An LMS Adaptive Array Using a Pilot Signal

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A least mean square (LMS) adaptive array requires a reference signal. When the desired signal contains a pilot signal, it may be used as the reference signal. In this paper the steady-state performance of an LMS adaptive array in which the pilot signal is used as the reference signal is examined. It is shown that the LMS adaptive array occasionally suppresses the desired signal. The loop gain, which is an important parameter, is also considered.

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I. INTRODUCTION

A least mean square (LMS) adaptive array automatically tracks a desired signal and nulls interference signals [1]. Thus, the LMS adaptive array is useful in mobile communication systems in which the desired signal arrival angle is unknown at the receiver. The LMS adaptive array, however, requires a reference signal in order to control each weight. Methods of generating the reference signal have been proposed for several communication systems [1–4]. Let us assume that the desired signal contains a deterministic component which is fully known at the receiver. Then, the deterministic component may be used for the adaptive array reference signal [2–4]. Examples of the deterministic component are a carrier in an amplitude modulated signal and a pilot signal which is added to the transmitted communication signal.

The purpose of this paper is to describe the steady-state performance of the LMS adaptive array in which a pilot signal is used for the reference signal. The LMS adaptive array regards all the signals except for the reference signal (pilot signal) as interferences. In order to separate the pilot signal easily, the frequency of the pilot signal is usually different from that of the information signal. For various conditions which will be discussed later, the information signal can be suppressed by the LMS adaptive array. The problem of information signal suppression in an LMS adaptive array has not been previously considered and is described in detail here.

The information signal suppression problem may be circumvented by decreasing the array loop gain. However, since lower values of loop gain yields poorer interference suppression performance, it is important to choose an appropriate value for loop gain. In this paper, we obtain an upper bound for the loop gain which keeps the output signal-to-noise ratio (SNR) above the required value when no interference signal is present. This result is useful for setting the loop gain in practical systems.

This paper is organized as follows. Section II describes the structure of the LMS adaptive array in which the pilot signal is used for the reference signal. Section III describes the steady-state performance of the LMS adaptive array. Information signal suppression is discussed in detail. Section IV describes the upper bound of the loop gain. Section V contains conclusions.

II. THE LMS ADAPTIVE ARRAY

Let the desired signal consist of two portions as shown in Fig. 1. One is the information signal which is to be transmitted. The other is the pilot signal which is known at the receiver. We assume that both are narrowband signals and that they are statistically independent of each other. We represent the frequencies of the information and pilot signals by f_a and f_p, respectively. We define the value r as

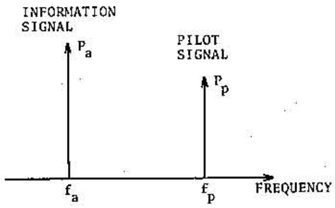

Fig. 1. Spectrum of the desired signal.

$$r = \frac{f_p - f_a}{f_a}. \tag{1}$$

Also, we express the powers of the information and pilot signals as P_a and P_p, respectively.

The pilot signal is either a pure tone or a tone with fully known modulation.

We consider the N-element LMS adaptive array shown in Fig. 2. In this paper we use complex-valued quantities. The pilot signal is used for the reference signal. In a case where the pilot signal is a pure tone, the reference signal may be generated by a phase-lock loop. In the case where the pilot signal is modulated by a PN code, the reference signal may be generated either by a delay-lock loop or by a tau-dither loop [5]. In the following considerations, we assume that the reference signal (pilot signal) is obtained at the output of the LMS adaptive array.

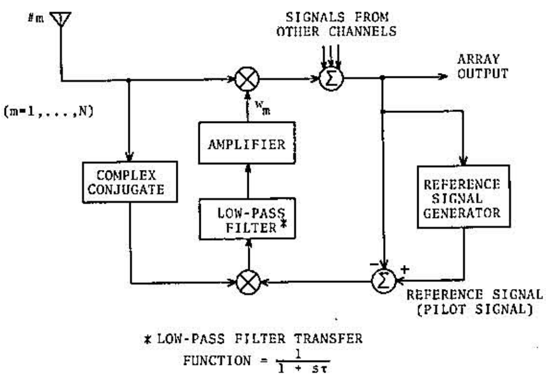

Fig. 2. LMS adaptive array.

