

## **An investigation of the regional variations and frequency dependence of anelastic attenuation in the mantle under the United States in the 0.5–4 Hz band**

Zoltan A. Der, Thomas W. McElfresh and

Anne O'Donnell *Teledyne Geotech, 314 Montgomery Street, Alexandria, Virginia 22314, USA*

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**Summary.** Studies of teleseismic *P*- and *S*-wave amplitudes and spectra in the 0.5–4 Hz band show large variations in the attenuative properties of the upper mantle under the United States. The data indicate that attenuation is greatest under the south-western United States including, but not confined to, the Basin and Range province. The lowest attenuation prevails under the north central shield regions. The north-eastern part of the country, consisting of New England and possibly including a larger area along the eastern seaboard, is characterized by moderate attenuation in the mantle.

The level of the high-frequency energy in short-period seismic waves and the differences between *Q* values derived from short- and long-period data indicate that *Q* is frequency dependent. The form of frequency dependence of  $t^*$  compatible with the data in the 0.5–4 Hz range does not allow a rapid decrease of  $t^*$  with increasing frequency. Rather it supports a gradual decrease covering the broader 0.1–4 Hz range. The curves of  $t^*$  versus frequency, for shield-to-shield and mixed shield-to-western United States type paths are parallel with an average difference of 0.2 s in  $t^*$  in the short-period band, but may diverge towards the long-period band. For both curves  $t_p^*$  is below 1 s. For shield-to-shield paths  $t_p^*$  must be below 0.5 s at 1 Hz.

### **Introduction**

Anelastic attenuation in the mantle is closely related to the evolution of continents and oceans and is thus of great geophysical interest. Before we can understand the attenuation properties of the mantle there are a number of facts which must be established. We must put limits on the absolute value of energy losses due to attenuation, delineate the frequency dependence of absorption, outline regional variations and depth distribution of anelasticity and correlate these with other relevant geophysical variables. In this paper we shall attempt to do this for the mantle under the contiguous United States.

We shall discuss anelastic attenuation in terms of the quantity  $t^* = \int (dt/Q)$  where *Q* is the quality factor and the integral involving travel time *t* is taken along the seismic ray path.  $t^*$  is a path-dependent quantity, being a function of the *Q* variation in the Earth as a function

of depth and region. It also appears to be frequency dependent.  $t^*$  is a convenient parameter for characterizing attenuation because it changes little with epicentral distances greater than  $25^\circ$ . It can, therefore, be used as a single (although frequency-dependent) parameter to characterize regional differences for various types of paths described in terms of upper mantle structures under the source and receiver. For distances less than about  $20\text{--}25^\circ$ ,  $t^*$  increases with epicentral distance due to the fact that the body waves gradually penetrate the low-velocity–low- $Q$  layer in the upper mantle;  $t^*$  decreases slightly with distance beyond  $25^\circ$  as the angle of penetration of the upper mantle becomes steeper (Der & McElfresh 1977).

The contributions from the low-velocity zone (LVZ) that is coincident with the low- $Q$  layer, dominate the integral, and thus  $t^*$  becomes a property that depends almost entirely on the types of upper mantle structures the ray path crosses. Thus:

$$t^* \approx \int_D \frac{dt}{Q} + \int_U \frac{dt}{Q} + t_r^* \quad (1)$$

where  $D$  is the integral along the downgoing leg of the ray path through the LVZ, and  $U$  is the upgoing part.  $t_r^*$  is the contribution from the rest of the path through the crust and the lower mantle. The relative sizes of the three terms in equation (1) depend on the upper mantle  $Q$  structure. In regions with well-developed low-velocity–low- $Q$  layers, the up- and downgoing terms through the upper mantle dominate the contribution to the total attenuation. In case the low- $Q$  layer in the upper mantle is absent, the relative sizes of the upper and lower mantle contributions will be roughly proportional to the relative travel times, and the upper mantle contributions will be relatively small.

We shall limit our study to  $t^*$  measured at teleseismic distances and classified with respect to the types of upper mantle  $Q$  structure that the ray path crosses. We thus ignore second-order variations with epicentral distances that are small compared to the regional variations. Alternate representations of attenuation are less convenient. For example, if the average  $Q$  were used it would have to be specified as a function of travel time and other parameters. It should be understood therefore that  $t^*$  in this report always refers to values measured at teleseismic distances unless it is clearly associated with near distances.

In principle, the measurement of  $t^*$  should be simple. If one knows the amplitude of any sinusoidal signal component at the source one could measure the amplitude of the same component at a distant observation point and after correcting for geometrical spreading effects in the Earth, which at teleseismic distances are reasonably well known,  $t^*$  could be found directly from the ratio of the two amplitudes  $A$  at the receiver and the source respectively, by the formula

$$A = \exp(-\pi f t^*). \quad (2)$$

Frequency-dependence of  $t^*$  causes no difficulty with this approach since  $t^*$  can be computed separately for each frequency component. Unfortunately, absolute signal amplitudes at the source are greatly uncertain and near receiver effects also cause great fluctuations in the absolute signal levels observed at a receiver in the short-period band. The simple approach outlined above is therefore not practical.

Most published  $t^*$  estimates pertaining to the short-period band were derived by another method, by utilizing the fall-off rates of spectral ratios derived from the observed and estimated source spectra, or indirectly, by attempting to match observed waveform shapes with synthetic seismograms but disregarding the absolute amplitude. Such methods involve the comparisons of the spectral shapes, i.e. the measurement of the decrease of high-frequency content in the signal relative to the lower frequencies. This approach to measuring attenuation is made attractive by the fact that the spectral shapes of the seismic sources do not change rapidly with the event magnitude (Aki 1967; von Seggern & Blandford 1972) and the shapes of observed spectra, as discussed elsewhere in this report, are not sensitive to local

focusing that affects absolute amplitudes of signals severely. Practically all  $t^*$  estimates obtained by such methods assumed a constant, frequency-independent  $Q$ . If  $Q$ , or  $t^*$ , is frequency-dependent all such results will be biased and will yield an apparent  $t^*$ ,  $\bar{t}^*$  which is related to the absolute  $t^*$  in equation (2) by the formulae

$$\bar{t}^* = -\frac{1}{\pi} \frac{d(\log A)}{df} = t^* + f \frac{dt^*}{df} \quad (3)$$

where  $A$  is a spectral ratio, and  $f$  is frequency. All terms in these formulae are assumed to be functions of frequency throughout this paper. The slopes of the logarithm of the spectral amplitude ratio in a linear frequency plot,  $d(\log A)/df$ , is used usually to determine  $\bar{t}^*$ , and  $t^*$  can be determined in principle by solving the differential equation (3) using some appropriate absolute values of  $t^*$  at some frequency as well as the values of  $\bar{t}^*$  determined for each frequency. In practice since spectral ratios determined from data are quite variable in shape, only averages of  $\bar{t}^*$  over limited frequency bands can be obtained. Although the spectral ratios  $A$  are corrected for instrument response, the fact that  $\bar{t}^*$  has to be averaged over limited frequency bands results from the fact that spectral ratios have to be smoothed heavily for stability in the measured slope and the limited bandwidth of instruments does not allow the estimation of  $\bar{t}^*$  outside the main band of instruments due to system noise. Empirical  $t^*$  versus frequency relationships can be derived by drawing curves that conform with the measured average of  $\bar{t}^*$  in various frequency bands and some bounding values of absolute  $t^*$  usually at the long-period end of the spectrum. In this paper we shall use this approach.

In the rest of the paper we shall use the notation  $\bar{t}^*$  for averages of this quantity in various frequency bands. In each case the frequency band in question will be clearly implied. We are somewhat reluctant to introduce a new symbol  $\bar{t}^*$ , but we find it necessary in order to connect new frequency-dependent models of  $Q$  to historical  $Q$  estimates, and because  $\bar{t}^*$  is an easily and directly measurable quantity, while  $t^*$  is not, at least in the short-period band if  $Q$  depends on frequency. Besides, the differences in the frequency and waveform characteristics of signals due to attenuation can be more easily described in terms of an average  $\bar{t}^*$  over the band of an instrument than in terms of a frequency-dependent  $t^*$ . For the case of frequency-independent  $Q$ ,  $\bar{t}^*$  and  $t^*$  are the same.

Similar arguments apply to estimates of path  $t^*$  differentials. Assuming a constant  $Q$ , we obtain relative  $\bar{t}^*$  that may also be biased by the frequency dependence of the absolute  $t^*$  differentials between the paths in question.

Although equation (3) allows great differences between  $\bar{t}^*$  and  $t^*$  for rapidly changing  $Q$  with frequency, the available evidence discussed in more detail in the rest of this paper indicates that the frequency dependence is fairly gradual and the difference between  $\bar{t}^*$  and  $t^*$  is not likely to exceed a few tenths of a second. Moreover,  $\bar{t}^*$  appears to be fairly constant over wide frequency bands as evidenced by the long history of attenuation studies in limited frequency bands using a constant frequency-independent  $Q$ , in which no indications of frequency dependence were detected.

The reader may regard the notation  $\bar{t}^*$  as a device to identify the results which have been obtained by comparing the relative amplitudes of various spectral components and thus may be biased relative to the absolute  $t^*$  by the frequency dependence of  $Q$ .

### Factors influencing measurements of $t^*$ in the short-period band: constraints imposed on the regional variations and frequency dependence by short-period data

Short-period waves are subject to many well-known disturbing factors that complicate the measurements of attenuation. Therefore, we find it necessary to discuss the methodology

of measuring attenuation in the short-period band and to make the case that the effects of disturbing factors are small compared to the effect of  $t^*$  if the data from recording systems with high dynamic range are interpreted in the frequency domain.

While the function  $F = \exp(-\pi ft^*)$  describing diminution of wave amplitudes due to attenuation causes only a minor modification of spectra and amplitudes in the long-period band (0.001–0.1 Hz), it is an extremely sensitive function of  $t^*$  and frequency  $f$ , in the short-period (0.5–5 Hz) band. For example, a constant  $t_p^* \approx 1$  s implies a decrease of a  $P$ -wave spectrum by a factor of more than  $10^4$  between 1 and 4 Hz, effectively ruling out any observation of 4 Hz  $P$ -wave energy with all seismic recording systems currently in use. Since  $t_s^* = 4t_p^*$  the same formula gives a decrease of  $10^4$  of 2 Hz amplitude relative to 0.5 Hz. A constant  $t_s^*$  of 2 in this frequency range would thus make impossible the observation of 2 Hz  $S$ -waves from deep focus earthquakes.

For a given event, relative attenuation of a frequency component in the  $S$ -wave compared to that of  $P$  is governed by the factor  $\exp(-3\pi ft_p^*)$  that, again, decreases very rapidly with increasing frequency. Utilizing the extreme sensitivity of the spectral content of short-period body waves to  $Q$ , limits on the possible ranges of  $t^*$  can be easily stated in terms of the existence or non-existence of detectable signal energy at the high-frequency end of the spectrum. Such arguments will be repeatedly presented in this paper. Similarly, the reader can easily convince himself that other perturbing factors discussed in more detail below cannot change the spectra of short-period body waves in a way that is comparable in importance to  $Q$ .

We shall show that the high  $t^*$  values found by analysing long-period data cannot be valid in the short-period band even around 1 Hz, because they would lead to discrepancies of several orders of magnitude in the spectra of short-period  $P$ - and  $S$ -waves, and the waveforms would also have much longer periods than observed.

To observe the high-frequency seismic energy in the 2–5 Hz range, instrumentation with response peaked at high frequencies and with sufficient dynamic range is necessary. The LRSM, SDCS and SRO systems are suitable for this since their instruments peak around 3 Hz, and their dynamic range is that of either the FM analogue tape for LRSM (normally 40 dB) or the digital systems (80 dB) for SDCS and SRO. The instrumentation and recording systems of modern arrays should be equally suitable for this kind of analysis. On the other hand, the paper recordings of the WWSSN system are not. That system is peaked at lower frequencies and has a low transport speed so that even frequencies between 1 and 2 Hz often become blurred. Furthermore, it is very hard to digitize these records with any accuracy, and some advanced processing techniques such as deconvolution cannot be performed on them in the short-period range.

