An Iterative Blind Source Separation Method for Convolutive Mixtures of Images

Marc Castella and Jean-Christophe Pesquet

Université de Marne-la-Vallée / UMR-CNRS 8049
5 bd Descartes, Champs-sur-Marne
77454 Marne-la-Vallée CEDEX 2, France
{castellm,pesquet}@univ-mlv.fr

Abstract. The paper deals with blind source separation of images. The model which is adopted here is a convolutive multi-dimensional one. Recent results about polynomial matrices in several indeterminates are used to prove the invertibility of the mixing process. We then extend an iterative blind source separation method to the multi-dimensional case and show that it still applies if the source spectra vanish on an interval. Based on experimental observations we then discuss problems arising when we want to separate natural images: the sources are non i.i.d. and have a band limited spectrum; a scalar filtering indeterminacy thus remains after separation.

1 Introduction

Due to its numerous applications such as passive sonar, seismic exploration, speech processing and multi-user wireless communications, blind source separation (BSS) has been an attractive and fruitful research topic for the last few years. Independent Component Analysis (ICA) has also found interesting applications in image processing, but in this context, the original framework of instantaneous mixture has been mainly considered (e.g. [1, 2]).

However mixtures may be more complicated in practice, and the spread of each source over several pixels may require to address the general model of convolutive mixtures [3]. The single-channel blind deconvolution of images has been extensively studied and solutions have been proposed which usually involve some regularization and use some prior information [4]. Multichannel acquisition, which provides several different blurred version of a single image, also allows to improve the image restoration quality. This Single Input/Multiple Output (SIMO) case has been extensively studied (see e.g. [5, 6]) and is not addressed here. However, little attention has been paid to the general Multiple Input/Multiple Output (MIMO) case, where independent sources are mixed on different sensors.

This paper considers MIMO convolutive mixtures of independent multi-dimensional signals: the problem is described in Section 1. Two main difficulties arise in the 2D case: finding invertibility conditions for the 2D mixing process and ability to deal with non i.i.d. sources which may have band-limited (or rapidly decaying) spectra. The former problem is discussed in Section 3. The latter one is addressed in Section 4 which presents the separation method. Finally, simulation results in Section 5 show the validity of the proposed method and outline the specificities of convolutive source separation for images.

2 Problem statement

We consider $N \in \mathbb{N}^*$ two-dimensional signals which, for $i \in \{1, \ldots, N\}$ are denoted by $(s_i(\boldsymbol{n}))_{\boldsymbol{n}\in\mathbb{Z}^2}$. Though our theoretical results apply to the general p-dimensional case, we will be more particularly interested in images. For the sake of readability, we shall equivalently use either a two-dimensional notation (n_1, n_2) or a boldface character $\boldsymbol{n}$. The N former signals are referred to as *source* signals, which generate $Q \in \mathbb{N}^*$ *observation* signals according to the following 2D convolutive mixture model:

$$\boldsymbol{x}(n_1, n_2) = \sum_{(k_1,k_2)\in\mathbb{Z}^2} \boldsymbol{M}(k_1, k_2)\boldsymbol{s}(n - k_1, n - k_2) = \sum_{\boldsymbol{k}} \boldsymbol{M}(\boldsymbol{k})s(\boldsymbol{n} - \boldsymbol{k}). \quad (1)$$

We use here vector notations where $\boldsymbol{s}(\boldsymbol{n}) = (s_1(\boldsymbol{n}), \ldots, s_N(\boldsymbol{n}))^T$ and $\boldsymbol{x}(\boldsymbol{n}) := (x_1(\boldsymbol{n}), \ldots, x_Q(\boldsymbol{n}))^T$ are respectively the source and observation vectors, and $(\boldsymbol{M}(\boldsymbol{k}))_{\boldsymbol{k}\in\mathbb{Z}^2}$ is a set of $Q \times N$ matrices which corresponds to the impulse response of the mixing system. BSS aims at inverting the above described process, with no precise knowledge about the mixing process or the sources. The *separating* system is modeled as a linear convolutive structure and reads:

$$\boldsymbol{y}(n_1, n_2) = \sum_{(k_1,k_2)\in\mathbb{Z}^2} \boldsymbol{W}(k_1, k_2)\boldsymbol{x}(n - k_1, n - k_2) = \sum_{\boldsymbol{k}} \boldsymbol{W}(\boldsymbol{k})x(\boldsymbol{n} - \boldsymbol{k}) \quad (2)$$

