

An investigation of uncertainties in the QCD NLO predictions of the inclusive jet cross section in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV and 630 GeV

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Abstract. Uncertainties in the NLO calculation of the inclusive jet cross section due to the choice of renormalization scale, parton distribution functions and clustering algorithm are explored. These are found to be similar in size to the current experimental uncertainties of the measured inclusive jet cross section at $D\bar{O}$ and CDF.

1 Introduction

Measurement of the inclusive jet cross section in proton anti-proton ($\bar{p}p$) collisions constitutes a strong test of the predictions of perturbative quantum chromodynamics (QCD). Deviations of the theoretical cross section from the experimentally observed cross section may be evidence for physics beyond the Standard Model. In particular, the presence of quark compositeness would enhance the cross section at high values of transverse energy (E_T).

Recently, the inclusive jet cross section has been measured by the CDF [1] and $D\bar{O}$ [2] Collaborations with systematic uncertainties ranging from 10% to 40% as function of E_T . With the improvement in the experimental accuracy of this measurement, it is worth investigating the accuracy of the next to leading order (NLO) QCD predictions [3–5].

In this paper the uncertainties in the NLO QCD inclusive jet calculations are explored using two available programs: JETRAD [4] a complete $\mathcal{O}(\alpha_s^3)$ event generator, and EKS [5] a complete $\mathcal{O}(\alpha_s^3)$ analytical calculation of the inclusive jet cross section. Both programs require the selection of a renormalization and factorization scale (typically chosen to have the same value, μ), a set of parton distribution functions (PDF), and a jet clustering algorithm.

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2 Discussion of the theoretical predictions

If QCD could be calculated to all orders the results would be independent of the choice of renormalization scale. Because the inclusive jet cross section has been calculated only to NLO ($\mathcal{O}(\alpha_s^3)$), the choice of renormalization scale does affect the result. The authors of JETRAD have provided several choices for the renormalization scale, we have chosen to investigate a scale proportional to the E_T of the leading jet after parton clustering ($\mu = AE_T^{\max}$, where A is a constant typically chosen to lie in the range $0.25 \leq A \leq 2$). The authors of EKS prefer an alternative definition of the renormalization scale: the E_T of each jet in the event ($\mu = AE_T^{\text{jet}}$)¹. An alternative scheme (available in JETRAD) uses the center-of-mass energy of the two outgoing partons as the renormalization scale ($\mu = C\sqrt{\hat{s}} = C\sqrt{x_1x_2s}$ where C is a constant chosen to lie in the range $0.25 \leq C \leq 1$, $x_1 = \sum E_{T_i}e^{\eta_i}/\sqrt{s}$, $x_2 = \sum E_{T_i}e^{-\eta_i}/\sqrt{s}$, $i = 1 \dots n$ where n is the number of jets in the event, $\eta = -\ln[\tan(\theta/2)]$ and θ is the polar angle relative to the proton beam)². The effect of these scale choices on the inclusive jet cross section is discussed.

¹ A version of EKS that uses the renormalization scale $\mu = AE_T^{\max}$ is also available

² The choice of renormalization scale $\mu = C\sqrt{\hat{s}}$ is somewhat unnatural for the inclusive jet cross section which is dominated by t -channel exchange. It has been included to study the effect of an extreme choice of scale and for comparison with previous two-jet mass analyses [6]

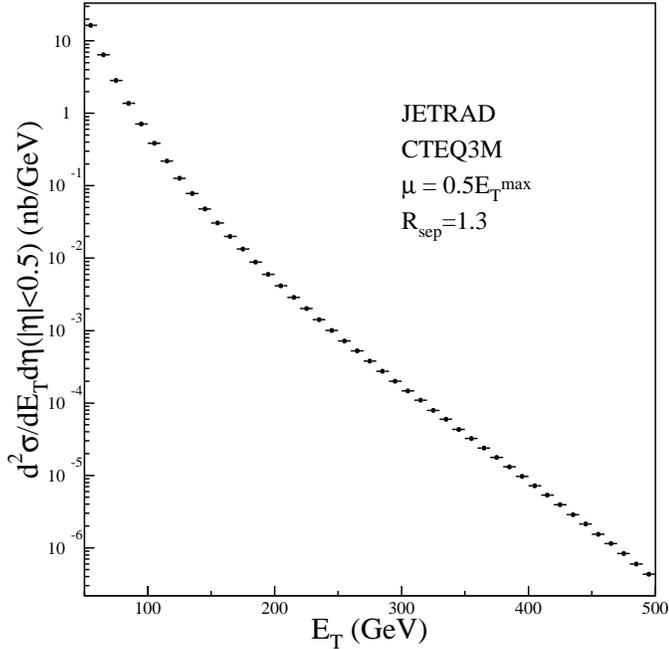


Fig. 1. The reference NLO calculation of the inclusive jet cross section at $\sqrt{s} = 1.8$ TeV. JETRAD calculation for the pseudorapidity range $|\eta| < 0.5$, $\mu = 0.5E_T^{\max}$, $\mathcal{R}_{\text{sep}} = 1.3$ and the CTEQ3M PDF evolved using the CTEQ method

