An Investigation of Subgrade Differential Settlement on the Dynamic Response of the Vehicle Slab-Track System

Zhang, Xiaohui; Burrow, Michael; Shunhua , Zhou

[Link to publication on Research at Birmingham portal](#)

An Investigation of Subgrade Differential Settlement on the Dynamic Response of the Vehicle Slab-Track System

Xiaohui Zhang[1], **Michael Burrow**[2], and **Shunhua Zhou**[1]

[1] Key Laboratory of Road and Traffic Engineering of the Ministry of Education, Tongji University, People's Republic of China

[2] School of Civil Engineering, University of Birmingham, UK

Corresponding author: Xiaohui Zhang

Email: 1988xiaohui@tongji.edu.cn   tj_zxh@qq.com

Tel.: +86 13651976796   +44 07871621415

Mailing address:

 Room 438

 School of Transportation Engineering, Tongji University

 No. 4800, Cao'an Road

 Jiading District

 Shanghai

 China.

Abstract: The construction of non-ballasted slab railway track on existing subgrade soils, or on embankments, is at an early stage of development in Chinese railways. Developing appropriate standards for the allowable amount of subgrade differential settlement which takes into account the dynamic response of the train-track system is one of a number of issues that need to be addressed. To inform the development of such standards, a model based on the theory of vehicle-track coupling dynamics, which considers the self-weight of the track structure, was built to investigate how differential settlement, in terms of the amplitude, wavelength and position of the settlement along the track can affect various railway performance related criteria including ride quality, stability, vehicle safety and potential damage to the train wheel and the rail (i.e. forces at the wheel/rail contact and in the fasteners). The performance of the model was compared favorably with other widely used models described in the literature. The analysis of the study to inform design standards using the developed model demonstrated that the magnitude of the differential settlement influences passenger comfort the most compared to the other performance criteria. There exists for the CRTS I track-form considered a particular wavelength (8m for the specific conditions considered) which results in all measures of performance being at their maximum values. Further, the longitudinal position of the settlement waveform in relation to the joints between two concrete slabs, a factor which is not considered in design standards, was shown to influence component deterioration, passenger comfort and safety. The greatest propensity to cause component damage occurs when the beginning or end of the differential settlement waveform corresponds with the inter-slab joint of a concrete base. Accordingly, it is recommended that current design standards should be modified to specify appropriate combinations of amplitude, wavelength and position of the differential settlement which give acceptable measures of performance.

Key words: CRTS I slab-track; subgrade differential settlement; vehicle-track coupling dynamics; structure self-weight; dynamic response

1 INTRODUCTION

China has embarked on an extensive programme of building high speed railway lines and the current length of its high speed network of approximately 11,000 km (with a similar length planned), accounts for about half of the world's total. The majority of these lines have been built using non-ballasted track-forms on bridges, through tunnels or on pile reinforced embankments. Indeed slab-track because of its superior stability and low maintenance requirements also is increasingly becoming the track-form of choice in China for many other lower speed lines. However because of the expense required to build slab-track on engineered structures, less expensive solutions are being investigated including constructing the slab-track directly on the existing subgrade soils or on embankments where necessary. However, a major issue for such slab-track systems, no matter the line speed, concerns the allowable amount of differential settlement as this directly affects safety, passenger comfort and damage to the wheel-rail. Differential settlement can occur in slab-track particularly on soft subgrade soils due to changes in track support condition. Japan has developed the differential settlement control standards for non-ballasted railway, of 12.5mm for a 20 m length of track (i.e. a chord length of 20m) [1].In China, the Suining-Chongqing railway sets 20mm for a chord length of 20m as the control standard [2].

A number of authors have undertaken studies using numerical models of the train-track system to determine the limits of differential settlement (as a function of the amplitude and wavelength of the settlement) of slab track systems according to a number different limiting performance criterion (e.g. safety). For example, from the point of view of the concrete base, Zhou [3] and Chen P [4] utilizing a