Changes in short-period body wave amplitudes and spectra can be brought about by the effect of near-surface sedimentary layers that can cause increases in amplitudes at 1 Hz by factors of 2 or 3 as well as relative enhancement of high frequencies. Crustal amplification can be estimated (Der, McElfresh & Mrazek 1979) and corrected for to some degree. We have found, having calculated responses of numerous crustal models, that flat layered models of most crustal structures do not appreciably change the average spectral slopes used for estimating  $t^*$ . Since flat layered models are especially apt to possess resonances due to multiple reflections of body waves, the absence of major biases in the spectral slopes in flat layered structures makes a biasing effect from non-parallel-layered structures quite unlikely. In such structures the transit times of multiple reflections are non-constant and directionally varying, excluding the possibility of sharp resonances. We do not claim that crustal structures do not constitute a perturbation of spectral fall-off rates, but the available evidence suggests that this effect is small compared to the effect of a change in  $t_p^*$  of the order of 0.1 s at most sites. For hard rock sites, such effects are probably even smaller.

Another effect larger than that of local near surface layering, is due to lateral inhomogeneities in the crust. The recent COCORP studies, for example (Schilt *et al.* 1979), show a large degree of lateral variability of seismic wave velocity in the crust. The velocity inhomogeneities manifest themselves in teleseismic studies as large, azimuthally varying, relative amplitude anomalies for closely spaced sensors. The relative amplitude patterns are repeatable for events at similar azimuths and distances, and the intersite modifications of waveforms are repeatable and describable in terms of transfer functions (Filson & Frasier 1972; Chang & von Seggern 1980; Butler & Ruff 1980; Lay, Minster & Ruff 1979). Focusing by near source velocity variations causes broad world-wide variations in amplitude that are modified at each site by receiver inhomogeneities. The result is that although amplitude patterns from limited source regions observed at common stations are remarkably stable, relative amplitudes from sources at slightly different locations have a large scatter. These phenomena were recognized at LASA, NORSAR and other arrays and have been analysed extensively (Berteussen *et al.* 1975; Capon 1974; Capon & Berteussen 1974; Christofferson 1975; Dahle 1975; Dahle *et al.* 1975; Haddon & Husebye 1978; Hadley 1979; Chang & von Seggern 1980; and many others). Despite the difficulty in predicting amplitude fluctuations at individual sites, regional averages of magnitude residuals are still meaningful (Der *et al.* 1979).

While the wave shapes and amplitudes of teleseismic *P*-waves vary considerably with even small distances across arrays such as NORSAR, the slopes of the corresponding spectra appear to be quite stable in the 0.5–4 Hz range. For instance, at NORSAR the standard deviation of amplitude measurements is 0.4 expressed in ten base logarithm units, while the standard deviation of the spectral slopes (line fits to the spectral fall-off in a logarithmic amplitude-linear frequency scale) is only 0.06 s expressed in apparent  $t^*$  units. Considering NORSAR to be located on a fairly representative piece of the crust, these figures reflect the relative stability of the two types of data.

The stability of spectra relative to waveforms and amplitudes can also be explained in terms of some fundamental theorems of time series analysis i.e. Wold's theorem (Robinson 1967) that states that spectra of random or quasi-random superpositions of elementary wavelets approach the spectrum of the elementary wavelet. In practice even superpositions of a few wavelets usually produce spectra quite similar to that of the elementary wavelet.

The effects of surface reflections above the source can also bias the individual  $t^*$  measurements, especially if  $t^*$  is to be measured from the ratios of observed and source spectra. For earthquakes, *pP* and *sP* phases may be present. If a large number of events is used, the arguments of random superpositions of pulses can be applied to show that the resulting bias is small compared to the *Q* effect. For explosions, *pP* may distort the spectra. However, fitting of explosion spectra with variable reflection coefficients, *Q* and other parameters usually shows that the effective surface reflection coefficient is much smaller than one would expect assuming ideal reflection on a free surface. The causes may be that the effective reflection coefficient is diminished by an uneven surface or that the waveshapes are distorted by spall or multipathing. In most cases the spectral nulls expected from prior knowledge of the source depth and the uphole velocities cannot be found at all in the spectra. Except for extremely shallow explosions the *Q* effect can easily be separated from the effect of *pP*. For events of moderate depth with surface reflection delays exceeding 1 s relative to the primary *P*-wave, a scalloping of the spectra appears that does not interfere with the estimation of  $t^*$  since the overall fall-off rate of the spectrum is unchanged in the 0.5–4 Hz band.

For differences in  $t^*$  estimates between stations with data from shallow earthquakes, the surface reflections constitute a relatively minor perturbation if many events are used with varying source depths, and the differences of take-off angles from a given event to the various stations are small. In the case of a network of stations it is advantageous to avoid using

relative  $Q$  estimates from widely separated stations. We have found that averages from many earthquakes, regardless of source depths, yield the same relative  $t^*$  estimates as do better controlled sources such as explosions or carefully selected deep earthquakes. For estimates of absolute  $t^*$ , it is better to utilize explosions and deep earthquakes. For the most part, however, the  $Q$  effect overwhelms most other factors and extreme care in modelling the details of the source does not yield appreciable dividends in the 0.5–4 Hz band in improving the accuracy of attenuation measurements.

Local scattering is another process which might affect the spectra of  $P$ -waves taken over long time windows. Band pass filtering of teleseismic  $P$ -waves usually shows, however, that the envelope shape is essentially the same, within a factor of 2, for all bands in the 0.5–4 Hz range for the first 10 s into the signal (Der, McElfresh & O'Donnell 1980a). Any variability introduced by scattering would be small relative to the drastic effect of even moderate changes in  $t_p^*$ . In all short-period  $Q$  estimates, there may be some contribution from scattering, however, and the  $Q$  estimates in this paper may be lower than that in reality. Since one of the main purposes of this paper is to present data in favour of higher  $Q$  values than previously accepted, the fact that our  $Q$  estimates are high even including scattering further strengthens the case for higher  $Q$  in the short-period band.

Non-linearities in the recording systems can, in principle, also introduce high frequencies into recordings of signals that could conceivably cause errors in the estimation of  $t^*$  leading to too low values (Sacks 1980). We have concluded (Der *et al.* 1980a), however, that the level of high-frequency ( $f > 2$  Hz) energy in the recordings of short-period signals is considerably above the level that could be attributed to non-linearity. Fourier analysis of steady state calibration signals, where the input level is considerably above the amplitude of most real signals, shows harmonics well below the spectral level of recorded teleseismic signals. In addition, even moderate clipping, the most common form of non-linearity, is easily recognized and cannot cause the changes in dominant signals periods shown in this paper.

A non-linearity that would be hard to detect could be caused by a large low-frequency signal outside the main pass band of the instrument that would excite a high-frequency resonance in some part of the seismometer. This vibration could, in theory, be mistaken for a seismic signal. Since many short-period body waves are severely attenuated as compared to the long-period motion this possibility needs to be ruled out. However, examination of teleseismic 14–20 s Rayleigh waves from very large events as seen on short-period systems shows that if the short-period systems are driven by large long-period surface waves the high-frequency content of the seismogram does not increase during the time the long-period signal dominates the trace. This effectively rules out generation of short-period energy by non-linearities in significant amounts.

The fact that none of the perturbing factors listed above can be adequately modelled in the 0.5–4 Hz band causes scatter in all observations of short-period wave properties. This has to be reduced by some kind of averaging process if we desire to measure  $Q$  in the mantle. Experience indicates that the shapes of short-period wave spectra, especially the average fall-off rates of these with frequency, are primarily determined by the  $Q$  in the mantle. Regional variations of the spectral characteristics of short-period  $S$ -waves are quite pronounced and can be seen without any sophisticated analysis by inspecting the dominant period of these waves. Variations of  $P$ -wave spectra are more subtle, but become clear if averages of the spectral ratios of about twenty events are taken. While several of the factors discussed above significantly affect the short-period wave properties, the sensitivity of these waves to  $Q$  is even more important, thus allowing one to measure mantle  $Q$  in spite of the many uncertainties in source spectra and local structures.

Aside from the greater statistical stability of spectra, our preference for spectral arguments is also based on the fact that amplitudes of short-period body waves may be

*systematically biased* due to near receiver focusing (Chang & von Seggern 1980) while spectra appear to be relatively unaffected by such factors.

### Constraints imposed on the regional variations and frequency dependence by short-period data

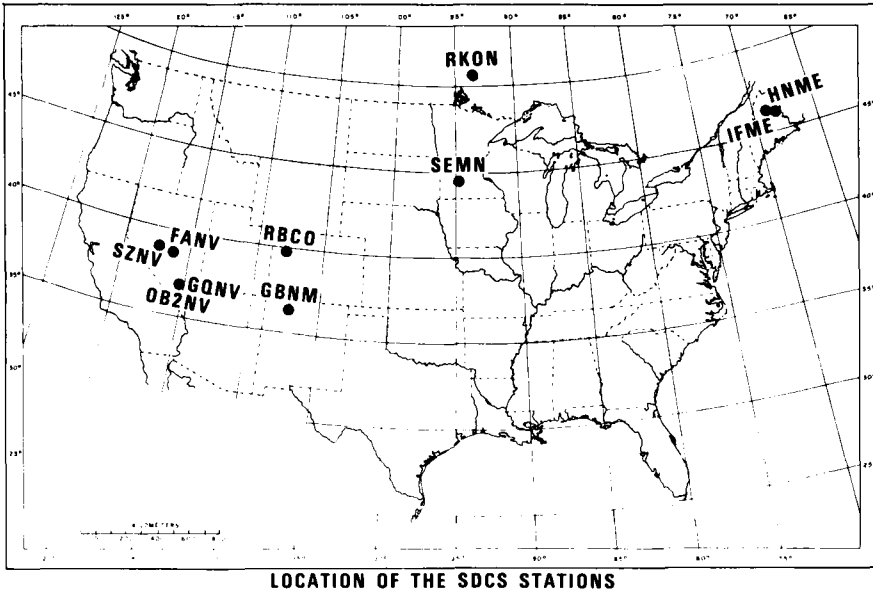
#### GENERAL REMARKS

This section will summarize work on short-period attenuation in the United States. Some of the data have been discussed already in the literature and we shall discuss in addition the results of the SDCS experiment, an extensive project to measure anelastic attenuation in the mantle under various US test sites. The areal coverage of the stations used for the experiment is limited and so is the regional detail provided, but since it covered numerous test sites in the western United States, the information provided is important for estimating yields of nuclear explosions. The most important data provided by the experiment is the large amount of spectral information in the 0.5–4 Hz range. Presented in a condensed form in terms of averaged relative  $t_p^*$  estimates, the amount of short-period spectral information included in this paper exceeds that of any other paper in the literature. Following the discussion of *P*-wave analyses we shall summarize data provided by short-period *S*-waves from deep earthquakes and outline regional variations of *Q* implied by the variations of their amplitudes and frequency content.

Following the discussion of the relative, regional variations of *Q* under the United States we shall summarize some spectral characteristics of short-period body waves that limit the allowable range of the *absolute*  $t^*$ . These constraints will be used in the latter part of the paper to outline the frequency dependence of  $t^*$ .

#### THE SDCS EXPERIMENT

A considerable amount of the data we used for this paper comes from stations of the SDCS (Special Data Collection System) project, funded by the Defense Advanced Research Projects



LOCATION OF THE SDCS STATIONS

Figure 1. Map of the SDCS and LRSM stations discussed.

Agency (DARPA). The seismic stations were deployed at the sites shown on the map in Fig. 1. These stations were occupied for an average span of six months each, in various overlapping configurations to allow the determination of relative magnitude and spectral characteristics among all stations. Two of the sites discussed OB2NV, and GQNV, were located at the Nevada Test site on the granite of the Climax and Gold Meadows stocks respectively.

The stations recorded digitally with a 20 cps rate, or on analogue tape. Descriptions of the stations are given in Table 1. All SDCS stations were equipped with essentially the same type of short-period instrumentation having the overall LRSM short-period system response, peaking in the 3–4 Hz range. Through regular calibrations the instrument responses were kept within 10 per cent of the specified desired responses. The same calibration procedures were applied to the standard LRSM stations analysed in this study. Of all the stations analysed, only HNME had a slightly different response, being less peaked towards the higher frequencies. We corrected for this difference in the spectral analysis, but the trace amplitudes and dominant wave periods at this station are not directly comparable to the rest of the stations. For further details we refer the reader to a more complete report (Der *et al.* 1980a). Since not all stations operated at the same time it was necessary to determine some inter-station magnitude,  $\bar{t}_p^*$  and amplitude differentials indirectly, i.e. through relative measurements relative to a third station. In estimating the confidence limits of the differentials in the various quantities this was taken into account.

Although the areal coverage of the SDCS stations is limited, the high data quality allowed us to measure  $P$ -wave spectral characteristics with a much greater accuracy than has been previously possible. The confidence limits for  $\bar{t}_p^*$  estimates (0.05 s or better at the 95 per cent confidence level) are narrower than in any other set of  $t^*$  measurements in the geophysical literature.

