE, and CONVICTION. Tables reporting results for the covariates ACHIEVEMENT and VH are then provided. Table 2 provides means and standard deviations for all three dependent variables in both assignment cases regardless of the environment used. Recall that two different assignment cases were used in this study. Class A received the instruments in the dynamic environment in the odd assignment case and Class B received the instruments in the dynamic environment during the even assignment case.

Table 2

Means and standard deviations for both assignment cases for the three dependent variables in both environments

Dependent	Odd Assignment Case		Even Assignment Case	
Variable	Mean	S.D.	Mean	S.D.
RELEVANT	1.3374	0.95512	1.5469	0.81262
FALSE	0.1315	0.22820	0.1964	0.30440
CONVICTION	8.1667	1.53885	8.5231	1.54195

Note that in this table, there is no distinction between the environments used, therefore the means represent an average over both environments. The means in the odd assignment case represent both the means of Class A, conjecturing in the dynamic environment, and Class B, conjecturing in the static environment. The roles are reversed with the even assignment case. It should not be surprising that the scores are quite comparable, since they include both cases.

Table 3 provides means for each variable within a specific environment. Differences in means are apparent when considering the two different environments. RELEVENT and CONVICTION is higher in the dynamic environment and FALSE is lower in the dynamic environment. This is true in both assignment cases. Further analysis will determine the statistical significance of these differences.

Table 3

Means in terms of environment for each of the dependent variables

Dependent Variable	Odd Assignment Case		Even Assignment Case	
	Class A*	Class B	Class A	Class B*
RELEVANT	1.80	0.92	1.21	1.93
FALSE	0.02	0.238	0.31	0.06
CONVICTION	8.8	7.6	7.90	9.20

* Group using the dynamic geometry environment

Tables 4 and 5 provide means and standard deviations for the covariate variables ACHIEVEMENT and VH. The mean achievement of 33.3 in Class A corresponds with the 16th percentile when compared with national data and the mean score of 36.8 corresponds with the 26th percentile when compared with national data (College Entrance Examination Board, 2004).

Table 4

Means and standard deviations for the covariate ACHIEVEMENT

ACHIEVEMENT	Class A	Class B
Mean	33.3	36.8
Standard Deviation	6.3	7.5

In Class A, 19 of the 20 subjects were assessed at van Hiele Level 1 with one subject at Level 2. In Class B, 17 of the 21 subjects were assessed at van Hiele Level 1 with four subjects at Level 2.

Table 5

Means and standard deviations for the covariate VH

VH	Class A	Class B
Mean	1.05	1.19
Standard Deviation	0.22	0.42

Interrater Reliability

Each of the lab instruments used in this study as well as the van Hiele geometric reasoning instrument was assessed by the researcher and by an outside grader experienced in the teaching of secondary school geometry. There were no differences in the assigned scores for the van Hiele instrument and only small differences in the lab instruments. Percentage of agreement was calculated for each instrument for both RELEVANT and FALSE, and they ranged from 92% to 100%. When a score for any of these variables was found to differ, the mean score was used in the analysis.

Correlations of Variables

The following tables show the zero order Pearson correlations for all variables in each of the two different assignment cases. Statistically significant correlations are indicated for the .05 and .01 levels. These zero order correlations do not take into account collinearity aspects of the data; therefore, they should not be interpreted as "unique effects" (Shannon & Davenport, 2001).

Table 6

Zero order Pearson correlation coefficients for the odd assignment case

	RELEVANT	FALSE	CONVICTION	ENVIRONMENT	GENDER	ACHIEVEMENT	VH
RELEVENT		-.191	.695**	.474**	.154	.332*	.161
FALSE	-.191		-.184	-.501**	-.253	.382*	.259
CONVICTION	.695**	-.184		.445**	.333*	.111	.151
ENVIRONMENT	.474**	-.501**	.445**		.267	-.255	-.215
GENDER	.154	-.253	.333*	.267		-.093	.102
ACHIEVEMENT	.332*	.382*	.111	-.255	-.093		.518**
VH	.161	.259	.151	-.215	.102	.518**	

*p<.05 **p<.01

This correlation matrix indicates a significant correlation of the independent variable ENVIRONMENT with the three dependent variables: RELEVANT, FALSE, and CONVICTION. In a later section, sequential regression analysis will be used to explore the unique contribution of the independent variable ENVIRONMENT on each of the dependent variables as well as the unique effects of the covariate variables on each dependent variable.

There is also a strong positive correlation between the dependent variables CONVICTION and RELEVANT indicating that those students that have more relevant conjectures are more confident in those conjectures. The results also indicate that higher achieving students tend to make more relevant conjectures and somewhat surprisingly, more false conjectures as well. Note the strong correlation between van Hiele level and mathematics achievement, as well as the correlation between GENDER and CONVICTION, indicating that males have a higher confidence in their conjectures.

Table 7

Zero order Pearson correlation coefficients for the even assignment case

	RELEVANT	FALSE	CONVICTION	ENVIRONMENT	GENDER	ACHIEVEMENT	VH
RELEVANT		-.246	.455**	.419**	-.053	.412**	.147
FALSE	-.246		-.517**	-.427**	-.036	-.048	-.026
CONVICTION	.455**	-.517**		.415**	-.022	.262	.116
ENVIRONMENT	.419**	-.427**	.415**		-.267	.255	.215
GENDER	-.053	-.036	-.022	-.267		-.093	.102
ACHIEVEMENT	.412**	-.048	.262	.255	-.093		.518**
VH	.147	-.026	.116	.215	.102	.518**	

*p<.05 **p<.01

This matrix shows many of the same results hold for the odd assignment case. The independent variable ENVIRONMENT correlates strongly with the three dependent variables; RELEVANT, FALSE, and CONVICTION. CONVICTION correlates strongly with RELEVANT as in the other case. In this case, however, ACHIEVEMENT correlates significantly with RELEVANT, but not significantly with FALSE, and there is not a significant correlation between GENDER and CONVICTION. In the next section, sequential regression analysis will be used to explore unique contributions of the geometric environment as well as the covariates.

Regression Analysis

A sequential regression analysis was used. In step one of the regression analysis the variables of ACHIEVEMENT, GENDER, and VH were entered. In step two of the analysis the ENVIRONMENT variable was added as a predictor. This analysis will show the unique effects of the ENVIRONMENT variable after the other variables have been accounted for. Since there are three dependent variables to consider and two separate assignment cases for each of the dependent variables, the results of six sequential multiple regressions are presented.

Relevant Conjectures

The variable RELEVANT measured the average number of relevant conjectures made per lab activity for each subject. The results of this variable in the odd assignment case yielded an overall mean of 1.34 relevant conjectures per lab activity with a standard deviation of .955. Table 8 presents the results of the sequential regression analysis. Step one of the regression analysis yielded R = .377 which was not statistically significant, $F(3, 35) = 1.928$, $p = .143$. The $R^2 = .142$ indicated that approximately 14% of the variance in RELEVANT could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added yielding R = .698 which was statistically significant, $F(4, 34) = 8.072$, $p <.001$. The change statistics produced by the addition of ENVIRONMENT were statistically significant with R = .588, $F(1, 34) = 22.886$, and $p <.001$. The R^2 change of .345 explained the variance of the dependent variable RELEVANT over and above the covariates. Thus the unique contribution of

ENVIRONMENT is responsible approximately 35% of this variance, which is statistically significant.

Table 8

Regression analysis for the variable RELEVANT in the odd assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.060	.440	.368	.135	.005**
VH	.295	.105	.086	.007	.491
GENDER	-.102	-.054	-.049	.002	.694
ENVIRONMENT	1.242	.649	.588	.346	<.001**

*p<.05 **p<.01

The results of the RELEVANT variable in the even assignment case yielded an average of 1.55 relevant conjectures per lab activity with a standard deviation of .813. Table 9 presents the results of the sequential regression analysis. Step one of the regression analysis yielded R = .393 which was not statistically significant, $F(3, 35)$ = 2.129, p = .114. The R^2 of .154 indicated that approximately 15% of the variance in RELEVANT could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added in yielding R = .509 which was statistically significant, $F(4, 34)$ = 2.966, p = .033. The change statistics produced by the addition of ENVIRONMENT were statistically significant with R = .323, $F(1, 34)$ = 4.787, and $p < .036$. The R^2 change = .104

explained the variance of the dependent variable RELEVANT over and above the covariates. Thus the unique contribution of ENVIRONMENT is responsible approximately 10% of this variance, which is statistically significant.

Table 9

Regression analysis for the variable RELEVANT in the even assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.036	.312	.261	.068	.087
VH	-.169	-.071	-.058	.003	.698
GENDER	-.015	-.009	-.008	<.001	.955
ENVIRONMENT	.581	.362	.323	.104	.036*

*p<.05 **p<.01

False Conjectures

The variable FALSE measured the average number of false conjectures made per lab activity for each subject. The results of this variable in the odd assignment case yielded an average of 0.1315 false conjectures per lab activity with a standard deviation of .228. Table 10 presents the results of the sequential regression analysis. Step one of the regression analysis yielded an R = .461 which was statistically significant, $F(3, 35)$ = 3.149, p = .037. The R^2 = .213 indicated that approximately 21% of the variance in FALSE could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added yielding R = .588 which was statistically significant, $F(4, 34) = 4.494$, p =.005. The change statistics produced by the addition of ENVIRONMENT were statistically significant with R = .365, $F(1, 34) = 6.928$, and

p <.013. The R^2 change of .133 explained the variance of the dependent variable FALSE over and above the covariates. Thus the unique contribution of ENVIRONMENT is responsible approximately 13% of this variance, which is statistically significant.

Table 10

Regression analysis for the variable FALSE in the odd assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.008	.241	.201	.040	.156
VH	.035	.052	.043	.002	.761
GENDER	-.049	-.107	-.097	.009	.490
ENVIRONMENT	-.184	-.409	-.365	.133	.013*

*p<.05 **p<.01

The analysis of the FALSE variable in the even assignment case yielded an average of 0.1964 false conjectures per lab activity with a standard deviation of .304. Table 11 presents the results of the sequential regression analysis. Step one of the regression analysis yielded R = .050 which was not statistically significant, $F(3, 35)$ = 0.030, p = .993. The R^2 = .003 indicated that less than 1% of the variance in FALSE could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added yielding R = .448 which was not statistically significant, $F(4, 34) = 2.129$, p =.099. The change statistics produced by the addition of ENVIRONMENT were statistically significant with R = .445, $F(1, 34) = 8.407$, and p =.007. The R^2 change of .198 explained the variance of the dependent variable FALSE over and above the covariates. Thus the unique contribution of ENVIRONMENT is responsible approximately 20% of this variance, which is statistically significant.

Table 11

Regression analysis for the variable FALSE in the even assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.001	.012	.010	<.001	.947
VH	.088	.098	.080	.006	.606
GENDER	-.110	-.182	-.165	.027	.289
ENVIRONMENT	-.300	-.498	-.445	.198	.007**

*p<.05 **p<.01

Conviction

The variable CONVICTION measured the average self-rated conviction in all conjectures made in every lab activity for each subject. The results of this variable in the odd assignment case yielded an average conviction of 8.1667 with a standard deviation of 1.54. Table 12 presents the results of the sequential regression analysis. Step one of the regression analysis yielded R = .386 which was not statistically significant, $F(3, 35)$ = 2.044, p = .126. The R^2 = .149 indicated that approximately 15% of the variance in CONVICTION could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added in yielding R = .566 which was statistically significant, $F(4, 34)$ = 4.009, p =.009. The change statistics produced by the addition of ENVIRONMENT were statistically significant with R = .414, $F(1, 34)$ = 8.578, and p =.006. The R^2 change of .171 explained the variance of the dependent variable CONVICTION over and above the covariates. Thus the unique contribution of ENVIRONMENT is responsible approximately 17% of this variance, which is statistically significant.

Table 12

Regression analysis for the variable CONVICTION in the odd assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.038	.172	.144	.021	.317
VH	.670	.147	.121	.015	.400
GENDER	.600	.196	.178	.032	.217
ENVIRONMENT	1.410	.464	.414	.171	.006**

*p<.05 **p<.01

The results of the CONVICTION variable in the even assignment case yielded an average conviction of 8.5231 with a standard deviation of 1.54. Table 13 presents the results of the sequential regression analysis. Step one of the regression analysis yielded R = .265 which was not statistically significant, $F(3, 35) = 0.880$, $p = .461$. The $R^2 = .070$ indicated that approximately 7% of the variance in CONVICTION could be explained by a combination of mathematics achievement, van Hiele level, and gender. In step two of the analysis, the ENVIRONMENT variable was added in yielding $R = .444$ which was not statistically significant, $F(4, 34) = 2.084$, $p =.105$. The change statistics produced by the addition of ENVIRONMENT were statistically significant with $R = .356$, $F(1, 34) = 5.366$, and $p =.027$. The R^2 change of .127 explained the variance of the dependent variable CONVICTION over and above the covariates. Thus the unique contribution of ENVIRONMENT is responsible approximately 13% of this variance, which is statistically significant.

Table 13

Regression analysis for the variable CONVICTION in the even assignment case

Variable	B Unstandardized Coefficients	Beta Standardized Coefficients	R Semipartial Correlation	R^2	Sig.
ACHIEVEMENT	.048	.218	.144	.021	.244
VH	-.419	-.092	-.075	.006	.628
GENDER	.327	.106	.097	.009	.534
ENVIRONMENT	1.215	.399	.356	.127	.027*

*p<.05 **p<.01

Conviction of Relevant versus False Conjectures

In the previous section, student conviction in the correctness of their conjectures was averaged for all of a participant's conjectures, including the false conjectures. Having a higher conviction on false conjectures is not desirable; therefore, a further analysis of CONVICTION comparing the averages of relevant versus false conjectures was conducted using a repeated measure ANOVA. In this analysis, the students' conviction in their relevant conjectures was compared with their conviction in their false conjectures. This analysis was grouped by the type of environment the students were using. Students that had made no false conjectures could not be included in this analysis. Table 14 summarizes the conviction means that were compared.

Table 14

Conviction scores for relevant and false conjectures for each environment

	Dynamic Environment	Static Environment
Relevant Conjectures	9.5	8.6
False Conjectures	6.7	6.9

The effect within subjects was significant, with $F(1, 27) = 4.561$, $p = .042$ indicating that students' conviction in relevant conjectures is significantly higher than their conviction in false conjectures. The effect between subjects was not significant, with, $F(1, 27) = .843$, $p = .367$. However, a post-hoc t-test on relevant conjectures by environment resulted in $t = 2.629$, $p = .014$, indicating that conviction in relevant conjectures is significantly higher in the dynamic environment. The t-test on false conjectures by environment resulted in $t = -0.350$, $p = 0.729$, showing no significant difference in the conviction of false conjectures within the two environments.

Collinearity of Predictors

Table 15 shows the tolerance and variance inflation factor (VIF). The tolerance shows the percentage of each variable that is not related to the other variables and the VIF is merely the reciprocal of the tolerance. With relatively high tolerance levels and VIF levels close to one there is not a problem with collinearity among predictors in this study (Shannon and Davenport, 2001).

Table 15

Collinearity results for the independent variables for each sequential regression model

Model		ACHIEVEMENT	GENDER	VH	ENVIRONMENT
1	Tolerance	.706	.950	.701	.796
	VIF	1.416	1.053	1.427	1.256
2	Tolerance	.700	.823	.669	.796
	VIF	1.429	1.216	1.495	1.256

Summary

In all six multiple regressions the variable ENVIRONMENT was a significant predictor of the dependent variable of concern. As a result, the null hypotheses were rejected since all results show that students made significantly more relevant conjectures in the dynamic environment, significantly fewer false conjectures in the dynamic environment, and have significantly greater conviction in their conjectures in the dynamic environment. In this study, the only difference between the static and dynamic environments provided was the opportunity to alter the figures by dragging in the dynamic environment. Therefore, it was concluded that the opportunity to alter figures by dragging results in students making more relevant conjectures, fewer false conjectures, and also results in a greater conviction in their conjectures. These differences were shown to be statistically significant by sequential regression analysis.

Zero-order Pearson correlation coefficients between the covariates and the dependent variables found some significant correlations, however, the regression analysis which examined unique effects of the covariates as predictors of the dependent variables rendered some of these relationships insignificant. The zero order correlation between GENDER and CONVICTION was significant in the odd assignment case but the subsequent linear regression found that the unique contribution of a student's gender was not a significant predictor of conviction in that assignment case. A similar result occurred between the variables ACHIEVEMENT and FALSE in the odd assignment case. Once again the regression analysis showed the unique contribution of a student's mathematics achievement not to be a significant predictor of the number of false conjectures that student will form, even though the zero order correlations were significant.

One covariate was found to be significant. The unique contribution of the variable ACHIEVEMENT was found to be highly correlated with RELEVANT, but only in the odd assignment case. Therefore, there is some evidence to support the claim that students with higher mathematics achievement levels tend to make more relevant conjectures regardless of the environment.

CHAPTER V: QUALITATIVE RESULTS

This chapter will analyze six qualitative themes that were developed prior to and during the course of the study. The instruments used to collect qualitative data include:

- The student conjectures created during the eight lab activities.
- A short participant survey with open ended items that was administered to all participants after the conclusion of the lab activities.
- A series of ten individual interviews also performed after all of the conjecturing labs had been concluded.
- The researcher's field notes taken during the lab activities and student interviews. These notes were recorded in journal form by the researcher.

