CEF 2006
Limassol, 23.6.2006

JUSTUS-LIEBIG-UNIVERSITÄT GIESSEN

An Objective Function for Simulation Based Inference on Exchange Rate Data

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Empirical Validation of Agent-Based Models

1. Introduction
2. The principle of indirect estimation/validation
3. Benchmark characteristics of FX data
4. The objetive function
5. Examples
6. Outlook: Optimization

Simulation of Agent Based Models

- Explicit aggregation of individual decisions
- Heterogeneity
- (Social) Interaction
- Feedback from aggregates
  (e.g. rational expectations, market outcomes etc.)

Modelling the Agents

- Based on econometric estimates
- Based on experimental evidence
- Assuming endogenous behaviour (interaction, learning, etc.)

Modelling the Markets

- Implicit market clearing
  (Kirman 1991,1993)
- Market maker with triggered adjustment
  (Lux 1998, Lux and Marchesi 1999)
- Explicit demand and supply functions
  $\rightarrow$ market clearing
- …

The Validation Issue

"For microsimulation models even the simplest form of aggregate validation is either difficult or impossible."

Stoker (1993)

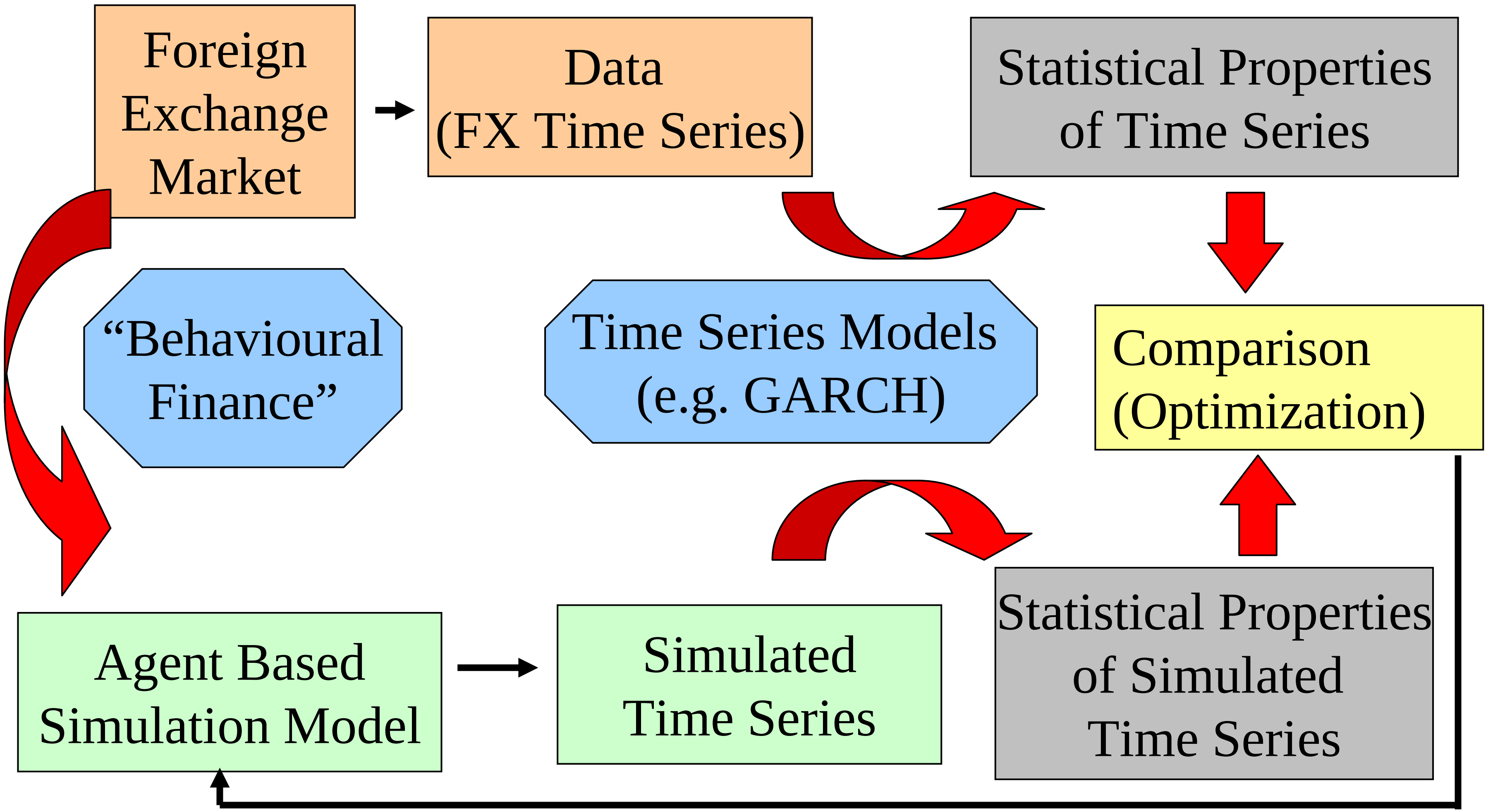
"Validation […] remains a very weak area for the class of models described here."

LeBaron (2000)

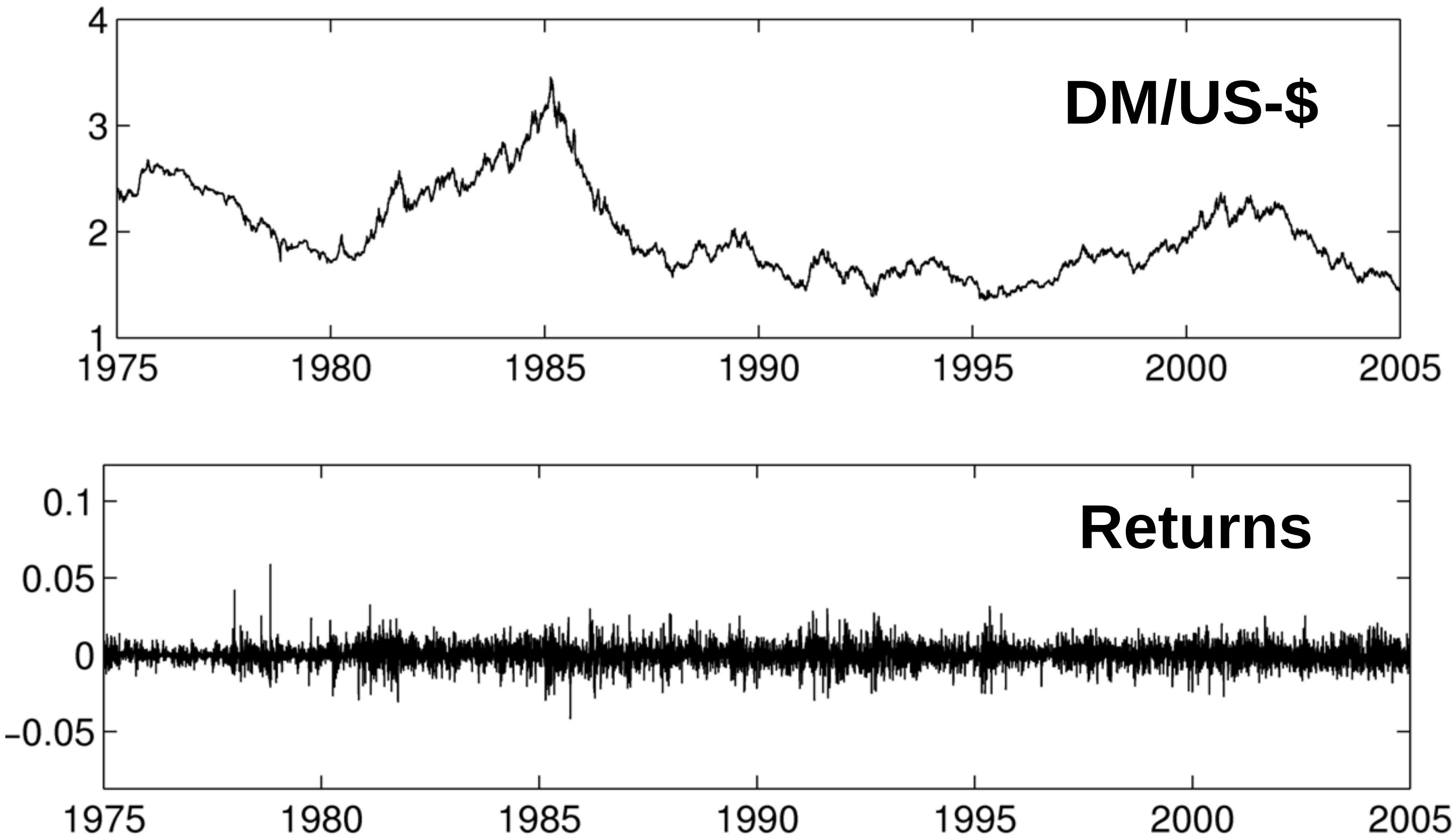
First Approaches

- Gilli and Winker (2003)
- Alfarano, Wagner and Lux (2004)
(Model allowing for a closed form solution)