where $(\boldsymbol{W}(\boldsymbol{k}))_{\boldsymbol{k}\in\mathbb{Z}^2}$ is the impulse response of the separating filter of size $N \times Q$ and $\boldsymbol{y}(\boldsymbol{n}) = (y_1(\boldsymbol{n}), \ldots, y_N(\boldsymbol{n}))^T$ is the separation result. Ideally, $\boldsymbol{y}(\boldsymbol{n})$ reduces to the original source vector, up to a permutation and a scalar filtering indeterminacy. Some assumptions have to be made in addition to the aforementioned convolutive model for the source separation task to be achievable:

A.1 The source processes $(s_i(\boldsymbol{n}))_{\boldsymbol{n}\in\mathbb{Z}^2}, i \in \{1, \ldots, N\}$ are statistically mutually independent and stationary.
A.2 The mixing system is stable (i.e. its impulse response is summable) and admits a summable inverse.

Assumption A.1 is a key assumption in BSS and ICA, whereas A.2 is necessary to be able to separate the sources. Invertibility conditions for multivariate systems are discussed in detail in Section 3.

Let us further emphasize that we do not require the sources to be i.i.d. and, contrary to other separation methods in the non i.i.d. context, we do not exploit the spectral diversity of the sources to realize source separation. Indeed, images may exhibit similar spectral characteristics. In most of the works dealing with convolutive mixtures in the same context, sources are generally supposed to be i.i.d.[3]. From the fact that the sources are non i.i.d., it follows that each source can only be recovered up to a scalar filtering, in addition to the well-known permutation ambiguity. The scalar filtering issue does not appear in instantaneous mixtures, as it reduces to a scaling factor ambiguity.

3 Invertibility conditions

The considered separation method is valid for all kind of filters, both with infinite impulse response (IIR) and with finite impulse response (FIR). However, considering FIR filters allows us to provide simple conditions for the invertibility of the mixing process.

3.1 Finite Impulse Response assumption

We assume in the following:

A.3 The mixing filter is FIR and $\boldsymbol{M}(k_1, k_2) = \mathbf{0}$ if $k_1 \notin \{0, \dots, L_1 - 1\}$ or $k_2 \notin \{0, \dots, L_2 - 1\}$.

In the 1D case, it is well known that, under primeness conditions, the mixing system admits an inverse (see [7] and references therein). This result is based on results concerning polynomial matrices and extends to the multivariate case. Let us first define the following z-transform of the mixing system:

$$\boldsymbol{M}[z_1, z_2] := \sum_{(k_1,k_2)\in\mathbb{Z}^2} \boldsymbol{M}(k_1, k_2) z_1^{-k_1} z_2^{-k_2} = \boldsymbol{M}[\boldsymbol{z}] = \sum_{\boldsymbol{k}} \boldsymbol{M}(\boldsymbol{k}) \boldsymbol{z}^{-\boldsymbol{k}}. \quad (3)$$

The z-transform of the separating system is defined in the same way and is denoted by $\boldsymbol{W}[\boldsymbol{z}]$. Equations (1) and (2) can then be formally written:

$$\boldsymbol{x}(\boldsymbol{n}) = \boldsymbol{M}[\boldsymbol{z}]\boldsymbol{s}(\boldsymbol{n}) \quad \text{and} \quad \boldsymbol{y}(\boldsymbol{n}) = \boldsymbol{W}[\boldsymbol{z}]\boldsymbol{x}(\boldsymbol{n}). \quad (4)$$

The goal of BSS consists in finding $\boldsymbol{W}[\boldsymbol{z}]$ such that the global transfer function $\boldsymbol{G}[\boldsymbol{z}] := \boldsymbol{W}[\boldsymbol{z}]\boldsymbol{M}[\boldsymbol{z}]$ is diagonal up to a permutation. Conditions for the existence of such an inverse are discussed in the next section.