The standard Snowmass clustering algorithm [7] combines two partons into a single jet if they are both within $\mathcal{R} \equiv \sqrt{\eta^2 + \phi^2} = 0.7$ of their E_T weighted center (where ϕ is the azimuthal angle). An additional constraint on the parton clustering is applied which requires that the two partons be closer than $\mathcal{R}_{\text{sep}} = 1.3 \times \mathcal{R}$ [8]³. The value of \mathcal{R}_{sep} has been chosen to match the experimental jet splitting/merging parameters used in the jet clustering algorithms [8]. The effect of using \mathcal{R}_{sep} will be discussed.

The uncertainty in the calculation of the inclusive jet cross section resulting from parton distribution functions will be divided into three parts.

The first is due to the choice of the PDF family. This choice is associated with the selection of data used to determine the PDF and the functional form used in the fits. The variation of the inclusive jet cross section has been studied using a selection of modern PDFs: CTEQ3M [9], CTEQ4M, CTEQ4HJ [10] and MRSA' [11].

The second category of PDF uncertainties results from the value of the strong coupling constant (α_s) used in the PDF. Usually a free parameter in the PDF fit, α_s can be fixed to a pre-determined value. The effect of α_s variation on the PDF is examined by using the CTEQ4A series in which α_s is fixed to values ranging from 0.110 to 0.122 at M_Z . In comparison, $\alpha_s(M_Z) = 0.116$ for CTEQ4M.

Finally, JETRAD and EKS use different strategies to evolve the PDF in x and Q^2 (where x is the momentum fraction carried by the parton and Q is the characteristic energy scale of the process, typically chosen to be the momentum transfer). The JETRAD program uses the strat-

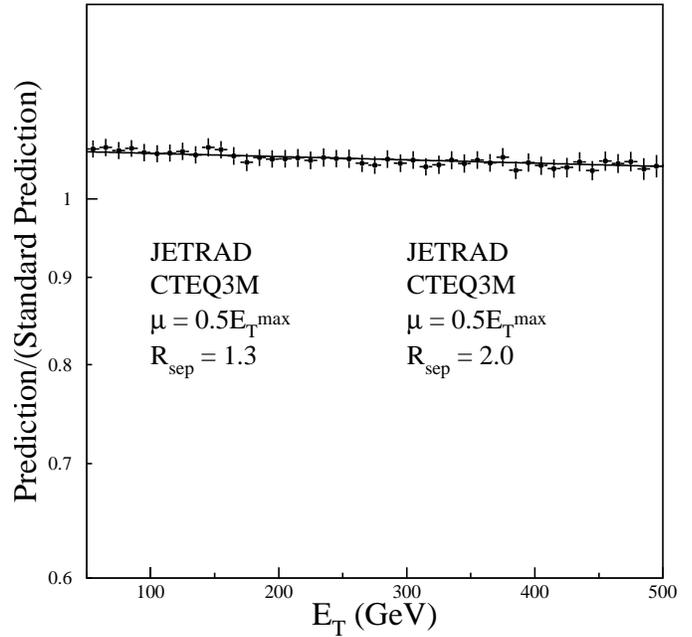


Fig. 2. A comparison between the JETRAD calculations of the inclusive jet cross section with $\mathcal{R}_{\text{sep}} = 2.0$ and 1.3 at $\sqrt{s} = 1.8$ TeV

egy as implemented by the MRS [11] group for evolving all PDFs (JETRAD also uses CTEQ PDFs generated using the CTEQ evolution package). EKS implements the PDFs by interpolating from a table of values that were generated directly from the original PDFs. While these implementations are theoretically identical, small differences can be produced by the numerical accuracy of the program.

The uncertainties will be determined at $\sqrt{s} = 1.8$ TeV and 630 GeV and compared to a reference model. For this study, the reference model will be the JETRAD calculation for the pseudorapidity range $|\eta| < 0.5$, $\mu = 0.5E_T^{\max}$, $\mathcal{R}_{\text{sep}} = 1.3$ and the CTEQ3M PDF evolved using the CTEQ method (see Fig. 1). The comparisons with other theories will be given by:

$$R = \left(\frac{\text{Prediction}}{\text{Reference Prediction}} \right).$$

This ratio is fitted with a third degree polynomial yielding a smooth curve. In most cases the ratio can be fitted with a resulting χ^2 per degree of freedom less than one.