In the following discussions we shall frequently use the abbreviations WUS, EUS and NEUS to designate the western, eastern and north-eastern United States respectively.

#### P-WAVE AMPLITUDE ANOMALIES

It would be expected that in the regions underlain by a low- $Q$  mantle, the  $P$ -wave amplitudes from observed teleseisms would be reduced. This has been confirmed in several studies of magnitude variations across the US (Evernden & Clark 1970; Cleary 1967; North 1977; Booth, Marshall & Young 1974; Butler & Ruff 1980). All the various studies show a negative bias in amplitudes in most of the WUS relative to the EUS as a whole even though the methods of data selection and amplitude measurement vary, and the reported bias values for common stations also differ considerably. A statistical study (Der *et al.* 1979) of the residuals of Booth *et al.* (1974) showed that crustal amplification alone, although it contributes significantly to individual station residuals, cannot explain the total variance, and that an average residual magnitude differential of about 0.3 magnitude units exists between stations grouped into EUS and WUS populations. A recent study by Butler & Ruff (1980) deals with  $P$ -wave amplitude variations from nuclear explosions and a selected set of deep earthquakes across the United States. The amplitude patterns vary greatly among the various source regions thus clearly demonstrating the near source and azimuthally varying station focusing effects discussed previously. After averaging over various source regions, the data also demonstrate clearly that much of the WUS is characterized by low signal amplitudes relative to most of the central and eastern US. Again, with few exceptions, all results show a pattern of high values in central US and low values in the western US.

Due to the wider geographical coverage of the WWSSN stations in the Butler & Ruff study, additional features of the regional amplitude patterns are also evident besides the EUS–WUS



Table 1. SDCS station descriptions.

| Station | Location          | Operating period | SP components*                         | Recording† | Crustal material         | N lat | W long | Elevation (m) |
|---------|-------------------|------------------|--|------------|--------------------------|-------|--------|---------------|
| FANV    | Faultless site    | 6/77–9/77        | Z, N, E                                | A          | Alluvium & tuff          | 38.64 | 116.22 | 1920          |
| GBNM    | Gasbuggy site     | 5/77–8/78        | Z, N, E                                | A          | Sediments; igneous dykes | 36.69 | 107.23 | 2164          |
| GQNV    | Gold Meadows, NTS | 10/77–12/77      | Z, N, E                                | A          | Granite                  | 37.33 | 116.21 | 2057          |
| HNME    | Houlton, Me       | 2/75–3/76        | Z, N, E                                | D          | Slate                    | 46.16 | 67.99  | 213           |
| HNME    | Houlton, Me       | 4/76–8/78        | Z, N, E                                | A          | Slate                    | 46.16 | 67.99  | 213           |
| OB2NV   | Climax Stock, NTS | 8/76–2/78        | Z, N, E                                | D          | Granite                  | 37.23 | 116.06 | 1542          |
| RBCO    | Rio Blanco site   | 12/77–1/79       | Z, N, E                                | D          | Alluvium & sediments     | 39.81 | 108.36 | 1996          |
| RKON    | Red Lake, Ontario | 3/75–3/78        | Z, R <sub>nts</sub> , T <sub>nts</sub> | D          | Granite                  | 50.84 | 93.67  | 365           |
| IFME    | Island Falls, Me  | 11/77–8/78       | Z, N, E                                | A          | Granite                  | 46.03 | 68.20  | 232           |
| SZNV    | Shoal Site,       | 1/63–2/63        | Z, R <sub>319</sub> , T <sub>49</sub>  | A          | Granite                  | 39.20 | 118.38 | 1606          |
| SEMN    | Sleepy Eye, Mn    | 1/62–6/63        | Z, R <sub>73</sub> , T <sub>163</sub>  | A          | Granite                  | 44.41 | 94.67  | 244           |

\* Subscripts refer to radial and transverse orientations; nts indicates orientation toward the Nevada Test Site.

† A = analogue; D = digital.

areal difference. The low amplitude anomaly in the north-eastern United States that possibly extends along the eastern seaboard is clearly seen. The same anomaly was previously indicated in the studies of Solomon & Toksöz (1970) and Der, Masse & Gurski (1975). One somewhat less clearly defined feature is the maximum of  $P$ -amplitudes in a narrow strip along the Pacific Coast. A much broader feature of this kind is shown in the study of Solomon & Toksöz (1970) of long-period waves. Thus, all published studies of  $P$ -wave magnitudes show similar patterns of broad regional variations that require an explanation other than simple crustal amplification under the individual sites.

While we accept the results of Butler & Ruff (1980) on the regional patterns of amplitude distribution in the United States of teleseismic waves after averages over many events are taken to eliminate focusing at the receivers and sources, we do not agree with their contention that source,  $Q$  and receiver effects are indecipherable. The lack of even crude crustal corrections in their study make their statements about  $Q$  unreliable. Butler & Ruff (1980) conclude on the basis of their plots that there is no EUS–WUS differential in magnitudes. If one considers the fact that their figures do not reflect the uneven sampling of the United States by the stations they used, this conclusion appears to be inaccurate. The north-eastern United States is overrepresented among the EUS stations of Butler & Ruff and for this reason the plotted data gives the visual impression that tends to confirm their assertion. The available evidence indicates on the other hand that while most of the WUS is characterized by high attenuation in the upper mantle and most of the EUS by low attenuation, two narrow regions along both coasts appear to have only intermediate attenuation in the underlying mantle.

Amplitudes of  $P$ -waves from SDCS data sets essentially conform to the broad pattern outlined above. In general terms, the data consist of approximately 600 events recorded over the course of two years. Magnitude differentials between pairs of stations were computed by averaging over 40–200 events on the time interval over which both stations were in operation. Amplitudes at the sedimentary sites RBCO, GBNM and FANV were corrected for crustal amplification. The crustal corrections were computed by using a flat layered shallow structural model of the crust under the station and predicting the amplification of a synthetic  $P$ -wave pulse of predominantly 1 Hz at the surface relative to that measured over a granitic half-space. The procedure is described in more detail by Der *et al.* (1979). No corrections were applied to the granite or hardrock stations at RKON, HNME, IFME, SZNV, SEMN and GQNV. Fig. 2 shows crust-corrected relative magnitude residuals among the SDCS stations that give, as regional averages, low magnitudes in the WUS, and high magnitudes at RKON (shield). At IFME the magnitudes are much closer to those in the WUS.

The size of the average regional EUS–WUS  $m_b$  differential, 0.2–0.3 magnitude units, limits the possible range of regional variation in  $t_p^*$  to a few tenths of a second in the short-period band. A larger regional variation in  $t_p^*$  would cause much larger regional average differences in  $m_b$ . Although the above results are stated in terms of  $m_b$ , a parameter commonly used to determine yield of nuclear explosions, they are more meaningful stated in terms of raw trace amplitudes corrected only for gain levels and distance. The computation of  $m_b$  involves some quite peculiar interactions between wave periods and instrument response that tend to reduce and obscure the attenuation effect. Since the attenuated waves have somewhat longer dominant periods than the unattenuated ones, the instrument ‘correction’ for the dominant wave period will, in general, overcorrect the attenuated signal in a manner that reduces the magnitude differential. Division by the period in the  $m_b$  calculation enhances the differential again but not enough to compensate for the effect of the ‘correction’ for wave period. The problem with the  $m_b$  procedure is that it assumes that one deals with a monochromatic signal when in reality one does not. It can be demonstrated on

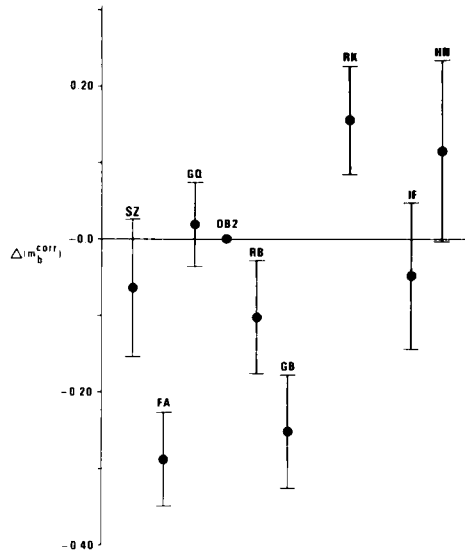


Figure 2. Crust corrected magnitudes measured at SDCS stations relative to OB2NV. The bars denote 95 per cent confidence limits. The stations are arranged in the order of decreasing longitude (west to east).

synthetic seismograms that the unattenuated signal may end up with a lower  $m_b$ . In the case of RKON, the average period of the  $P$ -waves we analysed is 0.7 s. At OB2NV it is 0.9 s. A plot of the histogram for differentials in base ten logarithms of trace amplitude, Fig. 3, gives 0.27 units, a figure also arrived at by Hart *et al.* (1979). Their magnitude residual at HNME is also comparable to ours.

It appears therefore that in spite of the differences in data selection and methodology, the results of the various studies are remarkably similar, and the various methods of presenting regional variations in  $P$ -wave energy show basically the same regional picture. Fig. 4 shows  $P$ -wave trace amplitude levels relative to OB2NV at the various SDCS stations after correction for estimated crustal amplification and distance. In this figure, we excluded IFME

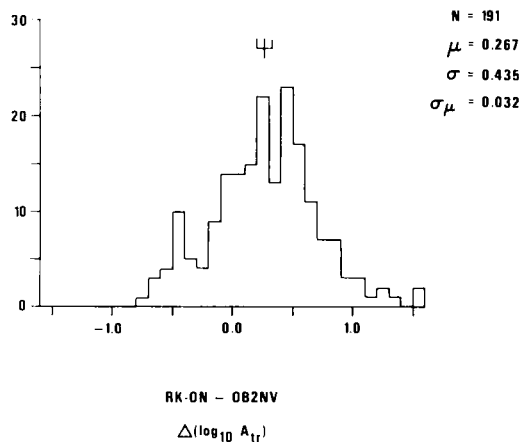
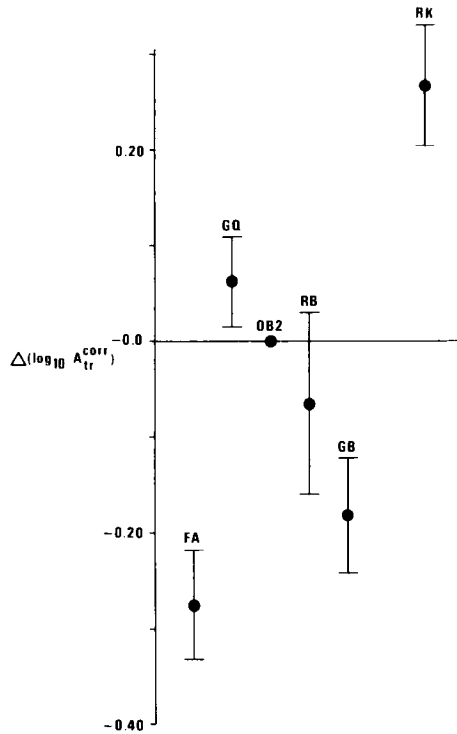


Figure 3. Histogram of trace amplitude differentials between RKON and OB2NV. Vertical line and bracket shows the location of the mean and its 95 per cent confidence limits.



**Figure 4.** Crust corrected trace amplitude differentials among selected SDCS stations expressed in  $\log_{10}$  (amplitude) units relative to OB2NV. Ninety-five per cent confidence limits are indicated by bars. Differentials for IFME and HNME were not computed due to the different instrument responses at the stations.

and HNME because the instrument response was different at HNME from the rest of the SDCS stations and IFME measurements were tied in using HNME. Since the magnitude calculation makes a crude correction for different instruments, only magnitude anomalies are quoted for these two stations in this paper. These amplitudes show essentially the same picture for the remaining stations as the magnitudes. The details of interstation variability of the amplitudes could not be associated with any known factor in a deterministic fashion, but such variability is not at all surprising in view of the lateral variability in the structure of the crust discussed above. In the following presentation of *SH*-wave amplitude differentials we shall also use trace amplitudes.

We have examined the magnitude residuals obtained from events in various source regions for indications of possible source region bias that may be due to preferred orientations of sources and other directional features. We were unable to find any indications of such effects, and it appears that the method of averaging magnitude residuals from many earthquakes does not have a demonstrable source region related bias, except for azimuth-dependent focusing near the receivers.