3.2 Primeness properties and invertibility

Invertibility properties of the mixing system rely on primeness properties of the polynomial matrix $\boldsymbol{M}[\boldsymbol{z}]$. Although some theoretical results may be found in the literature [8, 9] for multi-dimensional signals, some of them are not easily accessible. The ring of Laurent polynomials in indeterminates $\boldsymbol{z} = (z_1, \dots, z_p)$ and with coefficients in $\mathbb{C}$ is denoted by $\mathbb{C}[\boldsymbol{z}]$. Primeness properties in $\mathbb{C}[\boldsymbol{z}]$ are somewhat more complicated than in the case of polynomials in one indeterminate as there exist four distinct notions of primeness. We will be particularly interested in the following definition:

Definition 1. *A polynomial matrix* $\boldsymbol{M}[\boldsymbol{z}] \in \mathbb{C}[\boldsymbol{z}]^{Q\times N}$ *is said to be* right zero prime *if* $Q \geq N$ *and the ideal generated by its maximal order minors is the ring* $\mathbb{C}[\boldsymbol{z}]$ *itself.*

An equivalent definition of right-zero coprimeness can be obtained after slight modifications of known results:

Property 1. A polynomial matrix $\boldsymbol{M}[\boldsymbol{z}] \in \mathbb{C}[\boldsymbol{z}]^{Q\times N}$ is *right zero prime* if and only if its maximal order minors have no common zero in $(\mathbb{C}^*)^p$.

The invertibility of the mixing system is ensured by the following property [8]:

Property 2. A polynomial matrix $\boldsymbol{M}[\boldsymbol{z}] \in \mathbb{C}[\boldsymbol{z}]^{Q\times N}$ is right zero prime if and only if it has a polynomial left inverse, or equivalently if and only if there exists $\boldsymbol{W}[\boldsymbol{z}] \in \mathbb{C}[\boldsymbol{z}]^{Q\times N}$ such that $\boldsymbol{W}[\boldsymbol{z}]\boldsymbol{M}[\boldsymbol{z}] = \boldsymbol{I}_N$

The above property provides a necessary and sufficient condition for the mixing system to be invertible. Since however the mixing system is supposed to be unknown, an interesting point would be to know if a randomly generated polynomial matrix is likely to have a polynomial left inverse or not. The answer was partially given in [9]:

Property 3. If the $Q \times N$ polynomial matrix $\boldsymbol{M}[\boldsymbol{z}]$ has coefficients drawn from a continuous density function and if $\binom{Q}{N} := \frac{Q!}{N!(Q-N)!} > p$, then $\boldsymbol{M}[\boldsymbol{z}]$ is almost surely invertible.

In particular, for images $p = 2$ and one can see that a mixing system with coefficients driven from a continuous density function is almost surely invertible as soon as there are more sensors than sources ($Q > N$). Finally, bounds on the order of the separating filter have been given [9]: although they are quite large, they give a maximum order for a possible separating system.

4 Separation method

In this section, we will see how a 1D iterative separating method can be used for the separation of multi-dimensional sources. Among the possible approaches, iterative and deflation-like methods appear especially appealing as they allow the separation of non i.i.d. sources. In addition, they do not present spurious local maxima, unlike many global MIMO approaches.

4.1 An iterative approach

Contrast function for the extraction of one source We first consider the extraction of one source and denote by $\boldsymbol{w}[\boldsymbol{z}]$ one row of the separating system $\boldsymbol{W}[\boldsymbol{z}]$. Let $\boldsymbol{g}[\boldsymbol{z}] := \boldsymbol{w}[\boldsymbol{z}]\boldsymbol{M}[\boldsymbol{z}]$ denote the corresponding row of the global system. Contrast functions are a practical tool to tackle BSS as they reduce it to an optimization problem: by definition, a contrast function is maximum if and only if separation is achieved. Since we consider here an iterative approach and since the global filter $\boldsymbol{g}[\boldsymbol{z}]$ is a Multiple Input/Single Output (MISO) one, a contrast is maximum if and only if the global scalar output is a scalar filtered version of one source. Consider the following function:

$$J(\boldsymbol{w}) := \left|\mathrm{Cum}^4[y(\boldsymbol{n})]\right| \tag{5}$$

where $y(\boldsymbol{n}) := \boldsymbol{g}[\boldsymbol{z}]s(\boldsymbol{n})$ is the global scalar output corresponding to $\boldsymbol{w}[\boldsymbol{z}]$ and where $\mathrm{Cum}^4[.]$ denotes the fourth-order auto-cumulant. It has been proved that the function J (which depends on $\boldsymbol{w}$ or equivalently on $y(\boldsymbol{n})$) constitutes a contrast for both i.i.d. [10] and non i.i.d. sources [11], if it is maximized under the constraint:

C.1 $\mathrm{E}\{|y(\boldsymbol{n})|^2\} = 1$.