3 Inclusive jet cross section at $\sqrt{s} = 1.8$ TeV

The difference between using the standard Snowmass clustering algorithm and the modified algorithm with $\mathcal{R}_{\text{sep}} = 1.3$ is shown in Fig. 2. The effect ranges from 8% at 50 GeV decreasing to 5% at 500 GeV. Because the value of \mathcal{R}_{sep} is selected to reflect the experimental clustering algorithm, the uncertainty resulting from its use is much smaller than 5%. A more appropriate variation of the value of \mathcal{R}_{sep} is

³ The Snowmass algorithm corresponds to $\mathcal{R}_{\text{sep}} = 2.0 \times \mathcal{R}$

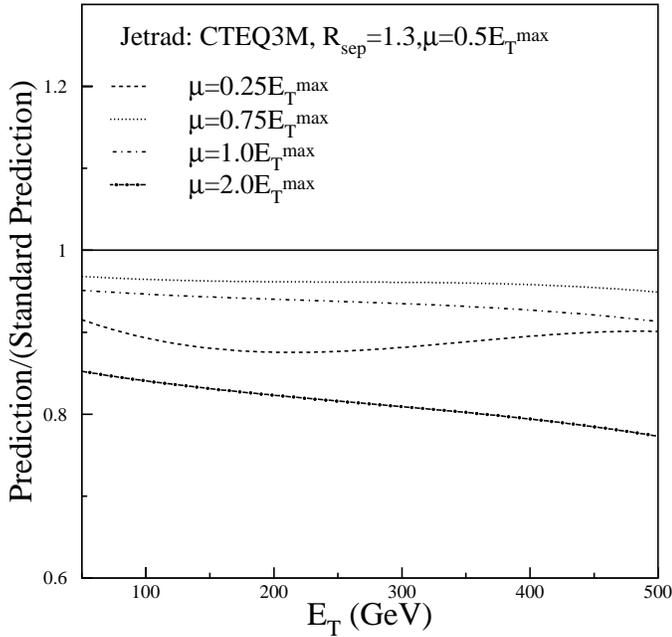


Fig. 3. A comparison between the JETRAD calculations of the inclusive jet cross section with $\mu = 0.25, 0.75, 1.0$ and $2.0 E_T^{\max}$ compared with $\mu = 0.5 E_T^{\max}$ at $\sqrt{s} = 1.8$ TeV

from 1.2–1.4 resulting in variations in the cross section of less than 1% [8]⁴.

The effect on the cross section due to the choice of the renormalization scale was studied by using JETRAD with several different values of μ . First, the cross section was calculated using $\mu = 0.25, 0.75, 1.0$ and $2.0 E_T^{\max}$. These are compared to the cross section with $\mu = 0.5 E_T^{\max}$ in Fig. 3. The cross section is largest⁵ for $\mu = 0.5 E_T^{\max}$ and is reduced by 5–10% with some E_T dependence for $\mu = 1.0 E_T^{\max}$ and $0.75 E_T^{\max}$. The cross section is approximately 10% below the reference model for $\mu = 0.25 E_T^{\max}$ and 15–20% below the reference model for $\mu = 2.0 E_T^{\max}$ with some E_T dependence.

Figure 4 compares the predictions for alternative choices of μ , E_T^{jet} and $\sqrt{\hat{s}}$. The choice $\mu = 0.5 E_T^{\text{jet}}$ (calculated with EKS⁶) is compared to $\mu = 0.5 E_T^{\max}$ and shows a 5% difference at an E_T of 50 GeV dropping to less than 1% at 500 GeV. Also shown in Fig. 4 is a comparison of the JETRAD calculations for $\mu = 0.25, 0.5, 1.0 \sqrt{\hat{s}}$ compared with $\mu = 0.5 E_T^{\max}$. The effect is approximately 20–25% at 50 GeV decreasing to 10% at 500 GeV with a strong dependence on the choice of scale used. In summary, there are only small differences between the scale choices of E_T^{jet} and E_T^{\max} but large differences when the scale is changed from E_T to $\sqrt{\hat{s}}$.

⁴ The uncertainty resulting from the choice of \mathcal{R}_{sep} will not be considered in the remainder of this note

⁵ The inclusive jet cross section at $\mu = 0.5 E_T^{\max}$ is a maximum since this is a point of minimum sensitivity for the calculation, see [12] for a discussion