#### P-WAVE SPECTRAL ANOMALIES

Due to the high sensitivity of the high-frequency body waves to variations of  $t^*$ , spectral shapes are strong diagnostics of regional variations of attenuation. While body wave magni-

tude measurements involve mostly 1 Hz energy, our spectra of  $P$ -waves utilize energy up to 4 Hz in a wide band that is considerably more sensitive to variations of  $Q$  than 1 Hz energy.

In previous papers (Der & McElfresh 1976a, b, 1977), we compiled  $\bar{t}_p^*$  estimates for a group of paths crossing the mantle under the United States, mostly for nuclear explosions. Paths not crossing under the WUS were associated with signals of significant energy in the 3–4 Hz range. Paths crossing the mantle under the WUS showed an absence or great reduction of high-frequency energy. The  $\bar{t}_p^*$  for shield paths was around 0.2 and for paths crossing the upper mantle under the WUS it was in the 0.4–0.5 range. None of our data implied an apparent  $t_p^*$  equal to or larger than 1 s, although it is possible that some paths from the WUS to other tectonic regions might have  $\bar{t}_p^* \approx 1$  s. Spectral ratios computed at the SDCS stations fall into the pattern outlined above. The relative  $\bar{t}^*$  were computed from spectral ratios of the first 7 s of the  $P$ -waves from common events at various stations. The  $P$ -wave windows were tapered with a Parzen window and Fourier transformed. The Parzen window has very low spectral side lobes, thus eliminating any deleterious truncation problems. We obtained  $\bar{t}_p^*$  from smoothed spectral ratios by fitting a straight line to the log (amplitude)–linear frequency plots.

The noise spectra prior to the  $P$ -wave arrival were also computed and the sections of the spectral ratios where the power ratio of signal-to-noise exceeded 3:1 were fitted with straight lines in a log-ratio linear frequency coordinate system. For the overwhelming majority of the events, the S/N ratio was considerably better than the 3:1 limit. Instrumental corrections were applied for station pairs with unmatched instruments; for the rest, the instrument responses cancel. Only the slopes of spectral ratios were utilized in estimating  $\bar{t}^*$ , since the absolute amplitude ratios were extremely variable.

Since some stations were operational only for periods not exceeding six months, all events with sufficient S/N ratios were utilized. In this we had little choice since the variability of spectral shapes, although less than that of amplitudes, had to be overcome to arrive at statistically stable estimates of relative  $\bar{t}^*$ . Fig. 5 shows the range of  $\bar{t}^*$  measurements with their 95 per cent confidence limits relative to the NTS station LB2NV, on granite.

The figure shows that all stations in the western United States are similar in that  $P$ -waves observed there have less high-frequency content (higher  $t^*$ ) than the shield station, RKON.

The two stations in the north-eastern US, HNME and IFME, have a significantly lower  $\bar{t}^*$ , that is, less attenuation than the WUS stations. While the amplitude data presented earlier

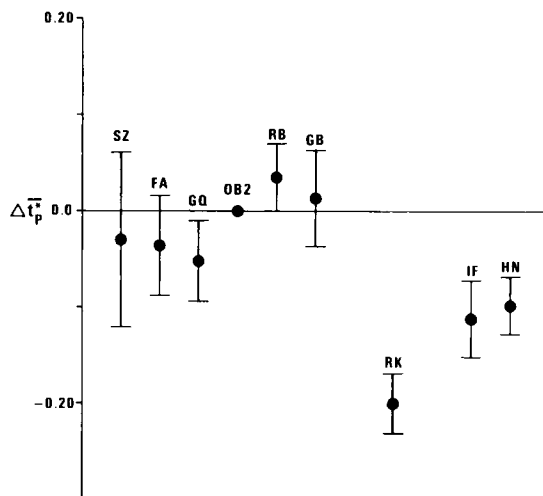


Figure 5.  $\bar{t}^*$  measured at the SDCS stations relative to OB2NV. Bars denote 95 per cent confidence limits.

do not show a significant difference between the WUS and NEUS stations in amplitudes, spectra show a significant difference indicating that the stations HNME and IFME occupy an intermediate position with respect to mantle  $Q$  between the WUS stations and RKON. Comparing the scatter in  $\bar{t}_p^*$  measurements to those of  $m_b$ , one finds that the  $\bar{t}_p^*$  scatter is considerably less compared to the amplitude anomalies we are measuring. Fig. 6 shows a histogram of  $\bar{t}_p^*$  differentials between OB2NV and RKON. Out of the 90 relative  $\bar{t}_p^*$  measurements, only six show more absorption at RKON than at OB2NV, and the standard deviation of the population is 0.15. The mean  $\bar{t}_p^*$  differential is  $0.20 \pm 0.03$  (95 per cent confidence limits). For comparison, the  $m_b$  differentials in Fig. 7 have a standard deviation of  $0.45 m_b$  units for the same station pair, standard deviation for the population of relative  $m_b$  measurements, indicating the inherent instability of wave amplitudes in the short-period band. The relative  $\bar{t}_p^*$  values of the order of 0.2 s arrived at for the OB2NV-RKON pair agree well with those we obtained previously by computing  $\bar{t}_p^*$  and their differential from ratios of observed  $P$ -wave spectra and estimated source spectra for nuclear explosions (Der & McElfresh 1976, 1977). The tight confidence limits given here for  $\bar{t}_p^*$  are without precedent in the geophysical literature.

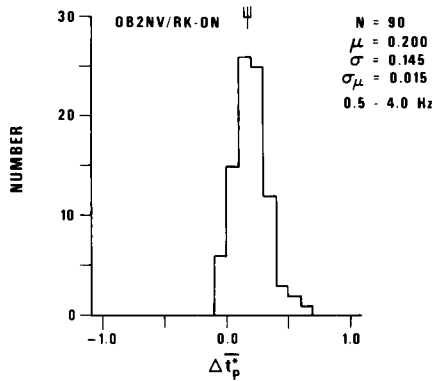


Figure 6. Histograms of  $\bar{t}_p^*$  differentials between OB2NV and RKON indicating a loss of high-frequency content in  $P$ -waves at NTS station OB2NV.

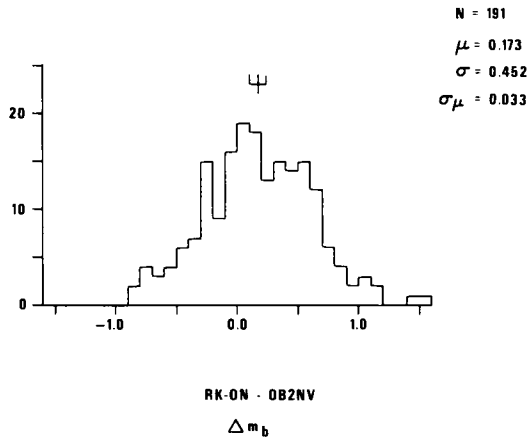


Figure 7. Histogram of  $m_b$  differentials for the RKON-OB2NV station pair. This figure demonstrates the extreme scatter in  $m_b$  measurements.

We have also fitted the spectral ratios in the 0.5–2 Hz range for the OB2NV–RKON station pair. The  $\bar{t}_p^*$  difference for this group of measurements is  $0.24 \pm 0.06$  s (95 per cent confidence limits). This result is not significantly different from the fits in the 0.5–4 Hz range, a fact that imposes some weak constraint on the frequency dependence of the  $\bar{t}_p^*$  difference between RKON and OB2NV. The  $\bar{t}^*$  differentials deduced in the 0.5–4 Hz band from spectral ratios are also reflected in the dominant periods of the signals (as seen in the time domain). The relative difference in the dominant periods of the *P*-wave for the RKON–OB2NV pair was found to be  $0.200 \pm 0.036$  s (95 per cent confidence limits). The measurement of *P*-wave periods was greatly facilitated by the digital displays and high time resolution films available for the SDCS stations. The average period read to RKON was 0.7 s while at OB2NV it was 0.9 s.

Again, we examined the possibility that frequency-dependent source directivity may bias our relative  $\bar{t}_p^*$  values. We were unable to find any pattern in the distribution of residuals in  $\bar{t}_p^*$  with respect to source regions that may indicate such effects. Our previous work using mostly nuclear explosions (Der & McElfresh 1977) as sources also yield  $\bar{t}_p^*$  comparable to those found in this study. We are therefore confident that the relative  $\bar{t}_p^*$  values in this study are not contaminated significantly by source directionality effects.

### S-WAVE AMPLITUDE ANOMALIES

An additional measure of regional attenuation is provided by amplitude anomalies of short-period *S*-waves. If regional *P* magnitude anomaly patterns are caused by variations of *Q*, the regional amplitude anomaly patterns of *S* should resemble those of *P*. This appears to be the case. In Fig. 8 we show averaged *SH* trace amplitude residual terms expressed in units of  $\log_{10}$  (amplitude) from seven deep earthquakes listed in Table 2. The *SH* amplitudes were read from film using the component closest to the transverse to the event (the seismographs

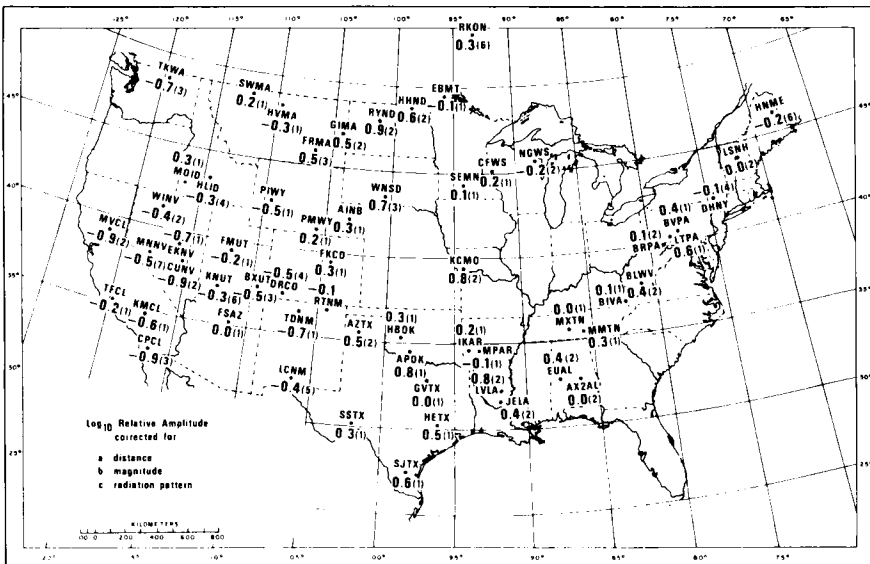


Figure 8. Map of relative *SH*-wave trace amplitudes (in units of base ten logarithms) for seven deep earthquakes listed in Table 2. The amplitudes were normalized with respect to event magnitude and corrected for instrument gain and double couple radiation patterns. Small numbers in parentheses indicate the number of events considered for each station.

Table 2. Deep earthquakes analysed.

| No. | Date          | Origin time | Location           | Depth (km) | Latitude | Longitude | $m_b$ | Focal mechanism |       |        |
|-----|---------------|-------------|--------------------|------------|----------|-----------|-------|-----------------|-------|--------|
|     |               |             |                    |            |          |           |       | Rake            | Dip   | Strike |
| 1   | 1962 Dec. 08* | 21:27:18.0  | NE Argentina       | 620        | 27.0S    | 63.0W     | 7.0   | 241.21          | 85.02 | 164.75 |
| 2   | 1963 Nov. 09* | 21:15:30.0  | Western Brazil     | 600        | 9.0S     | 71.5W     | 7.0   | 258.10          | 66.89 | 104.01 |
| 3   | 1963 Nov. 10* | 01:00:38.8  | Peru-Brazil border | 600        | 9.2S     | 71.4W     | 5.6   | 259.92          | 61.47 | 126.60 |
| 4   | 1964 Mar. 18* | 04:37:26.9  | NW of Kuriles      | 438        | 52.5N    | 153.6E    | 5.6   | 270.00          | 85.00 | 50.00  |
| 5   | 1965 Nov. 03  | 01:39:03.1  | Peru-Brazil border | 583        | 9.1S     | 71.4W     | 6.2   |                 |       |        |
| 6   | 1966 Aug. 22* | 14:21:14.0  | Sea of Okhotsk     | 653        | 50.3N    | 147.7E    | 5.1   | 270.00          | 90.00 | 20.00  |
| 7   | 1966 Nov. 22* | 06:29:53.1  | Sea of Okhotsk     | 469        | 48.0N    | 146.8E    | 5.7   | 90.00           | 55.00 | 190.00 |
| 8   | 1967 Oct. 12* | 12:53:46.9  | NW of Kuriles      | 483        | 52.2N    | 152.5E    | 5.5   | 298.71          | 79.16 | 43.10  |

\* Events used for Figs 8 and 9.