We assume that the weight is controlled by a low-pass filter and amplifier instead of an integrator. We express the amplifier gain as G.

III. ARRAY PERFORMANCE

We assume for simplicity that each element is isotropic and that all mutual impedances are zero. Moreover, we assume that thermal noise components on different elements are independent of each other and that they have the same power P_n.

We express the complex weight for the mth element as w_m and the N-dimensional weight vector as W, that is,

$$W = [w_1 \quad w_2 \quad \ldots \quad w_N]^{\mathrm{T}} \tag{2}$$

where T denotes transpose.

Let ϕ_{am} and ϕ_{pm} be phases of the information and pilot signals on the mth element, respectively. We define N-dimensional phase vectors V_a and V_p as

$$V_a = [\exp(j\phi_{a1}) \ \exp(j\phi_{a2}) \ \ldots \ \exp(j\phi_{aN})]^{\mathrm{T}} \tag{3}$$

$$V_p = [\exp(j\phi_{p1}) \ \exp(j\phi_{p2}) \ \ldots \ \exp(j\phi_{pN})]^{\mathrm{T}}. \tag{4}$$

In this paper, we do not consider the random component of the complex weight, as we are interested only in the mean value of W. In the remainder of this paper, W denotes the ensemble average of the weight vector.

Let R_{xx} be the covariance matrix of the signals (the sum of the desired signal, interference signal, and thermal noise) from each element. Then, the steady-state weight vector is given by (5) to within a multiplicative constant:

$$W = (R_{xx} + I/G)^{-1} V_p^*. \tag{5}$$

The asterisk and I denote complex conjugate and an N×N identity matrix, respectively.

Here, we assume that no interference signal is present. Then, since the information and pilot signals are narrowband, R_{xx} is given by

$$R_{xx} = P_a V_a^* V_a^{\mathrm{T}} + P_p V_p^* V_p^{\mathrm{T}} + P_n I. \tag{6}$$

From (5) and (6), W is given by (7) to within a multiplicative constant:

$$W = \left(I + \frac{gS}{g+1} V_a^* V_a^{\mathrm{T}}\right)^{-1} V_p^*. \tag{7}$$

S and g are defined as

$$S = P_a/P_n \tag{8}$$

$$g = GP_n. \tag{9}$$

S represents the ratio of the information signal power to the thermal noise power. In the remainder of this paper, we refer to S as the input SNR. On the other hand, g represents the normalized loop gain in the LMS adaptive array. The matrix inverse in (7) is easily computed. Then, W is given by

$$W = V_p^* - \frac{hcNS}{hNS+1} V_a^* \tag{10}$$

where $h = g/(g+1)$ and (11)

$$c = V_a^{\mathrm{T}} V_p^*/N. \tag{12}$$

c is the spatial correlation coefficient of the information signal and the pilot signal through the adaptive array [6].

Using (10), the output thermal noise power and output information signal power are obtained as follows:

output thermal noise power

$$= \frac{1}{2} P_n W^{\dagger} W$$

$$= \frac{NP_n}{2}\left\{1 - \frac{2cc^*hNS}{hNS+1} + \frac{cc^*h^2N^2S^2}{(hNS+1)^2}\right\} \tag{13}$$

where † denotes Hermitian conjugate, and

output information signal power

$$= \frac{1}{2} W^{\dagger} (P_a V_a^* V_a^T) W$$

$$= \frac{N_c^2 c^* P_a}{2(hNS + 1)^2}. \tag{14}$$

As stated earlier, for certain conditions the information signal can be suppressed by an LMS adaptive array using a pilot signal for the reference signal. Thus, the output information signal power is very important. On the other hand, since the pilot signal is obtained at the output of the LMS adaptive array, the output pilot signal power is not so important. Therefore, we refer to the output information signal power as merely the output signal power in the remainder of this paper. Then, from (13) and (14), the output SNR is given by