Examination of these sources of data by the researcher yielded results for six qualitative themes concerning student conjectures and the use of dynamic geometry software. Four of these themes were developed before the study and were motivated by the literature sited in Chapter II. The survey and interview instruments were developed to address these *a priori* themes. The two additional themes concerning dynamic language and student dragging tendencies emerged during the course of the study. The theme concerning dynamic language led to an additional review of the literature, which was added to Chapter II. This chapter will report on the findings of each of four *a priori* themes as well as the two emergent themes. The *a priori* qualitative themes discussed are:

1. Student concepts of conjecture and proof
2. Student preferences concerning the geometry environment
3. Students' ability to form and find counterexamples in the dynamic environment
4. Students' conviction in computer generated output

The Emergent themes are:

5. Student dragging techniques
6. The use of dynamic language

Concept of Conjecture and Proof

Senk (1985) questions if students really understand the meaning of proof while Mingus and Grassl (1999) contend that even younger students are capable of forming proofs at some level given freedom to do so. Martin and Herel (1989b) found that students often use both inductive and deductive frames of reasoning when asked to form proofs, and Pandescia (2002) questioned the need for formal proof instruction when the use of dynamic geometry software can easily verify conjectures. This led to the questions: "What do students consider to be a proof?", "What is their concept of conjecture?", "How do they differ?"

This theme was explored using two primary data sources. First, the survey instrument asked the subjects to "describe what a conjecture is in your own words." Larson, Boswell, and Stiff (2001) define a conjecture as "an unproven statement that is based on observations" (p. 4). The participants had been exposed to this definition of conjecture during classroom instruction prior to the beginning of the study; however, students were expected to reply with a definition given in their own words.

The coding of this survey item resulted into two core categories: Students that expressed a concept of conjecture and those that expressed little or no concept of conjecture. The data was further analyzed by comparing the students' core category with the students' ability to form relevant conjectures demonstrated by the lab activities used in the study. The following key words and phrases emerged as codes to organize the students' responses.

1. Guess
2. Opinion
3. Observation
4. Hypothesis
5. Statement of Truth
6. Lack of Concept

The first five codes were used to classify responses that showed some concept of the working definition of conjecture. The sixth code was reserved for those responses that showed little or no concept of what a mathematical conjecture is. Approximately 80% of the subjects were classified as having a reasonable concept of conjecture. Consider some of the responses associated with the codes used to classify them.

Several of the subjects defined conjecture as some kind of guess: "A conjecture is a guess that you may or may not be able to prove." "A conjecture is a guess about a given item on a figure." "A conjecture is an educated guess." Others saw a conjecture as a personal opinion: "A conjecture is an opinion of what is true." "A conjecture is your own opinion made about a particular figure." "A conjecture is a fact or opinion about a given geometric figure."

Another common word used in subject responses was "observation" which is also present in the working definition. Consider the following responses: "A conjecture is a statement made by observing and testing." "A conjecture is made by observing and thinking about what you see looking at a figure." "A conjecture is your observation of what is in a figure." The word hypothesis also occurred several times in subject responses: "A conjecture is a hypothesis or judgment based on evidence." "A conjecture is your hypothesis about the facts of the figure". Once again these statements were accepted by the researcher as evidence of student understanding of conjecture.

The most common code indicated some kind of belief of truth about the figures. These responses were varied but did show an understanding of the concept of conjecture. Consider the following examples: "A conjecture is what you think is true about a problem." "A conjecture is what you conclude to be true." "A conjecture is your own words about what you think are true about the figure." "A conjecture is a fact that you think is true mathematically."

In total there were 33 responses similar to those mentioned that were categorized by these five codes and showed evidence of some concept of conjecture. Eight students that did not show evidence of the concept of conjecture on the survey instrument:
"A conjecture is a figure that a person could make a statement about and answer it."
"A conjecture is a theorem proven by statements." "A conjecture is a theorem of many

angles." "A conjecture is a figure with congruent sides." However, when averaging these students number of relevant conjectures across both environments it was found that these eight participants averaged 1.75 relevant conjectures per lab activity, which is higher than the overall average in both assignment cases. This is a surprising result and indicates that students can find relevant conjectures independent of their stated concept of conjecture.

Additional core categories related to this theme emerged from student responses to the interview. Interview participants were given conjectures about given figures and asked if they agreed or disagreed with those conjectures and how they would prove or disprove those conjectures. The purpose of these questions was to see if the participants made a distinction between inductive conjectures based on observation and facts proven by deduction.

To examine the concept of proof or conjecture, figure 8 was presented in the dynamic environment to ten participants interviewed individually.

Figure 1: The midsegment quadrilateral
A midsegment quadrilateral is formed by connecting the midpoints of the sides of a quadrilateral

Conjecture: The midsegment quadrilateral will always be a parallelogram.

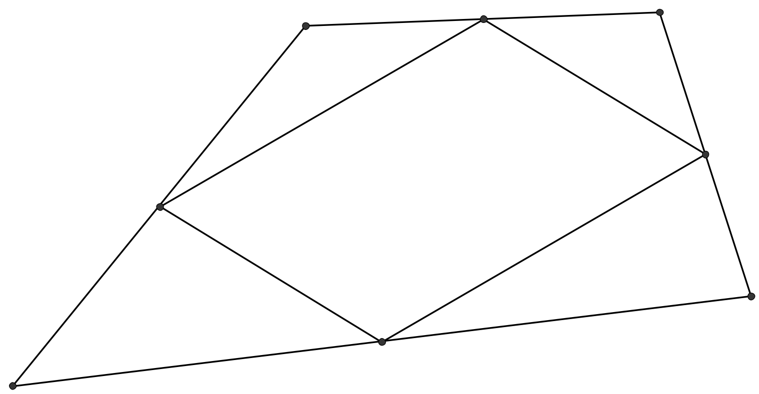

Figure 8. First situation posed to the interview participants in the dynamic environment along with the given conjecture.

Subjects were asked if they agreed with the stated conjecture. After some dragging and observation, all of the subjects agreed with the stated conjecture, which is in fact true. The subjects were then asked how they would prove the conjecture. Of the ten subjects none of them mentioned the idea of a purely deductive proof even though most of the targeted conjectures used in the lab activities were proven deductively as part of the course curriculum. Two of the participants did not indicate how they would prove the conjecture. Eight of the ten subjects indicated that they would use a combination of deduction and observation to prove the conjecture. The following excerpts from the interviews demonstrate this kind of "mixed" reasoning.

Mark is a relatively high-achieving male student when compared to the entire sample. He claimed that he would prove that the midsegment quadrilateral is always a parallelogram by observing that the opposite angles of the midsegment quadrilateral stay congruent throughout dragging in the dynamic environment.

Researcher: What you see on this first figure is called a midsegment quadrilateral; it is formed by connecting the midpoints of any general quadrilateral. A conjecture from a former student says that the midsegment quadrilateral will always be a parallelogram. Do you think this conjecture is true?

Mark: It looks like it here.

Researcher: How would you test your conclusion? (*Student begins to drag)*

Mark: To see if the opposite sides stay parallel as I drag around. Yes, I think it is true.

Researcher: Would you like any measures?

Mark: Yes

Researcher: What measures?

Mark: How about the angles? *(Pause while researcher measures and displays the interior angles of the midsegment quadrilateral)*

Researcher: How does this help with your conclusion?

Mark: Opposite angles are congruent as I drag around so I am convinced.

Researcher: How can you prove that the conjecture is true?

Mark: I don't know exactly how on paper but this proves it because for a parallelogram the opposite angles are always congruent.

Note the combination of deductive reasoning and inductive reasoning used by this student. Deductive reasoning is used by claiming since the opposite angles are congruent, the quadrilateral is a parallelogram. This is an accurate deduction. Inductive reasoning is used by observing that during the dragging process, the opposite angles remained congruent. Thus, this student drew no distinction between inductive and deductive reasoning and combined them into what he considered to be a “proof”.

Another example of this kind of reasoning was provided by Tasha. Tasha is a relatively low achiever when compared to the rest of the sample. She was somewhat reluctant to use the dragging utility during the interview. However, she demonstrated the same kind of inductive and deductive combination in her reasoning.

> *Researcher*: What you see on this first figure is called a midsegment quadrilateral; it is formed by connecting the midpoints of any general quadrilateral. A conjecture from a former student says that the midsegment quadrilateral will always be a parallelogram. Do you think this conjecture is true?
>
> *Tasha*: Yes
>
> *Researcher*: How would you test your conclusion?
>
> *Tasha*: To see if the opposite sides equal up.
>
> *Researcher*: Would you like to see the measures of those sides?
>
> *Tasha*: Yes, (*Pause while researcher measures and displays the measures of the sides of the midsegment quadrilateral)*
>
> *Researcher*: OK, what do you notice?
>
> *Tasha*: That the opposite sides are equal.
>
> *Researcher*: How can you prove that the conjecture is true?
>
> *Tasha*: I don’t know? Maybe just keep dragging. *(The student begins to drag slightly).* These opposite sides stay equal and so this must be a parallelogram.

Six other interview participants used a similar kind of reasoning by stating the observation that the opposite angles or sides remained congruent while using the dragging utility. They then claimed that these observations proved that a midsegment quadrilateral is a parallelogram. They used the inductive process of observation through the dragging process to conclude that these properties hold for all cases and followed that by a true deduction that if these properties are true for all cases then the midsegment quadrilateral is indeed a parallelogram. This combination of inductive and deductive reasoning did not pose a conflict for the students and gave rise to a discussion of what students understand to be proof.

To summarize the results of this theme, approximately 80% of the subjects demonstrated a reasonable concept of conjecture when compared with the working definition; however, those who did not respond with a reasonable definition of conjecture averaged more relevant conjectures than the sample as a whole. This result indicates that the "skill" of finding relevant conjectures is independent of the subjects' ability to articulate a reasonable concept of conjecture. Students tend to use a combination of deductive and inductive reasoning when asked to prove conjectures if the dynamic environment is available. Although using inductive observation by the dragging process is not considered a valid proof, the subjects see it as proof and shy away from a purely deductive path to proof when the dynamic environment is available. The results indicate that although most of the participants were able to articulate a reasonable concept of conjecture, they lacked an understanding how this relates to mathematical proof.

Student Preferences

Furinghetti, Olivero, and Paulo, (2001) suggest presenting students with dynamic explorations to foster conjecturing. Arcavi and Hadas (1996) list several positive implications of the use of dynamic geometry software including the enhancing of understanding and insight. Haddas and Hershkowitz (1999) claim that discovery and conviction are greatly enhanced by dynamic geometry software. However, do students feel that dynamic geometry software is aiding them in the forming of conjectures? What features of the software do the students claim aids them the most in the forming of conjectures?

The catagories for the theme of student preferences emerged from two primary data sources: the student survey and the researcher's journal. On the student survey subjects were asked the questions: "What are the differences between the static and dynamic environments?" and "Which environment, static or dynamic, is better for conjecturing? Why?" As in the preceding theme a number of key words and phrases were identified as codes to sort and analyze the responses to these questions.

All of the participants correctly identified the differences in the two environments as the ability to drag, cause movement, or change. Consider the following excerpts: "In the dynamic environment you can move the figure and see changes while in the static environment you can't move it." "The static environment has shapes that can't be moved but in the dynamic environment you can drag segments and points to see the measurements change." "In the static environment you just have to look at it and figure it out, but in the dynamic environment you can do hands on movements on the figure."

All of the participants stated that the dynamic environment was better for conjecturing. Coding the responses yielded the following categories used to classify the reasons why the students felt the dynamic environment was better for conjecturing.

1. Interactive Qualities
2. Facilitate Conjecture
3. Improve Conviction

The category of Interactive Qualities referred to responses that indicated that the dynamic environment was superior because of the ability to interact with the figure by dragging and to observe change within the figures and their measures. "The dynamic environment is better because when you drag you can see what changes and what does not." "The dynamic is better because you can play with it and see how the shape changes." Of the forty-one participants surveyed, eighteen of the responses fell in this coded category making it the most common of the three.

The category of Facilitate Conjecture referred to responses in which the subjects felt the dynamic environment was better because it was easier to find relevant conjectures in this environment. In many of these responses, the subject expressed the concept of having to do less to get more. Consider the following responses: "The dynamic is easier because you don't have to think so hard, and you can observe the patterns formed when dragging instead of figuring out just one shape." "It's a lot easier when you can move to make new shapes instead of studying one still shape." "You can figure out more when you drag to get new figures, and you don't have to rack your brain on one figure." Twelve of the responses fit into this category and interestingly, ten of those twelve were male subjects.

Ten of the 41 subjects indicated that the dynamic environment was better because it improved their confidence in their conjectures. This improvement in conviction is the last coded category used for this analysis. “Getting hands on experience increases your confidence level in the conjectures.” “In the dynamic environment you can make sure that your conjectures are right.” “In the dynamic environment you have the ability to move shapes around to prove your conjectures.” Notice this participant considered the testing of a conjecture by dragging equivalent to proving conjectures. Gender also had an effect but this time nine of the ten subjects that were classified under this category were female students.

To summarize, about half of the participants considered the dynamic environment superior to the static environment simply because of the ability to interact with the figures by the dragging process. Many male students tend to favor the dynamic environment because it facilitates the conjecturing process whereas many females favor the dynamic because it increases their conviction in their conjectures.

Student preferences toward the dynamic environment were also apparent during the course of the study and were noted by the researcher in field notes written in the researcher’s journal. The subjects typically spent more time on the figures while in the dynamic environment. The static environment seemed to bore many of the students and some of the figures were left blank with no conjectures even attempted. During the discussions after the lab activities, the class who used the dynamic environment usually had a more lively discussion and all of the targeted conjectures were found by at least one of the participants.

Forming and Finding Counterexamples

Arcavi and Hadas (1996) claim that dynamic geometry aids in the finding of counterexamples while de Villiers (1996) considers the finding of counterexamples an important component of the overall proof process. NCTM (2000) also claims that students should learn to understand the need for counterexamples to false conjectures. Does the use of dynamic geometry software aid in the finding of counterexamples?

In this theme students' ability to use dynamic geometry software to find counterexamples in order to disprove false conjectures was examined. The second figure shown to the students during the interviews was that of a general quadrilateral with angle measures shown. This figure was accompanied with a false conjecture: "A quadrilateral will have at most two obtuse angles." Students were asked if they thought the conjecture was true or false. The results provide insight on the subjects' ability to form a negation, find a counterexample, and use the dragging utility to find that counterexample.

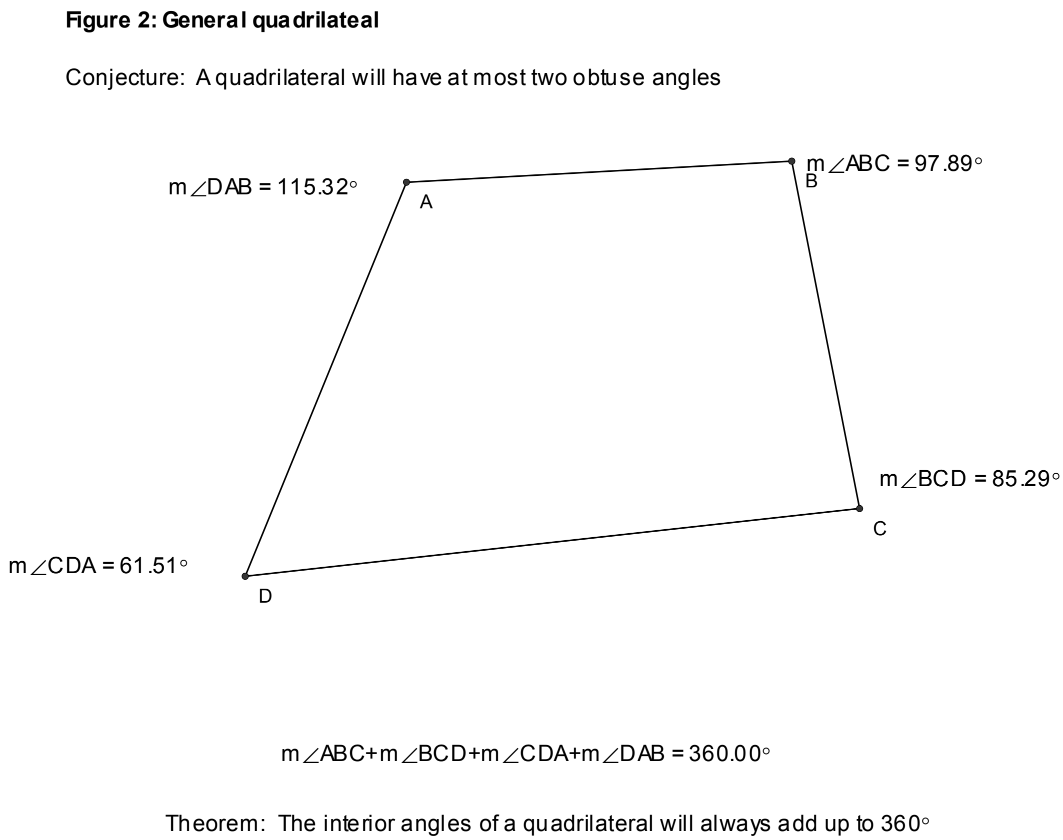

Figure 9. A general quadrilateral with a false conjecture and true theorem

Of the ten individual interviews only four of the participants were able to form the negation and find a counterexample to disprove the false conjecture even though logical statements and their negations had been covered in the curriculum of the course. The six participants who claimed that the conjecture was true either used flawed logic, were not able to form a proper negation to the conjecture, or demonstrated a weak dragging technique which convinced them that a counterexample did not exist. The following excerpts from three different interviews will demonstrate these three phenomena.

Chris is an average achiever who demonstrated flawed reasoning which led to the incorrect conclusion that the conjecture was true. Consider the following excerpt.

Researcher: On this next figure we have a general quadrilateral with all four interior angles measured. A former student conjectured that a quadrilateral will have at most two obtuse angles. Do you agree with this conjecture?

Chris: *(Pause)* Yea, I agree.

Researcher: You don't want to drag at all?

Chris: No.

Researcher: OK, How would you prove or disprove this conjecture.

Chris: Well the most it could have would be two because if it had three it would be over 360. I don't think it is possible to have more than two because each obtuse angle would have to have an acute angle to cancel it out to 180 degrees, so two obtuse and two acute angles will give you 360.

