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Yingbo Hua, Tapan K. Sarkar, D.D. Weiner

Institutions: University of Melbourne

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Authors

Hua, Y
Sarkar, TK
Weiner, DD

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An L-Shaped Array for Estimating 2-D Directions of Wave Arrival

Yingbo Hua, *Member, IEEE*, Tapan K. Sarkar, *Senior Member, IEEE*, and Donald D. Weiner, *Member, IEEE*

Abstract—For the estimation of two-dimensional (2-D) directions of plane wave arrival, an L-shaped array of sensors has 37% better accuracy potential than the conventional cross array. Computationally efficient maximum likelihood algorithm for the L-shaped array is available to achieve its full potential.

I. INTRODUCTION

UNIFORM linear arrays (ULA's) for estimation of wave arrival have been studied extensively [1]–[3]. It is understood that the ULA can provide only the one-dimensional (1-D) information of wave arrival. For two-dimensional (2-D) wave arrival information, a 2-D array of sensors is needed. A conventional 2-D array is the cross array, which consists of two ULA's connected orthogonally at the middle of each array. The cross array is important because of its simple structure so that the computationally efficient maximum likelihood (ML) algorithm [2], [3] can be designed to achieve its full potential. (Note that the Cramer–Rao bound (CRB) of the cross array can be shown to be the same as the CRB of each ULA in the cross array so that the ML algorithm for the ULA is also optimum for the cross array.) On the other hand, another conventional array called circular array has a much more complicated structure so that there is no efficient ML algorithm available for multiple wave direction finding. In this paper, we present another simple structured 2-D array, called the L-shaped array. The L-shaped array consists of two ULA's connected orthogonally at the one end of each ULA.

In Section II, we show that the Cramer–Rao bounds (CRB's) of the estimated wave directions based on the L-shaped array are about 37% smaller than those for the cross array. CRB indicates the accuracy potential because it is the (reachable) lower bound on variance of any unbiased estimate [4].

In Section III, an efficient ML algorithm is developed utilizing the ULA structure inherent in the L-shaped array.

II. L-SHAPED ARRAY

To appreciate the accuracy potential and the unique structure of the L-shaped array for estimation of 2-D wave arrival, we compare it with several other 2-D arrays as shown in Fig. 1. All sensors in Fig. 1 are identical and omnidirectional, and they are located on a uniform grid (so that the CRB can be derived). The arriving waves are assumed to be plane waves. The array output

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Y. Hua is with the Department of Electrical Engineering, University of Melbourne, Parkville, Victoria 3052, Australia.
T. K. Sarkar and D. D. Weiner are with the Department of Electrical Engineering, Syracuse University, Syracuse, NY 13244-1240.
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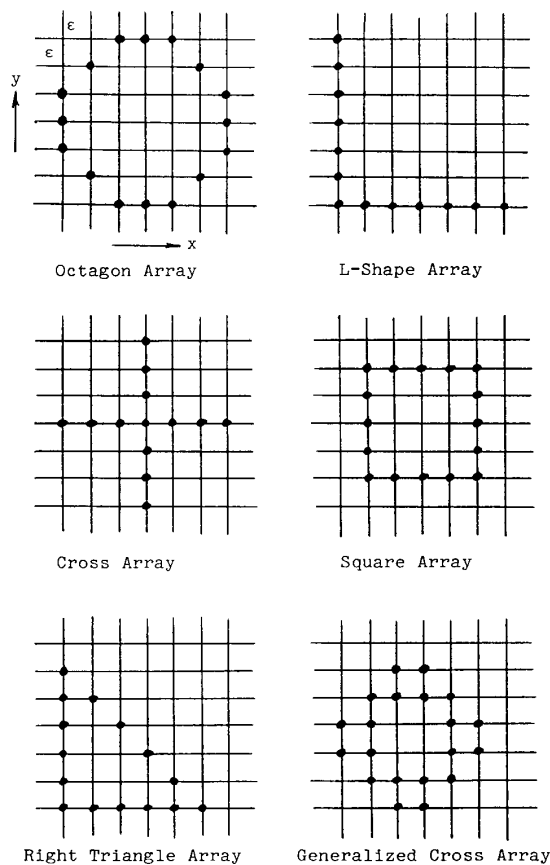