⁶ JETRAD does not implement $\mu = A E_T^{\text{jet}}$

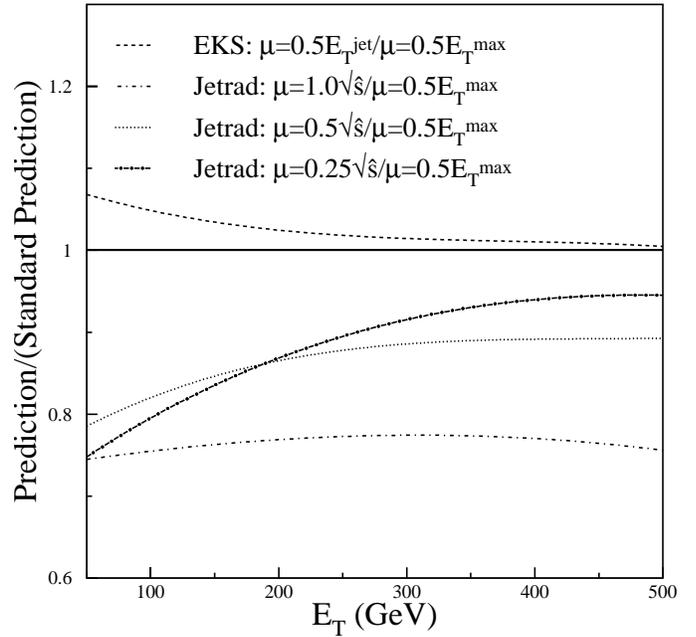


Fig. 4. A comparison between the EKS calculation of the inclusive jet cross section with $\mu = 0.5 E_T^{\text{jet}}$ and $\mu = 0.5 E_T^{\max}$. Also shown is a comparison between the JETRAD calculations of the inclusive jet cross section with $\mu = 0.25, 0.5$ and $1.0 \sqrt{\hat{s}}$ compared with $\mu = 0.5 E_T^{\max}$ at $\sqrt{s} = 1.8$ TeV

The effect of different PDF choices is depicted in Fig. 5. The cross section calculated with CTEQ4M is a few percent lower than the cross section calculated using CTEQ3M. These differences result from a change in the parameterization used to model the gluon distribution⁷ of the CTEQ4M PDF and the inclusion of additional data sets. These data sets include more precise deep inelastic scattering (DIS) data from NMC [13] and HERA [14, 15], the inclusive jet cross section measured by CDF [1, 16] and the preliminary inclusive jet cross section measured by DØ [17]. These changes lead to a change in the optimal value of α_s from 0.112 to 0.116 [10]. The CTEQ4HJ PDF, which emphasizes recent CDF jet data to constrain the gluon distribution, shows a decrease in the cross section at low E_T of approximately 5% and an increase in the cross section at 500 GeV of approximately 25%. The final comparison is made using MRSA', which uses a slightly different parameterization and input data to CTEQ3M. This results in cross sections that are similar at high and low E_T and approximately 5% higher at 300 GeV. These differences are caused by the gluon distributions of the PDFs (see Fig. 6). Note that the variations in the cross section calculations due to the choice of PDF are limited by the similarity of the parameterizations used to model the gluon distributions in the PDFs (which are not well constrained by experiment). The choice of an alternative parameterization of the gluon PDF could lead to larger uncertainties.

⁷ CTEQ3M uses a more restrictive gluon parameterization: $G(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} (1+A_3 x)$ while CTEQ4M uses: $G(x, Q_0) = A_0 x^{A_1} (1-x)^{A_2} (1+A_3 x^{A_4})$ [10]

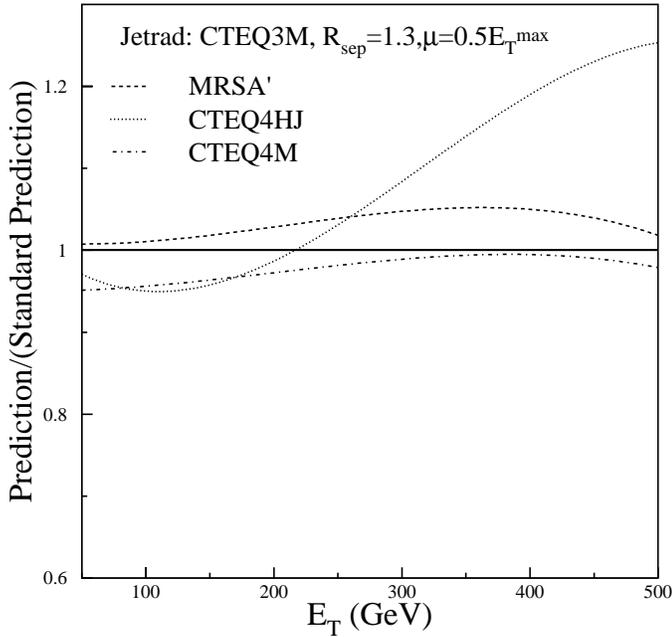


Fig. 5. A comparison between the JETRAD calculations of the inclusive jet cross section with the CTEQ4M, CTEQ4HJ and the MRSA' PDF compared with the calculation using CTEQ3M at $\sqrt{s} = 1.8$ TeV