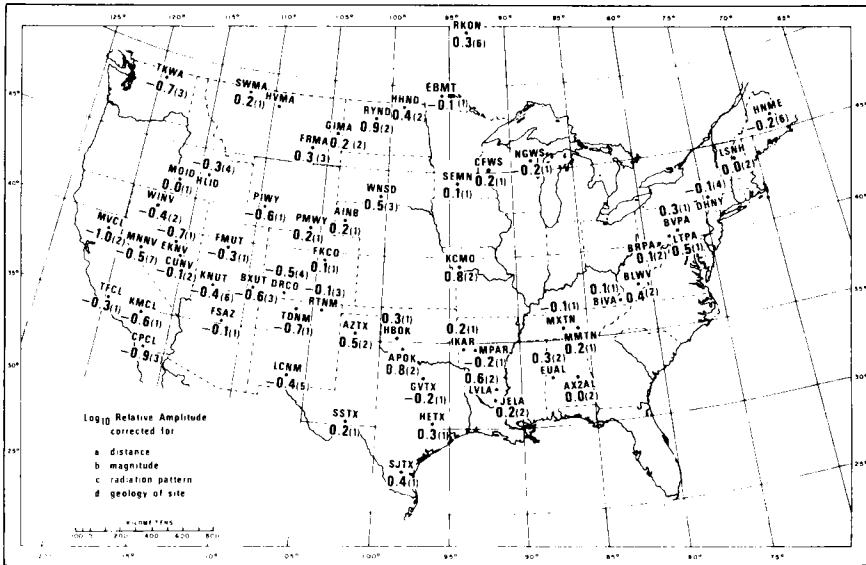


Figure 9. Map of relative *SH*-wave trace amplitudes (expressed in units of base ten logarithms) for seven deep events. In addition to the corrections in Fig. 8, approximate corrections for local crustal amplification were also applied. Small numbers in parentheses indicate the number of events considered for each station.

were oriented radially and transversely with respect to the azimuth to NTS). We have not used any data from stations close to the nodal lines of the *SH* radiation pattern to avoid overcorrections due to imprecision in the focal plane determinations. This also reduces the chance for major errors due to the misorientation of a few instruments relative to the true transverse direction. Since the *SH* component never becomes too small relative to the *SV*, no major overestimation of the *SH* component was possible due to misorientation of the instrument by inclusion of too much of the *SV* component. *SH* could have been underestimated, on the other hand, by a factor of  $\cos(\pi/4)$  in a few cases, but this factor is insignificant compared to the order-of-magnitude areal variations due to *Q* and the usual site effects due to focusing. The misorientation may have the most effect at a few stations in Texas and in southern California due to NNW–SSE direction of the sources. The amplitudes were also normalized with respect to event magnitude for each event. The map clearly shows a pattern similar to that of *P*-waves (Booth *et al.* 1974). Adjustment of amplitudes for crustal amplification effects on *SH* using flat layered models of the crust under LRSM stations (Fig. 9) does not remove the anomalies. The approach used to adjust amplitudes is identical to that presented by Der *et al.* (1979). This procedure in which a short-period (1 s) *SH* pulse is applied to a layered model of the crust, tends to overstate the crustal effect because the actual *SH* pulse is longer at most stations and is less sensitive to crustal effects. It is significant, therefore, that this correction does not eradicate the regional amplitude anomalies of *SH*-waves, thus indicating that the anomalies are not due to the crustal structure. The regional pattern of amplitudes clearly indicates a low amplitude region in the south-western United States, but the western part of the country as a whole is also characterized by low amplitudes compared to the east. Indications of a reduction in amplitudes in the north-eastern United States, if it exists, is not as pronounced as in the SWUS. There is considerable scatter undoubtedly due to effects similar to those affecting *P*-amplitudes.

## S-WAVE SPECTRAL ANOMALIES

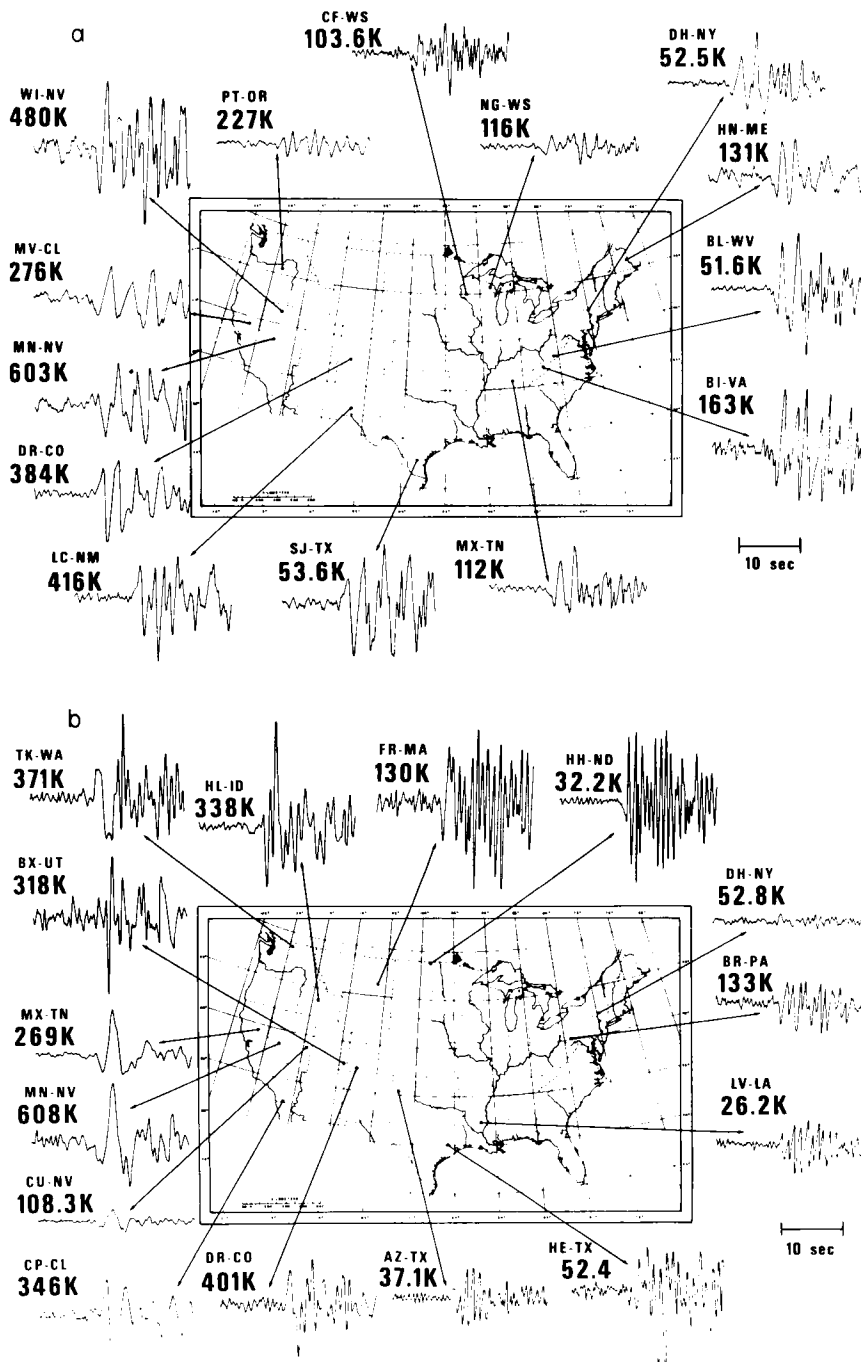
Since, according to most evidence, the relation  $t_s^* \sim 4t_p^*$  also appears to be valid for regional changes in  $t^*$  (Solomon & Toksöz 1970; Der, Smart & Chaplin 1980b), spectral changes in  $S$  should be quite noticeable from region to region. This appears to be true. In Fig. 10(a, b) we show tracings of short-period  $S$ -waves at various LRSM stations across the US. Since one of the (perpendicular) horizontal components of the LRSM stations is oriented towards NTS, the tracings were done on the component closest to the transverse direction ( $SH$ ) to the event. Although a few of these components may be misoriented by as much as  $45^\circ$ , the figures show a clear tendency for the stations in the south-western US to show lower amplitudes (instrument gains are given above each trace) and lower frequencies. These are typical examples of this phenomenon and are not accidents of fault plane orientation and directivity. All  $S$ -waves from earthquakes which have high frequencies at the source, as evidenced by the presence of high-frequency energy in the east, show this phenomenon regardless of fault plane orientation. For some events, signal energy may be seen up to 2.5 Hz in the EUS. Inspection of the frequency content of these waveforms (and the many other examples not shown here) enables one to outline the regional variations of  $Q$  under the United States). The stations in the north central United States have the highest frequency content, and those in the Basin and Range, as well as in the south-western United States in general, have the lowest. The north-eastern United States and the rest of the western United States are intermediate in mantle  $Q$ .

The recent study of Lay & Helmberger (1980) also presents short-period  $SH$  waveforms for a large number of South American events displaying striking differences in the frequency content of signals at WWSSN stations that have essentially similar regional distribution across the United States. This also confirms our interpretation of these differences as  $Q$  effects rather than source directionality or site effects. This demonstrates the validity of our claim that the frequency content of teleseismic short-period signals,  $S$  especially but  $P$  also, is determined by mantle  $Q$  and not by the many extraneous factors discussed above.

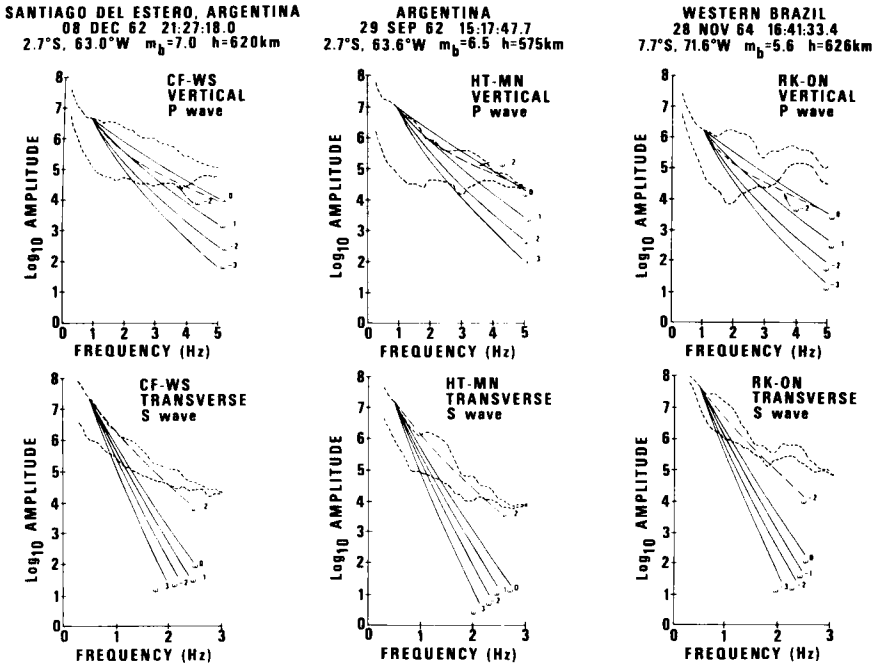
## SUMMARY OF THE REGIONAL STUDY OF SHORT-PERIOD BODY WAVE AMPLITUDE AND SPECTRAL ANOMALIES

In spite of the extreme variability and scatter in the short-period data, detailed analysis of amplitude and spectral anomalies shows that these anomalies are caused by lateral variations of  $Q$  for body waves in the mantle under the United States. Within the general pattern of regional variations, the Nevada Test Site (NTS) appears to occupy a position similar to several other WUS stations. The stations HNME and IFME appear to have characteristics intermediate between the WUS stations and those in the north central parts of the country.

Although the  $t_p^*$  variations we found in the short-period band of about 0.2 s are smaller than the 0.5 s found by Lay & Helmberger (1980), these values may not be inconsistent with each other. One must remember that the result of 0.5 was obtained in the 0.03–1 Hz band while ours was defined in the 0.5–4 Hz band, and the data set of Lay & Helmberger also included some stations with possibly extremely low mantle  $Q$  along the southern Rocky Mountains. The various studies of amplitude and spectral anomalies of short-period teleseismic  $P$ - and  $S$ -waves show that the amplitude anomalies of both types of waves agree in magnitude with the relative  $t^*$  derived from spectral slopes (Der & McElfresh 1977; Der *et al.* 1979) in the 0.5–4 Hz band within the limits imposed by uncertainties of crustal structures and other factors.