Fig. 1. Six array configurations on uniform $x-y$ grid.

vector can then be written as

$$y(t) = \sum_{k=1}^D a_k(t) z(\alpha_k, \beta_k) + n(t) \quad (1)$$

with

$$y(t) = [y_1(t), \dots, y_N(t)]^T \quad (2)$$

$$z(\alpha_k, \beta_k)$$

$$= \begin{bmatrix} \exp [j2\pi(\epsilon/\lambda) \cos(\alpha_k) x_1 + j2\pi(\epsilon/\lambda) \cos(\beta_k) y_1] \\ \dots \\ \exp [j2\pi(\epsilon/\lambda) \cos(\alpha_k) x_N + j2\pi(\epsilon/\lambda) \cos(\beta_k) y_N] \end{bmatrix} \quad (3)$$

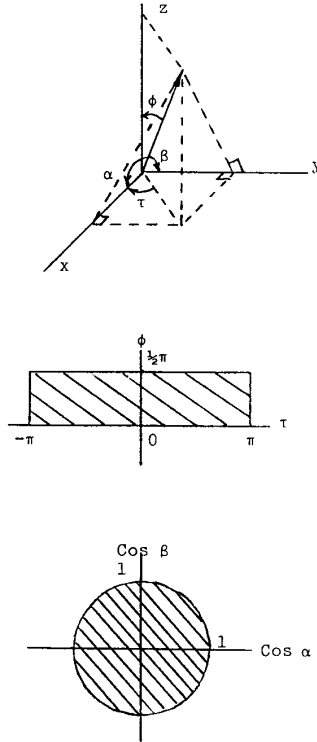


Fig. 2. Relation between two coordinate systems of a 2-D direction.

where t denotes the t th snapshot of the total T ; D denotes the number of waves; $a_k(k)$ is the complex amplitude of the k th wave, the $N \times 1$ vector $\mathbf{z}(\dots)$ is called the steering vector; α_k and β_k are the two angles of the k th wave with respect to the x and y axes as shown in Fig. 2; ϵ is the spacing in the x - y grid; λ is the wavelength; (x_i, y_i) is the coordinate of the i th sensor of the total N ; and $\mathbf{n}(t)$ is the noise vector.

Computing the CRB [4] involves the inversion of a Fisher information matrix of the dimension equal to the number of unknowns. In order to be able to derive the CRB analytically, we consider the case where $T = 1$, $D = 1$, and the noise is the white Gaussian with the variance $2\sigma^2$. Then, there are the four unknowns: α_1 , β_1 , the magnitude and the phase of $a_1(1)$. Now, computing the CRB's of the estimates of α_1 and β_1 for each of the 2-D arrays in Fig. 1 becomes straightforward. Note that the 4×4 Fisher information matrix is a sparse matrix which can be inverted analytically. In Table I, the CRB's of $\cos \alpha_1$ and $\cos \beta_1$ are listed for each array. The CRB's on $\cos \alpha_1$ and $\cos \beta_1$ are equal because of the symmetry of all the arrays considered. The simple expressions given in the table also requires $N \gg 1$.

It is seen from this table that the CRB of the L-shaped array is significantly ($\cong 37\%$) smaller than that of the conventional cross array!

It is also seen from the table that the octagon array (which is close to the circular array in structure) has only $\cong 5\%$ smaller CRB than the L-shaped array.

III. MAXIMUM LIKELIHOOD ESTIMATION (MLE)

It is known that the CRB can be achieved by the MLE. For the L-shaped array, a computationally efficient ML algorithm is developed in the following way.

TABLE I
CRB($\cos \alpha_1$) = CRB($\cos \beta_1$)

Array Type	CRB($\cos \alpha_1$) = CRB($\cos \beta_1$)
Octagon Array	$\frac{57}{\delta N^3}$
L-Shaped Array	$\frac{60}{\delta N^3}$
Cross Array	$\frac{96}{\delta N^3}$
Square Array	$\frac{96}{\delta N^3}$
Right Triangle Array	$\frac{108}{\delta N^3}$
Generalized Cross Array	$\frac{192}{\delta N^3}$

$\delta = 2 \text{SNR}_1 (2\pi\epsilon/\lambda)^2$ and $\text{SNR}_1 = |a_1|^2 / 2\sigma^2$. $2\sigma^2$ is the variance of the white noise. Note that $\text{CRB}(\alpha_1) = \text{CRB}(\cos \alpha_1) / \sin^2 \alpha_1$ and $\text{CRB}(\beta_1) = \text{CRB}(\cos \beta_1) / \sin^2 \beta_1$.