three-dimensional finite element model (FEM) suggested that for an assumed settlement waveform of wavelength of 20m, the differential settlement should be limited to less than 15mm, depending on the tensile strength of the concrete slab. On this basis, Chen RP [5] considered the effect of temperature change on the deformation and tensile stress of the track plate and concrete base, and suggested a stricter limiting value for subgrade differential settlement. However Zhou [3], Chen P [4] and Chen RP [5] did not consider functional aspects of the track such as safety and passenger comfort. Liu [6] developed an FEM using beam, shell and spring elements to simulate a train traversing a foundation subject to differential settlement, but their model is unable to consider track geometry irregularities. In terms of passenger comfort, Cai CB [7] for an assumed settlement waveform of wavelength of 20m, utilizing a conventional vehicle and slab track coupled model and found that amplitudes not greater than 20mm would keep the vertical acceleration of the vehicle body to within acceptable limits. By means of a similar method, Cai XP [8] analyzed the dynamic characteristics of the train-track system due to subgrade differential settlement. However, both t Cai CB [7] and Cai XP [8] did not consider the effect of the train speed on the differential settlement limits. Utilizing a numerical dynamic model of the train slab-track system which assumed that the subgrade differential settlement was entirely translated into the rail surface differential settlement, Han [9] related differential settlement limits to train speed. However, the assumption made by Han regarding subgrade settlement and rail surface settlement results in track support conditions which differ significantly from those found in practice. Xu [10-11] developed a train-track model in which the vehicle was represented as a numerical multi-rigid vehicle system, and the track support system was represented as a three-dimensional FEM. Using this model Xu calculated the dynamic response of different kinds of non-ballasted track systems and suggested standards for the subgrade differential settlement of waveforms of a length of 20m.

A limitation of the studies described above is that they focus on limits of differential settlement solely as a function of the amplitude of the differential settlement and do not consider other potentially important parameters associated with settlement such as the wavelength and position of the differential settlement in relation to the slabs which make up many non-ballasted track systems. In addition, conventional train slab-track coupled dynamic analysis methods assume that the subgrade stiffness and damping is zero in the area where the differential settlement takes place. In such cases where the self-weight of the track structure is not considered, the track structure settles only when the train arrives (see Figure 1(a)). In reality the differential settlement already exists before the train arrives (see Figure 1(b)). Consequently, modeling the track in this way results in an incorrect dynamic response of the system and can therefore lead to incorrect estimates of allowable differential settlement.

(a) [Insert Figure 1(a).]

(b) [Insert Figure 1(b).]

Fig.1 Track structure diagram before the train arrives: (a) model that doesn't consider self-weight, (b) reality

To address the issues described above apparent in the identified existing studies, a model based on the theory of vehicle-track coupling dynamics [12], which considers the self-weight of the track structure, was developed to investigate allowable subgrade differential settlement as a function of the amplitude and wavelength and position of the settlement waveform. To determine the allowable settlement limiting performance criteria which consider stability, safety and potential damage to the train wheel and the rail were considered.

2 THE VEHICLE SLAB-TRACK COUPLED MODEL

2.1 Calculation model

The vehicle dynamics model is based on multi-rigid system dynamics theory described in the literature [12]. The rail is modeled as a simply supported Euler beam with self-weight; the track plate and the concrete base are modeled as the free-free Euler beam with self-weight; the emulsified cement asphalt mortar (CA mortar) and the subgrade are regarded as discrete spring-damping systems. In real environments under cyclic and environmental (temperature) loading, the CA mortar may tear from the track plate and the concrete base. Therefore to account for this, the stiffness and damper values in the proposed model were set to zero when subjected to tension. Through mechanical analysis, the Euler beam oscillation differential equation in the vertical direction considering the self-weight can be written as follows:

$$EI \frac{\partial^4 y(x,t)}{\partial x^4} + m_r \frac{\partial^2 y(x,t)}{\partial t^2} = F(x,t) + m_r g \qquad （1）$$

where $y(x,t)$ is the vertical displacement of the Euler beam; m_r is the mass of per unit length of the Euler beam; EI is the flexural rigidity of the Euler beam cross section and $F(x,t)$ is the external force.

The partial differential equations of the vertical vibration of rail, track plate and concrete base can be obtained by determining the external forces of the structures. Solving the fourth order partial differential equations requires the Ritz method [13], and the basic form of second order ordinary differential equations of the modal coordinates of rail, track plate and concrete base can be obtained. These are as follows:

Rail:

$$\ddot{q}_k(t) + \frac{E_r I_r}{m_{rr}} \left(\frac{k\pi}{l_r}\right)^4 q_k(t) = -\sum_{i=1}^{N} F_{rsi}(t) Y_k(x_i) + \sum_{j=1}^{4} P_j(t) Y_k(x_{wj}) + \frac{g}{k\pi} \sqrt{2m_{rr}l_r}(1-\cos k\pi) \quad k = (1 \sim NM) \quad (2)$$

Track plate:

$$E_s I_s l_s \beta_{sk}^4 T_k(t) + m_s l_s \ddot{T}_k(t) = \sum_{i=1}^{n_0} F_{rsi}(t) X_k(x_{si}) - \sum_{i=1}^{n_0} F_{cai}(t) X_k(x_{si}) + m_s g GS(k) \quad k = (1 \sim NS) \quad (3)$$