Note that this student did not even attempt to form or find a counterexample. He was convinced in the truth of this conjecture by using the same logic that leads to the conclusion that a triangle must have at most one obtuse angle. However, this student failed to realize that the quadrilateral has one more degree of freedom thus allowing for three obtuse angles with room to spare for a forth acute angle. Two other participants used the same kind of flawed logic when asked how they would prove the conjecture. However, they at least attempted to form or find a counterexample before concluding the conjecture was true.

Two of the participants had difficulty forming a negation of the conjecture and thus were not able to find a proper counterexample because they did not know what to look for. Beth is an average achiever who had difficulty understanding the concept of counterexample. Consider the following excerpt from the interview.

Researcher: On this next figure we have a general quadrilateral with all four interior angles measured. A former student conjectured that a quadrilateral will have at most two obtuse angles. Do you agree with this conjecture?

Beth: I don't know. There is two here but you could have right angles that would make it false.

Researcher: Would that make it false?

Beth: Well they would not be obtuse.

Researcher: How could you prove your conclusion?

Beth: I will drag some more?

Researcher: So what are you looking for?

Beth: Angles that are greater than 90. (*Student stops dragging when the quadrilateral has only one obtuse angle and two right angles.)*

Researcher: So what about the conjecture?

Beth: Is it false then?

This student reached the proper conclusion but did not actually form a proper negation and thus did not find a valid counterexample. One other student failed to understand the concept of counterexample and simply gave up on the conjecture without ever saying if it was true or false.

Three of the participants had no problem forming the negation of the conjecture and knew what to look for when they began dragging; however, they failed to actually find a counterexample. Consider this excerpt with an average-achieving student named Joe.

Researcher: On this next figure we have a general quadrilateral with all four interior angles measured. A former student conjectured that a quadrilateral will have at most two obtuse angles. Do you agree with this conjecture?

Joe: I agree but I'm not positive.

Researcher: How would you test that conjecture?

Joe: Well if three angles are obtuse then the conjecture is false but I don't think that is possible.

Researcher: You don't think it is possible to have three obtuse angles in a quadrilateral?

Joe: Can I drag?

Researcher: Sure.

Joe: *(Does some dragging).* I don't think you can have three because the sides won't connect so I think the conjecture is true.

Joe reached the wrong conclusion because of a weak dragging technique which was a common observation by the researcher. He simply gave up too soon and did not make the fine adjustments to the figure to find a counterexample. The next section of this chapter will discuss student dragging techniques in detail.

The four students who formed and found a counterexample to this conjecture all used the dragging utility to find the counterexample instead of logically deducing a counterexample. In other words, no student started off by saying that the conjecture was false by citing a counterexample. They all had to convince themselves by finding a counterexample through the use of the dragging utility.

In the following excerpt Juan disproved the conjecture by exploring using the dragging utility, even though he intuitively thought the conjecture was true at the onset.

Researcher: On this next figure we have a general quadrilateral with all four interior angles measured. A former student conjectured that a quadrilateral will have at most two obtuse angles. Do you agree with this conjecture?

Juan: I think so because two obtuse and two acute angles will give you 360 like on the figure.

Researcher: How would you test your conclusion?

Juan: By forming different quadrilaterals and measuring their angles.

Researcher: So what are you looking for?

Juan: A shape that does not have two obtuse angles but has three obtuse angles.

(*Student begins dragging and stops when the figure shows three obtuse angles.)*

Researcher: So what does this show?

Juan: This shows that the conjecture is not true.

Researcher: Does this disprove the conjecture?

Juan: Yes, by showing a quadrilateral with more than two obtuse angles.

Researcher: What if the shape only had one obtuse angle or none?

Juan: That would not disprove it because the conjecture said at most two obtuse angles.

This student overcame his initial incorrect reasoning that each obtuse angle has to be paired with an acute angle. He formed a proper negation and used the dragging utility to find a counterexample, thus proving the conjecture false.

To summarize this theme, it was found that less than half of the interviewed participants were able to disprove a false conjecture. Some students who used flawed logic, were not able to form the proper negation to the false conjecture, or simply had weak dragging skills and did not find a counterexample even after searching for one. Of those that did disprove the conjecture, the dragging utility was essential to their arrival at a counterexample.

Conviction in Computer Output

Mudely and de Villiers (1999) examined students' conviction in the output generated by dynamic geometry software and found that students were very confident in the output even though the result was surprising. They used the surprising result as a motivation to prove. However, consider the case where students are confronted with a surprising result generated by dynamic geometry software that was in conflict with a known theorem stated by the teacher or the textbook. De Villiers (1992) showed that students tend to hold more conviction in the authority of the teacher and the textbook than they do on their own conjectures. However, what level of conviction will they hold in dynamic geometry software, relative to other sources of information? If students see something on the screen that contradicts what they have learned from the instructor or the text book, how will they rationalize that contradiction?

The second situation presented to the interview participants also stated the true theorem, "The interior angles of a quadrilateral will always add up to 360 degrees." This theorem is true for all quadrilaterals and had been proven by the instructor and accepted by the students as part of previous classroom instruction. However, Geometer's

Sketchpad gives angle measures only between 0 and 180 degrees. Therefore, if a quadrilateral is dragged so that it is concave, forcing an interior angle greater than 180 degrees, the software package will default to measuring the angle that measures less than 180 degrees on the exterior of the quadrilateral and produce the false result that the sum of the interior angles of a concave quadrilateral is less than 360 degrees. Figure 10 illustrates this deceiving result.

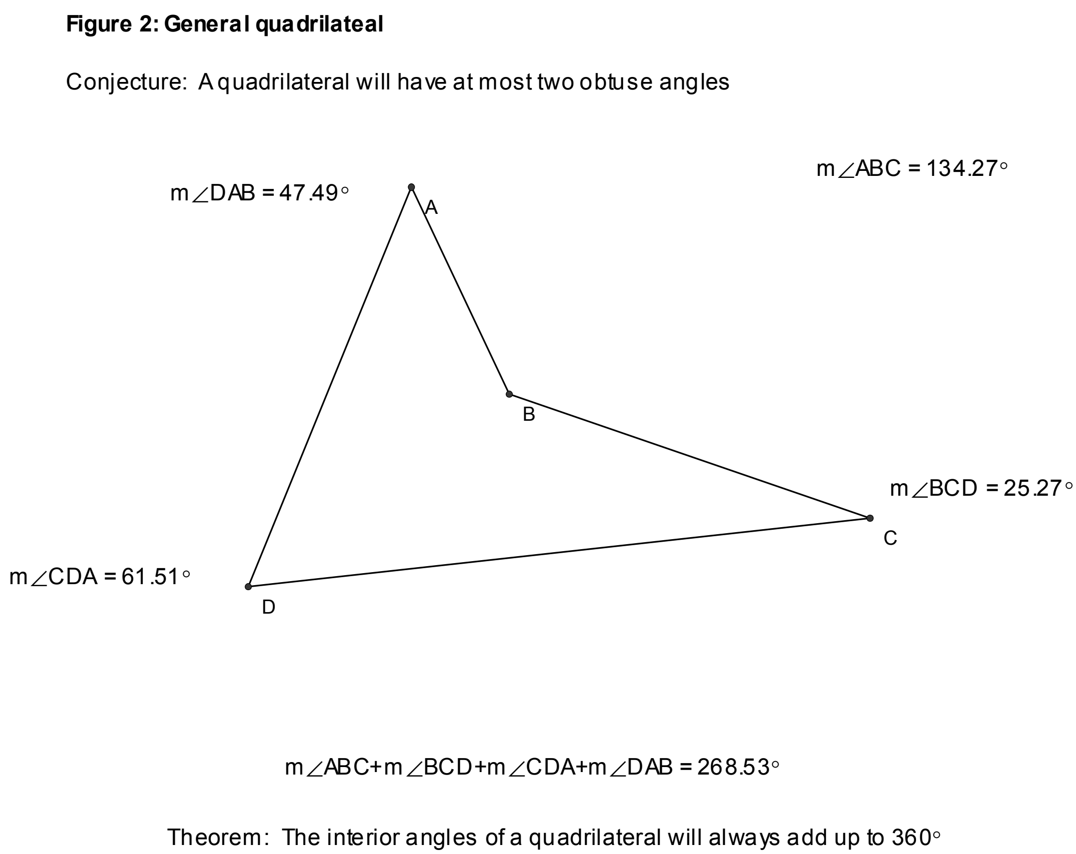

Figure 10. A concave quadrilateral with angle measures shown, leaving the impression that the sum of its interior angles are less than 360 degrees.

This figure illustrates that the output shows angle ABC as an angle measuring 134.27 degrees rather than the interior angle that would measure 225.73 degrees (360-134.27). Of the ten interview participants, nine of them believed that the sum of the interior angles was indeed not equal to 360 degrees when the quadrilateral was concave. The following excerpts from participant interviews are examples of modifying a given theorem falsely in order to rationalize the computer's visual output. The first excerpt is from the interview of Tamika, a relatively high achiever and the only freshman in the class.

> *Researcher*: OK, here is a theorem from the text book. The sum of the interior angles for any quadrilateral is 360 degrees. Do you think this is true for all quadrilaterals?
>
> *Tamika*: I'll drag to see. *(Student drags until a concave quadrilateral is formed.)*
>
> *Researcher*: What just happened?
>
> *Tamika*: It got too far in.
>
> *Researcher*: What do you mean?
>
> *Tamika*: It's like not convex but concave.
>
> *Researcher*: So should the theorem be changed?
>
> *Tamika*: Yes
>
> *Researcher*: How?
>
> *Tamika*: By using a biconditional.
>
> *Researcher*: What do you mean?
>
> *Tamika*: Like saying the interior angles of a quadrilateral add up to be 360 if and only if the quadrilateral is convex.

Note that this participant observed under what conditions the output was not producing a sum of 360 degrees. She then questioned the theorem rather than the output and made a false modification to the theorem. However, this modification was consistent with the computer's visual output. The next excerpt is from Juan.

Researcher: OK, on the same figure scroll down to see that the sum of all the interior angles is equal to 360. This is actually a theorem from the book. Do you think that this will always be true? *(Student drags and the shape goes concave)*

Juan: It's concave and not 360.

Researcher: So is the theorem wrong?

Juan: It's not wrong, however it only works on a normal quadrilateral not a concave one.

Researcher: Should the theorem be changed?

Juan: Yes, I would specify the kind of quadrilateral. The concave ones add up to be less than 360.

Once again the subject failed to examine the output carefully and instead questioned the theorem. Note that the modification once again is false relative to the definitions used in the course but consistent with the generated output. The sum will always appear to be less than 360 degrees when the quadrilateral is concave. Only one of the interview participants noticed the inconsistency in the output and was able to figure out why the output was reading less than a 360 degree sum. Consider this excerpt from Mark's interview.

Researcher: OK, on the same figure scroll down to see that the sum of all the interior angles is equal to 360. This is actually a theorem from the book. Do you agree with this theorem?

Mark: Yes.

Researcher: Can we confirm it with the computer? *(Student begins dragging until shape goes concave.)*

Mark: Here it goes less than 360.

Researcher: What happened?

Mark: Oh, it's measuring the outside angle.

Researcher: Do you think the theorem is wrong?

Mark: No, because it is measuring the wrong angle and if you add up the angle that is greater than 180 it will probably still work.

Mark was the only student that was able to rationalize the inconsistent output and noticed right away what angle measure was really being displayed.

The results of these interviews show that students tend not to question computer output even when it contradicts what the class has proven deductively and what the textbook has stated as fact. They also failed to closely observe the interior angles that were formed in the concave quadrilateral. One of these angles is indeed greater than 180 degrees. Instead, students may falsely alter the facts in order to rationalize the output generated by the computer even though they were aware of the fact that angles can measure more than 180 degrees. In this experiment, close examination of the angle measures themselves reveals the inconsistency. However, only one student closely examined these measurements.

Dragging Tendencies

The theme of dragging tendencies was the also an additional emergent theme developed during the course of data collection. This theme was drawn from the researcher's journal, based on careful observation of students during the eight lab activities and the ten individual interviews. The researcher identified three different kinds of dragging. They were designated as Random Dragging, Dragging for Pattern Recognition, and Dragging for a Specific Purpose. This section will define and discuss these three kinds of dragging.

The category of Random Dragging was especially prevalent during the first two lab activities. This category is characterized by a considerable amount of random dragging where students moved the figures around without really looking at how the measures changed. This dragging technique is characterized by sporadic and unpredictable motions by the user. At first glance it may seem like this kind of dragging served no useful purpose and was employed simply to amuse the user; however, the students made comments that suggested that they were using the dragging utility to test the limits of the figure and how different points and segments behaved during the dragging process. Students would often ask why different points moved in a different manner. The researcher always replied that these different motions were due to the construction of the figures. Therefore, random dragging did serve the purpose of testing the robustness of the figure during the dragging process.

The category of Random Dragging was frequently followed by Dragging for Pattern Recognition within a lab session. Students would drag in less sporadic motions with pauses to observe the figure and the measures of its components. During this kind of dragging students were looking for some kind of pattern. For example, a student may deform the parallelogram to a new parallelogram and observe that the opposite sides are congruent. Doing this several times and stopping to observe that the opposite sides are always congruent would lead the student to conjecture that the opposite sides of a parallelogram are always congruent. This dragging technique is used to find conjectures and is characterized by smaller deformations with pauses for observation.

The third category of dragging was Dragging for a Specific Purpose. For example, several of the subjects were observed dragging the parallelogram into a rectangle or dragging the transversal of the parallel lines so that all angles would be right angles. This kind of dragging sometimes was evidenced in the conjectures themselves. "If two parallel lines are intersected by a transversal then there will be four acute angles and four obtuse angles unless all of the angles are right angles." This conjecture shows evidence that the user attempted to see what would happen when the angles were right. Without this last qualifying statement about right angle the conjecture would have been false. This third level of dragging was also observed during the interviews where students were searching for a counterexample. Those who found the counterexample displayed this technique. The technique is characterized by careful meticulous adjustments to the figure by slight drags until the figure displays what the user intends.

In some sense, these categories might be considered to be levels of dragging, representing increasing levels of sophistication in their use and understanding of dragging. Most of the participants in this study achieved and operated on the second level of dragging, Dragging for Pattern Recognition. Many of the participants did not use dragging characterized by Dragging for Specific Purpose; only four of the ten interview participants were able to find a counterexample for a given false conjecture. It must be noted that the participants of this study did not receive any particular instruction on dragging techniques and developed their own personal style of dragging independently.

Dynamic Language

The theme of dynamic language was developed during the course of data collection as an "emergent" theme. However, this theme also suggested in studies by Jones (2000) and Mariotti (2001), both of which observed the formation of dynamic language among students using dynamic geometry software, although these studies were not identified until after the fact.

The categories in this theme address how students phrase their conjectures in the two different environments. It was noted during the study that many of the subjects used language that reflected the dynamic environment itself. By examining the conjectures produced for all of the eight lab activities it was determined that the conjectures made in the dynamic environment tended to be more general in their phrasing while the conjectures made in the static environment tended to be more specific to the figure that was presented. The more general conjectures tended to have the word "always" embedded in the phrasing of the conjecture: "If a triangle is isosceles then there will always be two congruent angles." "If a parallelogram is a rectangle then the diagonals are always the same length." "If two parallel lines are intersected by a transversal then corresponding angles will always be congruent." The word "always" used in this context was much more frequent in the responses made in the dynamic environment.

In the static environment it was common for the subjects to give responses that referred directly to the figure often by using the labeled parts of the figure. For example: "If a quadrilateral is a parallelogram then angles DAB and DCB are congruent, and so are angles ADC and ABC." This is a relevant conjecture indicating that the opposite angles of a parallelogram are congruent but using a particular figure. These kinds of responses were much more common in the static environment.

After examining the responses from all lab activities, two categories were identified that reflected the dynamic environment itself: Move or Drag, and Change or Stay the Same. These codes used to develop these categories were only found embedded in the responses from the dynamic environment and reflect the environment itself.

The "Move or Drag" category referred to conjectures which incorporate the student's action during the formation of the conjecture. Consider the following responses: "If a triangle is equilateral then all the angles will be congruent anyway you move it." "If a triangle is a midsegment triangle then it will divide the area by four and the perimeter by two no matter how you move the points." "If a parallelogram is a rectangle then the diagonals will be congruent as you drag different points." Note that these conjectures make more sense if you consider the dynamic environment in which they arose.

The "Change or Stay the Same" category refers to the figure itself transforming into a different shape. Embedded within the conjectures is what the subjects see changing or not changing within the figure during deformations. Consider the following responses: “If a parallelogram is a rhombus then the diagonals will change size but they will stay perpendicular.” “If a figure is a triangle then the angles may change but they will always add up to 180.” “If an angle is an inscribed angle then when you change the measure of the angle the arc will stay twice as big.” Once again these conjectures refer to the dynamic environment in which they were discovered. The words “change” or “stay the same” never occurred in the responses found in the dynamic environment.

Several conjectures were classified by both categories. These conjectures referred to the subject’s action and the subject’s observation. Consider the following responses: “If a parallelogram is a square then when you drag the square to change its size the diagonals stay congruent and all the angles stay the same.” “If a trapezoid is isosceles then when you move the points the angles change but the top angles stay congruent and the bottom angles stay congruent.”

The presence of dynamic language was not consistent throughout the lab activities. Lab 1 (parallel lines and transversals) and Lab 4 (parallelograms) were found to have very few conjectures that contained dynamic language while Lab 5 (rectangles and squares) and Lab 3 (midsegments) had numerous examples of dynamic language. In the examples sited, all of the conjectures were relevant; however, dynamic language also occurred in irrelevant conjectures. For example consider the following irrelevant conjectures: “If a segment is a trapezoid midsegment then when you drag the segment the measurement changes.” “If a quadrilateral is inscribed in a circle then when you change its shape and size the angles still add up to be 360 degrees always.”

The analysis of this theme shows that the environment not only has an effect on the number of relevant conjectures, false conjectures, and the conviction in those conjectures, but also has an effect on the phrasing of the conjectures themselves. Conjectures formed in the dynamic environment tend to be more general often using the word “always”. These conjectures often use language that reflects the environment itself referring to the action taken by the participant, the changes observed, or both. The conjectures formed in the static environment tend to be more specific to the figure used and contain no language of action or change.