**Figure 10.** (a) Film tracings of short-period *S*-waves from a deep earthquake in Argentina (27.0°S, 63.0°W) on 1962 December 8 at 21:27:18.0. Pointers show the location of recording LRSM stations on the map. Since one of the (perpendicular) horizontal components of the LRSM stations is oriented towards NTS, the tracings were done on the component closest to the transverse direction (*SH*) to the event. (b) Film tracings of short-period *S*-waves from a deep earthquake on the Peru–Brazil border (9.3°S, 71.5°W) on 1962 November 10 at 01:00:38.8. Pointers show the location of recording LRSM stations on the map. Since one of the (perpendicular) horizontal components of the LRSM stations is oriented towards NTS, the tracings were done on the component closest to the transverse direction (*SH*) to the event. Magnification of the instrument is indicated above each trace.



**Figure 11.** Instrument corrected  $P$ - and  $S$ -waves amplitude spectra for deep events observed on hard rock at shield sites. Lower curves show the spectra of noise background preceding the arrival of the respective phases. The solid curves show the expected fall-off of spectra assuming various rates of source spectrum decrease  $\omega^{-n}$  for  $t_p^* = 0.5$  s and  $t_s^* = 2$  s; one-half the values for shallow events commonly used in synthetic simulations. The dashed curve corresponds to  $t_p^* = 0.2$  and  $t_s^* = 0.8$  with  $\omega^{-2}$  falloff of source spectra.

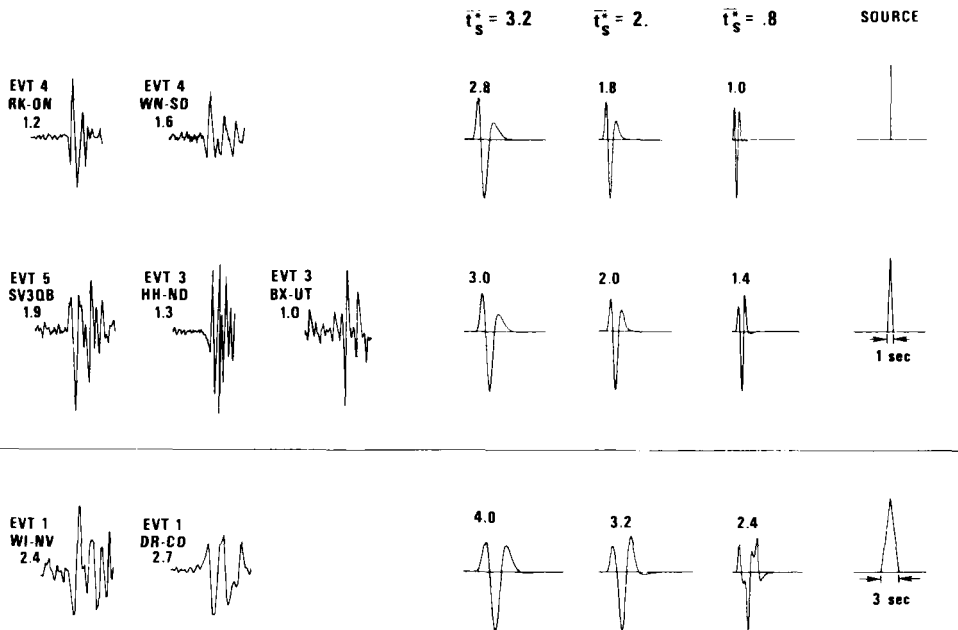
#### LIMITS ON $\bar{t}^*$ IMPOSED BY SPECTRA FROM DEEP EARTHQUAKES AND EXPLOSIONS

The values of acceptable  $\bar{t}^*$  can be delimited by examining the highest frequencies in the spectra observed at high- $Q$  sites. A problem of special interest is the mantle  $Q$  under shields. For shield paths,  $t_p^*$  as high as 1 s have been claimed in the 0.1–1 Hz band (Langston 1976; Given & Helmberger 1981). For deep events observed at shield sites, Lay & Helmberger (1980) and Burdick (1978) found a  $t_s^* = 3.2$  s in, roughly, the 0.1–1 Hz band. To examine what  $t_p^*$  and  $t_s^*$  should be at frequencies at and above 1 Hz we have computed a set of spectra from deep earthquakes for  $P$ - and  $S$ -waves observed at hard rock sites in the southern edge of the Canadian shield. Fig. 11 shows some representative examples of these. Noise spectra preceding the arrival of the respective phases are also plotted. For  $S$ -phases the noise spectra also include the scattered energy preceding the  $S$ -arrival. The  $P$ -spectra up to at least 5 Hz are above the background noise and the  $S$ -wave spectra up to at least 2 Hz are above the background noise. The solid curves show the expected fall-off of spectra assuming a rate of source spectrum decrease of  $\omega^{-n}$  for  $t_p^* = 0.5$  and  $t_s^* = 2$  s, one-half the values for shallow events commonly used in synthetic simulations. Since the events in question are large,  $\omega^{-2}$  most likely characterizes source spectrum. The discrepancy between the observed and predicted spectra with  $\omega^{-2}$  exceeds two orders of magnitude at the high-frequency end for both  $P$ - and  $S$ -waves. Even if one assumes an  $\omega^0$  fall-off rate for the  $S$ -wave sources, which corresponds to a delta function source, the discrepancy of the 2 Hz amplitude level with the line computed for  $\omega^0$  with  $t_s^* = 2$  s is about a factor of 200.

The dashed lines superposed on the spectra correspond to  $\bar{t}_p^* = 0.2$  and  $\bar{t}_s^* = 0.8$  assuming a  $\omega^{-2}$  fall-off in the source spectra. Even at these low  $\bar{t}^*$  values the spectra do not fit well and lower attenuation is indicated. We must also add that even a frequency-dependent  $t_p^* = 0.5$  and  $t_s^* \approx 2$  at 1 Hz is inconsistent with these data (with  $\omega^{-2}$  fall-off) since the  $t^*$  needs to change faster than  $\omega$  and current physical  $Q$  models do not allow for this (Minster 1978a, b).

Arguments based on spectral fall-off rates are, of course, somewhat imprecise. The  $\omega^{-2}$  fall-off argument is for large unspecified frequencies (Brune 1970), and smaller fall-off rates are possible. On the other hand, observational evidence is such that spectra actually fall-off as  $\omega^{-2}$  for events of the size analysed here, and theoretical modelling using triangular or trapezoidal source pulses also implies  $\omega^{-2}$ . It appears therefore that the spectra presented here cannot be reconciled with the  $\bar{t}_s^* \approx 2$  just mentioned for any currently accepted source model, theoretical or observational, even for an  $\omega^0$  fall-off in the source spectra. These spectra clearly demonstrate that the discrepancy is prevalent in the vicinity of 1 Hz and that the high  $\bar{t}^*$  values frequently used in many studies are certainly not valid for these data.

It also appears that even if one adapts the time domain approach, a good case can be made against the  $\bar{t}_s^*$  value 3.2. In Fig. 12 we show observed and synthetic  $SH$ -waveforms for several events and assumed source time histories. The top set of traces shows  $SH$ -waveforms of about 1 s period from several South American deep earthquakes compared to synthetic waveforms for a triangular pulse of 1 s duration and a delta function both passed through the LRSM instrument response and the  $\bar{t}_s^*$  values of 3.2, 2.0 and 0.8. For the 1 s triangular source appropriate for these earthquakes of about 5.6 body wave magnitude, all synthetics have longer periods than the observed data. This source function was also used by Lay & Helmberger (1980). With  $\bar{t}_s^* = 3.2$ , or even  $\bar{t}_s^* = 2$ , such short-period, 1 s, observed waveforms



**Figure 12.** Comparison of observed (left) and synthetic (right)  $SH$ -waveforms for various deep South American earthquakes. The event number from Table 2, the LRSM station and the period of the first cycle is indicated above each observed waveform. The synthetic  $SH$  pulses were computed for three types of source waveforms, a delta function, and triangular pulses of 1 and 3 s duration respectively. The  $\bar{t}_s^*$  values of 3.2, 2.0 and 0.8 were used. The period of the first cycle is written above each synthetic pulse.

are not possible even with the assumption of a delta function which represents an extreme, unlikely example of a high-frequency source. The bottom set of traces shows the comparison of synthetic waveforms for a 3 s triangular source appropriate to a larger event with the observed waveforms from a magnitude 7, deep earthquake, again the  $\bar{t}_s^* = 0.8$  provides the best fit.

Note that the LRSM stations BXUT, WINV and DRCO are located in the WUS, ruling out the value of  $\bar{t}_s^* = 3.2$  even for this low- $Q$  region. While our spectral arguments based on Fig. 11 limit the value of  $\bar{t}_s^*$  in the 0.5–2 Hz range, the time domain simulation, on the other hand, constrains  $\bar{t}_s^*$  at lower frequencies, roughly in the 0.3–1 Hz range. The fact that both estimates give a  $\bar{t}_s^*$  much less than 3.2 indicates that this value is unacceptable for deep earthquake  $S$ -phases observed in the EUS. Since impulsive source functions are physically not very likely, the short-period  $SH$ -waves shown here rule out  $\bar{t}_s^*$  values of 2–3 s. However,  $SH$ -waveforms with considerably less high-frequency content between 0.3–2 Hz have also been observed.  $S$ -waves observed at close range also commonly contain less high-frequency energy than  $P$ -waves. It is therefore not permissible to attribute all or most of the spectral differences between  $P$  and  $S$  to anelastic attenuation as was done in the past in many time domain studies of waveforms, since source effects also contribute to these differences (Molnar, Tucker & Brune 1973; Hanks 1981). The reason for some of the high  $\bar{t}_s^*$  estimates besides the assumption of near equality of  $P$ - and  $S$ -wave spectra at the source was non-representative data selection that overlooked the high-frequency  $S$ -waves in search for 'simple' events (Burdick 1978).

Moreover, the spectral shapes of  $P$ -waves for practically all of the 600 events used for the relative  $\bar{t}_p^*$  studies in this paper are inconsistent with the assumption of  $\bar{t}_p^* = 1.0$  even if one assumes flat source spectra in the 0.5–4 Hz band. Assuming an  $\omega^{-2}$  fall-off and a corner frequency given by the  $\omega^{-2}$  scaling law of Aki (1967), the spectra imply a  $\bar{t}_p^*$  in the 0–0.6 range.

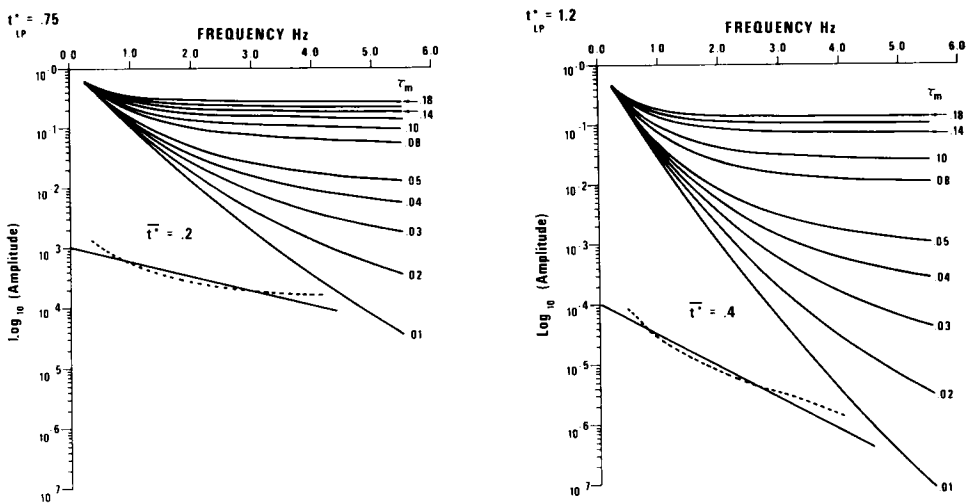
In addition to spectra from deep earthquakes, those of nuclear explosions are good sources of data for estimating the values of  $\bar{t}_p^*$ . For explosions, near-source measurements are often available to estimate the source spectrum from a reduced displacement potential. In the absence of near source data, source models such as those of von Seggern & Blandford (1972) or Mueller & Murphy (1971) have been used. In spite of the uncertainties in such data, the currently acceptable range of source models yield  $\bar{t}_p^*$  that are not significantly different from each other due to the high sensitivity of the spectra to  $Q$ . For instance, the  $\bar{t}_p^*$  estimates from Salmon (Der & McElfresh 1976a) were about the same regardless of whether near field gauge measurements, near regional spectra, or scaled, theoretical salt explosion spectra were utilized. The same was found in the case of other explosions (Der & McElfresh 1976b).