Under the Gaussian assumption of the noise, the ML estimates minimize the cost function:

$$J = \left\| \mathbf{y} - \sum_{k=1}^D A_k \mathbf{z}(\alpha_k, \beta_k) \right\|^2 \quad (4)$$

where $\|\cdot\|$ denotes the 2-norm and

$$\mathbf{y} = \begin{bmatrix} y(1) \\ \vdots \\ y(T) \end{bmatrix} \quad (5)$$

$$A_k = \begin{bmatrix} a_k(1) I_N \\ \vdots \\ a_k(T) I_N \end{bmatrix}. \quad (6)$$

I_N is the $N \times N$ identity matrix. The cost function J is a highly nonlinear function of the unknowns: $\{a_k(t), \alpha_k$ and $\beta_k; k = 1, \dots, D$ and $t = 1, \dots, T\}$. Based on the structure of the L-shaped array, the following iterative ML algorithm has been developed.

A. Outer Loop

Let A'_k and \mathbf{z}'_k be the previous estimates of A_k and \mathbf{z}_k , where $\mathbf{z}_k = \mathbf{z}(\alpha_k, \beta_k)$. Define for $k = 1, \dots, D$,

$$\mathbf{x}_k = A'_k \mathbf{z}'_k + w_k \left(\mathbf{y} - \sum_{k=1}^D A'_k \mathbf{z}'_k \right) \quad (7)$$

where $w_k, k = 1, \dots, D$, are nonnegative real values satisfying

$$\sum_{k=1}^D w_k = 1. \quad (8)$$

Then, for each k , the new estimates of A_k and \mathbf{z}_k are obtained by minimizing

$$J_k = \left\| \mathbf{x}_k - A_k \mathbf{z}_k \right\|^2 \quad (9)$$

with respect to $a_k(t)$, α_k , and β_k .

For multiple waves, the correct pairing between the estimated α and the estimated β obtained from the ULA's still needs to be done. The optimum pairing is to minimize J with respect to all possible pairings of α and β . A more computationally efficient pairing technique follows.

Computing the sample correlation matrix ($N \times N$) of $y(t)$:

$$R_y = 1/T \sum_{t=1}^T y(t)y(t)^H$$

$$= Z \left\{ 1/T \sum_{t=1}^T a(t)a(t)^H \right\} Z^H + 1/T \sum_{t=1}^T n(t)n(t)^H \quad (25)$$

where

$$a(t) = [a_1(t), \dots, a_D(t)]^T \quad (26)$$

$$Z = [z_1, \dots, z_D]. \quad (27)$$

Assume both the signal amplitude vector $a(t)$ and the noise vector $n(t)$ are ergodic random processes with the covariance matrices: R_a (of the full rank D) and $R_n = 2\sigma^2 I_N$, respectively. Then, for large T (i.e., asymptotically),

$$R_y \cong ZR_a Z^H + 2\sigma^2 I_N. \quad (28)$$

It can be shown that the D principal eigenvectors of R_y asymptotically span the same space as the columns of Z , and the $N-D$ nonprincipal eigenvectors are asymptotically orthogonal to each of the columns of Z . Let the N eigenvectors of R_y be denoted by $v_1, \dots, v_D, v_{D+1}, \dots, v_N$, which are corresponding to the decreasing order of the eigenvalues. Then, the pairing can be obtained by minimizing the following weighted sum of the inner products between the steering vector $z(\alpha, \beta)$ and the $N-D$ nonprincipal eigenvectors v_m , $m = D + 1, \dots, N$:

$$J'' = \sum_{m=1}^D w'_m |z^H(\alpha, \beta)v_m|^2 \quad (29)$$

(with respect to D possible choices of β_k , $k = 1, \dots, D$, for each α_k , $k = 1, \dots, D$) where w'_m are positive weights. J'' is asymptotically zero if the ML estimates of α_k and β_k are correctly paired.