Summary

The qualitative results of this study provide a description of the subjects experience over the six different themes explored. It was observed that most of the participants had a reasonably accurate concept of conjecture but that the ability to articulate a definition of conjecture was independent of the ability to form relevant conjectures about geometric figures. The interview subjects displayed a concept of proof that included a mixture of both inductive and deductive reasoning.

All of the participants stated that the dynamic environment was better for finding conjectures and an interesting gender effect was noted where many of the males preferred this environment because it facilitated the conjecture process; many of the females preferred the dynamic environment because it strengthened their conviction in their conjectures.

The results from the quantitative portion of this study indicated that the dynamic environment had a statistically significant effect on the students’ ability to form relevant conjectures and false conjectures as well as a statistically significant effect on the students’ conviction in their conjectures. A close qualitative examination of these conjectures indicates that the environment has an effect on the way in which students phrase conjectures. Conjectures formed in the static environment tend to be specific to the figure given while those formed in the dynamic environment are more general often using language of action and change.

Most of the interviewed subjects in this study had difficulty finding a counterexample to a false conjecture even when using dynamic software. Some used flawed logic to deduce that the conjecture was true. Others had difficulty articulating the negation of the conjecture, while some simply did not find a counterexample because of weak dragging technique. Dragging techniques themselves were classified into three distinct categories, each with its own characteristics and purposes.

The final theme examined students' conviction in the output generated by the computer software package. The results indicate that the students tend to question the authority of the instructor and the text book rather than question the output of the computer. A discussion of these themes in terms of meaning, implications, and recommendations for further research will be presented in chapter six of this report.

CHAPTER VI: DISCUSSION

This study focused on both quantitative and qualitative data concerning student conjectures using two different geometric environments: The static geometry environment in which a specific static figure was presented and the dynamic geometry environment which allowed students to deform the figure. The quantitative results of this study focused on statistical differences between predetermined and measurable variables in each of the geometric environments. These results were used to answer the specific research questions concerning differences in the participants' conjectures formed in the two environments. The qualitative results explored data categorized by six themes that delved deeper into the students' experiences, tendencies, and concepts developed while participating in this study. The only notable difference between the dynamic and static environments used in this study was the ability for the participants to employ the dragging utility in the dynamic environment. Because the ability to drag is a distinguishing factor of the dynamic environment, considerable attention was given to the act of dragging during this study.

This final chapter will discuss the implications of this study by reflecting upon the results from the two previous chapters. This chapter will also discuss the limitations of the research and suggestions for further research.

Implications

This study has yielded several results with possible implications and benefits for students as well as professionals in mathematics education. A discussion of the four research questions and six qualitative themes will be put into the context of potential benefits for students and implications for professionals in the field of mathematics education, including teachers of mathematics and educators in mathematics education.

Potential benefits for students

The quantitative results of this study support the use of dynamic geometry software as a classroom tool for student conjecturing in geometry. These results indicate that the use of dynamic geometry software aids in students making a significantly greater number of relevant conjectures about a given figure when compared to the use of a static representation of the same figure. The results also showed that the use of dynamic geometry software significantly decreases the students' likelihood of making false conjectures about geometric figures.

In this study, the researcher had already constructed the figures posed in both environments. The same measurements were also provided in each environment. Therefore, the ability to use the dragging utility in the dynamic environment was the only notable difference between the dynamic and static environments. This ability proved to be an important factor in making correct and relevant conjectures. The participants were not "coached" on dragging techniques and were left on their own to develop their own dragging techniques. Therefore, it is reasonable to assume, based on the results of this study, that simply given the ability to use the dragging utility in a dynamic environment, a students' ability to make relevant conjectures concerning geometric figures should increase, and the likelihood of the student making false conjectures about those same figures should decrease when compared to a traditional static setting.

Arcavi and Hadas (1996) explained, “Making sense of a situation while playing with the situation itself first, and then interpreting its representations, enhances both the understanding of the situation and of the representations” (p. 8). Students are able to form conjectures in geometry by recognizing and generalizing patterns during exploration. Pattern recognition is stimulated by the use of dynamic software which allows them to manipulate diagrams easily and focus on the patterns and relationships displayed on the screen (Glass, Deckert, Edwards, & Graham, 2001). It is not surprising that students were able to find significantly more relevant conjectures and stated significantly fewer false conjectures when given the ability to manipulate the figures in the dynamic environment.

Another benefit for the users of dynamic geometry software is the increase in the conviction of their conjectures. Results showed that when subjects were asked to rate their own conviction in the conjectures that they made in both the static and dynamic environments, those ratings were significantly higher in the dynamic environment. This result implies that students are more confident in their conjectures when they are able to use the dragging utility. Students have a continuum of figures to test their conjectures resulting in greater confidence in those conjectures. This view is shared by Mudaly and de Villiers (1999) who claimed that higher levels of conviction are attained when dynamic geometry software is used. They claim that learners would not likely achieve comparable levels of conviction using only pencil and paper methods.

In terms of the research questions posed by this study, the results of this study do support the hypotheses that students will make significantly more relevant conjectures and significantly fewer false conjectures in the dynamic geometry environment when compared to the static geometry environment. The results also support the hypothesis that the conviction of conjectures made in the dynamic geometry environment would be significantly greater than the conviction of those conjectures made in the static environment.

Further analysis indicated that students in the dynamic environment were significantly more confident in their relevant conjectures but not in their false conjectures. Thus, the reported increase in conviction occurred primarily with true conjectures. This is, of course, a desirable outcome. Although significantly fewer false conjectures are formed in the dynamic environment, we do not want the students to have a higher conviction in those false conjectures that were formed.

The forth exploratory question asked how the variables of gender, mathematics achievement, and entering van Hiele level of geometric reasoning are related to the students' ability to conjecture. It was found by regression analysis that mathematics achievement is a significant predictor of a student's ability to form relevant conjectures. This is not a surprising result; however, it was also found that in one of the assignment cases, the higher achieving students tended to make more false conjectures. This result was unexpected but can be somewhat explained by the fact that the higher achievers tended to make more conjectures in total. Therefore, their likelihood of making false conjectures may increase because of the higher number of total conjectures made.

It was shown that mathematics achievement was not a significant contributor to the students' conviction; however, there was a strong positive correlation between the number of relevant conjectures and the subjects' conviction in their conjectures. This correlation suggests that subjects tend to be more confident with conjectures that are relevant, which is also is not a surprising result.

Even though the gender variable was significantly correlated with conviction in the zero-order correlations, this correlation did not hold up in the regression analysis and thus it was concluded that gender did not significantly affect any of the dependent variables. The participants' van Hiele level of geometric reasoning also did not significantly affect any of the dependent variables. Therefore, the environment itself was shown to be the greatest predictor of a students' ability to form relevant conjectures, make fewer false conjectures, and increase the conviction of their conjectures.

This finding is important because it implies that the use of dynamic geometry software contributes to the students' ability to make good conjectures more than the students' level of mathematics achievement does. This implies that lower achieving students benefit by the use of dynamic geometry software. The use of dynamic geometry software may give them the opportunity to achieve results similar to higher achieving students.

The participants of this study were predominately African American, many from low socio-economic backgrounds, and most were relatively low achievers in mathematics with weak backgrounds in algebra and basic geometry skills. This finding suggests that the use of dynamic geometry software may make an important contribution to achieving equity among students of different backgrounds and achievement levels. The NCTM's

"Equity Principle" states that "technology can assist in achieving equity and must be accessible to all students" (NCTM, 2000, p. 12). The results of this study support that claim.

Implications for Professionals

The results of the six qualitative themes developed for this study yielded some interesting points for professionals in mathematics education and teachers of mathematics. Those themes included: Student concepts of conjecture and proof, student preferences concerning the geometry environment, language that reflects the dynamic environment, students' ability to form and find counterexamples in the dynamic environment, student dragging tendencies, and students' conviction in computer generated output.

Concept of Conjecture. Results from the survey and the lab instruments indicated that the ability to form relevant conjectures is independent of the subject's ability to articulate a concept or definition of conjecture. It was an unexpected result that those students who could not articulate a sensible concept of what a conjecture "is" actually averaged a greater number of relevant conjectures across the eight lab activities when compared to the entire sample mean of relevant conjectures. This result supports the notion that students may possess a mental concept of a mathematical process without knowing or understanding a definition of that process.

Tall and Vinner (1981) claim that a student's concept definition of a mathematical process and their concept image of that process are not the same. The concept image consists of all of the cognitive structures in the individual's mind related to that process, many of which cannot be expressed verbally. The concept definition is the individual's way of defining that concept verbally. The concept definition may include both formal and informal mathematical language. The concept definition then is merely a component of the total concept image.

When students were asked to explain what a conjecture was in their own words, they were expressing their concept definition. Some of those students who lacked the ability to articulate a definition of conjecture may still have had a strong concept image of conjecturing since they were able to form relevant conjectures unassisted.

Concept of proof. To address the concept of proof in terms of proving conjectures, selected students were interviewed and presented a true conjecture in the dynamic environment. These students were not told that the conjecture was true and were asked to indicate the truth of the conjecture. All of the participants indicated that the midsegment quadrilateral was indeed a parallelogram, as conjectured. When asked to tell how they would "prove" that conjecture, the participants overwhelmingly responded that they would use the dynamic dragging tool as part of their "proof". Participants indicated that they would use the dragging utility to show that either the opposite sides or the opposite angles remained congruent while distorting the figure by dragging. They then concluded that since these parts remained congruent, the quadrilateral must be a parallelogram. This mix of inductive observation and deductive reasoning was common for the interviewed participants of this study.

Martin and Harel (1989b) indicate that in everyday life, people consider "proof" to be "what convinces me." Indeed, the combination of inductive and deductive reasoning used by the participants may form a convincing argument. However, it is not mathematical proof. The question is, "Does the use of dynamic geometry software enhance or substitute the need for deductive proof?" Pandiscio (2002) raised this same question in his study of preservice teachers' conception of proof and the use of dynamic geometry. In his study, the sample of preservice teachers questioned the need for formal proof when dynamic geometry software was available. They were also concerned that the students would not see the need for proofs after being exposed to dynamic software. The results of this study support the idea that the use of dynamic geometry may substitute for deductive proof in the minds of the student. However, if the purpose for proof is to merely confirm the truth or validity of an argument, then perhaps deductive proof can be replaced with dynamic geometry software. de Villiers (1999) agreed that when dynamic geometry software is available for students, they have little need for further conviction or verification. However, de Villiers (1999) also argued that the use of dynamic geometry software can serve to enhance rather than substitute deductive proof.

de Villiers (1999) claimed that students may be motivated toward deductive proof by being asked the question, "Why?" The use of dynamic geometry software can certainly convince students that a particular statement is true or false but it does not necessarily shed any illumination on why the statement is true or false. de Villiers (1999) contended that beside verification, deductive proof serves the purposes of explanation, discovery, systematization, and intellectual challenge. Students may find that deductive proof does serve to explain why statements are true and also serves to systemize mathematical theorems by linking them together deductively.

The results of this qualitative theme concerning students' concepts of conjecture and proof indicated that many students do not have a strong concept of what mathematical proof really is and how inductive and deductive reasoning differ. They did not see a distinction between inductive and deductive reasoning and mixed the two together to form what they see as proof. Students should be aware of what proof is and how it differs from conjecture. Classroom teachers should emphasize this difference and point out that the verifications and discoveries found using dynamic geometry software are not proofs but rather conjectures not yet proven. The results of this study support the claim that dynamic geometry software may indeed be seen as a substitute for proof by the students. This implies that de Villiers' (1999) suggestions concerning further explanation and challenge be taken into account for teachers using dynamic geometry software as a classroom tool.

Hadas, Hershkowitz, and Schwarz (2000) suggested using dynamic geometry activities that lead to surprise or uncertainty and motivate students to question further and motivate them to explore deductive proof as a means of explanation. The study conducted by Marrades and Gutierrez (2000), where students were asked for justifications of a variety of conjectures may also be of help in understanding how inductive explorations with dynamic software may lead towards deduction. Recall that they found that even the best students would choose to justify using empirical means rather than deductive means, but that the use of dynamic geometry software helped prepare students for the transition to deductive proof. Marrades and Gutierrez (2000) claimed that this transition is slow and in fact the progress from van Hiele level two to van Hiele level four may take several years.

Student preferences. The theme that explored the student preferences in terms of the environment showed that all of the students preferred the dynamic environment mainly because of the ability to interact with the figure. Students would spend more time examining the figures and experimenting with the figures by using the dragging utility. The dynamic environment tended to be much more engaging to the students and much more enjoyable. Discussions after the lab activities were richer when dynamic geometry software was used when compared with the static environment.

The fact that the students prefer to work with dynamic geometry software and spend more time engaged in conjecturing activities when using dynamic geometry software implies that the teacher's role may indeed be changed from instructor to facilitator while dynamic geometry software is being used. Students may enjoy the freedom given by the use of dynamic geometry software without being told what to do by a teacher. They may work harder and show greater interest in the subject matter (Hannafin, 2001).

There was somewhat of a gender effect in terms of the reasons why the students preferred the dynamic environment. Results show that many of the male students preferred the dynamic environment because it facilitated the conjecturing process. They did not have to "think as much" or "figure it out." The dynamic environment provides a continuum of like figures that allow students to find patterns by observation while dragging. Many of the female students preferred the dynamic environment because it improved their conviction in the conjectures they made. It is reasonable to assume from the results of this study that the use of dynamic geometry facilitates the conjecturing process and results in greater conviction in conjectures. It is interesting that these two reasons for the preference of the dynamic environment fell into gender categories where males leaned toward the facilitation aspect and females leaned toward the conviction aspect. This may suggest that there is a qualitative gender effect concerning students' attitude toward the use of dynamic geometry software.

Dynamic language. In the theme concerning the language used in forming conjectures in the two different environments, it was found that many of the conjectures formed in the dynamic environment reflected the environment itself. Often the action of dragging or change was written into the conjectures formed from the dynamic environment. The conjectures in the dynamic environment were much more general as stated theorems are. This raises an important point raised by Martin and Harel (1989a). In their study of the role of figure in students' concepts of geometric proof, it was found that students often did not accept a proof for a general figure and would think that a proof had to be redone for each specific figure.

Dynamic software may aid with the understanding of the general figure rather than the specific. Dynamic geometry software transforms specific figures into general figures by the dragging process. Conjectures that are general may lead toward the acceptance of theorems and toward proof. Conjectures that were worded in terms of the specific figure shown in the static environment lead a reader of that conjecture to believe that the conjecture only applies to that "particular" figure and not necessarily to a general conclusion about figures of that "type".

Many of the conjectures formed in the dynamic environment reflected the environment itself. This dynamic language was also found by Jones (2001) who observed a group of students evolve their language around the use of dynamic geometry software. In his observations terms like "moving" and "dragging" became more frequent as students progressed through a unit that used dynamic geometry software as the key component of instruction. He found that by the end of the unit, explanations related entirely to the context of dynamic geometry software.

The fact that student's use of language that is more attuned to the general rather the specific is an important finding and implies that the use of dynamic geometry software may indeed help students toward a concept of the "generic" or general figure and help them along the road to deductive proof of general theorems. Teachers who incorporate dynamic geometry software in their instruction should foster the dynamic language used by the students into the classroom semantics.

Finding Counterexamples. In the theme that examined the use of dynamic geometry software to find counterexamples and disprove false conjectures, the results showed that many of the students struggled with the concept of the negation and forming a proper counterexample. Other students had difficulty finding the counterexample because of poor dragging technique. They were able to form the proper negation to the false conjecture but because of their weakness in using the dragging utility, they did not actually find a counterexample and concluded that the false conjecture was indeed true.

The results of this theme imply that greater attention should be given to the logic of negation and disproving conjectures. Many of the interviewed students could not form the proper negation that would disprove the false conjecture and claimed that the false conjecture was indeed true because it appeared true on the figure provided and remained true during their dragging process. It must be noted that the students in this study were not "coached" in terms of dragging techniques and developed their dragging styles independently. Some of the students were able to form a proper counterexample to the false conjecture but because of inadequate dragging techniques they claimed that the counterexample did not exist. This result implies that proper modeling or coaching in the uses of dynamic geometry might benefit students' abilities to get a maximum benefit from the tool. Class discussions on the use of the software, with student input, may be worthwhile for all of the students in the class.

Reliance on dynamic software may have substituted for the logical thought process in which the students could have arrived with a counterexample without the use of dynamic software or even without a figure being displayed. The conjecture stated that "A quadrilateral may not have more than two obtuse angles." With some thought, students can produce specific counterexamples even without a figure. For example, consider a quadrilateral with three angles measuring 100 degrees each and one sixty degree angle. This quadrilateral has three obtuse angles thus negating the conjecture. This quadrilateral does exist because the angle sum of 360 degrees is satisfied. Teachers should recognize that the use of dynamic geometry software is a powerful and useful tool but it does not replace logical thought.

Dragging categories. During this study, the researcher noted the way in which students used the dragging utility. During observation throughout the eight lab activities and the ten interviews three distinct types of dragging were noted, each with their own characteristics and purposes. The first is referred to as "random dragging". Random dragging was sporadic and fluid and used to test the robustness of the figure and how different components of the figure behaved during the drag. "Dragging for pattern recognition" was choppy with pauses to observe the measurements. This kind of dragging may indeed be necessary to arrive with conjectures based on the changes shown on the figure and its measures, pausing to observe and contemplate these changes.

"Dragging for a specific purpose" was characterized by small and careful motion used to produce a specific outcome. This kind of dragging may be necessary to find counterexamples to false conjectures and to test for specific properties of figures.

This classification of dragging tendencies should be useful to researchers observing students using dynamic geometry software and to teachers who are coaching their students in the use of the dragging utility. It must be noted that these different kinds of dragging tendencies were observed while the participants were finding and confirming conjectures. Other dragging techniques may exist when dynamic geometry software is used for other purposes such as construction.