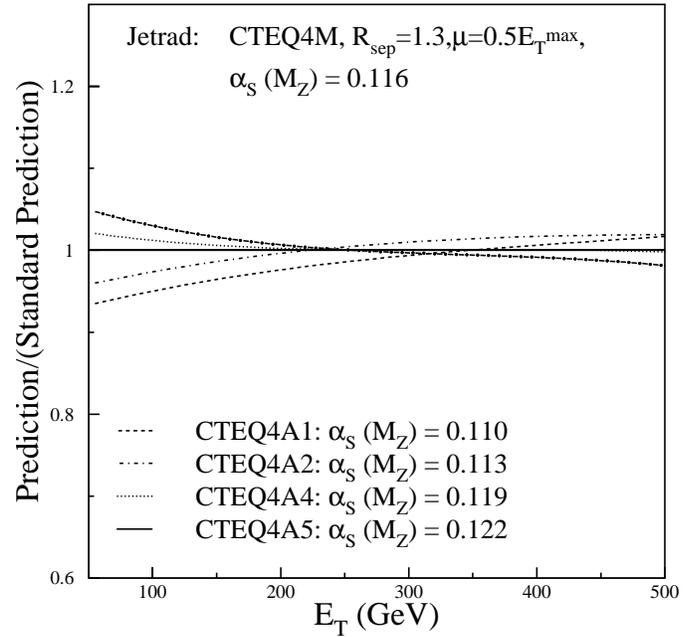


Fig. 7. A comparison between the JETRAD calculations of the inclusive jet cross section with the CTEQ4A series of PDFs compared with the calculation using CTEQ4M at $\sqrt{s} = 1.8$ TeV

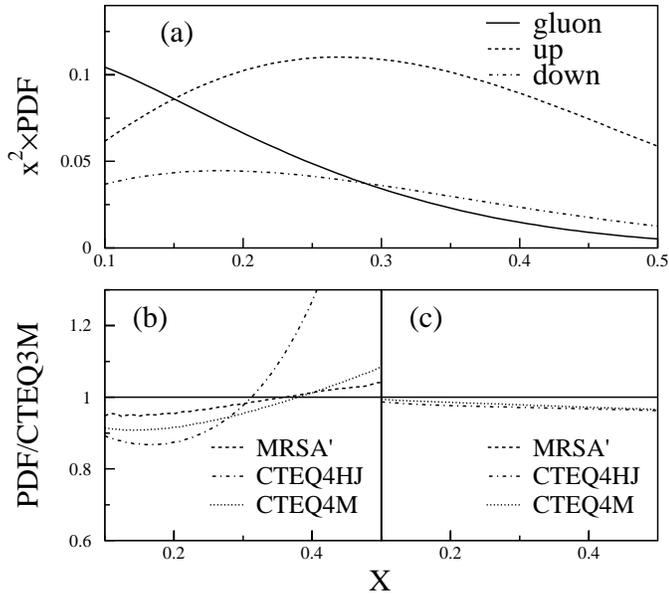


Fig. 6. A comparison of the PDFs used in this analysis. **a** The CTEQ3M PDF for $Q = 100$ GeV (the momentum transfer) as a function of the momentum fraction carried by the parton (x). **b** A comparison between the gluon distributions from the CTEQ4M, CTEQ4HJ and the MRSA' PDFs compared with the CTEQ3M PDF. **c** A comparison between the up quark distributions from the CTEQ4M, CTEQ4HJ and the MRSA' PDFs compared with the CTEQ3M PDF

The cross section change due to the variation of α_s in the PDF is shown in Fig. 7. Approximately $\pm 5\%$ changes are seen at low E_T which diminish as the E_T increases. The uncertainty in the PDF due to the choice of α_s is significantly smaller than the uncertainty due to the gluon distributions.

A comparison can be made between the EKS and JETRAD calculations for $|\eta| < 0.5$ using $\mu = 0.5E_T^{\max}$, $\mathcal{R}_{\text{sep}} = 1.3$ and the CTEQ3M PDF. As shown in Fig. 8, the two calculations differ at a level of 2-3% with some dependence on the E_T . This variation is due to the different evolution in x and Q^2 used by the two programs. Figure 9 shows the variation in the JETRAD predictions due to the different evolution methods. These differences lead to 5% difference in the cross section with some E_T dependence, similar to the differences between the EKS and JETRAD programs.

4 Inclusive jet cross section at $\sqrt{s} = 630$ GeV

The studies described in the previous section were repeated for $\sqrt{s} = 630$ GeV. The comparison between JETRAD and EKS with $\mu = AE_T^{\max}$ is given in Fig. 10. EKS produces a cross section that is 5% lower than the JETRAD cross section at 20 GeV and 10% higher at 150 GeV. These uncertainties cannot be fully explained by the difference in choice of PDF evolution.