The method has been used in the 1D case [11, 12]. In the 2D case, it can be implemented as follows: define the vectors $\boldsymbol{\mathcal{W}}$ and $\boldsymbol{\mathcal{X}}(\boldsymbol{n})$ which are composed of the terms $w_j(\boldsymbol{k})$ and $x_j(\boldsymbol{n}-\boldsymbol{k})$, respectively, when $\boldsymbol{k}$ varies in $\{0,\ldots,L_1-1\} \times \{0,\ldots,L_2-1\}$ and j varies in $\{1,\ldots,Q\}$. One can then write: $y(\boldsymbol{n}) = \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{X}}(\boldsymbol{n})$. The optimization of (5) is then be carried out with a batch, iterative algorithm, where constraint C.1 is imposed at each iteration by a re-normalization step. The optimization procedure can be written in a such way that $\boldsymbol{\mathcal{X}}(\boldsymbol{n})$ and $\boldsymbol{\mathcal{W}}$ are the only required inputs. This means that 1D separation procedures can be used under the appropriate modifications of the definition of $\boldsymbol{\mathcal{W}}$ and $\boldsymbol{\mathcal{X}}(\boldsymbol{n})$.

Extraction of the remaining sources After having separated one source, deflation approaches subtract its contribution from the observations by a least square approach. The former procedure is then applied again on a new observation vector. If P sources ($P < N$) have been extracted and $y_1(\boldsymbol{n}),\ldots,y_P(\boldsymbol{n})$ denote the obtained outputs, we alternatively suggest to carry out the optimization of J under the constraint:

C.2 $\forall i \in \{1,\ldots,P\}, \forall \boldsymbol{k} \quad \mathrm{E}\{y(\boldsymbol{n})y_i^*(\boldsymbol{n}-\boldsymbol{k})\} = 0$.

It can be proved that constraint C.2 prevents from separating twice the same source. Furthermore, C.2 is a linear constraint on $\boldsymbol{\mathcal{W}}$, which can hence easily be taken into account.

4.2 Validity of the method for sources with non positive definite auto-correlation

Natural images are highly correlated and their spectrum is mostly concentrated on low frequencies. We consider the limit case when the source spectrum is positive on a set Ω and vanishes on its complementary set $\overline{\Omega}$. Let us see the consequences when $\overline{\Omega}$ is with non zero measure. Writing $\boldsymbol{g}[\boldsymbol{z}] = (g_1[\boldsymbol{z}],\ldots,g_N[\boldsymbol{z}])$, let fix $i \in \{1,\ldots,N\}$ and define:

$$\|g_i\|_i = \Big(\sum_{\boldsymbol{k},\boldsymbol{l}} g_i(\boldsymbol{k})g_i^*(\boldsymbol{l})\gamma_i(\boldsymbol{l}-\boldsymbol{k})\Big)^{\frac{1}{2}} \tag{6}$$

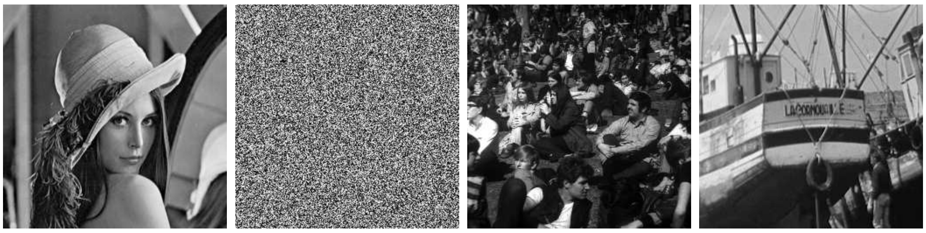


Fig. 1. Original images used as sources

where $\gamma_i(\boldsymbol{k})$ is the autocorrelation of the i-th source. If $\overline{\Omega} = \emptyset$ the sequence $\gamma_i(\boldsymbol{k})$ is definite positive and $\|.\|_i$ is a norm. On the contrary, if $\overline{\Omega}$ is with non zero measure, $\|.\|_i$ is a semi-norm only and the proof in [11] no longer applies, since it is based on the possibility to write $g_i[\boldsymbol{z}] = \|g_i\|_i \frac{g_i[\boldsymbol{z}]}{\|g_i\|_i}$ for any non-zero filter .