Conviction in the computer output. The final qualitative theme explored the students' conviction in the computers' output using dynamic geometry software. All but one of the interviewed participants believed the output of the computer without question when confronted with output that violated a known theorem. The dynamic geometry software package used in this study was Geometers Sketchpad. By default, Geometers Sketchpad measures angles between 0 to 180 degrees and does not consider angles greater than 180 degrees. When confronted with a concave quadrilateral in which an interior angle was greater than 180 degrees, nine out of ten students stated that the sum of the interior angles was less than 360 degrees. In reality, the software program was measuring an exterior angle by default and thus indicating a misleading result concerning the sum of the interior angles of the concave quadrilateral. It is interesting that the students made modifications to a known theorem to accommodate this anomaly.

de Villiers (1992) found that the majority of students based their conviction on authoritarian grounds rather than personal conviction when confronted with conjectures.

It is interesting to observe that in this study, the participants held conviction in the observed computer output over the authoritarian sources of teacher and textbook. It is important that instructors who use dynamic geometry software become acquainted with any anomalies created by the software and help their students see that they should not always accept the output generated by the software at face value. Students should question the computer output and learn to examine why these kinds of inconsistencies occur. Teachers may also turn these inconsistencies into learning opportunities for their students, asking the students to resolve these inconsistencies through discussion, close examination of the figures, and use of mathematical principles.

Limitations

Although this study yielded a variety of significant results, it was far from a purely experimental study conducted in a laboratory setting. Because of the "real world" setting of this study, there are some limitations that should be mentioned concerning the results.

One of the prime limitations to the results of this study was the fact that almost all of the participants entered the study with a van Hiele level of geometric reasoning at level one. Because of the low variability of the entering van Hiele levels of geometric reasoning it is difficult to state that this variable has no impact in the inductive process of conjecture even though none was found by the results of this study.

The samples used in this study were not random and were selected from the participating schools' registration process. A more controlled selection of participants using a random sample may produce different results. Likewise the setting of the classroom and computer lab was certainly less than ideal with both classes being overcrowded and sometimes difficult to accommodate.

The fact that the researcher was also the full time instructor of the classes in the study may limit the objective nature of study. In a sample of students not taught by the researcher, the researcher would not have been influenced by the daily classroom exposure to the participants during the entire semester. The researcher would not have been the full time instructor influencing the content knowledge of the participants and their conceptual understanding of geometry. Therefore, results may have been different if the researcher conducted this study on students taught by another instructor.

Suggestions for Further Research

This section offers some suggestions for further research concerning topics related to this study and topics that may stem from the results of this study. While examining the variables across the different lab activities, it was noted that there was no trend over time. Conjecturing abilities did not increase or decrease. Instead, some kinds of figures favored the dynamic environment while other figures showed no significant differences in the measured variables. Some of the lab activities produced good results in both environments while others did not. It would be interesting to examine what kinds of situations are most enhanced by the dynamic environment and what kinds of situations facilitate conjecture in general.

In this study, the dragging utility was highlighted, however, would the use of construction in the dynamic environment improve the students' ability to conjecture? The figures in this study were already constructed by the researcher who designed the instruments. It stands to reason that a greater appreciation of the tool and a better conceptual understanding of the figures could be attained by allowing the students to construct the figures by themselves with guidance by the instructor. Research concerning construction in the dynamic geometry environment as a means to improve conjecture and conceptual understanding of the definition of figure would be worthwhile.

Reproducing this study or a study similar to it using a sample of students with different demographical backgrounds would also be worthwhile. It would serve to strengthen and to extend the results of this study. It would be interesting to use a sample of higher achieving students from higher socioeconomic backgrounds and run a duplicate study using similar instruments and methodology. Comparing the results of both studies would be insightful. It would be interesting to see what aspects of both studies remained constant and what aspects differed.

Concluding Remarks

Participants of this study had the opportunity to explore and discover geometric truths in both static and dynamic environments without the aid of an instructor or text book. Allowing students to explore independently allowed them to build inductive reasoning skills and uncover truths or untruths on their own. The instruments used for this study provide a series of dynamic lab activities that are closely aligned with typical secondary geometry curricula and may be used in practical geometry instruction or in research studies such as this one.

This study yielded results supporting the use of dynamic geometry software in the classroom as well as interesting results concerning the interaction of gender, mathematics achievement, and van Hiele level with student ability to conjecture in geometry. Results indicate that even with little or no training, students may use the dragging utility available in a dynamic geometry environment to significantly improve their ability to conjecture in geometry and significantly increase the conviction in their conjectures, even without the ability to construct the figures on their own.

This study uncovered some of the potential weaknesses and misconceptions of students in the conjecture process and their use of dynamic software. It pointed out how students may blur the distinction between conjecture and proof or between inductive and deductive reasoning. It demonstrated how students misunderstand the act of counterexample and what it means to "disprove". The results of this study stress the importance of good dragging technique and the pitfalls of trusting output without question. This study also explored an important question concerning the role of proof with the availability of dynamic geometry software. Results indicate how students may see the use of dynamic geometry software as a replacement to deductive proof. Goldenburg and Cuoco (1996) stated the following concerning the use of dynamic geometry software:

> If the potential hazards and pitfalls seem larger and more numerous than one might have imagined, it is also clear that the opportunities to make new and important mathematical connections between classical geometric content and big ideas from other mathematical areas are too intriguing to ignore. So is the evidence in students' eyes and on their faces. We must come to understand the new terrain well, so that its roughness becomes a part of, and not a detraction from, its beauty. (p. 30)

This study has not only provided support for the use of dynamic software as an effective conjecturing tool, but has explored some of these mentioned “pitfalls”, classified distinct dragging techniques used by students while finding and testing conjectures, and explored interesting themes concerning the students experiences and perceptions using dynamic geometry software as a conjecturing tool.

de Villiers (1996) claimed that many of the teachers in the past have simply avoided the informal exploration of geometric relationships by construction and measurement with paper and pencil since they are so time consuming. de Villiers (1996) also claims that dynamic geometry software has revitalized the teaching of geometry in many countries where Euclidean geometry was in danger of being “thrown into the trashcan of history.” (p. 25).

The results and implications of this study help strengthen the claim that dynamic geometry software is a valuable tool that facilitates student conjecturing in geometry and promotes equity among students of different achievement levels. However, teachers who use dynamic software must be aware that the students may see the tool as a substitution for proof and must strive to use dynamic software to enhance the need for proof rather than replace it. The results of this study indicate that the use of dynamic software aids with the students’ concept of the general figure which is an important concept in the context of deductive proof. Finally, this study classified different techniques used with the dragging utility and uncovered several pitfalls that students may encounter when conjecturing using dynamic geometry software. It is important that these pitfalls be uncovered and addressed in order to make the use of dynamic geometry software in the classroom a worthwhile and productive experience for both instructors and learners of mathematics.

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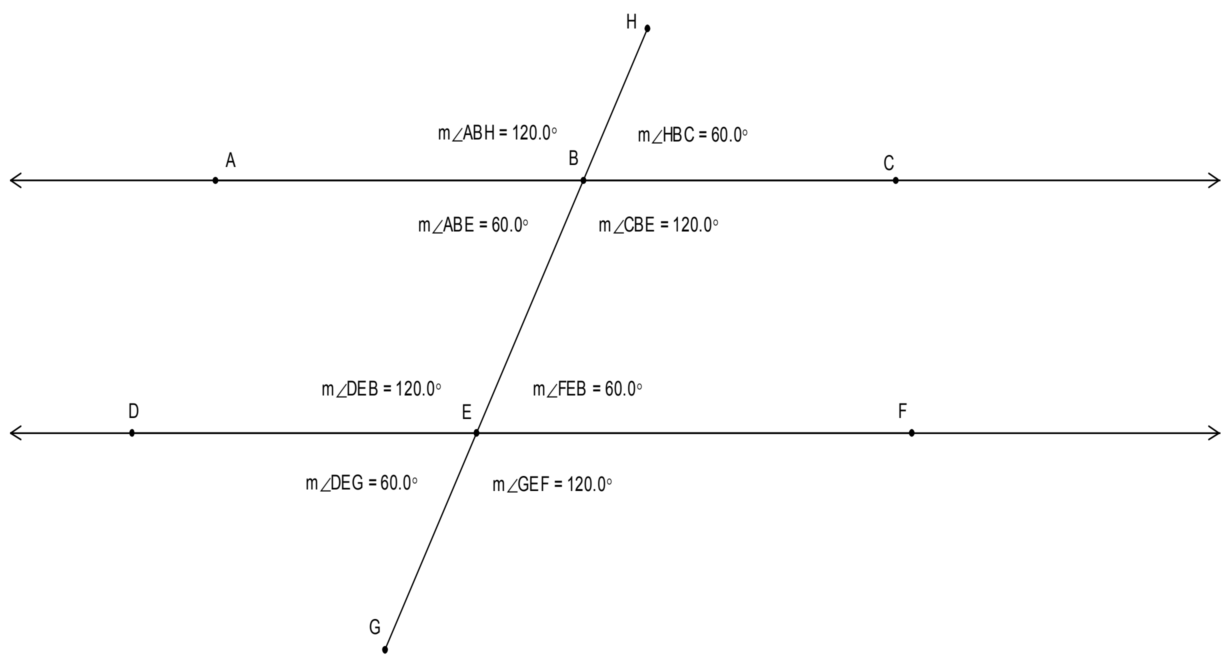
APPENDIX A

LAB ACTIVITIES WITH RUBRICS

Lab 1

In the following diagram lines AC and DF are **parallel** to each other.
Segment GH is a **transversal** of these parallel lines. Using the measurements provided make as many conjectures as you can about the angles formed by transversals and parallel lines.
We already know that vertical angles are congruent and linear pairs are supplementary. Now consider the relationships of the *corresponding, alternate interior, alternate exterior* and *consecutive* angles when you make your conjectures. After each conjecture indicate how sure you are that your conjecture is true for all transversals and parallel lines. Use a 1 – 10 scale with 10 being the most confident.



Conjecture(s): If two parallel lines are intersected by a transversal then:

Rubric Lab 1

This lab focuses on the angle pairs formed by transversals and parallel lines. Four conjectures are targeted: If parallel lines are intersected by a transversal then…

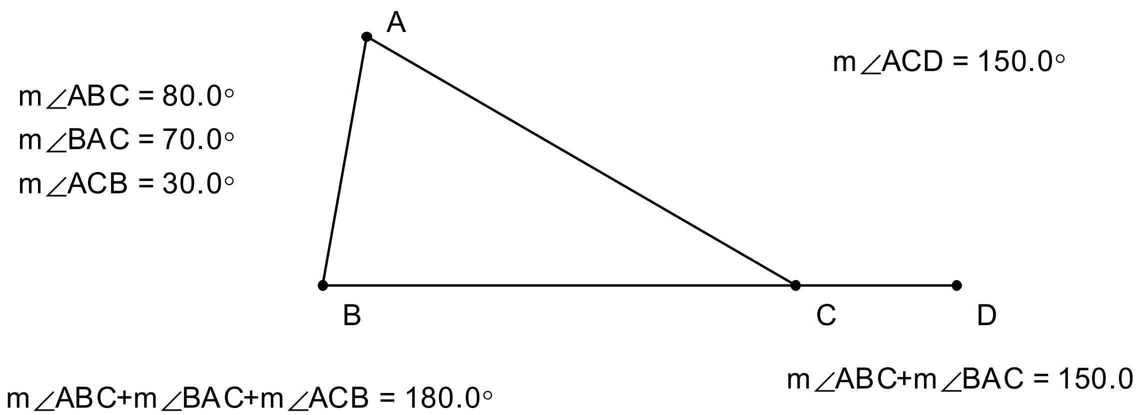
1) Corresponding angles are congruent.
2) Alternate Interior angles are congruent.
3) Alternate Exterior angles are congruent.
4) Consecutive (same-side interior) angles are supplementary.

True statements concerning the perpendicular case such as "If a transversal is perpendicular to one line then it will be perpendicular to the other line." will be counted as a relevant conjecture. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any true conjectures concerning vertical angles or linear pairs will be counted as irrelevant since these are theorems already covered in previous class lessens. The following chart has some examples taken from previous student responses.

Code	**Conjectures**
Relevant **R**	Consecutive interior angles have the same measure if and only if they are right angles. Alternate interior angles will always be the same. Corresponding angles are equal. For same side interior angles they equal up to 180.
Irrelevant **I**	Vertical angles are congruent. Lines AC and DF are parallel. Linear pairs are supplementary.
Ambiguous **A**	All alternate interior angles are two angles that lie between two opposite sides and intersect two or more coplanar lines on different points. If you take one alternate interior and one alternate exterior then they add up to be 180 degrees.
False **F**	You always get four acute angles and four obtuse angles. One consecutive angle is twice as big as the other. Same side interior angles are never congruent.

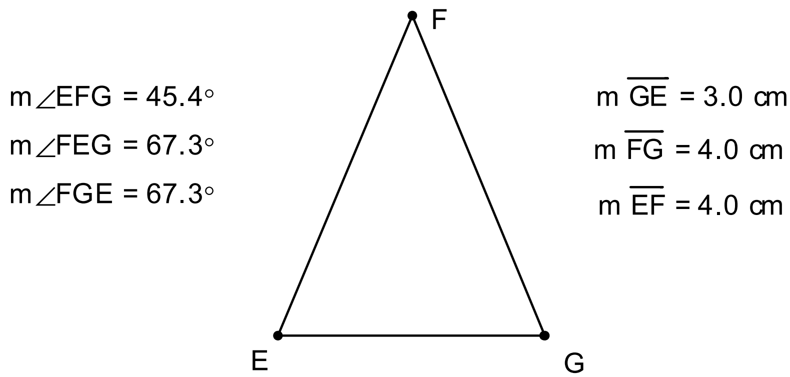
Lab 2

The measurements of the three interior angles of triangle ABC are shown.
The measurement of the exterior angle ACD is also shown.
Make as many conjectures as you can about the interior and exterior angles of triangles. After each conjecture rate how convinced you are that your conjecture is true for **all** triangles. Use a 1-10 scale with 10 being the most confident.



Conjecture(s): If a figure is a triangle then …

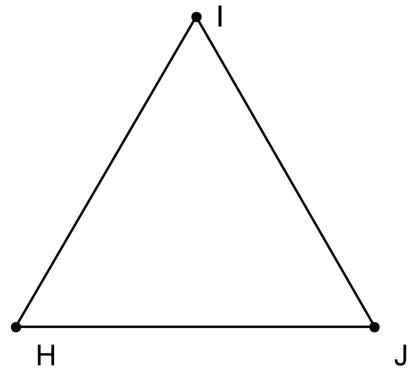
An isosceles triangle has at least two congruent sides. Triangle EFG is an isosceles triangle.
Make as many conjectures as you can about isosceles triangles.
After each conjecture rate how convinced you are that your conjecture is true for all isosceles triangles. Use a 1-10 scale with 10 being the most confident.



Conjecture(s): If a triangle is isosceles then …

Equilateral triangles have all three sides congruent. Triangle HIJ is equilateral. Make as many conjectures as you can about equilateral triangles.
After each conjecture rate how convinced you are that your conjecture is true for all equilateral triangles. Use a 1-10 scale with 10 being the most confident.

$m\,\overline{HI} = 3.9$ cm
$m\,\overline{IJ} = 3.9$ cm
$m\,\overline{JH} = 3.9$ cm



$m\angle IHJ = 60.00^\circ$
$m\angle HJI = 60.00^\circ$
$m\angle JIH = 60.00^\circ$

Conjecture(s): If a triangle is equilateral then …

Rubric Lab 2

This lab focuses on interior and exterior angles of triangles. Four conjectures are targeted using three different figures:

Figure 1: If a figure is a triangles then …

5) The sum of the measures of the interior angles is 180 degrees.
6) The measure of an exterior angle is equal to the sum of the measures of the two remote interior angles.

Figure 2: If a triangle is isosceles then …

3) The base angles are congruent.

Figure 3: If a triangle is equilateral then …

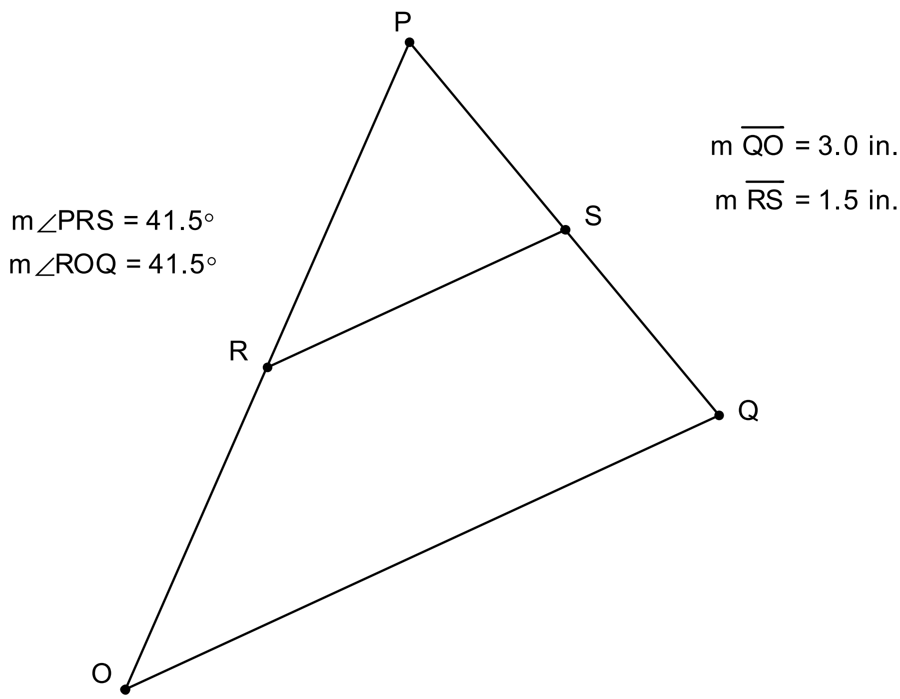
4) The triangle is equiangular.