The variations in the NLO calculations of the cross section due to the choice of renormalization scale and PDF are given in Fig. 11. These variations are slightly larger than those observed at $\sqrt{s} = 1.8$ TeV.

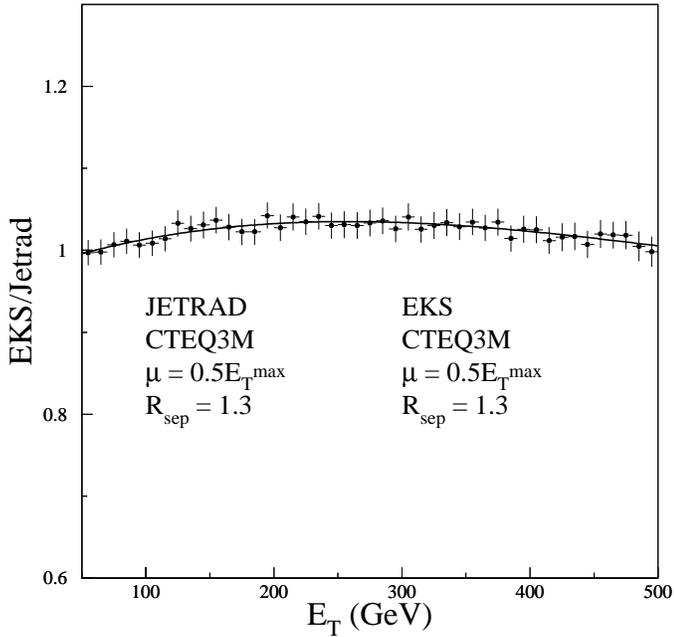


Fig. 8. A comparison between the JETRAD and EKS calculations of the inclusive jet cross section at $\sqrt{s} = 1.8$ TeV. The prediction parameters are $|\eta| < 0.5$, $\mu = 0.5E_T^{\max}$, $\mathcal{R}_{\text{sep}} = 1.3$ and the CTEQ3M PDF. The fit is to a third degree polynomial

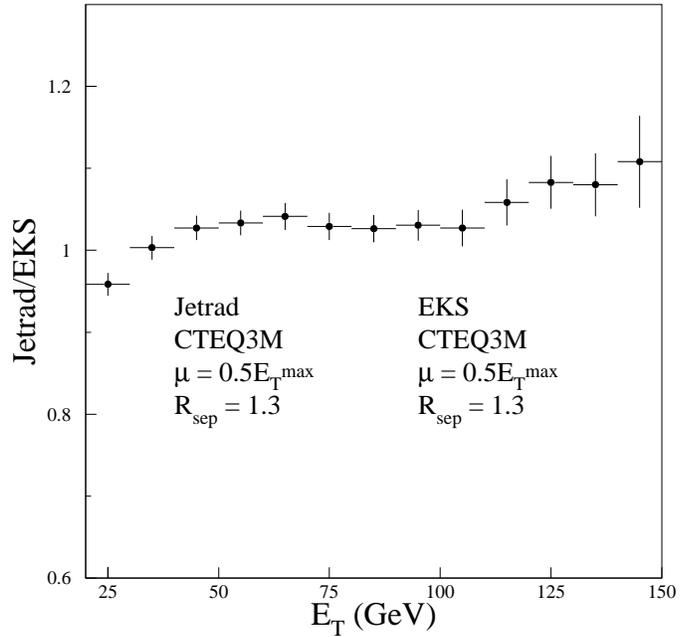


Fig. 10. A comparison between the JETRAD and EKS calculations of the inclusive jet cross section at $\sqrt{s} = 630$ GeV. The prediction parameters are $|\eta| < 0.5$, $\mu = 0.5E_T^{\max}$, $\mathcal{R}_{\text{sep}} = 1.3$ and the CTEQ3M PDF