Concrete base:

$$E_b I_b l_b \beta_{bk}^4 B_k(t) + m_b l_b \ddot{B}_k(t) = \sum_{i=1}^{m_0} F_{cai}(t) D_k(x_{bi}) - \sum_{i=1}^{m_0} F_{fdi}(t) D_k(x_{bi}) + m_b g GB(k) \quad k = (1 \sim NB) \quad (4)$$

where N, n_0 and m_0 are respectively the number of fasteners, the number of coordinate nodes of one track plate, and the number of coordinate nodes of one concrete base; $F_{rsi}(t)$, $F_{cai}(t)$ and $F_{fdi}(t)$ are respectively the fastener force, the CA mortar reaction force and the subgrade reaction force; $P_j(t)$ is the wheel-rail contact force of the wheel j; $q_k(t)$, $T_k(t)$ and $B_k(t)$ are respectively the regular modal coordinates of the rail, the track plate and the concrete base; Y_k, X_k and D_k are respectively the orthogonal function department of simply supported Euler beam of the rail, free-free Euler beam of the track plate and free-free Euler beam of the concrete base; β_{sk} and β_{bk} are respectively the constants of the track plate and the concrete base; NM, NS and NB are respectively the modal orders of the rail, the track plate and the concrete base; $GS(k)$ and $GB(k)$ are respectively the additional functions of self-weight of track plate and concrete base, and their values are:

$$GS(k) = \begin{cases} l_s & (k=1) \\ 0 & (k=2) \\ \dfrac{e^{l_s\beta_{sk}} - e^{-l_s\beta_{sk}}}{2\beta_{sk}} + \dfrac{\sin l_s\beta_{sk}}{\beta_{sk}} - C_k \left(\dfrac{e^{l_s\beta_{sk}} + e^{-l_s\beta_{sk}}}{2\beta_{sk}} - \dfrac{\cos l_s\beta_{sk}}{\beta_{sk}} \right) & (k>2) \end{cases} \quad (5)$$

$$GB(k) = \begin{cases} l_b & (k=1) \\ 0 & (k=2) \\ \dfrac{e^{l_b\beta_{bk}} - e^{-l_b\beta_{bk}}}{2\beta_{bk}} + \dfrac{\sin l_b\beta_{bk}}{\beta_{bk}} - C_k\left(\dfrac{e^{l_b\beta_{bk}} + e^{-l_b\beta_{bk}}}{2\beta_{bk}} - \dfrac{\cos l_b\beta_{bk}}{\beta_{bk}}\right) & (k>2) \end{cases} \qquad (6)$$

where C_k is the coefficient of the free beam.

The wheel-rail contact force is solved using the nonlinear elastic contact theory developed by Hertz [12]. The fastener force and the CA mortar reaction force can be obtained as follows:

$$\begin{cases} F_{rsi}(t) = C_{pi}[\dot{Z}_r(x_i,t) - \dot{Z}_s(x_i,t)] + K_{pi}[Z_r(x_i,t) - Z_s(x_i,t)] \\ F_{cai}(t) = C_{ai}[\dot{Z}_s(x_i,t) - \dot{Z}_b(x_i,t)] + K_{ai}[Z_s(x_i,t) - Z_b(x_i,t)] \end{cases} \qquad (7)$$

where $\dot{Z}_r(x_i,t)$ and $Z_r(x_i,t)$ are respectively the vertical velocity and displacement of the rail; $\dot{Z}_s(x_i,t)$ and $Z_s(x_i,t)$ are respectively the vertical velocity and displacement of the track plate; $\dot{Z}_b(x_i,t)$ and $Z_b(x_i,t)$ are respectively the vertical velocity and displacement of the concrete base; C_{pi} and C_{ai} are respectively the damping of the fastener system and the damping of the CA mortar; K_{pi} and K_{ai} are respectively the stiffness of the fastener system and the stiffness of the CA mortar.

In the model suggested herein, the differential settlement is simulated through the subgrade reaction force:

$$F_{fdi}(t) = \begin{cases} C_{fi}\dot{Z}_b(x_i,t) + K_{fi}[Z_b(x_i,t) - z(x_i)] & else \\ 0 & when \ Z_b(x_i,t) \le z(x_i) \end{cases} \qquad (8)$$

where $\dot{Z}_b(x_i,t)$ and $Z_b(x_i,t)$ are respectively the vertical velocity and displacement of the concrete base; $z(x_i)$ is the subgrade differential settlement value; C_{fi} is the damping of the subgrade; K_{fi} is the stiffness of the subgrade.