2. The Principle of Indirect Estimation/Validation



3. Benchmark Characteristics of FX Data



3. Benchmark Characteristics of FX Data

a. Unconditional return distribution
b. Conditional distribution(s) of returns
c. Robustness of the characteristics

3. Benchmark Characteristics of FX Data

a. Unconditional return distribution
   - Empirical moments
   - Tests for normality/comparison with empirical distribution (Kolmogorov-Smirnov)
   - Fat tails

b. Conditional distribution(s) of returns

c. Robustness of the characteristics

3. Benchmark Characteristics of FX Data

a. Unconditional return distribution

b. Conditional distribution(s) of returns

- BDS-test
- Autocorrelation of $|r_t|^d$
- ARCH, GARCH, EGARCH,…
- Long memory, Stationarity

c. Robustness of the characteristics

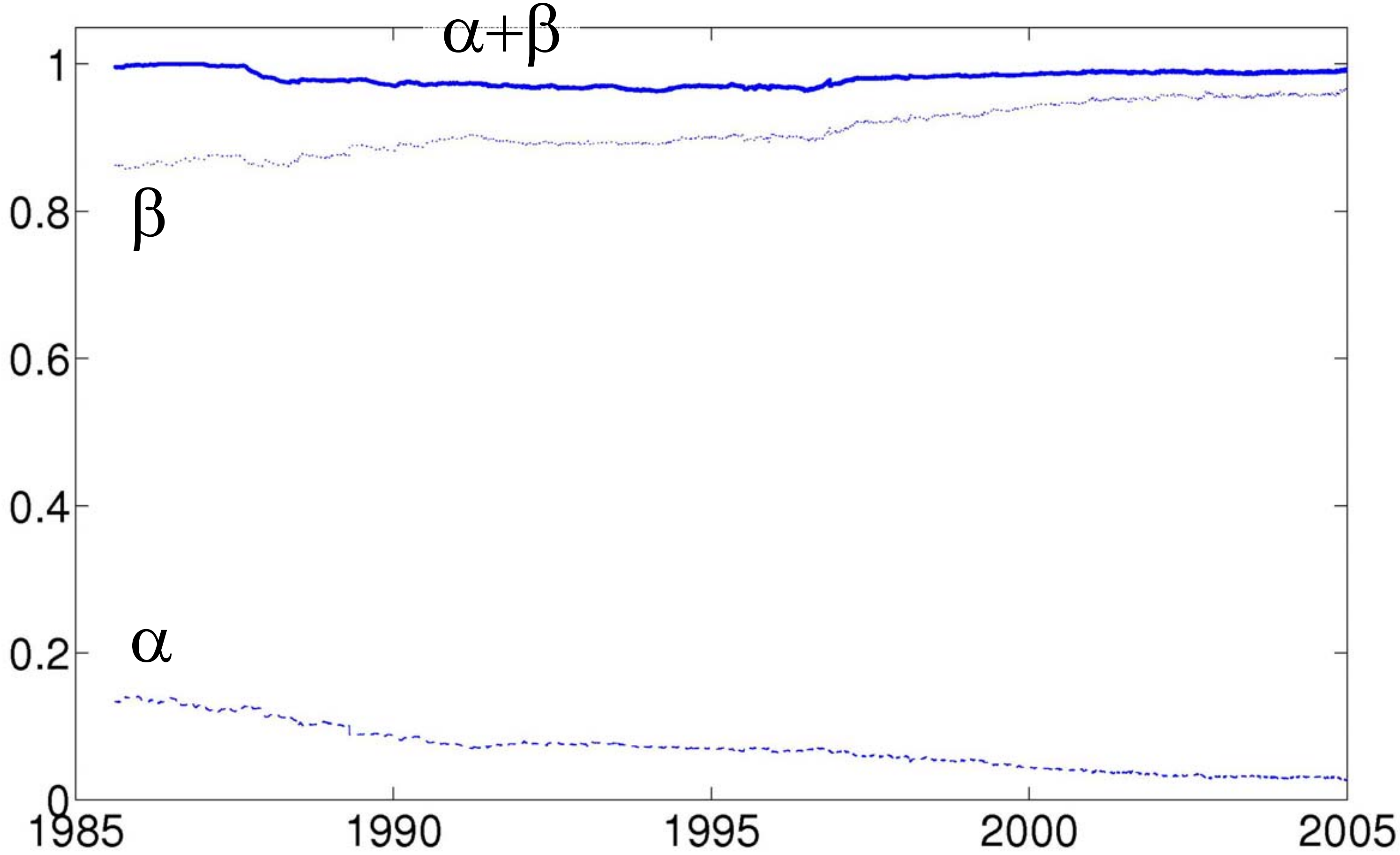
3. Benchmark Characteristics of FX Data

a. Unconditional return distribution
b. Conditional distribution(s) of returns
c. Robustness of the characteristics
   - Subsamples
   - Bootstrap
   - Time aggregation

Robustness

- Only „stylized" facts should be considered
- Changes over time:
  - Rolling windows
  - Sample split
- Variance of stylized facts:
  - (Block) Bootstrap
- Properties under temporal aggregation

Rolling Windows: GARCH(1,1)



Bootstrap

Statistic	DM/US	Mean	5%	50%	95%	Std.dev.
Mean	-0.68E-4	-0.71E-4	-1.98E-4	-0.72E-4	0.49E-4	0.75E-4
Std.dev.	0.0065	0.0065	0.0064	0.0065	0.0066	0.0001
Skewness	0.0314	0.0255	-0.1592	0.0165	0.2241	0.1217
Kurtosis	5.9866	5.9503	4.9149	5.8387	7.4826	0.7985
JB-Statistic	2806	2957	1175	2534	6351	1673
KS-Statistic	0.0490	0.0513	0.0465	0.0512	0.0566	0.0031
Tail index (left)	3.6208	3.6300	3.3709	3.6272	3.9091	0.1656
Tail index (right)	3.5409	3.5602	3.2598	3.5607	3.8545	0.1809