Fortunately, one can consider working over the subset of filters which are identically zero on the frequency band $\overline{\Omega}$, and the proof in [11] can then be easily adapted. However, one can see that after separation, the part of the global filter which operates on the band $\overline{\Omega}$ is left free. This part has indeed no influence neither on the separator outputs nor on the contrast J. This may however lead to numerical difficulties with sources with limited band spectrum.

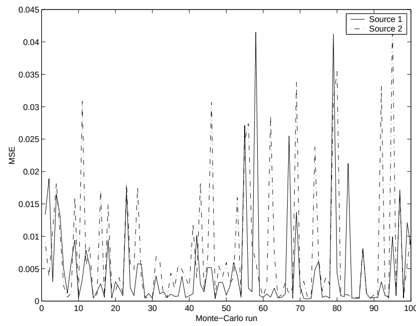


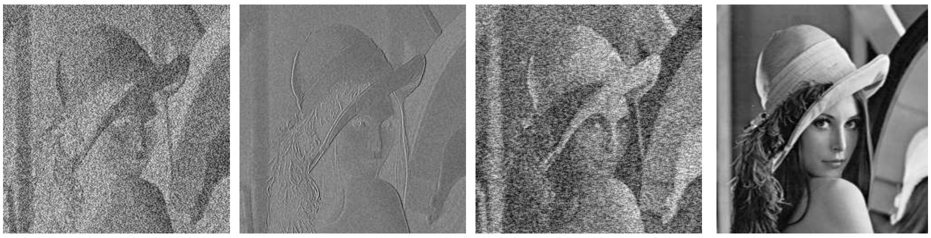
Fig. 2. MSE after separation of two uniform i.i.d. 2D sources.

5 Effectiveness of the procedure

The previous results have been tested on convolutive mixtures of images. In our experiments, there were 2 source images and 3 sensors, so that invertibility is almost surely guaranteed as soon as the coefficients of the mixing system are drawn from a continuous probability density function.

5.1 Simulation results with i.i.d. sources

We first verified the validity of our assertions with i.i.d. sources. The study was carried out on a set of 100 Monte-Carlo runs. The coefficients of the mixing systems were drawn randomly from a Gaussian zero-mean unit-variance distribution and the sources were i.i.d. uniform, unit-variance and zero-mean. The length of the mixing system was set to $L_1 = L_2 = 2$ whereas the length of the separator was set to $D_1 = D_2 = 5$. The images were of size 256×256. Results are plotted in Figure 2 and show the mean square reconstruction error (MSE) for each source an Monte-Carlo run. As the sources are i.i.d., the scalar filtering ambiguity is known to reduce to a simple delay. All MSE were below 0.043 and the mean value over all realization was 5.5×10^{-3}. Naturally, the invertibility of the mixing filter must be ensured in order to obtain good results. Hence the separator should be long enough and a shorter separator led in our experiments to degraded performances. These experimental results prove both the validity of Properties 2, 3 and the ability of the method to separate sources.



Observations Reconstructed image

Fig. 3. Separation of a natural image and a noise source

5.2 Natural images

The separation of a natural image and a noise image was tested on a filter of length $L_1 = 3, L_2 = 1$ and a separator of length $D_1 = 4, D_2 = 1$. The sources are the first two ones in Figure 1 and the mixtures are given in Figure 3. Another interesting example involving now two natural sources is given by the mixture of the last two images given in Figure 1 with a filter of length $L_1 = 2, L_2 = 1$ and a separator of length $D_1 = 2, D_2 = 1$. The observations are represented in Figure 4. As previously said, there is no guarantee to recover the original unfiltered sources at the output of the method: this is particularly well illustrated in Figure 4, where we roughly recognize high-pass filtered versions of the original sources. It is interesting to note however that in a certain number of experiments, other filtered versions of the original sources can be obtained as well, including filters which can be close to the identity. On the contrary, the noise in Figure 3 being i.i.d., it can be recovered up to a delay and scaling factor only.