Solutions of the vehicle model and the proposed slab track structure dynamics model considering self-weight all adopt the explicit integration method suggested in the literature [14].

2.2 Model verification

The model suggested herein was verified by a comparative analysis of the outputs calculated by the model with that: (1) computed by the railway vehicle and slab track vertical coupled software VICT for the vibration responses produced by a dipped welded joint excitation[12]; (2) computed by an FEM model for the static responses due to subgrade differential settlement regardless of wheel-rail contact force; (3) recorded in the literature[11] for the vibration responses due to subgrade differential settlement under a moving train load.

(1) VICT

Figure 2(a) shows the comparison of the wheel-rail contact force of the proposed model (with self-weight) and the VICT model (without self-weight) due to a dipped welded joint (5°) for a train travelling at a speed of 100km/h (i.e. without differential settlement). Note the stiffness of the fastening system is the dynamic stiffness from field test on the existing high speed railway lines. The difference in the wheel-rail contact force between the two models illustrates that the wheel-rail contact force is relatively unaffected by the structure's self-weight when there is no subgrade differential settlement. Figure 2(b) shows the comparison of the fastener force at the time when the wheel-rail contact force reaches a peak value. The lower part of Figure 2(b) shows that the difference in the resulting forces between the two models is approximately equal to 370 N per fastener, which is equal to the weight of the rail per fastener. Accordingly, this illustrates that the model proposed is

as accurate as the widely used VICT model, but at the same time can take into account of self-weight.

(a) [Insert Figure 2(a).]

(b) [Insert Figure 2(b).]

Fig.2 Comparison of the vibration responses of the case with no differential settlement: (a) wheel-rail contact force,

(b) fastener force

(2) FEM

The static responses due to subgrade differential settlement regardless of wheel-rail contact force can be calculated by assuming the wheel-rail contact force to be zero because the self-weight of the track structures and the concrete base is taken into consideration in the proposed model. Figure 3(a) shows the vertical displacement of the rail computed using the proposed model and an FEM suggested in the literature [3] due to differential settlement (assumed to be a cosine curve with a wavelength of 30m and an amplitude of 45mm) without train loading. By inspection of Figure 3(a), the vertical displacements in the two models show good agreement with a maximum difference of less than 5%. Figure 3(b) compares the fastener force of the two models in the area where the differential settlement takes place, in which the value that is greater than zero means the compressive force and the value that is less than zero means the tensile force. By inspection of Figure 3(b) the fastener forces computed by the two models show good agreement in magnitude and frequency. For example, the fastener forces in both models increase in magnitude rapidly at the inter-slab joints between adjacent concrete track plates and concrete base slabs.

(a) [Insert Figure 3(a).]

(b) [Insert Figure 3(b).]

Fig.3 Comparison of the static responses: (a) rail vertical displacement, (b) fastener force

(3) Existing literature

A comparison of the maximum forces and accelerations experienced by the vehicle body and track structural components due to subgrade differential settlement under a moving train load was made between the proposed model and that calculated by a FEM described in the literature [11]. The comparison is shown in Table 1 for a train travelling at a speed of 350km/h, with a differential settlement wavelength of 20m and amplitude of 20mm. In both models track irregularities were incorporated using the German railway spectra of low irregularity [12].

Table 1 Comparison of dynamic responses

Item	Proposed model	literature [11]	difference
The maximum acceleration of vehicle body (m/s^2)	1.323	1.217	8.7%
The maximum wheel-rail contact force (kN)	90.213	85.628	5.3%
The maximum fastener compressive force (kN)	47.934	46.352	3.4%
The maximum fastener tensile force (kN)	15.271	13.265	15.0%
The maximum CA mortar pressure (MPa)	0.138	0.128	7.8%

The differences in the dynamic responses are within 15%. The difference may be partly explained by the fact that the models are unlikely to have exactly the same track irregularity function and this would have resulted in different dynamic responses. The possible differences in track irregularity functions between the two models results from the randomized process associated with converting a railway track irregularity from the frequency to the time domain for use within the model.

It is evident from the above comparisons with the other models available in the literature that the proposed model suggested herein can calculate with a sufficient degree of accuracy the dynamic response of the railway vehicle and track structure components due to subgrade differential settlement. Compared to the VICT, the model takes the effect of track structure self-weight into consideration, which is more representative of the actual situation when the differential settlement occurs, and compared to the 3D FEM model, the model is much more efficient computationally but is still capable of computing the dynamic response of the railway vehicle and the track structures components to a sufficient degree of accuracy.