True statements concerning the number of interior angles that are obtuse or right will be counted as relevant. For example "A triangle can only have one obtuse (right) interior angle." is a relevant conjecture. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any statements about the exterior angle and its adjacent interior angle being supplementary will be counted as irrelevant. The following chart has some examples taken from previous student responses. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	The angles of the triangle add up to be 180. Two angles are the same for the isosceles. The equilateral has all angles 60 degrees. If you add up two of the triangle angles you get the outside angle ACD. If you get one obtuse angle inside then the other angles inside are smaller than 90. The other angles of a right triangle add up to 90 degrees.
Irrelevant **I**	ACB and ACD are linear pairs. The isosceles has two sides the same. ABC is a right triangle.
Ambiguous **A**	Angle ABC has been measures it will not change as much as the others. If you split angle ACD the measures would be close to each other.
False **F**	The exterior angles are obtuse. Isosceles triangles are acute triangles. The base angles will be greater than the angle at the top.

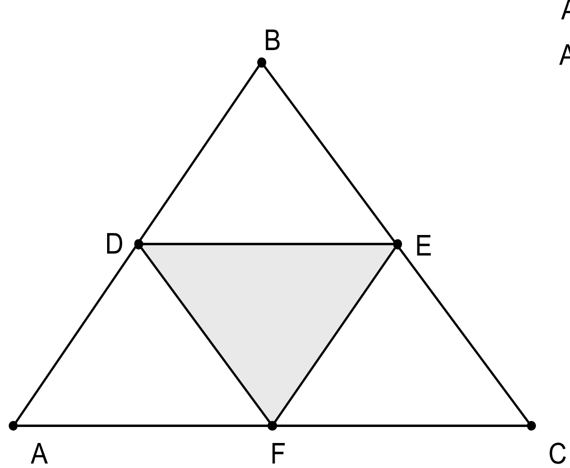
Lab 3

A **triangle midsegment** joins the midpoints of two sides of a triangle.
In the following figure segment RS is a triangle midsegment. Write as many conjectures as you can about triangles midsegments. Some measurements are provided with the figure. After each conjecture rate how confident you are that your conjecture is true for **all** triangle midsegments. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a segment is a triangle midsegment then …

Triangle DEF is formed by joining the midpoints of the sides of triangle ABC.
Triangle DEF is called a **midsegment triangle**.
Write as many conjectures as you can about midsegment triangles using the diagram and the measures provided.
For each conjecture rate how convinced you are that your conjecture will be true for **all** midsegment triangles. Use a 1-10 scale with 10 being the most confident.



Area $\triangle ABC = 2.0 \text{ in}^2$
Area $\triangle DEF = 0.5 \text{ in}^2$

$$\frac{(\text{Area } \triangle ABC)}{(\text{Area } \triangle DEF)} = 4.00$$

Perimeter $\triangle ABC = 6.6$ in.
Perimeter $\triangle DEF = 3.3$ in.

$$\frac{(\text{Perimeter } \triangle ABC)}{(\text{Perimeter } \triangle DEF)} = 2.00$$

Conjecture(s): If a triangle is a midsegment triangle then …

Rubric Lab 3

This lab focuses on the triangle midsegments. Four conjectures are targeted using two figures.

Figure 1: If a segment is a triangle midsegment then …

1) It is parallel to the remaining side.
2) Its measure is one half the measure of the remaining side.

Figure 2: If a triangle is a midsegment triangle then …

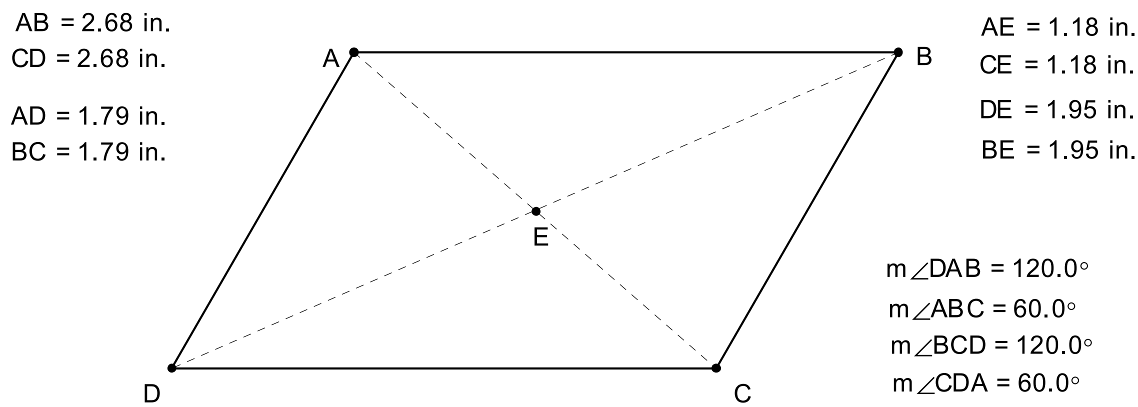
3) The area of the midsegment triangle is one forth the area of the original triangle.
4) The perimeter of the midsegment triangle is one half the perimeter of the original triangle.

True statements concerning the congruence of triangles found within figure 2 will be counted as relevant; Any statement concerning the similarity of the midsegment triangle with the original triangle will also be counted as relevant Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. If a student states that the midsegment is simply "smaller than" the remaining side, count it as irrelevant they should be more specific. The following chart has some examples taken from previous student responses. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	The bottom side is twice the size of the midsegment. There are three other triangles just like the middle triangle. The middle triangle is the same shape as the big triangle but one forth of the size. The midsegment has the same slope as OQ.
Irrelevant **I**	The points D, E, and F are midpoints. Triangle DEF is smaller than triangle ABC
Ambiguous **A**	You find the perimeter of both triangles by dividing them by each other. The midpoints of A, B, and C equal the perimeter of the inner triangle. The measurement of the area is more than the perimeter.
False **F**	The midsegment triangle is equilateral. All segments are congruent and all angles are the same.

Lab 4

A **parallelogram** is a quadrilateral in which both pairs of opposite sides are parallel.
Quadrilateral ABCD is a parallelogram.
Use the diagram below and the measurements provided to make as many conjectures you can about parallelograms.
Do not use theorems or postulates about quadrilaterals already discussed in class.
After each conjecture rate how convinced you are about your conjecture for **all** parallelograms. Use a 1 to 10 scale with 10 being the most confident.



Conjecture(s): If a quadrilateral is a parallelogram then…

Rubric Lab 4

This lab focuses on the properties of parallelograms. Four conjectures are targeted: If a quadrilateral is a parallelogram then …

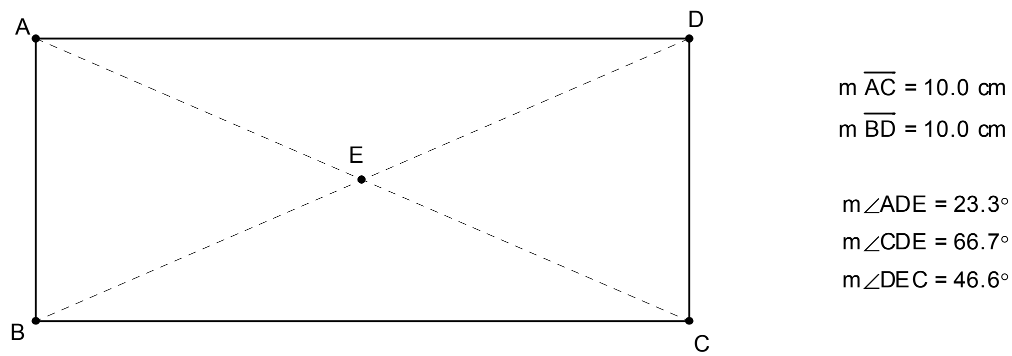
3) Opposite sides of a parallelogram are congruent.
4) Opposite angles of a parallelogram are congruent.
5) Adjacent sides of a parallelogram are supplementary.
6) The diagonals of a parallelogram bisect each other.

True statements concerning the congruence of triangles found within the figure will be counted as relevant; however, multiple statements concerning the congruence of triangles will count only as one relevant conjecture. For example, there are four different pairs of congruent triangles found within the figure. If a student states four different conjectures listing these pairs only one relevant conjecture should be awarded. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any statements about the congruence of the vertical angles resulting by the intersection of the diagonals will be counted as irrelevant. Any statements that hold true for a general quadrilateral such as "The sum of the measures of the interior angles is 360 degrees" or "The parallelogram has two diagonals" will be counted as irrelevant. The following chart has some examples taken from previous student responses. Remember that the conjectures should reflect the properties of parallelograms and not just quadrilaterals in general. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	The measure of AB and DC will stay the same. The angles across the diagonal are congruent. Line segment AEC bisects line segment DEB. BE and DE are equal and AE and CE are equal. The diagonals make congruent triangles. When a "perfect rectangle" the diagonals are congruent.
Irrelevant **I**	All angles add up to be 360 degrees. The opposite sides are parallel. Both of the diagonals intersect at E
Ambiguous **A**	The wider the shape the more it increases. The corresponding angles are congruent. The corresponding lines are congruent.
False **F**	The diagonals are congruent. The diagonals bisect the angles of the parallelogram. The diagonals are perpendicular. All sides are congruent.

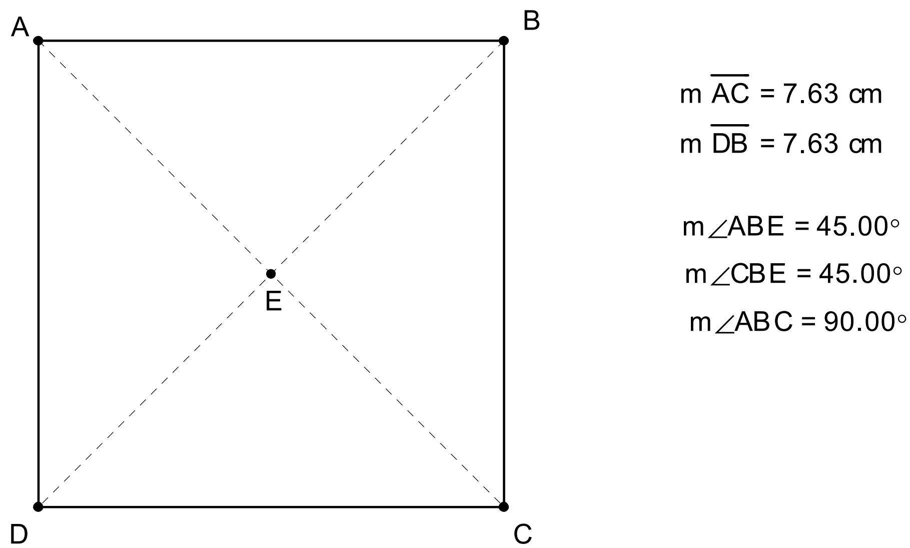
Lab 5

A **rectangle** is a parallelogram with four right angles. ABCD is a rectangle with diagonals shown dashed.
Write all of your conjectures about the diagonals of rectangles.
After each conjecture rate how confident you are that your conjecture is true for **all** rectangles. Use a 1- 10 scale with 10 being the most confident.



Conjecture(s): If a parallelogram is a rectangle then …

A **square** is a parallelogram with four congruent sides and four right angles. ABCD is a square with diagonals shown dashed.
Write as many conjectures as you can about the diagonals of a square.
After each conjecture rate how confident you are that your conjecture is true for **all** squares. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a parallelogram is a square then …

Rubric Lab 5

This lab focuses on the diagonals of rectangles and squares. Four conjectures are targeted using two figures

Figure 1) If a parallelogram is a rectangle then …

1) Diagonals are congruent.

Figure 2) If a parallelogram is a square then …

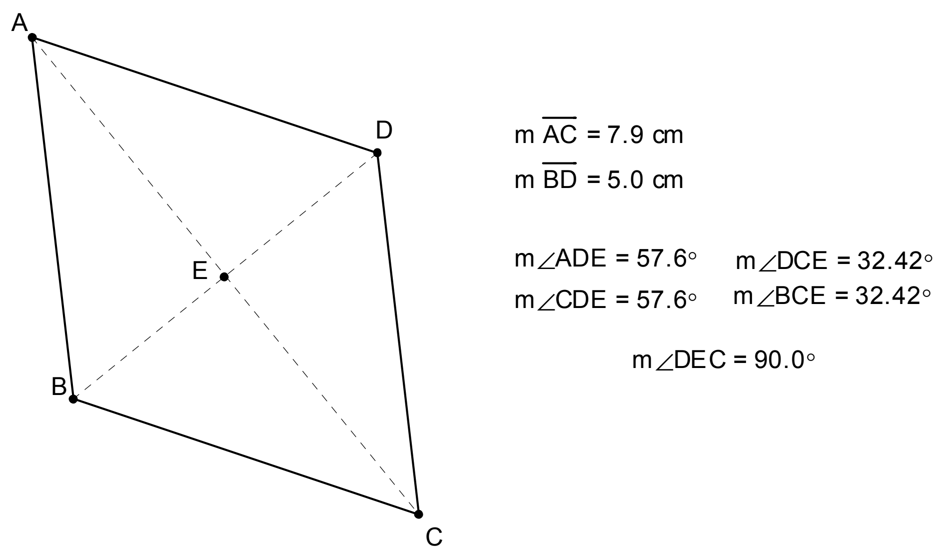
2) Diagonals are congruent.
3) Diagonals are perpendicular.
4) Diagonals bisect the interior angles of the square.

The conjectures in this lab should focus on the diagonals of rectangles and squares and not other properties that are inherent in these shapes. Any general properties of parallelograms or quadrilaterals should be counted as irrelevant. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any statements about the congruence of the vertical angles resulting by the intersection of the diagonals will be counted as irrelevant. Any statement concerning the congruence of the triangle pairs formed by the diagonals will be counted as irrelevant since this is a property of the general parallelogram. However, any statement that targets the fact that the diagonals form four congruent triangles will count as relevant since this is not a property of the general parallelogram but rather the rhombus and square. The following chart has some examples taken from previous student responses. Remember that the conjectures should reflect the properties of the diagonals of these shapes. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	For a rectangle the diagonal lengths are the same. The diagonals of the square are 90 degrees. The diagonals are cutting the angles of the square in half 45 degrees. The diagonals of a square are the same.
Irrelevant **I**	Rectangles and squares have 90 degree angles The opposite sides are parallel. The diagonals of a square cut the square in half.
Ambiguous **A**	I believe that all the diagonal angles are equal in a rectangle. The line bisects at the same point.
False **F**	The diagonals of a square form equilateral triangles. The diagonals of a rectangle bisect the angles of the rectangle.

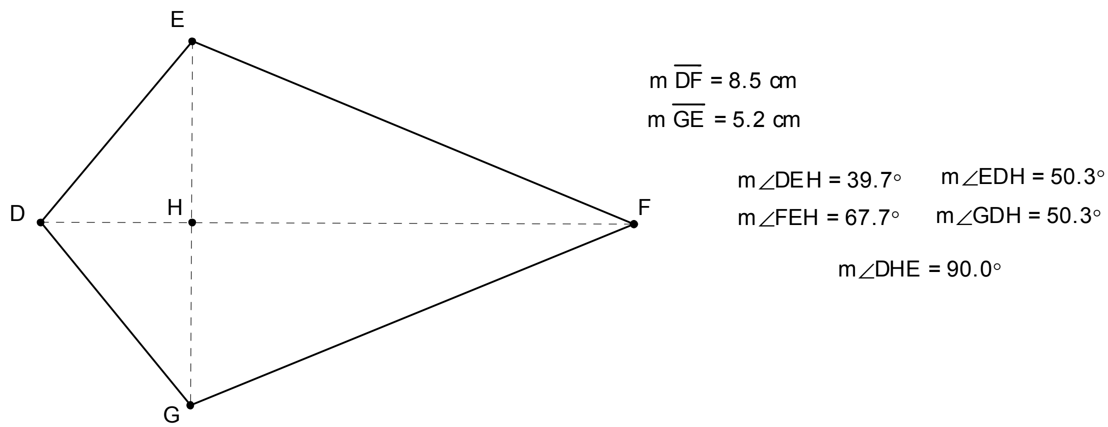
Lab 6

A **rhombus** is a parallelogram with four congruent sides. ABCD is a rhombus with diagonals shown dashed.
Write as many conjectures as you can about the diagonals of a rhombus.
After each conjecture rate how confident you are that your conjecture is true for **all** rhombi. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a parallelogram is a rhombus then …

A **kite** is a quadrilateral with two pairs of adjacent sides congruent but opposite sides not congruent.
DEFG is a kite with diagonals shown dashed.
Write as many conjectures as you can about the diagonals of a kite.
After each conjecture rate how confident you are that your conjecture is true for **all** kites.
Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a quadrilateral is a kite then …

Rubric Lab 6

This lab focuses on the diagonals of rhombi and kites. Four conjectures are targeted using two figures

Figure 1) If a parallelogram is a rhombus then …

1) Diagonals are perpendicular.
2) Diagonals bisect the interior angles of the rhombus.

Figure 2) If a quadrilateral is a kite then …

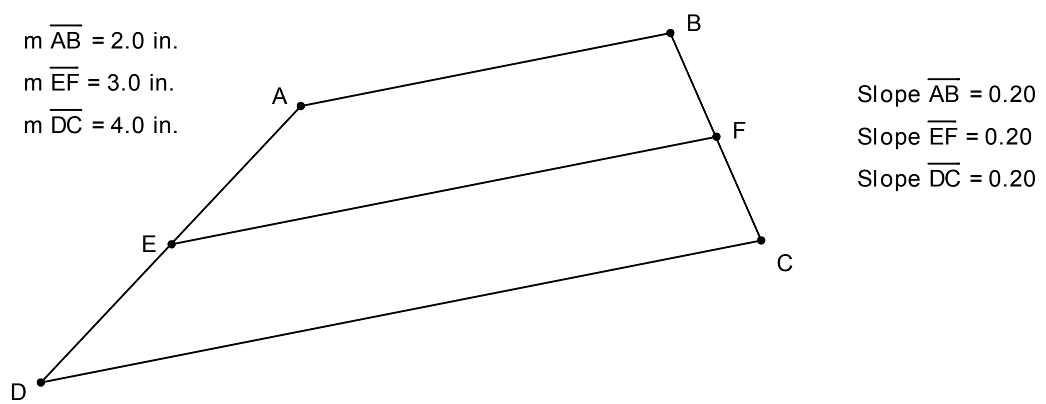
3) Diagonals are perpendicular.
4) Diagonals bisect one pair of interior angles of the kite.