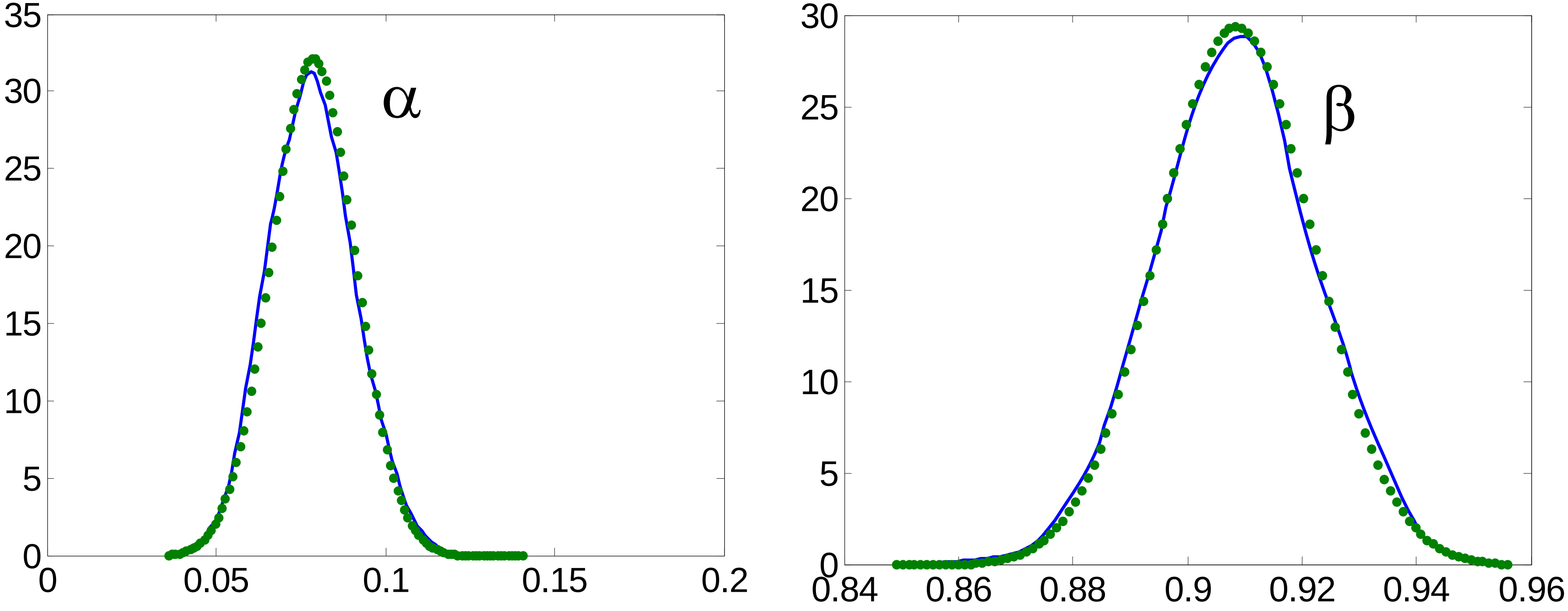
Block Bootstrap

Statistic	DM/US	Mean	5%	50%	95%	Std.dev.
BDS (dim. 2)	8.441	8.518	5.337	8.502	11.686	1.917
BDS (dim. 3)	11.425	11.425	7.836	11.426	14.978	2.191
BDS (dim. 4)	14.274	14.161	9.989	14.112	18.260	2.470
GARCH(1,1) α	0.071	0.078	0.059	0.078	0.099	0.012
GARCH(1,1) β	0.921	0.908	0.886	0.909	0.931	0.014
GARCH(1,1) $\alpha + \beta$	0.992	0.987	0.972	0.988	0.997	0.008
GPH $(m = 0.55)$ r_{DM}	0.091	0.044	-0.041	0.044	0.128	0.051
GPH $(m = 0.55)$ $\|r_{DM}\|$	0.386	0.360	0.255	0.361	0.447	0.058
GPH $(m = 0.55)$ r_{DM}^2	0.323	0.266	0.163	0.269	0.366	0.062
ADF (no drift) (DM)	-1.087	-1.546	-4.001	-1.406	0.404	1.393
ADF (with drift) (DM)	-1.297	2.026	0.061	2.039	3.910	1.146

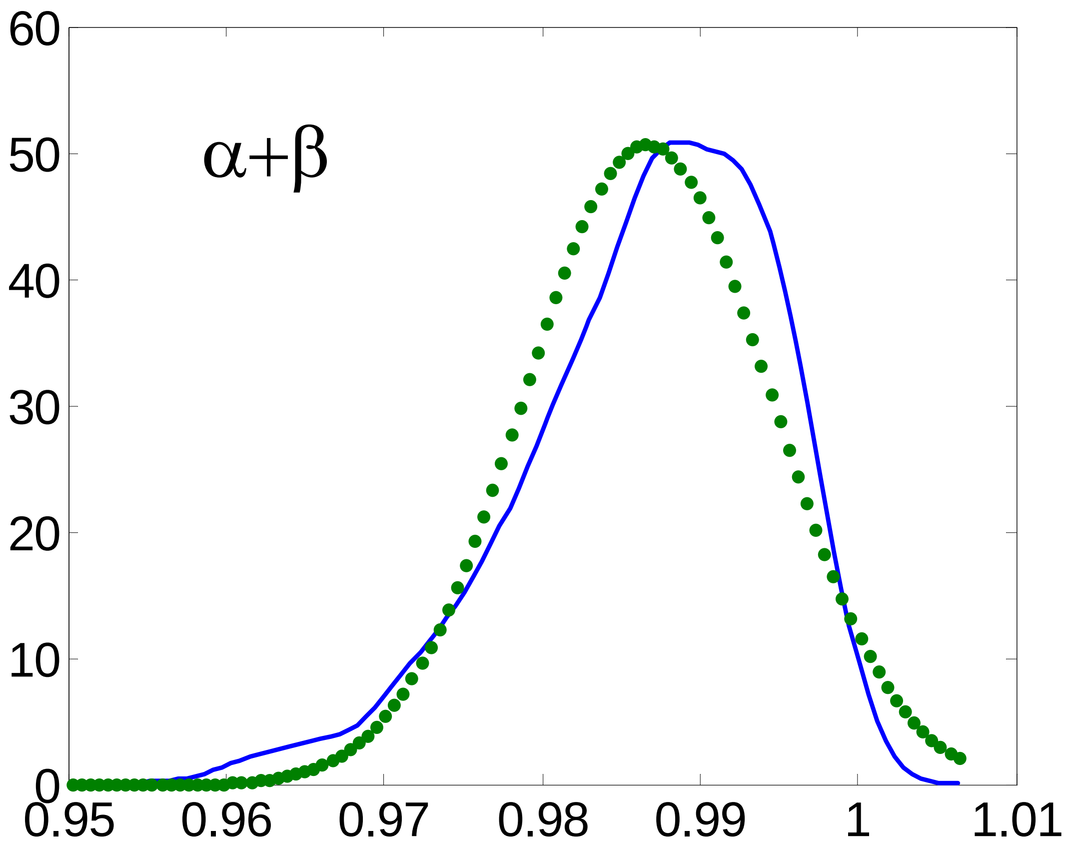
4. The Objective Function

- Estimation/validation should be based on
  - Relevant characteristics
  - Robust characteristics
  - Discriminating characteristics
- Use several characteristics
- Weighting of the characteristics?

Block Bootstrap: GARCH(1,1)



Block Bootstrap: GARCH(1,1)



A First Selection

- Unconditional distribution:
  - Mean
  - Standard deviation
  - Tail index
  - Kolmogorov-Smirnov statistic (empirical return distribution as benchmark)
- Conditional distribution:
  - GARCH(1,1): $\alpha + \beta$
  - GPH (m=0.5) estimate of degree of fractional integration
  - ADF-Test (with drift, no lags)