3   Subgrade differential settlement

In order to inform construction and maintenance standards and to suggest limits of differential settlement., a study was undertaken to determine functional performance criteria associated with train stability, safety and wheel-rail damage as a function of (i) the amplitude of differential settlement, (ii) the wavelength of differential settlement, and (iii) the position of differential settlement along the length direction of the track. The track was modeled as a straight section without any radius of curvature and the differential settlement was represented as a cosine function with a variable amplitude, wavelength and position along the track.

The measures of functional performance criteria chosen are:

(a). Stability: The maximum vertical acceleration of the train in keeping with a number of railway organizations was used as a measure of stability.   In Chinese design standards, an upper limit of the vertical acceleration 0.13g is specified [15].

(b). Safety: The likelihood of the derailment of a train is commonly measured by the axle load decrement ratio (PD) and an upper limit of 0.6 for the PD is specified in Chinese standards [15].

(c). Wheel and rail damage: The likelihood of excessive wheel or rail deterioration can be measured by the wheel-rail contact force and the maximum allowable value is suggested to be 250kN according to the British Rail research [16].

3.1 Amplitude of differential settlement

In order to study the influence of differential settlement on the three measures of track performance four values of the amplitude of the differential settlement, 10mm, 20mm, 30mm and 40mm were used together with three train speeds of 100km/h, 200km/h and 300km/h. The wavelength of the settlement profile was maintained at 20m. Note a wavelength of 20m is specified in a number of design standards [17].

Figure 4 shows the computed vertical acceleration for the four amplitudes chosen. It can be seen from the figure that the differential settlement causes the vehicle body to accelerate through two and half cycles. For all amplitudes the wavelengths of the first two complete cycles of acceleration are nearly constant with the speed of the vehicle, whilst the wavelength of the last half cycle (i.e. Δ) increases with speed, i.e. the distance that the vehicle body requires its acceleration to return to its initial state increases with the driving speed.

[Insert Figure 4.]

Fig.4 Vertical acceleration of vehicle body

Figure 5 shows the maximum vertical acceleration of the vehicle body as a function of the amplitude of the modeled differential settlement from which it can be seen that the maximum of the vertical acceleration increases with amplitude as expected, with a corresponding decrease in stability. The maximum acceleration for amplitudes of 10mm and 20mm for all three train speeds considered

is well below the maximum allowable limit stipulated in the Chinese standards (i.e.0.13g). However, for differential settlement amplitudes of 30mm and 40mm the stipulated limit is exceeded for speeds of 200km/h and 300 km/h.

[Insert Figure 5.]

Fig.5 Maximum vertical acceleration of vehicle body

Figure 6 shows the maximum computed PD value as a function of settlement amplitude. As is shown in Figure 6, the maximum PD value increases with the amplitude of the differential settlement, although the value in all cases is less than that stipulated in the Chinese standards.

[Insert Figure 6.]

Fig.6 Maximum axle load decrement ratio

Figure 7 shows the wheel-rail contact force for vehicle speeds of 100km/h and 300km/h. From Figure 7, the variation of the wheel-rail contact force, in the 20m section where the differential settlement has been modeled, increases with the amplitude of differential settlement and the fluctuation of the wheel-rail contact force is generally greater at a speed of 300km/h compared to a speed of 100km/h, although the maximum values in all cases however is much less than the 250kN suggested by British Rail.

[Insert Figure 7.]

Fig.7 Wheel-rail contact forces

3.2 Wavelength of differential settlement

As the concrete base in general has a higher stiffness than the soil subgrade, it resists to an extent subgrade settlement, and therefore prevents the rail from settling as much as the subgrade. The

wavelength of the differential settlement can affect this difference to a large extent and, ultimately, therefore the dynamic response of the system. Therefore to better understand the effect of the wavelength of the differential settlement, the functional performance of the system was investigated for 8 different wavelengths (5m, 6m, 7m, 8m, 10m, 15m, 20m and 30m) with an amplitude of 20mm at train speeds of 200km/h and 300km/h.

Figure 8 shows the maximum vertical acceleration as a function of the wavelength of subgrade differential settlement for train speeds of 200km/h and 300km /h. For a train speed of 200km/h it can be seen that the maximum vertical acceleration exceeds that stipulated in Chinese standards for wavelengths between 8m to 15m. When the train speed is 300km/h, the maximum vertical acceleration is equal to or exceeds the upper limit of 0.13g for the wavelengths investigated between 8m to 20m.

[Insert Figure 8.]