$$\text{output SNR} = \frac{(1/2)\, W^{\dagger} (P_a V_a^* V_a^T) W}{(1/2)\, P_n W^{\dagger} W}$$

$$= \frac{cc^* NS}{(1 - cc^*) h^2 N^2 S^2 + 2(1 - cc^*) hNS + 1}. \tag{15}$$

From (15), we may see the following.

When the input SNR is small ($S << 1$), the output SNR is proportional to the input SNR, that is,

$$\text{output SNR} \simeq cc^* NS. \tag{16}$$

When $S >> 1$ and $cc^* \neq 1$,

$$\text{output SNR} \simeq \frac{cc^*}{(1 - cc^*) h^2 NS} \tag{17}$$

is obtained.

In almost all cases where a pilot signal is used for the reference signal, $cc^* \neq 1$ holds. Thus, from (17), it may be said that when the input signal power is high, the output SNR is inversely proportional to the input SNR.

The reason the output SNR decreases when $S >> 1$ is because the LMS adaptive array regards all the signals except for the reference signal as interference to be suppressed. The strong information signal, uncorrelated with the pilot signal, is regarded as a strong interference signal and is suppressed.

The desired signal can be suppressed also by a Howells–Applebaum adaptive array when the desired signal arrives from the different angle pointed by the steering signal [7]. The desired signal suppression by both adaptive arrays is due to the equivalent cause; namely, having the information signal on one frequency and the pilot signal on another is electrically equivalent to having the information signal arriving from one angle and the steering signal pointing to another on the same frequency.

From (11), (12), and (15), it may be said that the information signal suppression depends on the input SNR, loop gain, array structure, pilot signal frequency, information signal frequency, and desired signal arrival direction.

From (15), we see that the information signal suppression can be reduced, either by setting cc^* close to 1, or by setting h close to 0. The former is realized by setting the pilot signal frequency near the information signal frequency. The latter is realized by decreasing the loop gain.

We express the wavelength of the information signal as λ_a. Let us consider a linear array whose element spacing is a half-wavelength ($\lambda_a/2$). Furthermore, let the desired signal arrive from spatial angle θ_d relative to broadside.

Then, we have

$$cc^* = \frac{1}{N^2} \frac{\sin^2\{(N\pi r \sin \theta_d)/2\}}{\sin^2\{(\pi r \sin \theta_d)/2\}}. \tag{18}$$

Using the results obtained above, numerical calculations were done for a four-element linear adaptive array whose elements are spaced a half-wavelength ($\lambda_a/2$) apart. At first, we assume that no interference signal is present.

Fig. 3 shows the output SNR versus input SNR for several values of θ_d. When the desired signal arrives from the broadside ($\theta_d = 0°$), the information signal suppression does not occur. This is because $cc^* = 1$ holds when $\theta_d = 0°$. When $\theta_d \neq 0°$, the output SNR is inversely proportional to input SNR for high values of input SNR. This is in agreement with the analytical results obtained previously. When $\theta_d = 90°$, the worst information signal suppression occurs. The reason for this is because cc^* becomes a minimum value when $\theta_d = 90°$.

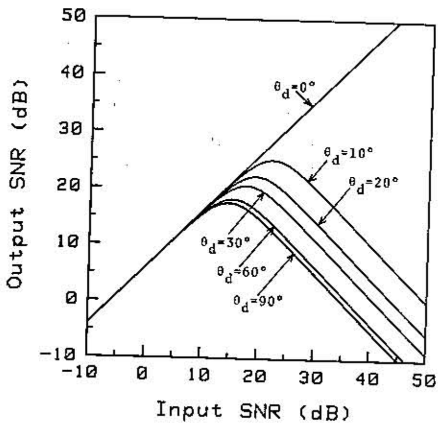

Fig. 3. Output SNR versus input SNR. No interference. $N = 4$, $r = 0.5$ percent, $g = 1$.