Moments and Variances

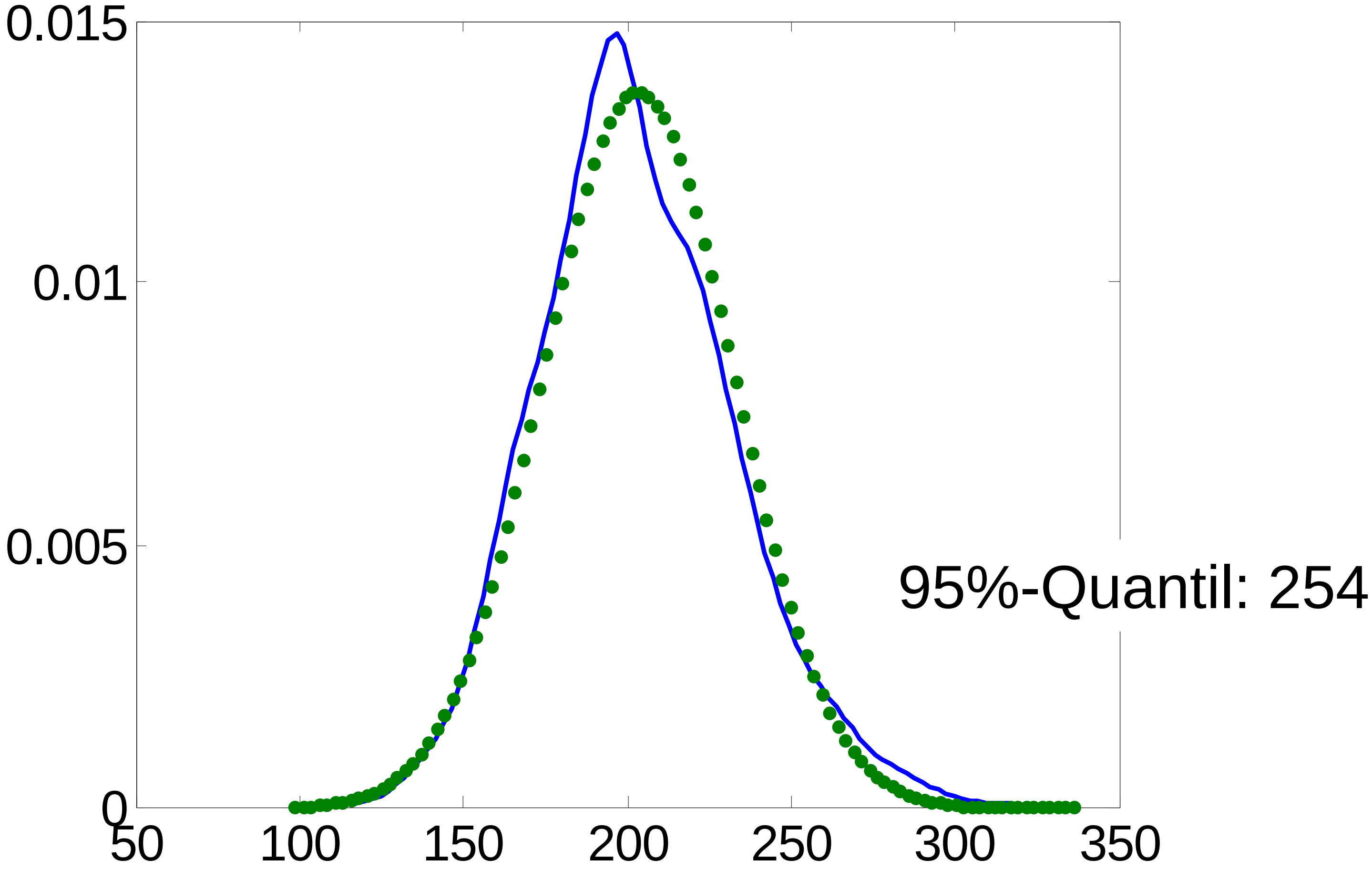
- **m** = [$m_1,\ldots,m_k$] vector of „moments“
- $\mathbf{m}^e$ estimates for real data
- Estimate of variance-covariance matrix of $\mathbf{m}^e$ by block bootstrap (250): Σ_{BB}
- **m** is approximated through simulation of the agent based model (mean or median): $\mathbf{m}^s$
- Monte-Carlo variance of $\mathbf{m}^s$ can be controlled by number of replications.

The Objective Function

- $f(\mathbf{m}^s) = (\mathbf{m}^s-\mathbf{m}^e)'\hat{\Sigma}_{BB}^{-1}(\mathbf{m}^s-\mathbf{m}^e)$
- Where $\hat{\Sigma}_{BB}$:

Mean	Std.dev.	KS-stat.	Hill (5%-10%)	GARCH $\alpha+\beta$	GPH (m=0.55)	ADF
$7.9 \cdot 10^{-9}$	$-8.7 \cdot 10^{-10}$	$-4.2 \cdot 10^{-8}$	$6.4 \cdot 10^{-6}$	$-3.5 \cdot 10^{-8}$	$5.2 \cdot 10^{-7}$	0.00005
$-8.7 \cdot 10^{-10}$	$6.1 \cdot 10^{-8}$	$-1.3 \cdot 10^{-7}$	$2.2 \cdot 10^{-6}$	$-4.2 \cdot 10^{-7}$	$-3.9 \cdot 10^{-6}$	$-1.1 \cdot 10^{-6}$
$-4.2 \cdot 10^{-8}$	$-1.3 \cdot 10^{-7}$	$4.7 \cdot 10^{-5}$	$4.1 \cdot 10^{-6}$	$1.2 \cdot 10^{-6}$	0.00002	0.00141
$6.4 \cdot 10^{-6}$	$2.2 \cdot 10^{-6}$	$4.1 \cdot 10^{-6}$	0.04113	-0.00026	-0.00189	0.04890
$-3.5 \cdot 10^{-8}$	$-4.2 \cdot 10^{-7}$	$1.2 \cdot 10^{-6}$	-0.00026	0.00006	0.00035	-0.00006
$5.2 \cdot 10^{-7}$	$-3.9 \cdot 10^{-6}$	0.00002	-0.00189	0.00035	0.00570	0.00491
0.00005	$-1.1 \cdot 10^{-6}$	0.00141	0.04890	-0.00006	0.00491	0.97167