Fig.8 Maximum vertical acceleration

In terms of the overall trend, the maximum vertical acceleration at the two train speeds considered increases rapidly with wavelength up to a maximum value of 8m. Thereafter at both speeds the acceleration decreases. The reason for the increase and then decrease of acceleration with wavelength of the differential settlement seen in Figure 8 may be explained as follows. When the wavelength of the differential settlement is small, the deflection of the concrete base is less than the magnitude of the differential settlement because of the flexure rigidity of the concrete base, resulting in a gap between the concrete base and the subgrade. As the wavelength of the differential settlement increases, the deflection of the concrete base increases correspondingly, and with it the vertical acceleration, until the base contacts the surface of the subgrade. However, the wavelength of the rail

deflection increases with the wavelength of the subgrade differential settlement and results in the dynamic response reducing correspondingly. The increase in vertical acceleration due to the former effect predominates at wavelengths of differential settlement up to 8m. At higher wavelengths the effect on the increase in the wavelength of the rail deflection on reducing the vertical acceleration predominates.

From the analysis above, at a wavelength of 8m is the wavelength the base contacts the surface of the subgrade (i.e. the gap between the concrete base and the subgrade is zero). Because the height of the gap depends on the flexure rigidity of the concrete base cross section, the length of the concrete base and the track structure design, the particular wavelength (8m for the specific conditions considered) which results in the acceleration being at its maximum can be considered to depend in part at least on the design of the track structure and the concrete base.

Figure 9 shows the axle load decrement ratio (PD) as a function of the wavelength of the differential settlement for train speeds of 200km/h and 300km/h. As is shown in the Figure 9, the change of the axle load decrement ratio with wavelength is similar to that of the vertical acceleration, and both reach a peak value at wavelengths of 8m. For a train speed of 200km/h the axle load decrement ratio exceeds the limit of 0.6 (i.e. the Chinese standards) for wavelengths of between 8m and 10m. When the vehicle speed is 300km/h, the axle load decrement ratio exceeds the limit of 0.6 for wavelengths of between 7m and 15m.

[Insert Figure 9.]

Fig.9 Axle load decrement ratio

Figure 10 shows the wheel-rail contact force for a train speed of 300km/h for all wavelengths considered. It can be seen that the impact of subgrade differential settlement on wheel-rail contact force has a process of weakness after a first enhancement. The maximum of the wheel-rail contact force of 221kN occurs when the wavelength is 8m which is less than that of 250kN suggested by British Rail.

[Insert Figure 10.]

Fig.10 Wheel-rail contact force

3.3 Position of differential settlement waveform

CRTS Ⅰ slab track consists of discrete concrete sections (see Figure 11) and therefore the longitudinal position of the differential settlement waveform with respect to these sections may influence the dynamic response of the system.

In order to study the impact of the longitudinal position of the differential settlement waveform, 8 different positions of the simulated differential settlement waveform were considered within the model. The first position is such that the start of the settlement waveform corresponds to the beginning of a concrete base section and the last position of the waveform corresponds to the end of the section. It should be noted that there is a horizontal gap between any two adjacent concrete bases, so the first position of the waveform and the last position do not correspond. The other 6 positions of the waveform were arranged at equidistant intervals as shown in Figure 11. For each case the maximum amplitude of the settlement was 20mm and the wavelength of the waveform was 20m (to match the existing Chinese railway standards for differential settlement). Excitation was provided by

a train travelling at a speed of 300km/h and the track was modeled as being smooth to simply the analysis (i.e. the track irregularity spectrum was not taken into consideration).

[Insert Figure 11.]

Fig.11 Position of subgrade differential settlement

Figure 12 shows the vertical acceleration of the vehicle body and the axle load decrement ratio for the eight different cases as the train passes the position where the subgrade differential settlement occurs.   For all eight cases it can be seen that the waveform and the magnitude of the acceleration curves do not change appreciably, and that the computed axle load decrement ratios in all cases are within a range of between 0.23 and 0.29 (i.e. there is approximately a 20% difference in the decrement ratio depending on the location of the settlement waveform).   Whilst values of the decrement ratios are less than that stipulated in the Chinese standard (i.e. 0.6) it should be noted that the Chinese standard includes the effects of track irregularities and therefore any corresponding dynamic effects.

(a) [Insert Figure 12(a).]

(b) [Insert Figure 12(b).]

Fig.12 Dynamic responses: (a) vertical acceleration of vehicle body, (b) axle load decrement ratio

Figure 13(a) shows the wheel-rail contact force for each of the eight positions considered.   As shown in the Figure 13(a), the minimum and maximum values of the wheel-rail contact force are relatively unchanged for the eight cases, but the waveforms of the contact force in each case are significantly different.   Further it may be seen that the wheel-rail contact forces change markedly at the inter-slab joint between two adjacent concrete bases because the rail support condition changes at these locations.   This may be further understood with reference to Figure 13(b) which shows the

forces within the rail fasteners due to differential settlement alone (i.e. without train loading) and Table 2 which shows the maximum compressive and tensile forces in the fasteners with and without train loading.