IV. CONCLUSION

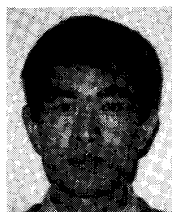
For 2-D wave direction finding, the L-shaped array has higher accuracy potential than the conventional cross array and many other simple structured arrays. Due to the property of the two ULA's in the L-shaped array, the maximum likelihood estimation of the wave directions can be implemented in a computationally efficient way.

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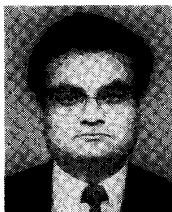
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Yingbo Hua (S'86-M'88) received the B.S. degree in control engineering from Nanjing Institute of Technology, China, in 1982, and the M.S. and Ph.D. degrees in electrical engineering from Syracuse University, Syracuse, NY, in 1983 and 1988, respectively.

He was a Graduate Teaching Assistant from 1984 to 1985, a Graduate Fellow from 1985 to 1986, a Graduate Research Assistant from 1986 to 1988, and a Postdoctoral Research Associate from 1988 to 1989 at Syracuse University. Since

February 1990, he has been a Lecturer with the University of Melbourne, Victoria, Australia. He has contributed over 30 articles on spectral analysis and array processing.



Tapan K. Sarkar (S'69-M'76-SM'81) was born in Calcutta, India, on August 2, 1948. He received the B.Tech. degree from the Indian Institute of Technology, Kharagpur, India, in 1969, the M.Sc.E. degree from the University of New Brunswick, Fredericton, Canada, in 1971, and the M.S. and Ph.D. degrees from Syracuse University, Syracuse, NY, in 1975.

From 1975 to 1976 he was with the TACO Division of the General Instruments Corpora-

tion. He was with the Rochester Institute of Technology, Rochester, NY, from 1976 to 1985. He was a Research Fellow at the Gordon McKay Laboratory, Harvard University, Cambridge, MA, from 1977 to 1978. He is now a Professor in the Department of Electrical and Computer Engineering, Syracuse University, Syracuse, NY. His current research interests deal with numerical solutions of operator equations arising in electromagnetics and signal processing with application to system design. He obtained one of the "best solution" awards in May 1977 at the Rome Air Development Center (RADC) Spectral Estimation Workshop. He has authored or coauthored more than 154 journal articles and conference papers and has written chapters in eight books.

Dr. Sarkar is a Registered Professional Engineer in the State of New York. He received the Best Paper Award of the IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY in 1979. He was an Associate Editor for feature articles of the *IEEE Antennas and Propagation Society Newsletter*, and he was the Technical Program Chairman for the 1988 IEEE Antennas and Propagation Society International Symposium and URSI Radio Science Meeting and an Associate Editor of the IEEE TRANSACTIONS ON ELECTROMAGNETIC COMPATIBILITY. He is an Associate Editor of the *Journal of Electromagnetic Waves and Applications*. He has been appointed U.S. Research Council Representative to many URSI General Assemblies. He is the Chairman of the Intercommission Working Group of International URSI on Time Domain Metrology. He is a member of Sigma Xi and International Union of Radio Science Commissions A and B.



Donald D. Weiner (S'54-M'60) received the S.B. and S.M. degrees from the Massachusetts Institute of Technology, Cambridge, in 1956 and 1958, respectively, and the Ph.D. degree from Purdue University, West Lafayette, IN, in 1964, all in electrical engineering.

He joined the faculty of Syracuse University, Syracuse, NY, in 1964 where he is currently a Professor of Electrical and Computer Engineering. He has performed extensive work in the application of nonlinear systems analysis tech-

niques and communications theory to electromagnetic compatibility problems. His present research deals with the detection of very weak radar signals embedded in non-Gaussian clutter and the use of matrix pencil techniques for system identification, spectral estimation and direction finding. He is the coauthor of *The Sinusoidal Analysis and Modeling of Weakly Nonlinear Circuits* with J. Spina (Van Nostrand Reinhold Co., 1980) as well as numerous technical papers.