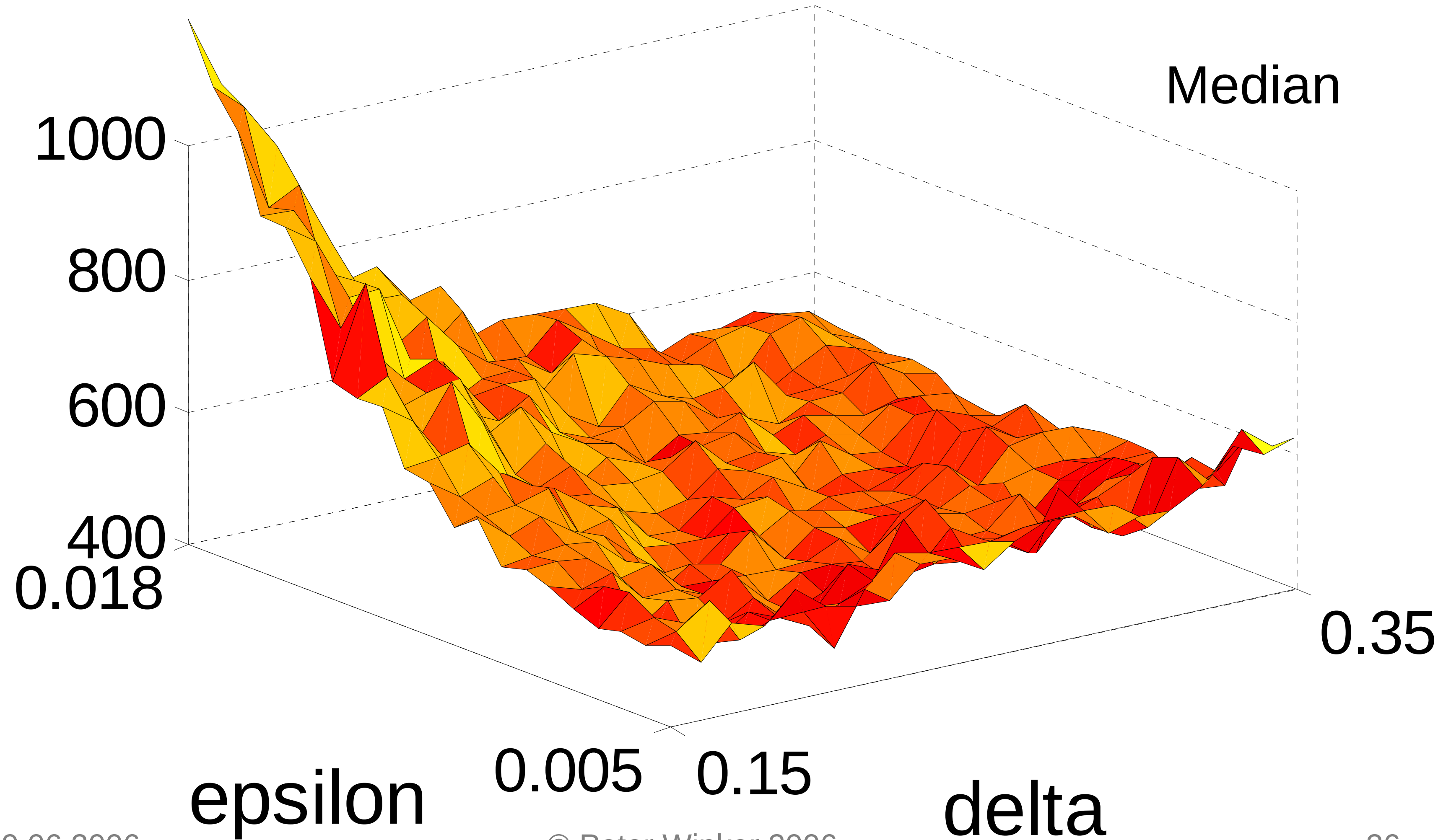
Block Bootstrap Results for f



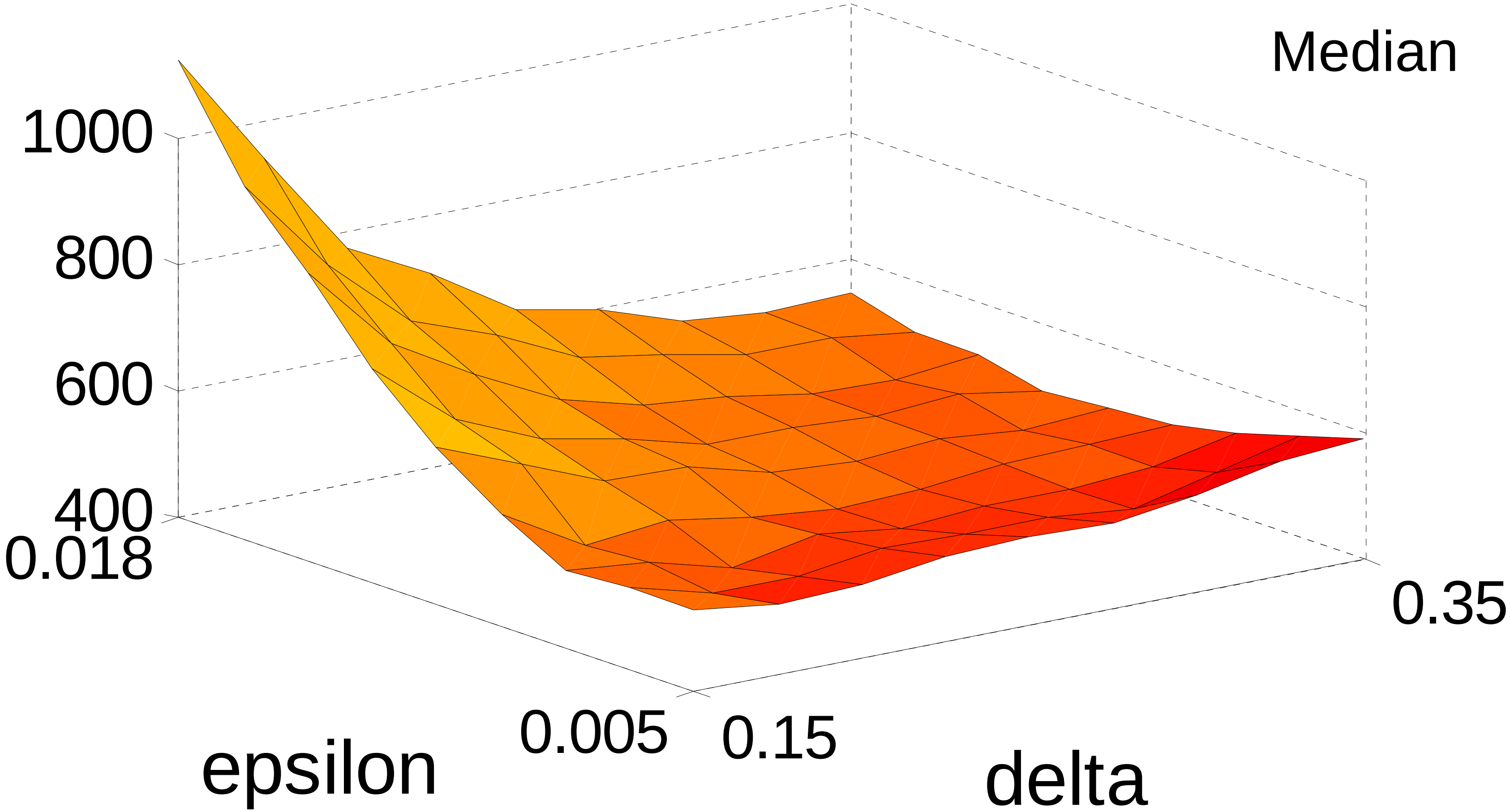
5. Examples

- Kirman (1993) (see Gilli and Winker, 2003)
  - Two types of agents
  - Switching random
- Lux (1998), Lux and Marchesi (1999, 2000)
  - Three types of agents
  - Switching depends on past success
  - Stepwise market adjustment
- Important parameters: Switching rates

Kirman: ε and δ (200 MC-Repl.)



Kirman: ε and δ (1000 MC-Repl.)



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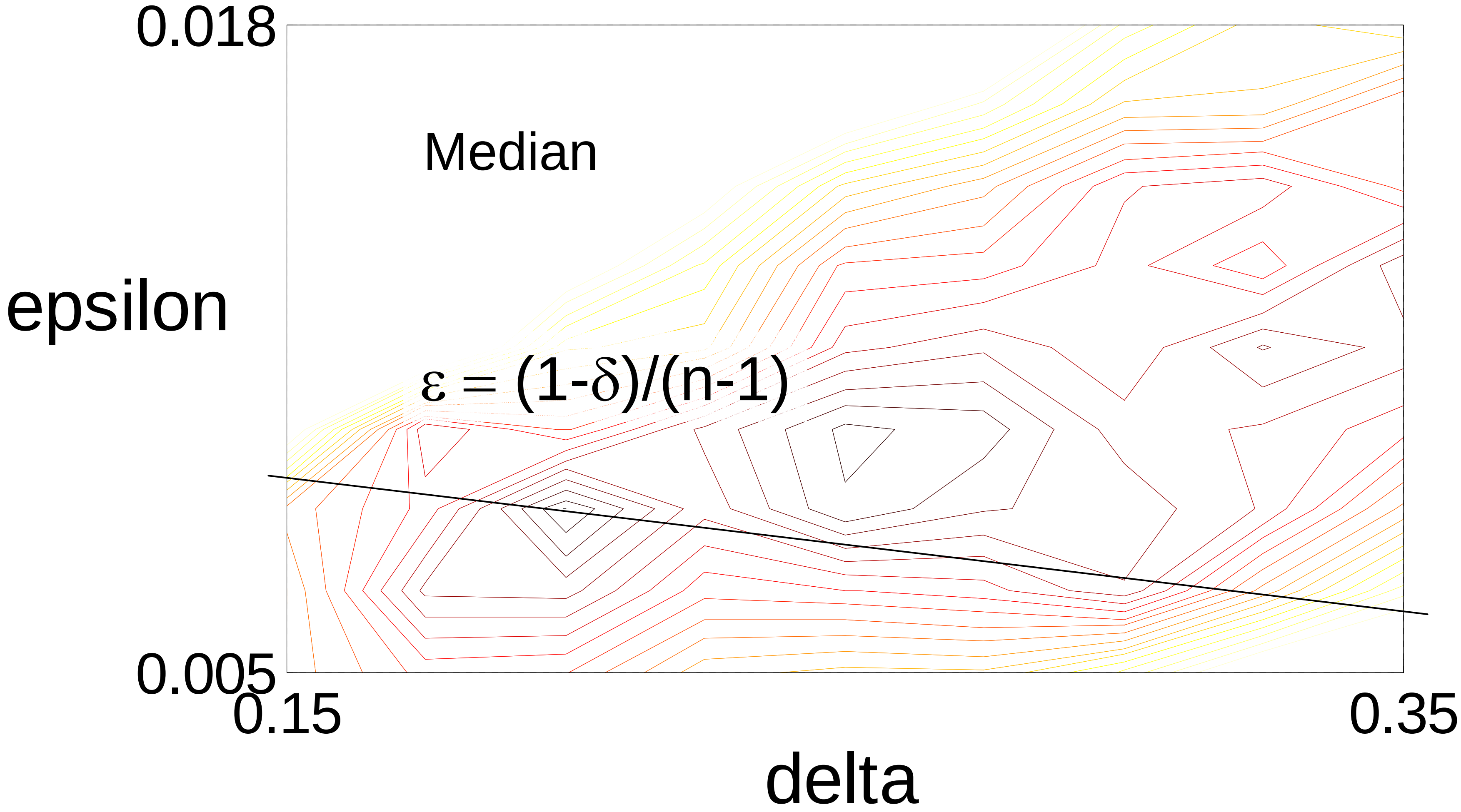