(a) [Insert Figure 13(a).]

(b) [Insert Figure 13(b).]

Fig.13 Wheel-rail contact force and fastener force: (a) wheel-rail contact force (b) fasteners force without train loading

Table 2 Maximum fastener compressive and tensile forces

Position of settlement waveform	Without train loading		As the train passes through	
	Compressive force /kN	Tensile force /kN	Compressive force /kN	Tensile force /kN
1	24.75	12.58	47.94	16.58
2	17.41	13.13	39.79	17.41
3	18.95	12.81	39.13	15.8
4	21.65	10.90	42.11	14.29
5	14.48	11.72	41.55	15.76
6	22.11	10.66	40.20	14.94
7	20.19	13.03	40.81	15.82
8	18.72	13.92	42.29	23.56

For all positions of the settlement waveform the maximum compressive force in a fastener is greater than the tensile force without train loading. The magnitudes of the compressive and tensile forces experienced by the fasteners are respectively greatest in case 1 and case 8 where the start of the settlement waveform corresponds to the inter-slab joint as described above. As the train passes through, the maximum compressive force in case 1 experienced by the fastener is between 13% and

22% greater than that in the other cases and the maximum tensile force in case 8 is 35% to 65% greater than that in the other cases.   Further analysis shows that some of the fasteners within the area of differential settlement are always in compression with and without train loading (see Figure 14(a)), whilst some fasteners experience both compressive and tensile forces as shown in Figure 14(b).   Those that experience both compressive and tensile cyclic loading forces are likely to have lower service lives than those which are subject to compressive forces alone [18].

(a) [Insert Figure 14(a).]

(b) [Insert Figure 14(b).]

Fig.14 Fastener forces of (a) fasteners subject to compressive forces alone (b) fasteners subject to both compressive and tensile cyclic forces

4 Concluding discussion

A railway vehicle and slab track vertical coupled dynamics numerical model was described in this paper and was shown to give similar results to three different existing railway vehicle/ track dynamic models in terms of the forces and accelerations experienced by the vehicle body and track structural components.   The proposed model has an advantage over these and other existing widely used dynamic models of the train track system in that it is more computationally efficient and therefore can be used in environments which do not have access to the computing facilities which are required to run similar FEMs (i.e. outside of research establishments).

Nevertheless, it is recognized that further refinement of the developed model is both desirable and necessary.   The construction of non-ballasted slab railway track on existing subgrade soils, or on

embankments, is at an early stage of development in Chinese railways, so it was not possible to compare the outputs of the model directly with field measurements. However, when such data becomes available verification of the model via the comparison of predicted settlements with those measured in the field should help to refine the model and the parameters adopted. Additional work is also required to refine the way in which the rail-sleeper fastening system has been modeled, since although the fastener force model has been developed according to the literature, whereby the fastener force is the same in tension and compression, the fastener force in reality exhibits some nonlinearity especially with loading frequency and temperature change.

Subgrade dynamics, which may impact the dynamic response of the slab track system, have not been accounted for in the proposed model since the emphases of the paper is on the ride quality, stability, vehicle safety and potential damage to the train wheel and the rail (rather than the subgrade). For this purpose, the literature suggests that a model, such as that proposed, can still give sufficient accuracy for the task at hand, albeit not including subgrade dynamics. However, further investigation is recommended to ascertain both how the dynamic response due to differential settlement can be incorporated and how the impact of including subgrade dynamics may affect the accuracy of the model.

The developed model was used to carry out a number of studies to inform railway design standards with respect to allowable subgrade differential settlement under CRTS I type slab track systems. These studies investigated the influence of the settlement waveform, in terms of its amplitude, wavelength and position on measures of track performance associated with passenger

ride quality, railway vehicle safety and track component damage.   The following findings may be drawn from the analysis.

(1) The dynamic response of the train-track system increases with the amplitude of the differential settlement and the train speed.   Whilst the amplitude of the differential settlement affects all of the criteria investigated, its greatest influence is on passenger comfort (vertical acceleration of the vehicle body).   When the differential settlement amplitude is greater than 20mm for a wavelength of 20m the limit stipulated in Chinese design standards is exceeded for speeds of 200km/h and 300 km/h.