Suppose that each source has a non convolutive contribution on one sensor. Then, it is possible to recover the sources by subtracting the output of the algorithm from the sensors by a least square approach. The results are given in Figure 4, and one can see that the original sources are well recovered. In other cases, one may resort to other image processing techniques in order to solve the remaining blind SISO deconvolution problem, when the sources have been separated.

Acknowledgment The authors are grateful to Professor Maria Elena Valcher from University of Padova for fruitful discussions.

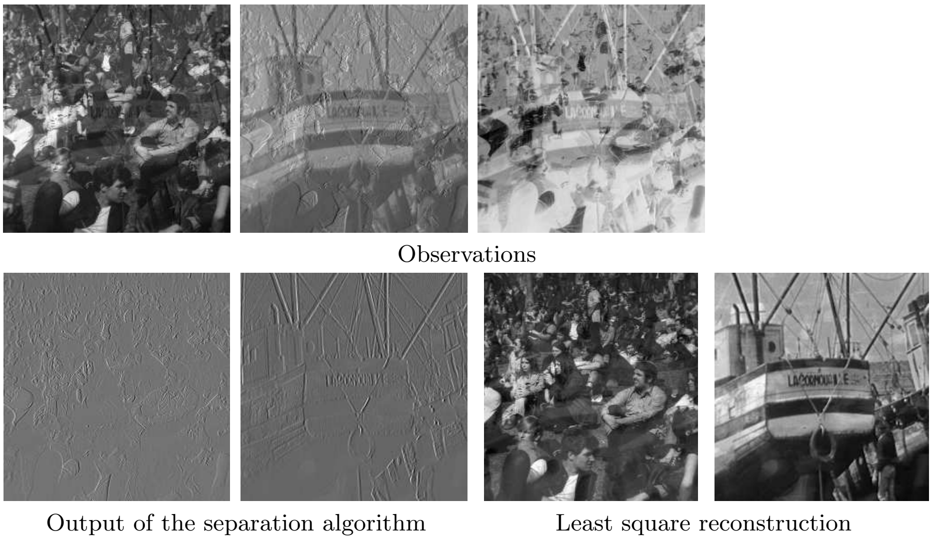


Fig. 4. Separation of two images

References

1. Cardoso, J.F.: Independent component analysis of the cosmic microwave background. In: Proc. of ICA'03, Nara, Japan (2003) 1111–1116
2. Hyvärinen, A., Hoyer, P.O., Hurri, J.: Extensions of ica as models of natural images and visual processing. In: Proc. of ICA'03, Nara, Japan (2003) 963–974
3. Comon, P.: Contrasts for multichannel blind deconvolution. IEEE Signal Processing Letters **3** (1996) 209–211
4. Kundur, D., Hatzinakos, D.: Blind image deconvolution. IEEE Signal Processing Mag. **13** (1996) 43–64
5. Giannakis, G.B., Heath, R.W.: Blind identification of multichannel FIR blurs and perfect image restoration. IEEE Trans. on Image Processing **9** (2000) 1877–1896
6. Šroubek, F., Flusser, J.: Multichannel blind iterative image restoration. IEEE Trans. on Image Processing **12** (2003) 1094–1106
7. Gorokhov, A., Loubaton, P.: Subspace based techniques for blind separation of convolutive mixtures with temporally correlated sources. IEEE Trans. Circuits and Systems I **44** (1997) 813–820
8. Fornasini, E., Valcher, M.E.: nD polynomial matrices with applications to multidimensional signal analysis. Multidimensional Systems and Signal Processing **8** (1997) 387–408
9. Rajagopal, R., Potter, L.C.: Multivariate MIMO FIR inverses. IEEE Trans. on Image Processing **12** (2003) 458–465
10. Tugnait, J.K.: Identification and deconvolution of multichannel linear non-gaussian processes using higher order statistics and inverse filter criteria. IEEE Trans. Signal Processing **45** (1997) 658–672
11. Simon, C., Loubaton, P., Jutten, C.: Separation of a class of convolutive mixtures: a contrast function approach. Signal Processing (2001) 883–887
12. Tugnait, J.K.: Adaptive blind separation of convolutive mixtures of independent linear signals. Signal Processing **73** (1999) 139–152