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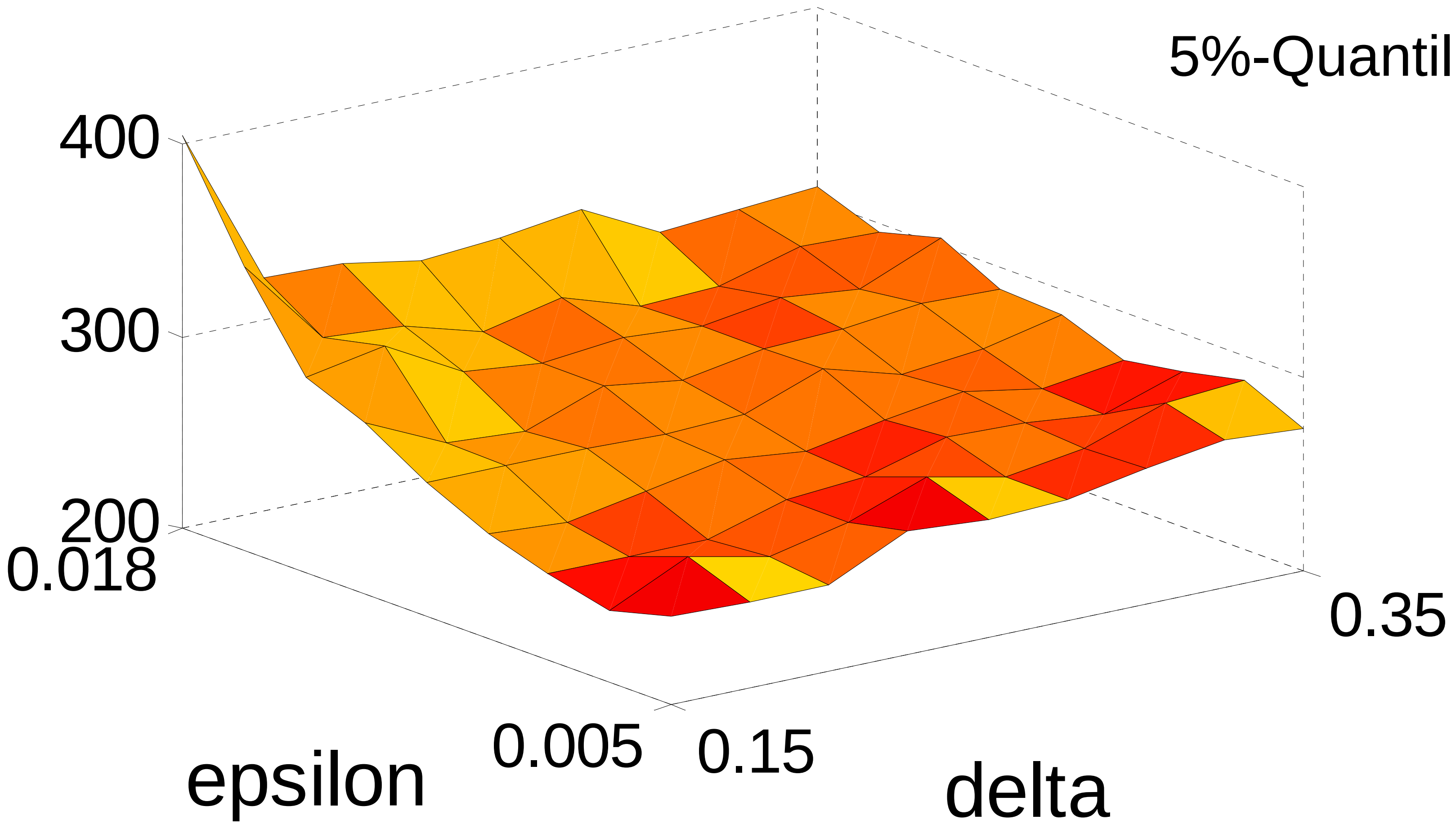

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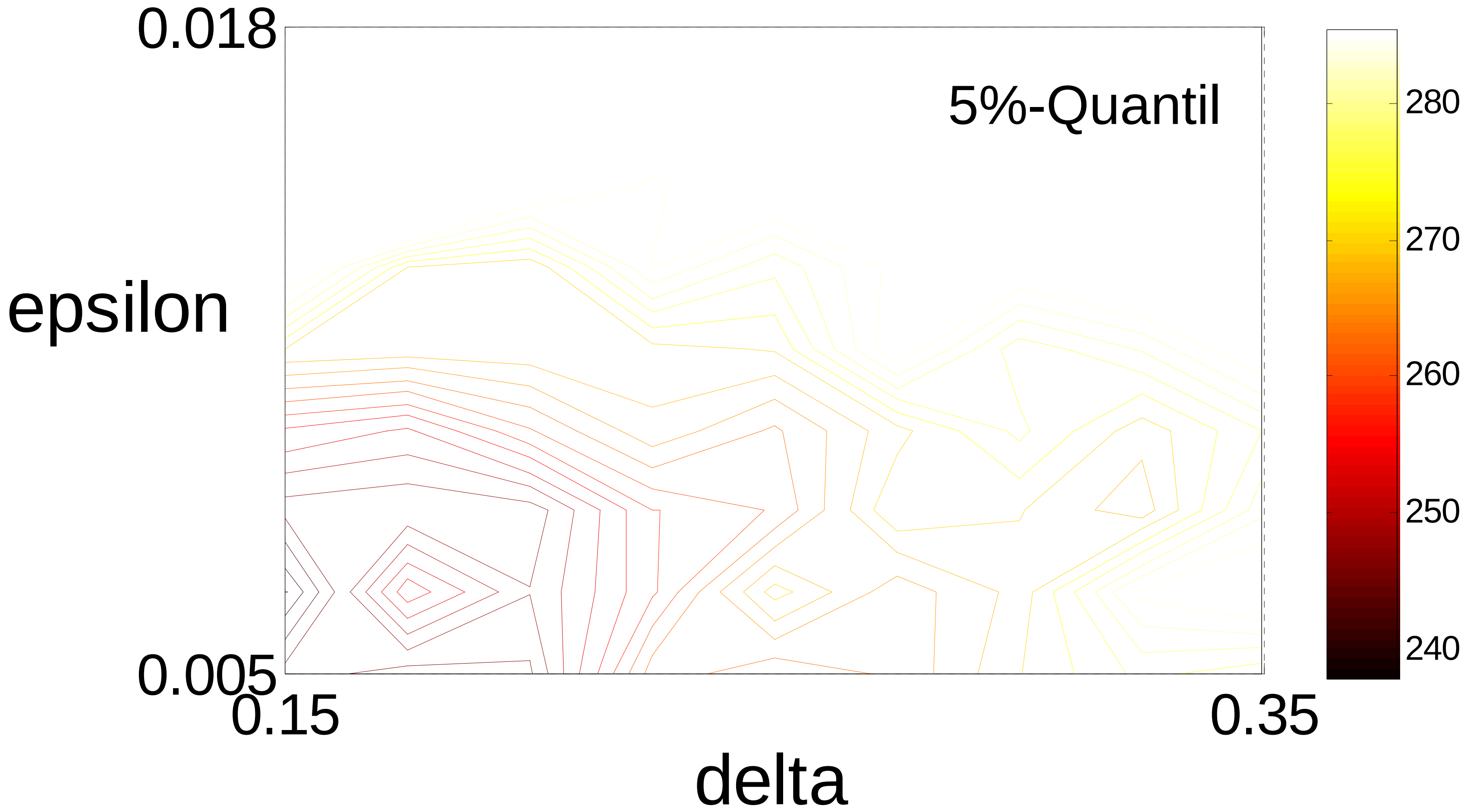
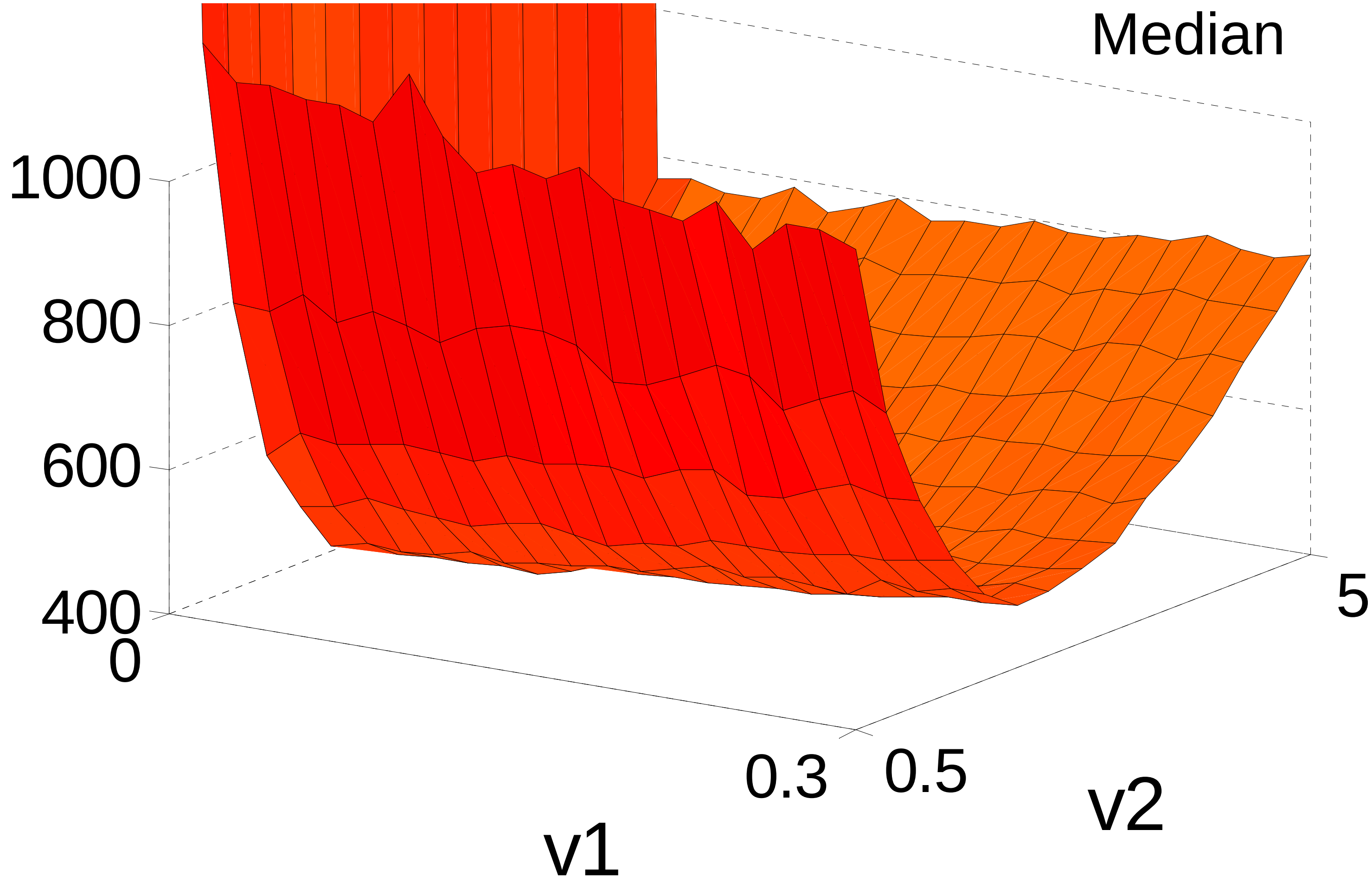