(2) The dynamic response of the system, in terms of stability, safety and damage to the wheel-rail, was shown to be a function of the wavelength of the subgrade settlement and that there exists for the slab-form considered a particular wavelength at which the measures of response are at their maximum values.   Therefore, for any particular slab-track form, the combinations of wavelength and settlement amplitude which cause the stability, safety and damage criteria to be at their maximum values should be used as design criteria, rather than the current situation where standards suggest a maximum allowable settlement amplitude for one wavelength only (e.g. 20 mm for a 20 m wavelength).

(3) CRTS I slab track consists of a number of discrete lengths of concrete slab and it was shown that the position of the settlement waveform in relation to the joints between two slabs influences greatly the wheel-rail and the fastener forces.   The position can also effect the magnitude of the measures of safety (PD) and passenger comfort by up to 20%.   The greatest potential to cause fatigue damage (and therefore possible early failure of track components) occurs when the beginning, or end, of the differential settlement waveform corresponds with the inter-slab joint of the concrete base.   Since such areas may be subject to water ingress and thereby softening of the underlying

subgrade promoting settlement, it is suggested that particular attention should be given to these areas in terms of monitoring condition and associated maintenance.

It may therefore be seen that when developing design standards for slab-track it is necessary to stipulate maximum values of the allowable settlement, in terms of its amplitude, wavelength and position (in relation to jointed slab-track) and that the allowable values should also be a function of train speed.

Acknowledgements

References:

［1］ **Zhou M**. The Influence of Slab Track and Train Operation Safety on Subgrade's Uneven Settlement. Master Thesis, Tongji University, 2011.

［2］ **Ministry of Railways of the People's Republic of China**. The design and technical condition of the non-ballasted track of the integrated test section of Suining-Chongqing railway. 2004.

［3］ **Zhou M, Gong QM, Wang BL,** and **Zhou SH**. Effect of differential settlement of subgrade on dynamic response of ballastless track. *Railway Standard Design*, 2010, **10**, 1-4.

［4］ **Chen P, Gao L** and **Ma MN.** Limited value of subgrade settlement and its influence on mechanical characteristics of ballastless track in high-speed railway. *Construction & Design for Project,* 2008, **05**, 63-66.

［5］ **Chen RP, Wang ZZ, Jiang HG** and **Bian XC**. Control standard of differential settlement in high-speed railway slab based on bending fatigue strength. *Journal of Zhejiang University (Engineering Science),* 2013, **05**, 796-802.

［6］ **Liu RB**. Analysis of Differential Settlement of Subgrade Effecting on Slab Track Dynamics. Master Thesis, Southwest Jiaotong University, 2008.

［7］ **Cai CB, Zhai WM,** and **Wang KY**. Calculation and Assessment Analysis of the Dynamic Performance for Slab Track on Sui-Yu Railway. *China Railway Science*, 2006, **27**, 17-21.

［8］ **Cai XP**. Dynamic characteristics of train-ballastless track under the conditions of differential settlement of subgrade. Advances in Environmental Vibration - Proceedings of the 5th International Symposium on Environmental Vibration, ISEV 2011, 2011, 647-652.

［9］ **Han YT** and **Yao L**. Analysis of the dynamic performance for slab track settlement on embankment. *Journal of Railway Engineering Society*, 2006, **27**, 17-21.

［10］ **Xu QY, Li B,** and **Zhou ZH**. Study on the limited value for the differential settlement of subgrade under CRTS-I type slab track. *China Railway Science*, 2012, **33**, 1-6.

［11］ **Xu QY, Li B,** and **Fan H**. Influence of differential settlement of subgrade on dynamic characteristic of train-ballastless track on subgrade coupling system. *Journal of Railway Science and Engineering*, 2012, **9**, 13-19.

［12］ **Zhai WM**. *Vehicle-Track Coupling Dynamics (Third Edition)*, 2007 (The Science Press, China)

［13］ **Li DX**. *Advanced Structure Dynamics (Second Edition)*, 2010 (The Science Press)

［14］ **Zhai WM**. Two simple fast integration methods for large-scale dynamic problems in engineering. *International Journal for Numerical Methods in Engineering*, 1996, **39**, 4199-4214.

［15］ **GB5599-85:1985**. Evaluation standard of the dynamic characteristic of railway vehicle.

［16］**Clark RA, Dean PA, Elkins JA** and **Newtons SG**. An investigation into the dynamic effects of railway vehicle running on corrugated rails. *J Mech Engng Sci*, 1982, **24**, 65-76.

［17］**Ministry of Railways of People's Republic of China.** Design specification of the integrated test section of Suining-Chongqing non-ballasted railway, 2004.

［18］**Shi J, Burrow M P N, Chan A H, et al.** Measurements and simulation of the dynamic responses of a bridge–embankment transition zone below a heavy haul railway line [J]. *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit*, 2013, **227**, 254-268.