The conjectures in this lab should focus on the diagonals of rhombi and kites and not other properties that are inherent in these shapes. Any general properties of parallelograms or quadrilaterals should be counted as irrelevant. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any statements about the congruence of the vertical angles resulting by the intersection of the diagonals will be counted as irrelevant. Any statement concerning the congruence of the triangle pairs formed by the diagonals of a rhombus will be counted as irrelevant since this is a property of the general parallelogram. However, any statement that targets the fact that the diagonals form four congruent triangles will count as relevant since this is not a property of the general parallelogram but rather the rhombus and square. Statements concerning the congruence of triangle pairs for the kite will be counted as relevant since a kite is not a parallelogram. The following chart has some examples taken from previous student responses. Remember that the conjectures should reflect the properties of the diagonals of these shapes. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	The diagonals of a rhombus make 90 degrees. The diagonals of a kite can be outside the shape when the kite is concave. The diagonals of a kite make right angles. One diagonal bisects the kite angles but the other diagonal does not. One diagonal is bisected but not the other for a kite.
Irrelevant **I**	Kites have one pair of congruent angles. The opposite sides of a rhombus are parallel and all sides are congruent. The diagonals of a rhombus cut the shape in half. For a rhombus diagonals bisect each other.
Ambiguous **A**	The angles of the kite are as not to be a parallelogram. Opposite points are congruent.
False **F**	The diagonals of a rhombus are not congruent. The diagonals of a kite are not congruent.

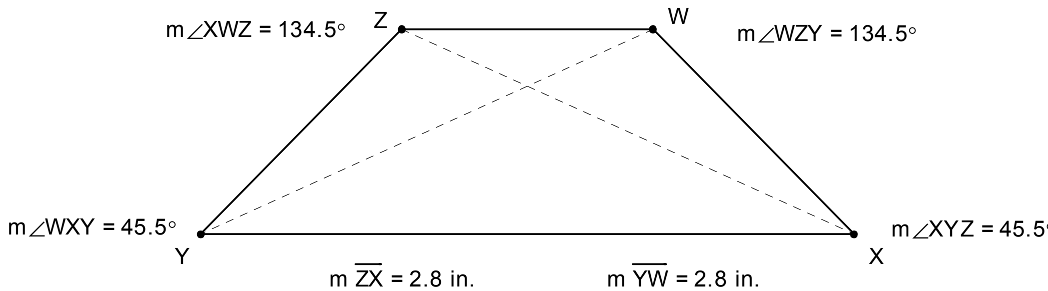
Lab 7

Segment EF is a **trapezoid midsegment**. It connects the midpoints of the legs of trapezoid ABCD.
Write as many conjectures as you can about trapezoid midsegments.
After each conjecture rate how confident you are that your conjecture is true for **all** trapezoid midsegments. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a segment is a trapezoid midsegment then …

An **isosceles trapezoid** has congruent legs. WXYZ is an isosceles trapezoid.
Write as many conjectures as you can about isosceles trapezoids.
After each conjecture rate how confident you are that your conjecture is true for **all** isosceles trapezoids. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a trapezoid is isosceles then …

Rubric Lab 7

This lab focuses on trapezoid midsegments and isosceles trapezoids. Four conjectures are targeted using two figures

Figure 1) If a segment is a trapezoid midsegment then …

1) It is parallel to the bases.
2) Its measure is one half the sum of the measure of the two bases.

Figure 2) If a trapezoid is isosceles then …

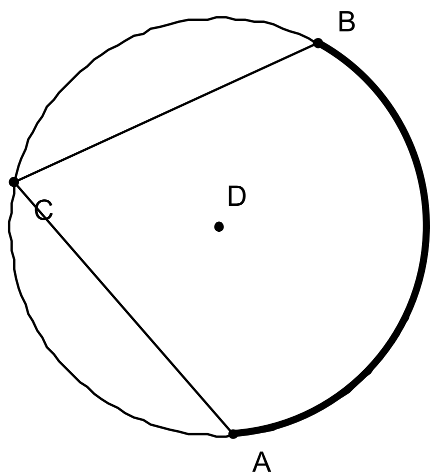
3) Diagonals are congruent.
4) Base angles are congruent.

The conjectures from figure one should focus only on the midsegment. Any statement concerning the adjacent angles being supplementary will be scored irrelevant because this is a property of the general trapezoid not the isosceles trapezoid. A statement concerning the opposite angles being supplementary for an isosceles trapezoid will be scored relevant. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. Any statements about the congruence of the vertical angles resulting by the intersection of the diagonals will be counted as irrelevant. The following chart has some examples taken from previous student responses. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

Code	**Conjectures**
Relevant **R**	The trapezoid midsegment is parallel to AB and DC. AB + DC divided by 2 is the midsegment. The diagonals will be equal in the isosceles trapezoid. The opposite angles will add up to 180 in the isosceles trapezoid. The isosceles trapezoid has two pairs of congruent angles. An isosceles trapezoid has two acute and two obtuse angles.
Irrelevant **I**	The legs are not parallel. The angles on the side are supplementary. Top and bottom sides are parallel.
Ambiguous **A**	Angle BAD is not congruent to any of the sides. AD is in a right angle.
False **F**	The trapezoid has two acute and two obtuse angles. The diagonals bisect each other. A trapezoid has all different angles unless it is isosceles.

Lab 8

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of that circle.
In the figure below Angle ACB is an inscribed angle in Circle D and Angle ACD has an intercepted arc AB. The measurements of the inscribed angle and its intercepted arc are shown. Write as many conjectures as you can about inscribed angles and their intercepted arcs. After each conjecture rate how convinced you are that your conjecture is true for **all** inscribed angles and their intercepted arcs. Use a 1 - 10 scale with 10 being the most confident.

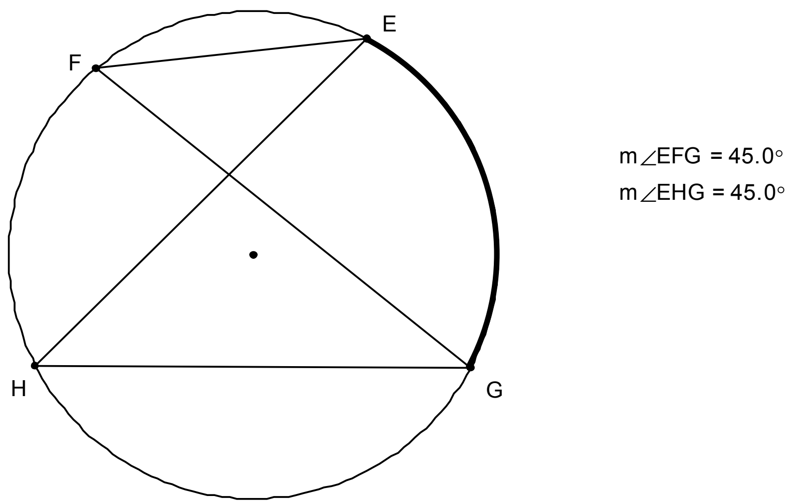


$m\,\overset{\frown}{AB} = 148.0°$

$m\angle ACB = 74.0°$

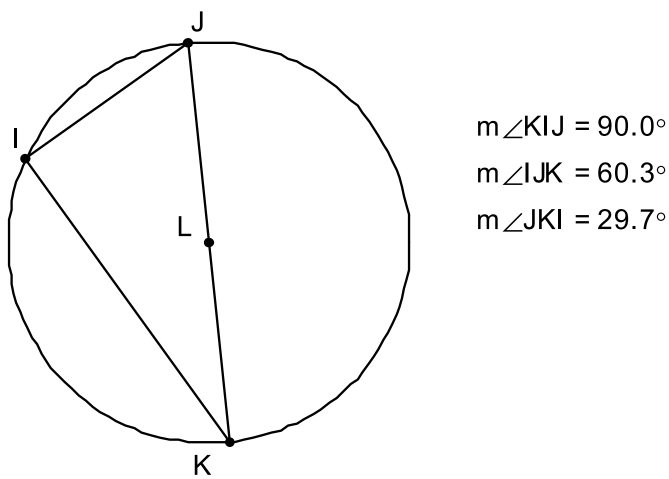
Conjecture(s): If an angle is an inscribed angle then …

In the figure below Angle EFG and Angle EHG are both **inscribed angles** who share the intercepted arc EG.
Write as many conjectures as you can about inscribed angles that have the same intercepted arc.
After each conjecture rate how convinced you are that your conjecture is true for **all** inscribed angles that have the same intercepted arc. Use a 1 - 10 scale with 10 being the most confident.



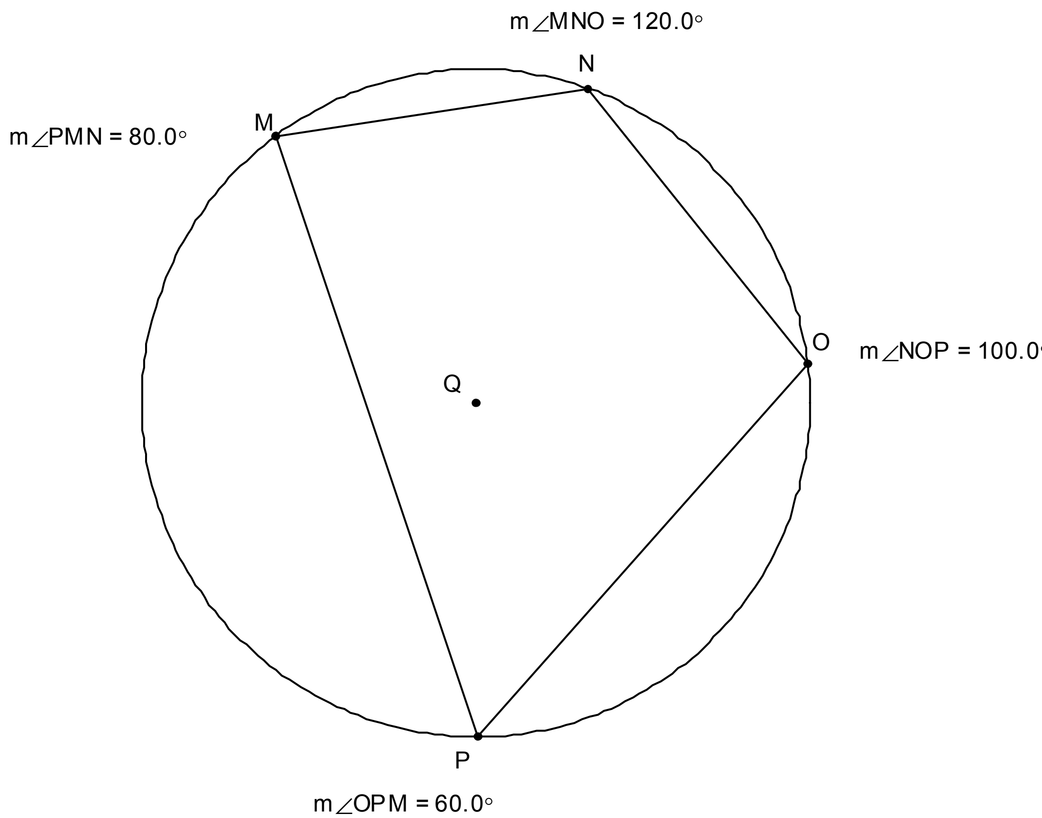
Conjecture(s): If inscribed angles have the same intercepted arc then …

In the figure below a right triangle is **inscribed** in a circle. Make is many conjectures as you can about inscribed right triangles. After each conjecture rate how convinced you are that your conjecture is true for **all** inscribed right triangles. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a right triangle is inscribed in a circle then …

In the figure below a quadrilateral is inscribed in a circle. Make is many conjectures as you can about inscribed quadrilaterals. After each conjecture rate how convinced you are that your conjecture is true for all inscribed quadrilaterals. Use a 1 - 10 scale with 10 being the most confident.



Conjecture(s): If a quadrilateral is inscribed in a circle then …

Rubric Lab 8

This lab focuses on inscribed angles. Four conjectures are targeted using four figures

Figure 1) If an angle is an inscribed angle then …

1) Its measure is half the measure of its intercepted arc.

Figure 2) If inscribed angles intercept the same arc then …

2) They are congruent.

Figure 3) If a right triangle is inscribed in a circle then …

3) The hypotenuse is a diameter of the circle.

Figure 4) *If a quadrilateral is inscribed in a circle then …*

4) The opposite angles are supplementary.

All relevant conjectures should be equivalent to the targeted conjectures above. Conjectures that are poorly worded may still be judged relevant if you determine that the statement is reasonable enough to be interpreted in proper mathematical language. The following chart has some examples taken from previous student responses. If you are unsure of what the student is trying to convey, score the conjecture as ambiguous.

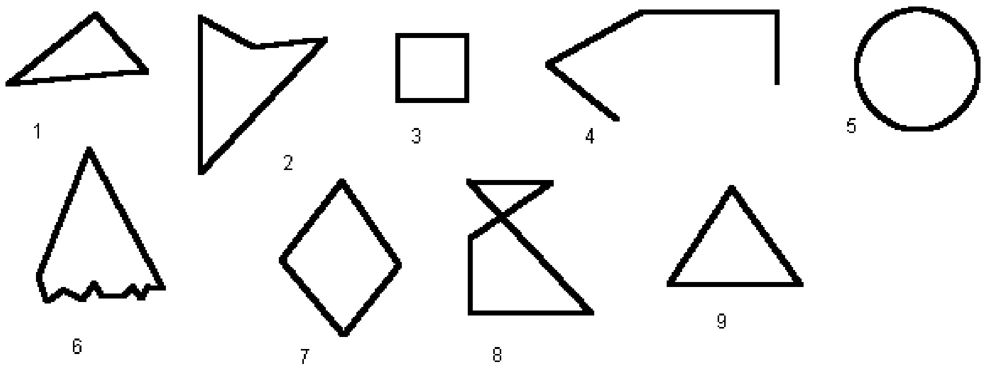
Code	**Conjectures**
Relevant **R**	The inscribed angle is half of the arc. The opposite angles of the quadrilateral add up to 180 degrees. The right triangle takes exactly half of the circle. Angles of the same arc are congruent.
Irrelevant **I**	Smaller arcs mean smaller angles. The inscribed angle is inside the circle. The right triangle has two angles that add up to 90.
Ambiguous **A**	The intersection of the inscribed angles makes the lines vertical. The thick arc is the most taken by the inscribed angle.
False **F**	Inscribed angles are acute angles. Inscribed angles intersect to make similar triangles.

APPENDIX B

VAN HIELE INSTRUMENT WITH RUBRIC

Item 1 name____

Write a **P** on the polygons, write an **N** on the nonpolygons, write a **T** on the triangles, and write a **Q** on the quadrilaterals. If necessary, you may write several letters on each shape



Write the numbers of the shapes that are not polygons and explain, for each of them, why it is not a polygon.

Write the numbers of the shapes that are triangles and explain, for each of them, why it is a triagle.

Write the numbers of the shapes that are quadrilaterals and explain, for each of them, why it is a quadrilateral.

Is shape 8 a polygon? Why?

Is shape 2 a triangle? Why?

Rubric Item 1

The correct answers for shapes 1 through 9 are as follows:

1) T, P

2) Q, P

3) Q, P

4) N

5) N

6) P

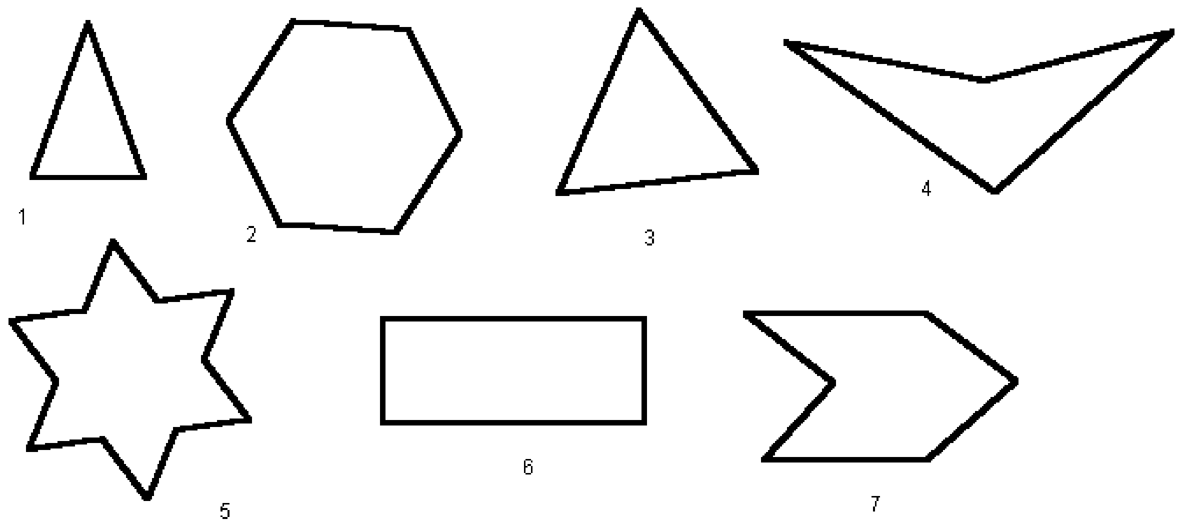
7) Q, P

8) N

9) T, P

This item can be answered with a level one response. For instance, a student wrote that shape 2 is not a polygon because "It does not follow any rule", and shape 7 is a quadrilateral because "It is a rhombus with its sides diagonal and parallel." Item one can also be answered with a level two response, when the reasons for classification are based on the number of sides and, in shapes 4, 5, and 6, on their openness or curvature (Gutierrez & Jaime, 1998 p. 35).