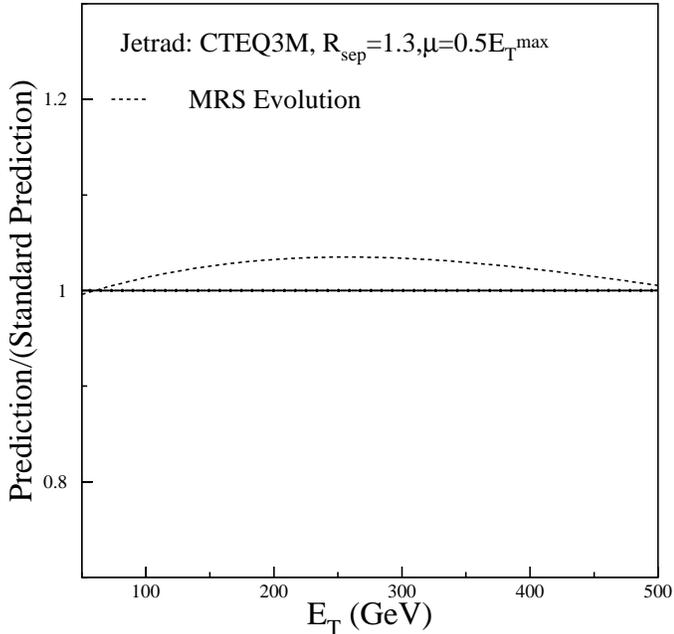


Fig. 9. A comparison between JETRAD calculation of the inclusive jet cross section with the CTEQ3M PDF using the MRS and the CTEQ evolution packages

5 The ratio of the inclusive jet cross sections at $\sqrt{s} = 1.8$ TeV and 630 GeV

Theoretical uncertainties in the NLO QCD predictions should be reduced in the ratios of the inclusive jet cross sections at $\sqrt{s} = 630$ GeV and 1.8 TeV as a function of the dimensionless quantity $X_T = 2E_T/\sqrt{s}$.

The variations in the ratio of the cross sections due to the choice of renormalization scale is approximately 15% with some dependence on X_T (see Fig. 12a and b). The uncertainty due to the choice of PDF is only a few percent (Fig. 12c) which compares to an uncertainty of up to 25% in the individual cross sections. The difference due to the variation of α_s is not reduced by measuring the ratio of the cross sections and is still at the 5% level (Fig. 12d).

6 Conclusion

The inclusive jet cross section, predicted using the available NLO programs, has significant uncertainties due to the choice of renormalization scale and PDF. The overall variation in the cross section can be as large as $\pm 30\%$. Except for alternative implementation of PDFs and the evolution strategies used, JETRAD and EKS appear to be identical.

The ratio of the inclusive jet cross sections at $\sqrt{s} = 630$ GeV and 1.8 TeV as a function of X_T has uncertainties of approximately 10–20%, which is much smaller than the variation of the cross sections.

Before the inclusive jet cross section can be used to test QCD or search for New Phenomena⁸, the theoretical predictions must improve. Most feasibly through improved measurement of the gluon distributions.

⁸ This result does not not eliminate the possibility of observing new physics that produce effects on the inclusive jet cross section that are significantly larger than the theoretical uncertainties

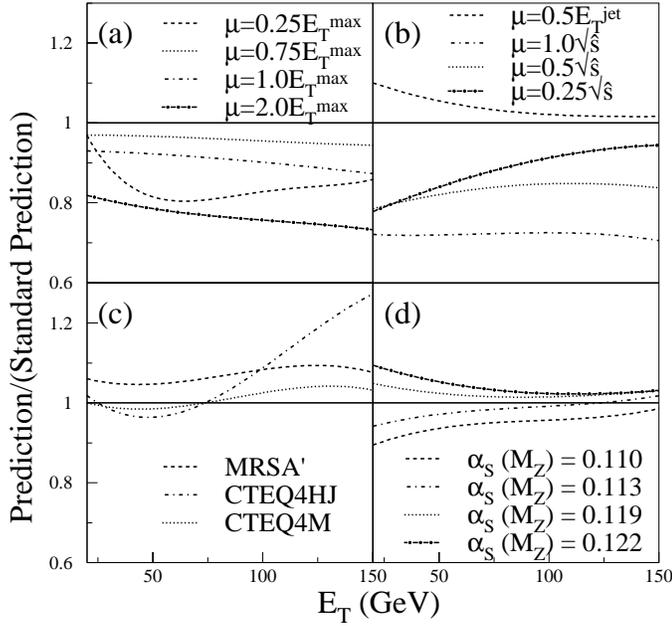


Fig. 11. The deviations of the inclusive jet cross section at $\sqrt{s} = 630$ GeV. **a** A comparison between the JETRAD calculations with $\mu = 0.25, 0.75, 1.0$ and $2.0E_T^{\max}$ compared with $\mu = 0.5E_T^{\max}$. **b** A comparison between the EKS calculation with $\mu = 0.5E_T^{\text{jet}}$ and $\mu = 0.5E_T^{\max}$. Also shown is a comparison between the JETRAD calculations with $\mu = 0.25, 0.5$ and $1.0\sqrt{s}$ compared with $\mu = 0.5E_T^{\max}$. **c** A comparison between the JETRAD calculations with the CTEQ4M, CTEQ4HJ and the MRSA' PDF compared with the calculation using CTEQ3M. **d** A comparison between the JETRAD calculations with the CTEQ4A series of PDFs compared with the calculation using CTEQ4M