In the following, it is assumed that $\theta_d = 90°$, namely, we deal with the worst case of the information signal suppression.

Fig. 4 shows the array patterns at the information signal frequency f_a for several values of input SNR. When the input SNR is small ($= 0$ dB), it is seen that the main beam is formed toward the desired signal arrival direction. In this case, the output SNR is proportional to the input SNR. When the input SNR is high ($= 40$ dB),

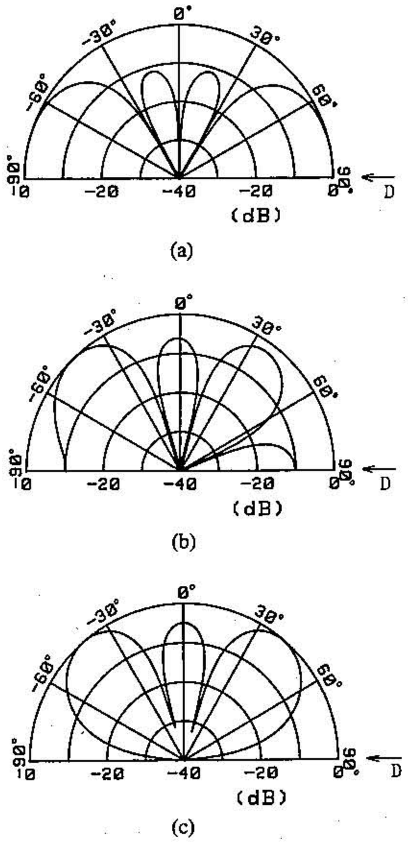

Fig. 4. Array patterns. No interference. D denotes the desired signal. $N=4$, $\theta_d=90°$, $g=1$, $r=0.5$ percent. Frequency is f_a. (a) $S=0$ dB. (b) $S=20$ dB. (c) $S=40$ dB.

we see that the LMS adaptive array nulls the information signal. The pattern null causes information signal suppression.

Fig. 5 shows the output SNR versus input SNR for several values of r defined by (1). The larger $|r|$ is, the worse the information signal suppression is seen. Thus, it is desirable that we set the pilot signal frequency as close to the information signal frequency as possible.

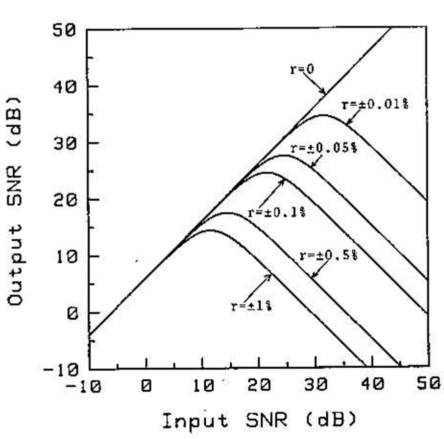

Fig. 5. Output SNR versus input SNR. No interference. $N=4$, $\theta_d=90°$, $g=1$.

Fig. 6 shows the output SNR as a function of the input SNR for several values of the loop gain g. Fig. 7 illustrates the effect of the loop gain on the array pattern.

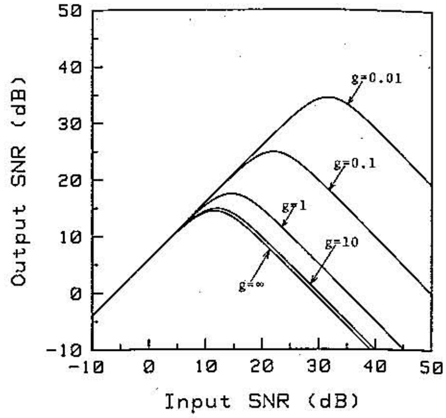

Fig. 6. Output SNR versus input SNR. No interference. $N=4$, $\theta_d=90°$, $r=0.5$ percent.