Summary for Kirman‘s model

- Other parameters fixed at „promising values“
- „Optimal“ combination of ε and δ is close to satisfy $\varepsilon = (1-\delta)/(n-1)$
- Values of objective function for „optimal“ parameter values are not significantly worse than for real data
- „Model cannot be rejected“

Lux: ν_1 und ν_2 (200 MC-Repl.)



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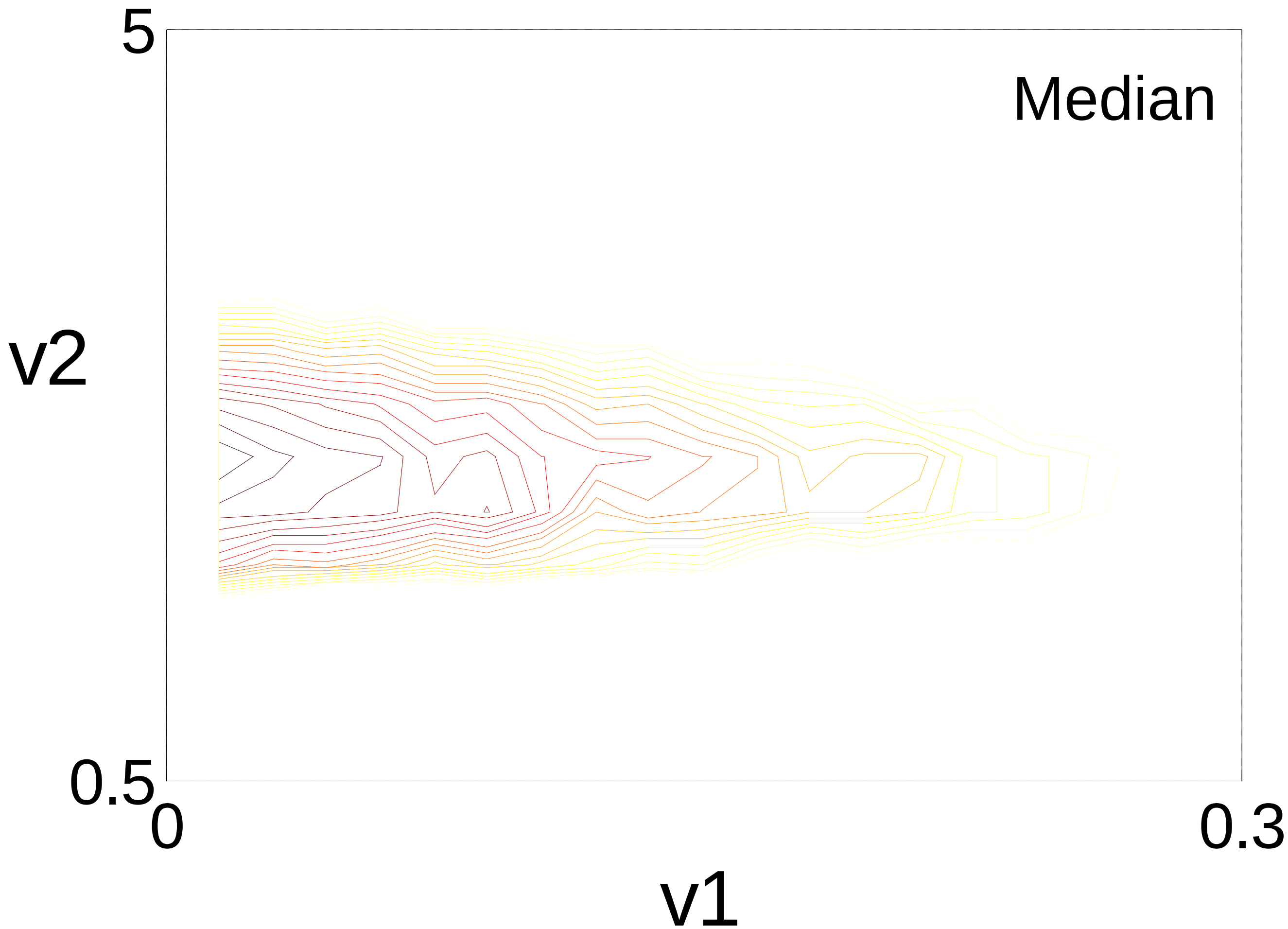