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References

1. F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. **77** (1996) 438
2. F. Nang (for the CDF and DØ Collaborations) *Jet Production at the Tevatron*, FERMILAB-Conf-97/192-E, published proceedings of the QCD and High Energy Hadronic Interactions, XXXII Rencontres de Moriond, Les Arcs, France, March 22-29, 1997
3. F. Aversa, et al., Phys. Rev. Lett. **65** (1990) 401
4. W.T. Giele, E.W.N. Glover and D.A. Kosower, Nucl. Phys. **B403** (1993) 633
5. S.D. Ellis, Z. Kunszt and D.E. Soper Phys. Rev. Lett. **64** (1990) 2121; Z. Kunszt and D.E. Soper, Phys. Rev. **D46** (1992) 192
6. F. Abe et al., (CDF Collaboration), Phys. Rev. Lett. **77** (1996) 5336
7. J. Huth et al., in proceedings of *Research Directions for the Decade, Snowmass 1990*, edited by E.L. Berger (World Scientific, Singapore, 1992)

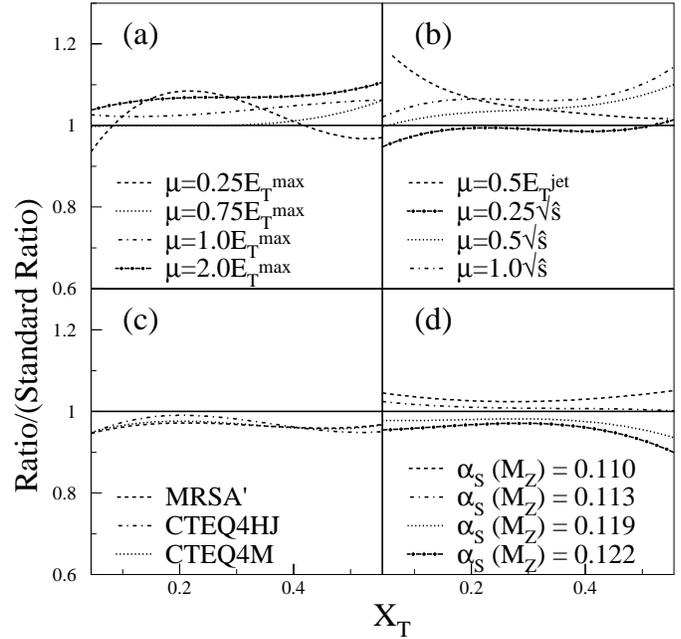


Fig. 12. The deviations of the ratio of inclusive jet cross sections at $\sqrt{s} = 1800$ GeV and 630 GeV. **a** A comparison between the JETRAD calculations with $\mu = 0.25, 0.75, 1.0$ and $2.0E_T^{\max}$ compared with $\mu = 0.5E_T^{\max}$. **b** A comparison between the EKS calculation with $\mu = 0.5E_T^{\text{jet}}$ and $\mu = 0.5E_T^{\max}$. Also shown is a comparison between the JETRAD calculations with $\mu = 0.25, 0.5$ and $1.0\sqrt{s}$ compared with $\mu = 0.5E_T^{\max}$. **c** A comparison between the JETRAD calculations with the CTEQ4M, CTEQ4HJ and the MRSA' PDF compared with the calculation using CTEQ3M. **d** A comparison between the JETRAD calculations with the CTEQ4A series of PDFs compared with the calculation using CTEQ4M

8. B. Abbott et al., FERMILAB-PUB-97/242-E
9. H.L. Lai et al., Phys. Rev. **D51** (1995) 4763
10. H.L. Lai et al., Phys. Rev. **D55** (1997) 1280
11. A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Lett. **B354** (1995) 154
12. D.E. Soper, *Jets and Partons*, hep-ph/9610434 (1996)
13. M. Arneodo et. al (NMC Collaboration) Phys. Lett. **B364** (1995) 107
14. S. Aid et al. (H1 Collaboration) Nucl. Phys. **B439** (1995) 471
15. M. Derrick e al. (ZEUS Collaboration) Z. Phys. **C65** (1995) 379
16. B. Flaugher (for the CDF Collaboration), *talk given at APS meeting*, Indianapolis, May 1996.; B. Flaugher (for the CDF Collaboration), *High E_T Jet Cross-sections at CDF*, FERMILAB-Conf-96/225-E, Presented at the 11th Topical Workshop on Proton-Antiproton Collider Physics, Padua, Abano Terme, Italy, 26 May–1 June 1996
17. G. Blazey (for the DØ Collaborations) *Preliminary measurement of the Inclusive Jet and Dijet Cross Section in pp Collisions at sqrt(s) = 1.8 TeV*, FERMILAB-Conf-96/132-E, published proceedings of the QCD and High Energy Hadronic Interactions, XXXI Rencontres de Moriond, Les Arcs, France, March 23-30, 1996