In the past,  $\bar{t}_p^*$  was thought to be strongly constrained because reduced displacement potentials of nuclear explosions at teleseismic distances required  $\bar{t}_p^* \sim 1$  for accurately predicting  $P$ -wave amplitudes. Recent work by Trulio (1978) indicates that a considerable amount of energy is lost in the region of non-linearity close to the explosion, and therefore the above constraint can be relaxed. The effect described by Trulio does not influence the spectral shapes of  $P$ -waves much and the  $\bar{t}_p^*$  derived from spectral ratios still appear to be correct. Studies of explosion spectra in the 0.5–4 Hz band consistently yielded a  $\bar{t}_p^*$  of the order of 0.1–0.2 for purely shield types of paths and values in the range of 0.4–0.5 for paths crossing the mantle under the WUS once and crossing a stable platform or shield on the other leg of the raypath (Frasier & Filson 1972; Filson & Frasier 1972; Der & McElfresh 1976a, b, 1977; von Seggern & Rivers 1979; Nojonen 1975). This provides a good independent check on our relative  $\bar{t}_p^*$  measurements presented in the paper. It is also revealing that the  $\bar{t}_p^*$  values we obtained from deep events were comparable to  $\bar{t}_p^*$  estimates from shallow

events along purely shield paths with source and receiver both on shields. A recent compilation of worldwide  $\bar{t}_p^*$  estimates of deep earthquakes observed in shields and along purely shield types of paths show little difference between the two groups of estimates (Der *et al.* 1982). This indicates that the low- $Q$  layer under shields is absent and  $Q$  is high throughout the upper mantle.

#### CONSTRAINTS ON THE FREQUENCY DEPENDENCE OF $t^*$ IMPOSED BY THE SHAPES OF SPECTRAL RATIOS AS FUNCTIONS OF FREQUENCY

If  $t^*$  (absolute or relative) changes rapidly with frequency, spectral ratios computed between the observed and source spectra to determine  $\bar{t}^*$ , or ratios of station spectra to measure relative  $\bar{t}^*$  should show marked peculiarities that would not have gone unnoticed over the long history of seismology. Plotted on a log-amplitude linear frequency scale, a  $t^*$  increasing with frequency would cause a marked upward curvature in the plot of measured spectral ratios. The fact that numerous researchers in the past used constant  $Q$  and did not notice any peculiar effects puts some, albeit loose, constraints on the rate with which  $t^*$  can vary with frequency. In Fig. 13, we show some curves of spectral ratios that result from the application of Minster's (1978a, b) absorption band model with varying  $\tau_m$ . As  $\tau_m$  increases the change of  $t_p^*$  with frequency and the curvature of the spectral ratios increases. None of the work in the short-period band shows any such indication of such rapid change of  $t^*$  with frequency. On a log (amplitude–linear frequency scale, the ratios of observed and estimated source spectra appear to be lines that fluctuate somewhat but have very little consistent curvature at teleseismic distances (Der & McElfresh 1976b; von Seggern & Rivers 1979; Nojonen 1975; and many others). This indicates a slowly varying or nearly constant  $t^*$  within the short-period band. The higher the original long-period value of  $t_p^*$ , the more



**Figure 13.** Observed-to-source spectral ratios for a set of absorption band models described by the formula of Minster (1978) using two asymptotic long-period  $t^*$  values of 0.75 and 1.2 and a set of  $\tau_m$  ( $\tau_m$  of 1000 was used). The bottom half of the figure compares the shapes of spectral ratios (dashed) implied by the upper bounds of  $t^*$  in Fig. 14 to solid lines implied by the apparent  $\bar{t}_p^*$  of 0.2 and 0.4 derived from the data for shield-to-shield and shield-to-WUS type paths (Der & McElfresh 1977). For an easier comparison of shapes the curves for the comparisons were displaced downward.

difficult it is to fit to the observations. Assuming a  $t_p^* = 1.2$  for long-period data and for a path from NTS to a shield, for instance, we need a curve with an average slope corresponding to a  $\bar{t}_p^*$  of 0.4–0.5 in the 0.5–4 Hz range. Such a curve would have an extremely great curvature and  $\tau_m$  of 0.05. Similarly, a possible curve for a high- $Q$  (shield) type path may be represented by the curves on the left in Fig. 13. In this case we need an average slope corresponding to the  $\bar{t}_p^*$  of 0.2. For this case one would need a  $\tau_m$  of 0.08. None of these frequency dependences implied by the Minster model are indicated by the data, but the disagreement between the data and the curves indicated would be more detectable for the case on the right of the figure with the higher  $t^*$  value in the low-frequency limit. Data at present do not support such curvatures in spectral ratios although we cannot rule out a small curvature. Spectral ratios in the 0.5–4 Hz range require a more gradual change of  $Q$  with frequency than the single band Minster model gives, although the asymptotic values of 1.2 and 0.75 are not unreasonable on the basis of published  $Q$  estimates in the long-period band. This indicates that even with the scanty information available the parameterization of the frequency dependence of  $t^*$  must be more complex than single Minster absorption band models used by Lay & Helmberger (1980) if one wants to model the frequency dependence beyond 1 Hz.

By the same token, relative spectral ratios are devoid of indications of rapid variations of relative  $\bar{t}^*$  differences among the stations we used. Spectral ratios naturally vary quite a bit, and it may be possible to find some such indications of frequency dependence in relative attenuation by statistical methods. A slight divergence of  $\bar{t}_p^*$  with decreasing frequency may be indicated by subtle differences in  $\bar{t}_p^*$  fitted in various frequency ranges for OB2NV and RKON. Thus, although there is quite a bit of uncertainty in the frequency dependence of  $Q$ , data do not favour very rapid variations of  $Q$  with frequency even if they are not incompatible with some physical models of attenuation.

### Constraints on the frequency dependence of $Q$ imposed by long-period data

Attenuation results in the long-period band applicable to the United States are numerous but quite confusing. Attenuation studies elsewhere in the world especially in shield areas may be assumed to be applicable to the EUS on the basis of similarities among shield regions worldwide. Global studies impose some limits on the possible variations of  $Q$ . Before fitting our short-period results to long-period  $t^*$  estimates to outline frequency dependence, some critical selection must be made among the numerous but mutually contradicting results.

A large number of  $\bar{t}_p^*$  and  $\bar{t}_s^*$  values in the recent literature come from studies involving time domain matching of synthetic seismograms with recorded data traces. These studies obtain the fits using complex multiparametric models containing various source parameters and mantle velocity models. The parameters are adjusted simultaneously until the time domain fits are deemed satisfactory by largely subjective criteria. These results cover the 0.03–1 Hz range in general. Strictly speaking these  $\bar{t}^*$  values are byproducts of source or mantle structure studies and their validity is not essential to most of the conclusions of time domain studies. We have expressed our reservations about this technique for estimating  $Q$  in the past (Der & McElfresh 1980). Since most workers using this technique obtained  $Q$  values that are in conflict with spectral studies, we consider their results concerning  $Q$  invalid in the short-period band.

A variant of the time domain method matches the relative amplitudes of short- and long-period recordings. We find the method acceptable for studying regional variations of  $Q$  if sufficiently large data bases are used to overcome the variability of short-period wave ampli-



tudes (Lay & Helmberger 1980). On the other hand, application of this method to small sets of stations can lead to invalid results. This is drastically demonstrated by the failure of Burdick (1978) to detect regional variations of  $Q$  under the United States, a phenomenon that is obvious from even casual inspection of short-period  $S$ -waves from deep earthquakes. Another illustration of this is a  $t_p^*$  estimate of 1.3 by Helmberger & Hadley (1981) from NTS to stable platform or shield regions in the 0.3–0.7 Hz range as indicated by the dominant periods of  $P$ -waves they analysed. To reconcile this measurement with the  $t_p^*$  of the order of 0.4–0.5 along similar paths determined by spectral methods (Der & McElfresh 1977; von Seggern & Rivers 1979; Noponen 1975; Frasier & Filson 1972) an unacceptably rapid variation of  $t_p^*$  with frequency in the short-period band would be required.

In our discussion of frequency dependence of  $t^*$  we shall consider the relative  $t^*$  differences determined by Lay & Helmberger (1981) and other studies using either spectral or combined time domain and spectral methods, but we will exclude all work using time domain waveform fitting alone. Although we do not have any direct proof that the long period  $Q$  estimates by these studies are wrong, the repeated discrepancies we have found between these and our work in the short-period band quoted makes us suspicious about the validity of all  $Q$  values obtained by such methods. Hanks (1981) point out that limitations in the source models used in many of these studies may cause  $Q$  estimates that are too low.

Measurements of the Earth's free oscillations are also valid indicators of the anelastic losses in an average earth, although extracting regional information from such data has barely begun (Jordan 1978; Silver & Jordan 1981; Dahlen 1980; Woodhouse 1980). On the average,  $t_p^* \sim 1$  and  $t_s^* \sim 4$  is required by such data (Anderson & Hart 1978; Sailor & Dziewonski 1978). It must be pointed out, however, that averaged  $Q$  models of the Earth may be biased by the grossly uneven regional distribution of anelastic losses in the Earth. If short-period  $Q$  data are any guide for this distribution, some areas behind island arcs, mid-ocean ridges and rift zones must dissipate a disproportionate amount of energy relative to their physical dimensions, biasing the  $Q$  of an averaged model towards values lower than for most of the Earth's volume. This possible bias makes it even more likely that the  $t_p^*$  involving shield type paths are considerably below 1 s throughout the seismic band. Moreover, some  $Q$  estimates obtained by spectral stacking methods in free oscillation studies may also be biased towards too low values by the lateral heterogeneities of the Earth (Sleep, Geller & Stein 1981).

Regional variations of  $Q$  in the long-period band were studied by Solomon & Toksöz (1970) and Solomon (1972) who give a regional  $t_p^*$  differential of  $\delta t_p^* \sim 0.5$  s or more between EUS and WUS. The studies of Lee & Solomon (1975, 1979) result in  $Q_\beta$  structures that imply a long-period  $t_p^*$  differential of only 0.25. Ray tracing through the  $Q$  models given by these authors would give  $t_p^*$  of the order of 0.6–0.7 in the eastern US (EUS) and close to 1 for an EUS–WUS path. The absolute  $Q$  values in these models are, however, rather uncertain due to the inherent difficulties of measuring  $Q$  of surface waves over short paths. In any case, these studies indicate that the upper mantle  $Q$  varies regionally also in the long-period band. The work of Nakanishi (1979) and Mills (1978) provides further indications that the upper mantle  $Q$  measured in the 150–300 s period range is high under the shields. It appears from Nakanishi's results that, on the average, anelastic losses under the shields are less than those associated with model MM8 of Anders, Ben-Menahem & Archaubeau (1965). At teleseismic distances, model MM8 gives a  $t_p^*$  of the order of 0.6–0.8 s, and thus those values should be considered as upper limits of absolute  $t_p^*$  in the long-period band for shield type paths.

Numerous measurements of  $Q_\beta$  averages in the mantle have been provided by multiple ScS phases. Unfortunately, there are no studies of ScS that could be clearly associated with purely shield type paths and none under the eastern United States. Nevertheless, the average  $Q_{ScS}$  values of 600 for the whole mantle by Kovach & Anderson (1964) and 580 by Sato &

Espinosa (1967) may be indicative of  $Q$  values in regions above the downgoing slab in South America that may have  $Q$  characteristics similar to shields (Sacks & Okada 1974). These  $Q$  values are considerably higher than those obtained from multiple  $ScS$  studies elsewhere (Sipkin & Jordan 1980), but the corresponding  $t_p^*$  at teleseismic distances in such structures would still be 0.4–0.5 s, twice the  $t_p^*$  from spectral ratios in the short-period band along similar paths.  $Q$  estimates obtained from core reflections may also be influenced by low- $Q$  regions near the core–mantle boundary and may not be directly applicable to the paths with epicentral distances less than  $85^\circ$  used in this study.

Indications of the frequency dependence of  $Q$  within the long-period band were found by Sipkin & Jordan (1979), Brune (1977), Yoshida & Tsujiura (1975) and Sato & Espinosa (1967) in other areas of the world.