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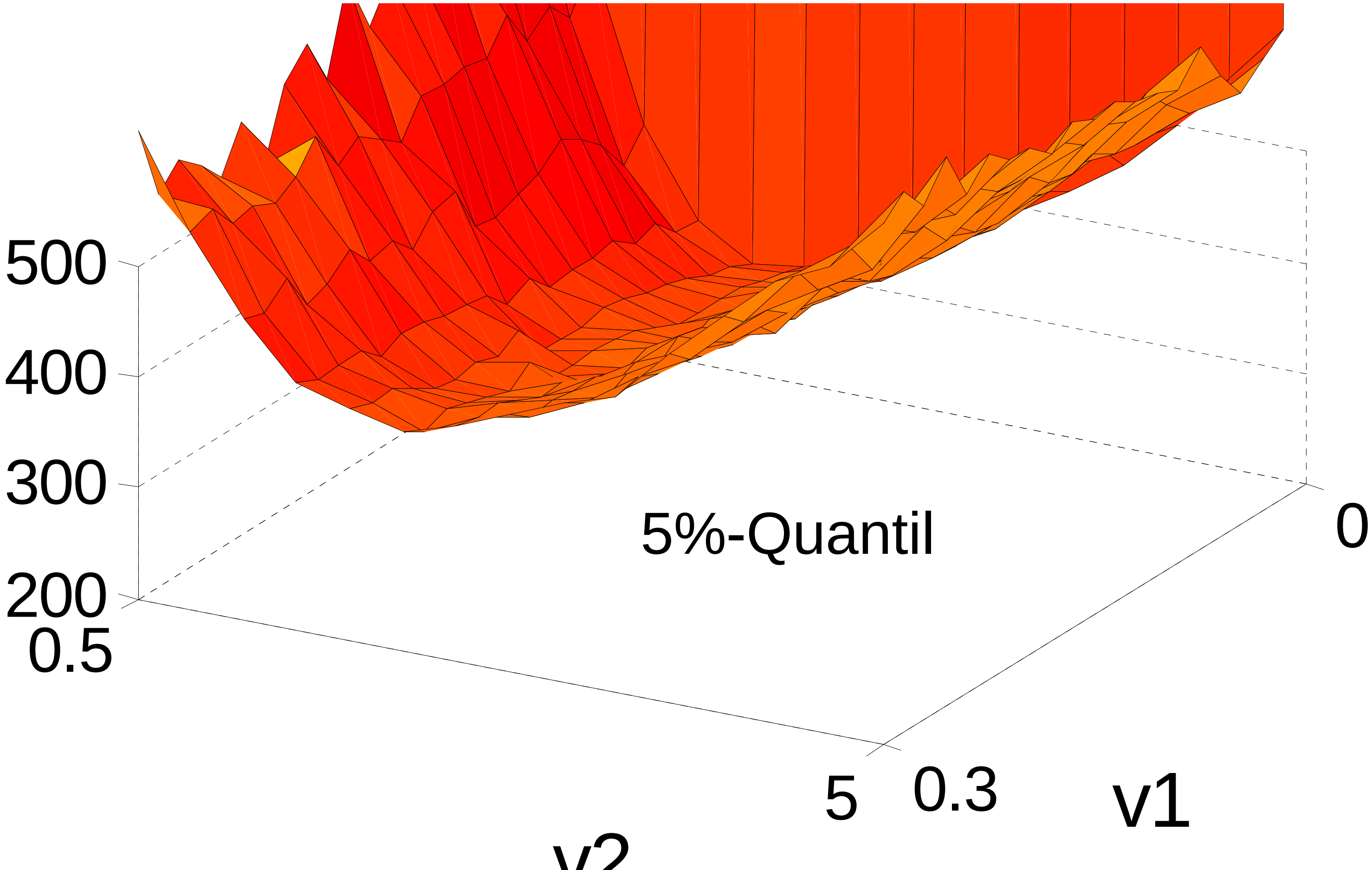
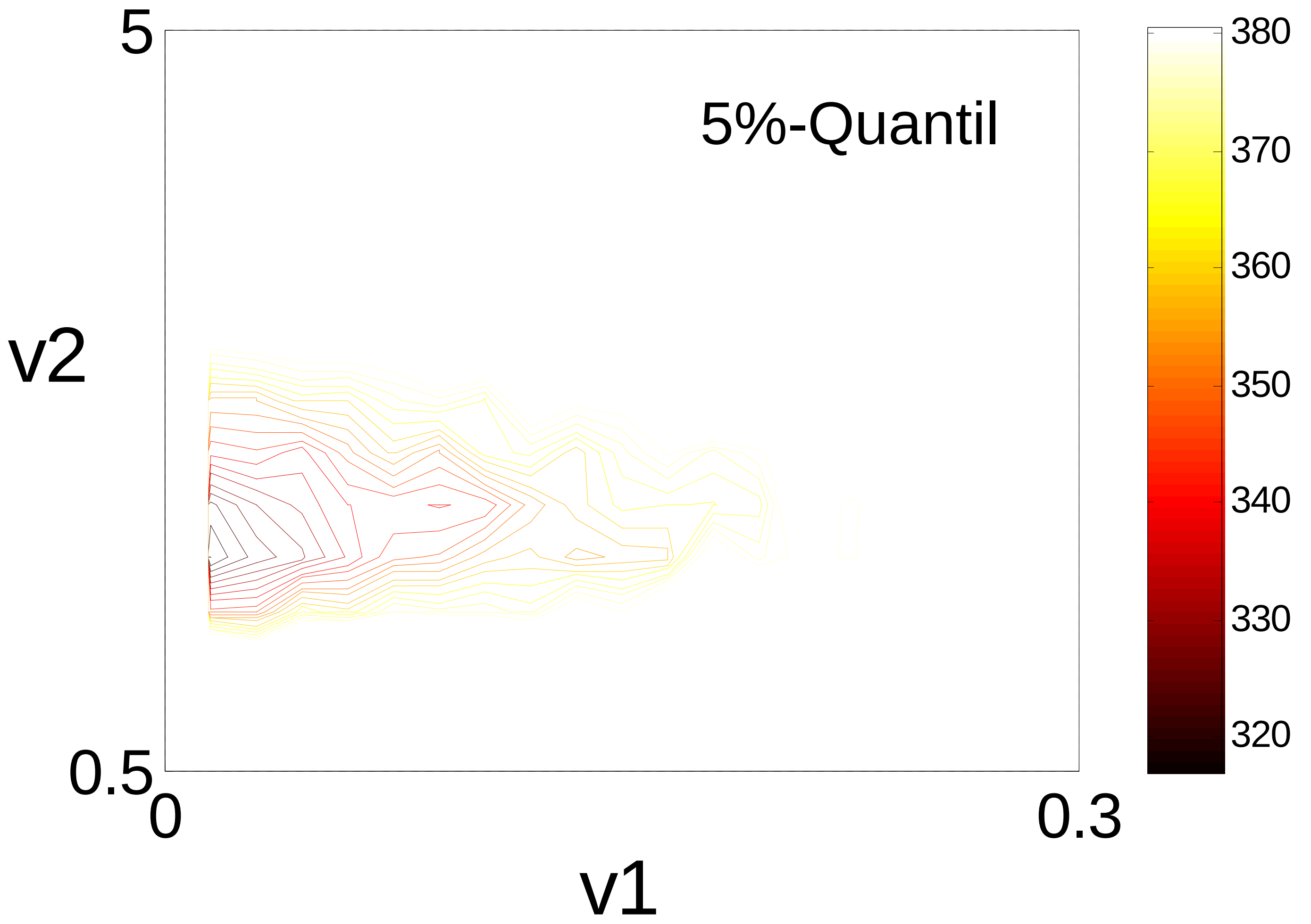


Lux: ν_1 und ν_2 (200 MC-Repl.)



Summary for Lux‘s model

- Other parameters fixed at „promising values“
- Identification of ν_1 appears to be difficult
- Values of objective function for „optimal“ parameter values are significantly worse than for real data
- Optimization required to find optimal values for all parameters!

Summary

- 95%-Quantil of objective function for data (from block bootstrap): 254
- 5%-Quantil of objective function for
  - Kirman model (ε, δ „optimal“): ~240
  - Lux model (ν_1,ν_2 „optimal“): ~ 320
  - Mod. Lux model (ν_1,ν_2 „optimal“): >> 100000
- Note: All other parameters are fixed (to promising, but probably suboptimal values)!

6. Outlook

- Validation of agent based models is important
- Simulation based inference appears feasible
- Selection of relevant/robust properties is crucial; additional statistics will be considered
- Parameters of models might be estimated

Optimization Methods

- Minimize f(**m**) over parameter space Ω
- Problems:
  - f(**m**) has to be approximated by f($\mathbf{m}^s$); how many replications to control MC error?
  - Objective function not globally convex?
- Approaches:
  - Response surface methods
  - Heuristic optimization (TA with Direct Search)