Item 2



Write an **R** on the regular polygons, an **I** on those that are irregular, a **V** on those that are concave, and an **X** on those that are convex.
If necessary, you may write several letters on each shape.

For polygons 2, 5, 6, and 7 explain your choice of letters or why you did not write any letter.

Rubric Item 2

The correct answers for shapes 1 through 7 are as follows:

1) I, X

2) I, V

3) R, X

4) R, X

5) I, X

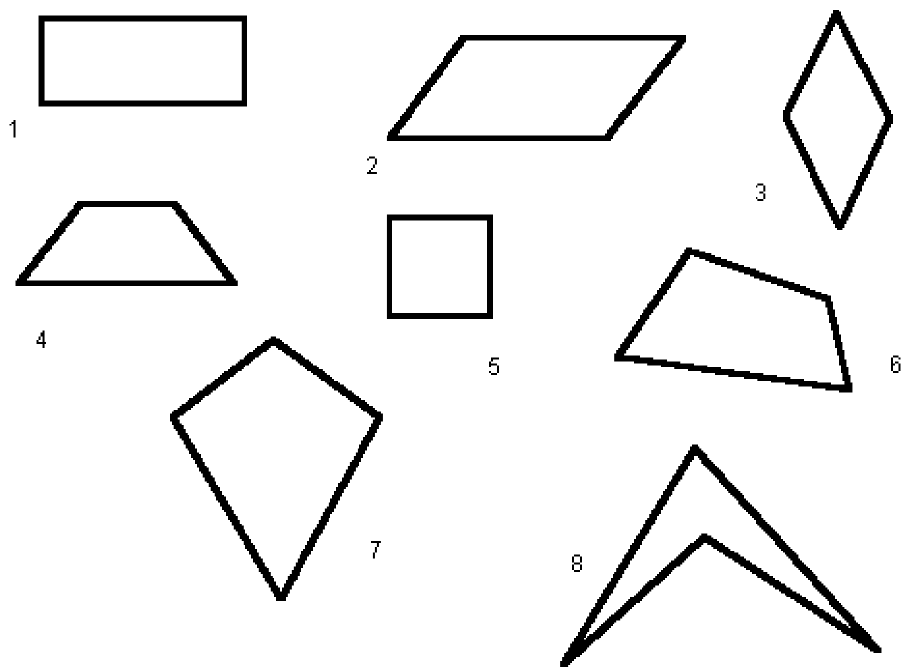
6) I, V

7) I, V

This item can be answered in level one or two. Some level one students may classify regular polygons as those that are "familiar" to them (1, 3, 4, and 5), and the irregular polygons as "estrange" shapes. Level two students should base their classification on the (in)equity of the angles and sides (Gutierrez & Jaime, 1998 p. 36).

Item 3

For each shape below place the letter **P** if the shape is parallelagram, **H** if it is a rhombus, **R** if it is a rectangle, and **S** if it is a square. You may use more than one letter for each shape.



Rubric Item 3

The correct answers for shapes 1 through 8 are as follows:

1) R, P

2) P

3) H, P

4) Blank

5) S, R, H, P

6) Blank

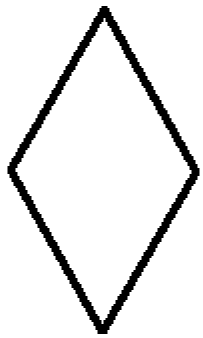
7) Blank

8) Blank

Students may answer with a level three response if the multiple combinations shown above are answered correctly indicating hieratical classification. Students with the correct one letter responses with 4, 6, 7, 8 left blank demonstrate level 2 understanding of definition.

Item 4

The shape below is called a rhombus. Make a list of all the properties that you find for this shape. (you can draw to explain the properties if you wish)



Write all the important properties which are shared by squares and rhombi.

Write all the important properties that are true for squares but not for rhombi.

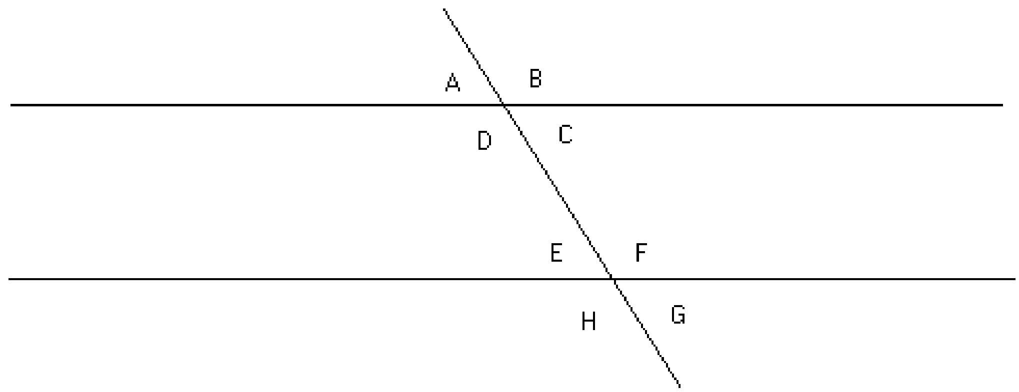
Write the important properties that are true for rhombi but not true for squares.

Rubric Item 4

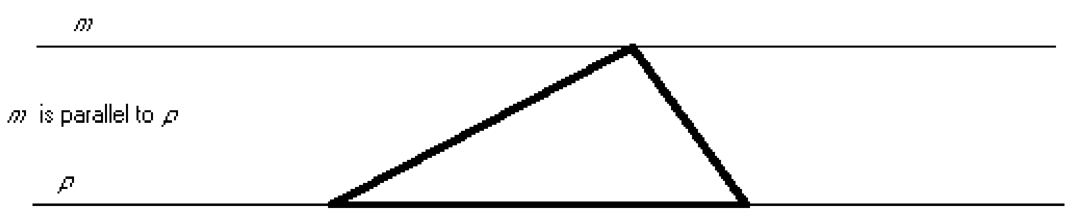
Level one students will find the number of sides the only property common to squares and rhombuses, a difference among these shapes may include a description such as “rhombuses are pointy but squares are not”. Students at level two will not differentiate properties that are shared by the two shapes from those that belong to only one. For example, the property of squares and rhombi having four sides may be mentioned as both a shared property and a differentiating property. Level two students may use exclusive classifications using lists of properties of angles, sides, and diagonals. The students in level three are able to justify either inclusive or exclusive classifications. For example a differentiating property would be “Squares have four right angles but rhombuses have two acute and two obtuse angles.” (Gutierrez & Jaime, 1998 p. 36).

Item 5

If you have two parallel lines cut by a transversal, all of the acute angles are congruent (A, C, E, G) and all of the obtuse angles are congruent (B, D, F, H).



Taking the above diagram into account, can you prove that the sum of the angles of a triangle is 180°? Consider the diagram below as an aid.



Rubric Item 5

Students who successfully complete a proof in any form are assigned a level three since this item uses a “hint” as part of the item. (Gutierrez & Jaime, 1998 p. 37).

Item 6

A *diagonal* of a polygon is a segment that joins two non adjacent vertices of the polygon.

Complete the statements (you can draw if you want):

1) In a 4-sided polygon, the number of diagonals which can be drawn from each vertex is___________________
and the total number of diagonals is________________

2) In a 5-sided polygon, the number of diagonals which can be drawn from each vertex is____________________
and the total number of diagonals is_________________

3) In a 6-sided polygon, the number of diagonals which can be drawn from each vertex is____________________
and the total number of diagonals is__________________

4) In a n-sided polygon, the number of diagonals which can be drawn from each vertex is ____________________
and the total number of diagonals is__________________

Prove your result for statement (4). (The total number of diagonals of an n-sided polygon)

Rubric Item 6

The correct answers for questions 1 through 4 are as follows:

1) 1 diagonal from each vertex, 2 total diagonals

2) 2 diagonals from each vertex, 5 total diagonals

3) 3 diagonals from each vertex, 9 total diagonals

4) n-3 diagonals from each vertex, n(n-3)/2 total diagonals

Level two may draw the polygons with their diagonals and count the number of diagonals answering the first three questions but are unsuccessful at question 4. Level three students are successful on question 4 with the answer above or an equivalent form of it. Level four students are able to justify the correct response in question 4 with a valid deductive proof. (Gutierrez & Jaime, 1998 p. 38).

APPENDIX C

PARTICIPANT SURVEY

1) Describe what a conjecture is in your own words?

2) Which environment, dynamic or static, is better for conjecturing? Why?

3) How do you prove a conjecture?

4) Do you do a lot of "dragging" when you are in the dynamic environment?

5) What is the purpose of dragging?

6) What is a "generic" figure?

7) Are the figures in Sketchpad generic figures?

8) Are the figures in the static environment generic?

APPENDIX D

INTERVIEW PROTOCAL

Student is seated in front of a computer and Figure 1 is opened.

Researcher: In this interview I will be asking you some questions about the figures shown on the computer screen. Feel free to use the mouse to drag at any time during the interview. This interview is being recorded and may be used in a published report but at no time will your real identity be published or made public.

This figure shows a midsegment quadrilateral that is formed by joining the midpoints of any quadrilateral. A former student conjectured that the midsegment quadrilateral will always be a parallelogram.

Do you agree with this conjecture?

How could you test your conclusion?

Would you like to have any measurements shown? Which measurements?

How could you prove or disprove this conjecture?

The figure of a general quadrilateral is then opened.

Researcher: A former student conjectured that there will be at most two obtuse interior angles for any quadrilateral.

Do you agree with this conjecture?

How could you test your conclusion?

How could you prove or disprove this conjecture?

Using the same figure and the sum of the interior angles measured. A theorem states that the sum of the interior angles for any quadrilateral is 360 degrees.

Do you agree with this theorem? (Encourage the student to drag until the quadrilateral is concave)

How do you resolve this contradiction?

Figure 1: The midsegment quadrilateral
A midsegment quadrilateral is formed by connecting the midpoints of the sides of a quadrilateral

Conjecture: The midsegment quadrilateral will always be a parallelogram.

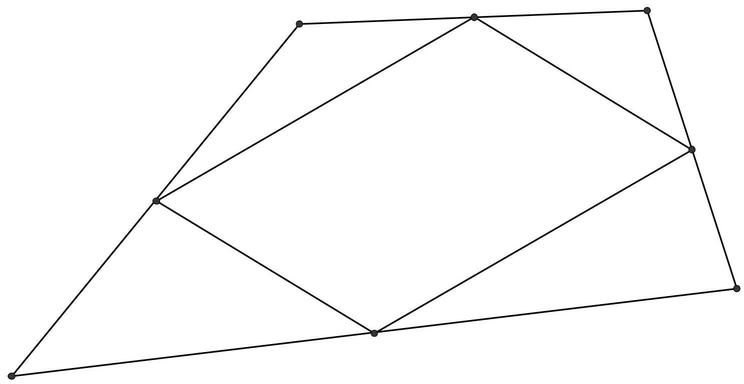
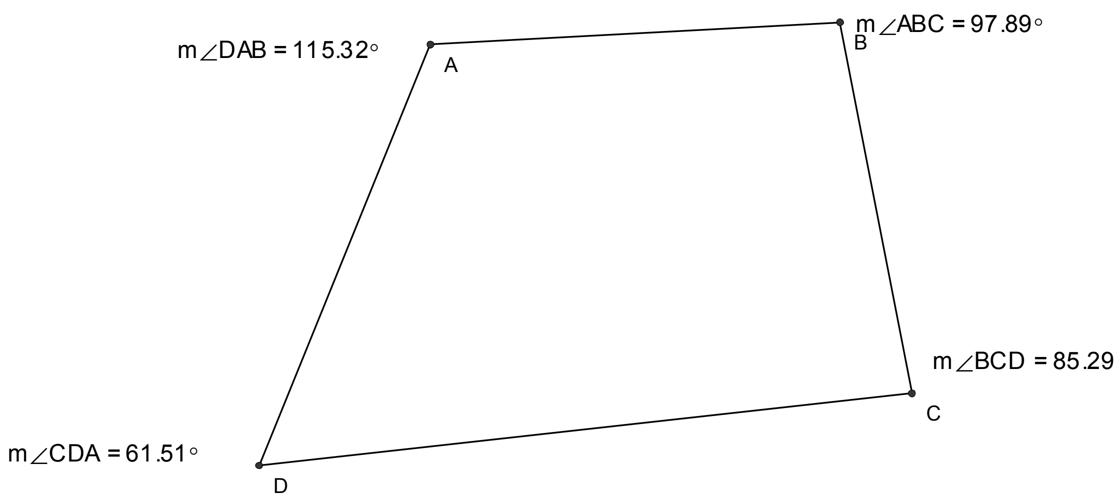


Figure 2: General quadrilateal

Conjecture: A quadrilateral will have at most two obtuse angles



$m\angle ABC + m\angle BCD + m\angle CDA + m\angle DAB = 360.00°$

Theorem: The interior angles of a quadrilateral will always add up to 360°

APPENDIX E

QUANTITATIVE DATA

The following tables show the results of all of the variables used in this study on both assignment cases. The first table shows the results of the odd lab activities where class A with 20 subjects conjectured in the dynamic environment and class B with 21 subjects conjectured in the static environment. The second table shows the results of the even lab activities with the environments reversed.

Input Data for the Odd Assignment Case

Subject	environment	relevant	false	conviction	gender	achievement	vh
A1	1	2.25	0	9.4	1	38	1
A2	1	2.75	0	10.0	0	37	1
A3	1	1.00	0	9.0	0	34	1
A4	1	1.00	0	5.0	0	31	1
A5	1	1.33	0	9.2	1	28	1
A6	1	1.33	0	6.7	0	32	1
A7	1	3.00	0	9.7	0	41	1
A8	1	2.50	0	9.1	1	32	1
A9	1	1.75	0	9.6	1	28	1
A10	1	3.50	0	9.3	0	36	1
A11	1	2.50	0	9.1	1	40	1
A12	1	1.25	0	8.6	1	24	1
A13	1	1.33	0	10.0	1	38	1
A14	1	2.67	0	9.4	1	46	2
A15	1	0.50	0	9.1	1	30	1
A16	1	0.33	0.33	6.8	1	32	1
A17	1	2.00	0	9.6	1	40	1
A18	1	2.25	0	9.8	0	26	1
A19	1	1.50	0	9.3	1	21	1
A20	1	1.25	0	8.0	0	32	1
B1	0	2.00	0	10.0	0	35	1
B2	0	1.75	.25	8.8	1		1
B3	0	1.75	0	9.2	1	45	2
B4	0	1.50	0	7.0	1		1
B5	0	0.00	0	6.6	0	22	1
B6	0	0.00	0	6.8	0	22	1
B7	0	2.00	0.5	7.5	0	41	1
B8	0	1.67	0.67	7.8	0	44	1
B9	0	1.33	0.67	7.8	0	45	2
B10	0	1.75	0.5	9.6	0	43	1
B11	0	0.00	0	5.5	0	42	1
B12	0	0.00	0	5.6	0	38	1
B13	0	0.00	0.67	7.6	0	36	1
B14	0	0.00	0.25	6.6	0	40	1
B15	0	1.00	0.5	8.2	0	42	2
B16	0	0.25	0	6.3	0	27	1
B17	0	2.00	0.25	9.2	1	44	2
B18	0	0.67	0	9.7	1	31	1
B19	0	0.75	0.25	5.8	0	37	1
B20	0	0.00	0.5	6.0	1	28	1
B21	0	1.00	0	6.0	1	38	1

Input Data for the Even Assignment Case

Subject	environment	relevant	false	conviction	gender	achievement	vh
A1	0	0.25	0.25	8.4	1	38	1
A2	0	1.00	0	8.8	0	37	1
A3	0	0.75	0	9.3	0	34	1
A4	0	0.67	0.67	4.5	0	31	1
A5	0	1.33	0	7.8	1	28	1
A6	0	1.50	0	7.5	0	32	1
A7	0	1.50	0	10.0	0	41	1
A8	0	1.33	0	9.1	1	32	1
A9	0	2.00	0	9.7	1	28	1
A10	0	2.67	0.33	7.2	0	36	1
A11	0	1.25	1.00	8.3	1	40	1
A12	0	0.75	0.25	8.2	1	24	1
A13	0	2.00	0	10.0	1	38	1
A14	0	1.50	0.50	10.0	1	46	2
A15	0	1.00	0	8.9	1	30	1
A16	0	0.75	0.50	6.1	1	32	1
A17	0	1.00	0.50	4.2	1	40	1
A18	0	1.50	1.00	9.6	0	26	1
A19	0	1.00	0.33	6.5	1	21	1
A20	0	0.50	1.00	4.6	0	32	1
B1	1	2.50	0	10.0	0	35	1
B2	1	2.67	0	9.6	1		1
B3	1	3.00	0	9.4	1	45	2
B4	1	2.75	0	9.5	1		1
B5	1	1.75	0	9.2	0	22	1
B6	1	1.00	0	8.2	0	22	1
B7	1	4.00	0	9.4	0	41	1
B8	1	1.50	0	10.0	0	44	1
B9	1	2.33	0.33	8.9	0	45	2
B10	1	2.75	0	10.0	0	43	1
B11	1	0.50	0	10.0	0	42	1
B12	1	2.00	0.25	8.6	0	38	1
B13	1	2.00	0..5	9.3	0	36	1
B14	1	2.00	0.25	8.7	0	40	1
B15	1	1.00	0	7.8	0	42	2
B16	1	0..50	0	8.0	0	27	1
B17	1	1.75	0	9.1	1	44	2
B18	1	2.00	0	10.0	1	31	1
B19	1	2.75	0	9.7	0	37	1
B20	1	1.00	0.5	8.8	1	28	1
B21	1	1.75	0	8.6	1	38	1