### Frequency dependence of $Q$

Strictly speaking, frequency dependence of  $Q$  can only be established if attenuation is measured in a wide frequency band using short- and long-period instruments over the same paths. Assumption of frequency dependence introduces a large number of additional parameters into studies of attenuation. It is possible that the form of frequency dependence changes from region to region, and the depth distribution of  $Q$  may be different at various frequencies (Solomon 1972; Lundquist & Cormier 1980). These uncertainties can only be resolved by further detailed studies. Studies that would give clear indications of frequency dependence across the United States from a single data set such as multiple  $ScS$  studies of Sipkin & Jordan (1979) in other regions, do not exist. We feel, however, that there is enough evidence to draw approximate limits on the frequency dependence of  $t^*$  in the 0.02–4 Hz band.

We wish to point out that we have found no clear indications of frequency dependence of  $t^*$  within the 0.5–4 Hz band. All of our findings of the observed spectral shapes from earthquakes and explosions, relative regional  $Q$  variations can be explained with frequency-independent  $Q$ . In the long-period band, direct indications of frequency dependence within the United States are also missing, but all  $Q$  estimates in the United States and in analogous tectonic elsewhere imply lower  $Q$  values in the long-period band than in the short-period band. In the following we shall outline the frequency dependence of  $t^*$  for two types of paths at teleseismic distances. The two path types are shield-to-shield and shield-to-tectonic. These typify the purely EUS and the EUS–WUS paths.

The results summarized in the preceding sections of this paper put several restrictions on any model describing the frequency dependence of  $t^*$  along the two types of paths. These are as follows:

- (1) The apparent  $t_p^*$  must be in the range of 0.1–0.2 for shield paths in the 0.5–4 Hz band.
- (2) The absolute  $t_p^*$  changes slowly with frequency in the same band.
- (3) The apparent  $t_s^*$  must be less than 1.6 between 0.3 and 2 Hz, twice the value derived from deep earthquakes, for shield-to-shield paths.
- (4) The absolute  $t_s^*$  cannot change faster than the frequency.
- (5) The apparent  $t_p^*$  differential between shield-to-shield and shield-to-tectonic paths must be of the order of 0.2 s in the 0.5–4 Hz range.
- (6) The apparent  $t_p^*$  differential between the two types of paths changes slowly with frequency.
- (7) The absolute  $t_p^*$  differential near 1 Hz must produce the observed magnitude and amplitude differentials (0.26 magnitude units or a factor near 2 in amplitude variation).
- (8)  $t_s^* = 4 t_p^*$  for both paths.

(9) The difference in apparent  $t_p^*$  between the two types of paths must be 0.5 s in the long-period band.

(10) The absolute  $t_p^*$  in the long-period band should be considerably less than 1 s, the worldwide average, for shield-to-shield type paths.

Fig. 14 shows two regional  $t^*$  versus frequency relationships that satisfy these constraints. These curves were drawn conservatively to set *upper bounds* of  $t_p^*$ . We made the shield-to-shield type  $t_p^*$  curve approach the value of 0.75 s at the long-period end, corresponding to ray tracing results from the models of Lee & Solomon (1975, 1979) and exceeding those implied by the work of Nakanishi (1979). The curves are drawn with a rapid decrease beyond 1 Hz to keep them as high as possible while still satisfying the constraints imposed by the apparent  $t^*$  from the data in the 0.5–4 Hz band, and also keeping the curvature in the spectral ratios reasonably low. The shield-to-shield curve can be fitted with an absorption band model described by Minster (1978) with the parameter  $\tau_m$  of 0.08. This curve gives a  $\bar{t}_s^*$  of about 1.6 in the 0.3–1 Hz range which is twice the value of the  $\bar{t}_s^*$  derived from deep earthquake data in Figs 11 and 12. Since the actual  $\bar{t}_s^*$  must be less than twice the value derived from deep earthquakes the curve for shield-to-shield paths must certainly be an upper bound. The  $t_p^*$  differential between the shield-to-shield and shield-tectonic curves at 1 Hz is dictated by the size of the regional EUS–WUS magnitude anomalies of the order of 0.26–0.33 magnitude units (Booth *et al.* 1974; Der *et al.* 1979). The shield-tectonic curve cannot be fitted with an absorption band model of the Minster type. A simple absorption band model starting out with high values at the long-period end would fall-off too rapidly to be compatible with the apparent constancy of  $Q$  in the short-period band and the  $t^*$  differential compatible with the  $m_b$  variations across the United States.

The  $t_p^*$  versus frequency relationships of Lundquist and Cormier (1980) were made to converge to 1 s at the long-period end. The models of Lay & Helmberger (1980) also converge to 0.8 at the long-period end. Our models on the other hand, diverge toward the lower frequencies, and in this they are in better agreement with available evidence that both the absolute values and the regional differentials of  $t_p^*$  become larger in the long-period band. The majority of the constraints in the long-period band can be satisfied by almost any pair of smooth curves that retain the relative vertical spacing of our proposed limiting curves while remaining below them.

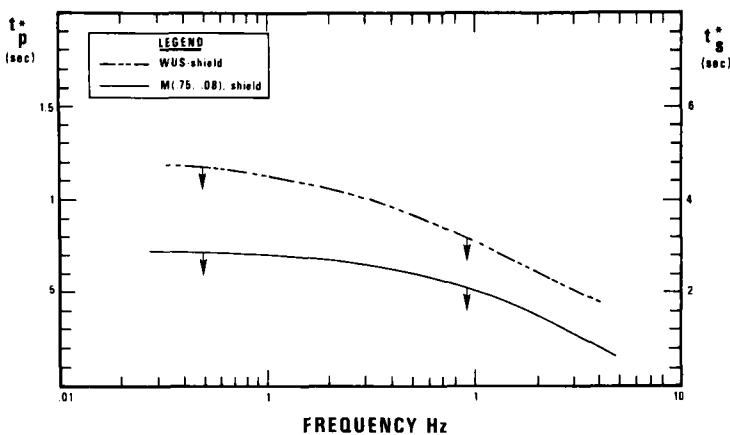


Figure 14. Proposed upper limits for  $t^*$  versus frequency curves for shield-to-shield and (shield-WUS or shield-tectonic) type paths. The shield-to-shield path curve can be fitted by a Minster type of absorption band model with a long-period  $t_p^*$  value of 0.75 and  $\tau_m = 0.08$ .

In spite of the fact that our upper bounds on absolute  $t^*$  as functions of frequency provide a marginal fit to the constraints listed above, the quality of fits to the conditions imposed by the *SH*-wave data from deep earthquakes and the near constancy of apparent  $t^*$  can be markedly improved by lowering the low-frequency limits by about 0.2 s, which is allowed by some of the available new evidence in the long-period band, and putting more of the change in  $t^*$  into the 0.1–1 Hz band, allowing the curves to level off above 1 Hz. Such curves would be equivalent to using  $\tau_m$  values in the 0.1–0.2 range in Minster's model, although a more gradual fall-off of  $t^*$  with frequency would be needed beyond 1 Hz. At this stage we prefer to set upper limits rather than to propose specific models the justification of which would require more data analysis. Moreover if we draw the  $t^*$  curves too low some absolute amplitude constraints derived from nuclear explosion data need to be resolved, since even Trulio's (1978) work does not explain amplitude losses due to near-source non-linear behaviour by a factor exceeding 2. Since some new work on  $Q$  indicates that attenuation was overestimated in the long-period band (Hanks 1981; Sleep *et al.* 1981), it is likely that the upper limit  $t^*$  curves given in this paper are not exceeded and that the frequency dependence of  $t^*$  may even be less strong than indicated by our curves.

Although the short-period data we analysed do not rule it out, we do not find the constant  $t_p^*$  assumption within the 0.5–4 Hz band very likely, and we favour a continuous change of  $t^*$  for the two path types within this band towards larger values in the long-period band although the rate of change may be less than that of our limiting curves. The differential attenuation curves of Solomon (1972) are in good agreement with our curves. There is no doubt that the following years will bring many new results concerning the specific, regionally dependent variations of  $Q$  versus frequency and depth for many areas of the world.

We now discuss briefly some worldwide measurements of  $Q$  in the context of frequency dependence and regional variation. Worldwide measurement of  $Q_{ScS}$  and mantle waves (Sipkin & Jordan 1979, 1980; Nakanishi 1979) shows that the regional variation of  $Q$  in the long-period band is similar to that observed in the short-period band (Barazangi, Pennington & Isacks 1975; Oliver & Isacks 1967; Solomon 1972). Attenuation is extremely high behind island arcs in certain areas, and under mid-ocean ridges and rift regions. It is low under shields and old ocean basins. This appears to be true throughout the 0.003–4 Hz band. In most regions of the Earth, attenuation measurements in the long-period band indicate  $Q$  values that would give  $t_p^* \sim 1$  and  $t_s^* \sim 4$ . Most  $Q$  models derived from free oscillation data also imply  $t_p^* \sim 1$  and  $t_s^* \sim 4$  (Anderson & Hart 1978). On the other hand, observations of high-frequency energy in the 3–5 Hz range are quite common over a wide variety of teleseismic paths (Asada & Takano 1963; Takano 1971; Kurita 1968; Felix, Gilbert & Wheeler 1971; Noponen 1975) if the observations were made using suitable instrumentation peaked at high frequencies and having suitable recording systems. A large number of worldwide spectral  $t_p^*$  estimates are given by Der *et al.* (1982), Sobel & von Seggern (1976, 1978); Sobel *et al.* (1977a, b), von Seggern & Sobel (1977) and von Seggern & Blandford (1977) indicating  $t_p^* < 0.6$  for most of the paths studied. Reports of similar observations are too numerous in the literature to quote them all. There is thus a clear conflict between short- and long-period  $Q$  measurements that, it appears, can be resolved only by assuming a frequency-dependent  $Q$  worldwide that doubles within the range of 0.01–2 Hz.

Essentially the same point was also made by many other researchers analysing short-period body wave data in the past (Takano 1971, Kurita 1968, and many others). The theoretical framework for discussion of the frequency dependence of  $Q$  is well developed by many researchers (Anderson & Hart 1978; Anderson *et al.* 1977; Minster 1978a, b 1980; Minster & Anderson 1981; Budianski & O'Connell 1976; Walsh 1969). Most proposed

physical models of dissipation in the Earth are inherently frequency dependent, and observational evidence for frequency dependence should come as no surprise.

### Conclusions

In order to investigate variations of  $Q$  in the mantle using short-period waves, a combined evaluation of amplitude and spectral data for both  $P$ - and  $S$ -waves is necessary. The instability of amplitude measurements due to near source and receiver focusing and unknown crustal responses suggests that more weight be given to spectral methods which are less sensitive to these factors.

Variations of  $Q$  under the contiguous United States are revealed by broad regional amplitude anomalies of teleseismic  $P$ - and  $S$ -waves. The regional amplitude anomalies correlate with spectral changes in both  $P$ - and  $S$ -waves; low amplitudes are accompanied by losses in high-frequency energy. These spectral changes are quite dramatic in  $S$ -waves from deep events as observed across the United States. The regional variations of teleseismic  $\bar{t}^*$  across the United States are of the order of 0.2 s for  $P$  and about 3–4 times that for  $S$ . This implies losses mostly or completely in shear deformation.

Mantle attenuation is greatest under the south-western United States including the Basin and Range province, and it is least in the shield region of the north central United States. As a whole, the mantle under the western United States is more attenuating than the mantle under the eastern United States. The north-eastern United States is characterized by mantle attenuation intermediate between the shield and the Basin and Range. These regional variations in  $Q$  correlate well with travel-time residuals and variations in the upper mantle LVZ derived from surface wave studies indicating that the LVZ is also a low- $Q$  region.

The amount of high-frequency energy in teleseismic  $P$ - and  $S$ -waves in the US and world-wide is incompatible with the values  $t_p^* \sim 1$  s and  $t_s^* \sim 4$  s derived from long-period attenuation studies and commonly used in long-period synthetic simulations. This indicates that  $Q$  is frequency dependent and doubles in value somewhere in the range of 0.01–2 Hz.

The upper limits of the possible values of absolute  $t_p^*$  can be drawn for the shield and shield-tectonic type of paths using short-period spectral and waveform data. These frequency versus  $t_p^*$  curves gradually rise towards the long-period band and diverge slightly. The values of absolute  $t_p^*$  are below 1 s for both types of paths at 1 Hz.

The value of  $t_p^*$  for shields probably never exceeds 1 s and must be below 0.5 at 1 Hz.

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