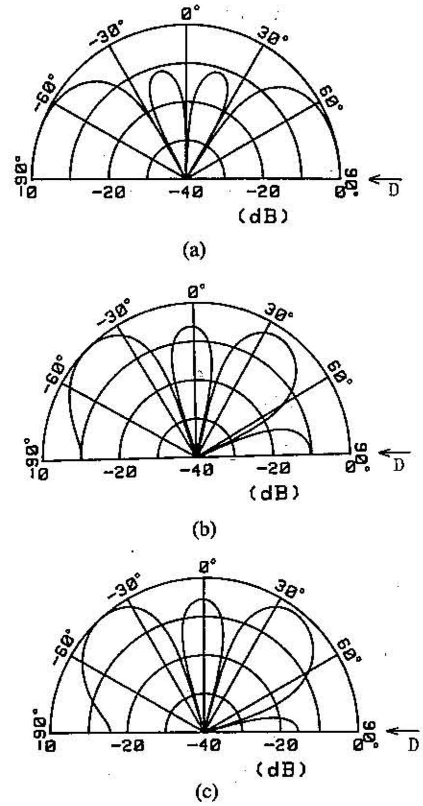

Fig. 7. Array patterns. No interference. D denotes the desired signal. $N=4$, $\theta_d=90°$, $r=0.5$ percent, $S=20$ dB. Frequency is f_a. (a) $g=0.01$. (b) $g=1$. (c) $g=\infty$.

From these curves, it is seen that the information signal suppression problem can be circumvented by decreasing the loop gain. A lower value of the loop gain, however, yields poorer interference suppression.

Next, we include the effect of an interference signal. It is assumed that the interference signal is narrowband and arrives from θ_i relative to broadside. Moreover, it is assumed that the interference signal frequency is the same as the information signal frequency f_a. Fig. 8 shows the steady-state output INR versus input INR for several values of loop gain. INR denotes the ratio of interference signal power to thermal noise power. From these curves, it is seen that when the loop gain is low, the interference

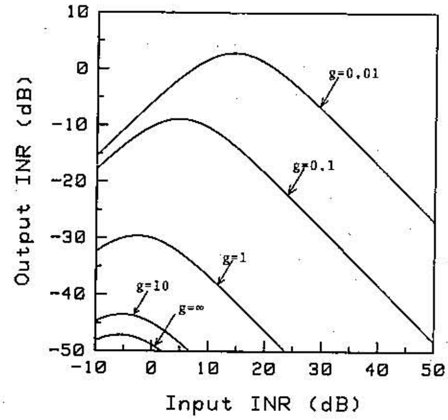

Fig. 8. Output INR versus input INR. $N = 4$, $\theta_d = 90°$, $\theta_i = 15°$, $r = 0.5$ percent, $S = 20$ dB.

suppression performance is degraded. Considerations on the loop gain are discussed further in Section IV.

IV. UPPER BOUND OF LOOP GAIN

As may be seen from the results obtained in Section III, the loop gain is very important in the LMS adaptive array. It is desirable to obtain the optimum loop gain which maximizes the output signal-to-interference-plus-noise ratio (SINR). Unfortunately, the output SINR depends highly on the radio environment. Thus, it is very difficult to obtain the optimum value. In this section, we obtain the upper bound of the loop gain which keeps the output SNR above the required minimum value in the case where no interference signal is present. We may say that the upper bound value gives maximum interference protection but eliminates the problem of desired signal suppression.

Since the loop gain is positive, $0 < h \leq 1$ holds. Furthermore, h is a monotone increasing function of g.

Assume that there is no interference signal. Let the required minimum output SNR be A. Then, from (15), we have

$$\frac{cc^*NS}{(1-cc^*)h^2N^2S^2 + 2(1-cc^*)hNS + 1} \geq A. \qquad (19)$$

Solving (19) with respect to h, we obtain the following results.

(1) When $cc^*NS \leq A$, (19) does not hold for $h > 0$. This means that when $cc^*NS \leq A$, the required minimum output SNR is not achieved for any loop gain.

(2) When $cc^*NS > A$, (20) and (21) are obtained:

$$h \leq h_u \qquad (20)$$

$$h_u = \frac{1}{NS}\left\{-1 + \sqrt{\frac{cc^*(NS-A)}{(1-cc^*)A}}\right\}. \qquad (21)$$

If $h_u < 1$, the corresponding value of g, that is,

$$g_u = h_u / (1 - h_u) \qquad (22)$$

is the upper bound of the loop gain.

If $h_u \geq 1$, there is no finite upper bound of g. Namely, any value of the loop gain is allowable to achieve the output SNR $\gtrsim A$.

The upper bound of g for a given input SNR, g_u, is given by (22). Now we consider the upper bound of g over a dynamic range of the input SNR. Assume that the input SNR varies from S' to S''. Also, assume that $S' > A / cc^*N$ holds. Then (19) must be satisfied for $S' \leq S \leq S''$. Thus the lowest value of g_u for $S' \leq S \leq S''$ is the allowable upper bound of the loop gain over the whole dynamic range of the input SNR. Differentiating (21) with respect to S, it is seen that h_u is a unimodal function of S so long as $S > A/N$. Let h_u' and h_u'' be the values of h_u which correspond to S' and S'', respectively. Furthermore, let g_u' and g_u'' be the values of g_u which correspond to S' and S'', respectively. Since h_u is the unimodal function of S, the lower value of either h_u' or h_u'' is the upper bound of h_u for $S' \leq S \leq S''$. Moreover, since h is the monotone increasing function of g, the lower value of either g_u' or g_u'' is the upper bound of the loop gain over a given dynamic range of the input SNR.

Fig. 9 shows the upper bound of g as a function of input SNR for several values of r. This figure assumes $A = 20$ dB. From these curves, the upper bound of g is easily obtained. Assume, for example, $r = 0.5$ percent and the input SNR varies from 15 dB to 30 dB. The upper bound of g for the input SNR = 15 dB is 0.2883. And the one for the input SNR = 30 dB is 0.0973. Thus 0.0973 is the upper bound of the loop gain which keeps the output SNR above 20 dB over the whole dynamic range of the input SNR in the case where no interference signal is present.

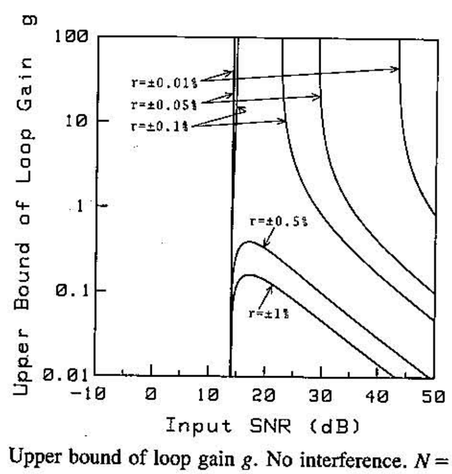

Fig. 9. Upper bound of loop gain g. No interference. $N = 4$, $\theta_d = 90°$, $A = 20$ dB.

V. CONCLUSIONS

This paper has discussed the steady-state performance of the LMS adaptive array in which a pilot signal is used for the reference signal. The results show the following.

(1) The LMS adaptive array occasionally suppresses the desired signal whose frequency is different from that of the pilot signal. This is because the LMS adaptive array regards the desired signal as the interference signal.

(2) Information signal suppression depends on the input SNR, loop gain, array structure, pilot signal frequency, information signal frequency, and desired signal arrival direction.

(3) Information signal suppression may be reduced by decreasing the loop gain. However, lower values of loop gain are undesirable as they result in degradation of interference protection. An upper bound for loop gain has been obtained giving maximum interference protection but eliminating the problem of desired signal suppression. This result is very useful for determining the loop gain in practical systems using a pilot signal.

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