An offshore risk analysis method using fuzzy Bayesian network

Ren J.[1], Jenkinson I.[1], Wang J.[1*], Xu D.L.[2], Yang J.B.[2]
[1]School of Engineering, Liverpool John Moores University
Byrom Street, Liverpool, L3 3AF, UK
[2] Manchester Business School, The University of Manchester
Manchester, M60 1QD, UK

Abstract. The operation of an offshore installation is associated with a high level of uncertainty because it usually operates in a dynamic environment in which technical and human and organisational malfunctions may cause possible accidents. This paper proposes a fuzzy Bayesian network (FBN) approach to model causal relationships among risk factors which may cause possible accidents in offshore operations. The FBN model explicitly represents cause-and-effect assumptions between offshore engineering system variables that may be obscured under other modelling approaches like fuzzy reasoning and Monte Carlo risk analysis. The flexibility of the method allows for multiple forms of information to be used to quantify model relationships, including formally assessed expert opinions when quantitative data are lacking in early design stages with a high level of innovation, or when only qualitative or vague statements can be made. The model is also a modular representation of uncertain knowledge due to randomness and vagueness. This makes the risk and safety analysis of offshore engineering systems more functional and easier in many assessment contexts. A case study of the collision risk between a Floating Production, Storage and Offloading (FPSO) unit and the authorised vessels due to human errors during operation is used to illustrate the application of the proposed model.

Key Words. Risk analysis, safety assessment, Bayesian networks, fuzzy number, fuzzy probability, offshore engineering systems

1. Introduction

An offshore installation is a complex and expensive engineering structure composed of many systems and is usually unique with its own design/operational characteristics (Wang et al., 2004). Offshore installations need to constantly adopt new approaches, new technologies, new hazardous cargoes, etc., each of which brings a new hazard in one form or another. Typical hazards include a reduction in the stability of the storage vessels, a fire in the offshore platform and loss of safety refuge in the platform. Over the past two decades, a number of serious accidents including the Piper Alpha accident and the Sleipner accident have attracted public concerns to offshore safety and reliability. The studies on how similar accidents may be prevented have been actively carried out at both the national and international levels. Lord Cullen's investigation into the explosion aboard the Piper Alpha platform which claimed the lives of 165 on board was published in November 1990 (UK Department of Energy, 1990). The report covered a complete range of issues from hardware design and integrity through day-to-day safety management. The inquiry was a milestone which marked a change in the safety regime in the UK offshore industry. Following Lord Cullen's report, a number of safety regulations have been approved (HSE, 1996). Recently, the industrial guidelines on a framework for risk-related decision support have been produced by the UK Offshore Operators Association (UKOOA) (UKOOA, 1999). At the moment, one of the major challenges on the practical application of formal offshore installation safety assessment is associated with the development of integrated and flexible approaches to facilitate its application

*: Corresponding author, email: j.wang@ljmu.ac.uk

while human and organizational elements significantly influence the safety of the offshore installation (Wang, 2006).

To reduce the likelihood of occurrence of accidents, it is essential that scenarios involving the potential loss of operational control be assessed at an early stage in the design of new facilities, in order to optimise technical and operational solutions. However, the operation of offshore systems is often associated with a high level of uncertainty because they usually operate in an ever-changing environment in which both technical and human and organisational malfunctions may contribute to a range of possible accidents. An efficient and effective safety assessment method is, therefore, needed to model the safety of offshore engineering systems.

The main issue in offshore risk analysis is how to deal with unpredictable and uncertain events. Uncertainty is mainly grouped into three categories: vagueness, randomness and ignorance. Vagueness is caused due to ill-defined concepts in observation or the inaccuracy and poor reliability of instruments used to make observations. Fuzzy set theory can be used to deal with vagueness. In maritime risk analysis, fuzzy reasoning approach has been developed to deal with problems associated with a high level of vagueness (Wang et al., 1995; Ren et al., 2005). This includes a subjective safety based decision-making method (Wang et al., 1996; Wang and Kieran, 2000), and an evidential reasoning approach based on Dempster-Shafer theory for risk modelling and decision making (Yang and Xu, 2002; Sii et al., 2001). Randomness is caused due to unpredictable events. It is about the certainty of whether a given element belongs to a well-defined set. Classical probability theory is often used to deal with randomness. Ignorance is caused due to weak implication, which occurs when an expert is unable to establish a strong correlation between premise and conclusion. As an application of probability theory, Bayesian network (BN) is a powerful tool both for graphically representing the relationships among a set of variables and for dealing with uncertainties in such variables. Unlike rule-based approaches for risk modelling e.g. approximate reasoning approaches, BN is capable of replicating the essential features of plausible reasoning in a consistent, efficient and mathematically sound way. Critically it is able to retract belief in a particular case when the basis of that belief is explained by new evidence (Pearl, 1988). BN has been used in many different domains.

However, both fuzzy logic based approximate reasoning and BN have limitations in modelling the safety of large engineering systems. The main limitation of fuzzy reasoning approaches is the lack of ability to conduct inference inversely. Feed-forward-like approximate reasoning approaches are strictly one-way, that is, when a model is given a set of inputs it can predict the output, but not vice versa. This may have limitation on the flexibility of a safety assessment method that focuses on exploring causal relationships among risk factors. For example, *failure consequence probability* (i.e. the probability that consequences happen given the occurrence of an event) is often used as an input variable in safety assessment. In order to estimate the value of *failure consequence probability*, it is assumed that the “consequence” is directly caused by the occurrence of an “event”. This may be true in some circumstances where personnel injury/loss (consequence), for instance, may be directly caused by a collision (event) between an FPSO unit and a shuttle tanker. In offshore safety assessment, however, an event-consequence pair is often indirectly inter-related. Personnel injury/loss (consequence) and bad weather (event) are such an indirectly inter-related pair. Current rule-based reasoning systems are not capable of finding out this type of causal relationship.

BN, on the other hand, is criticised with the utilisation of a probability measure to assess uncertainty. It requires too much precise information in the form of prior and conditional probability tables, and such information is often difficult or impossible to obtain. In particular, in dealing with indirect relationships, even domain experts may find that it is usually difficult to make precise judgments with crisp numbers, that is, to assign an exact number to the probability that consequences happen given the occurrence of an event. In certain circumstances, a verbal expression (e.g. "very unlikely") or interval value (e.g. (0.15, 0.20)) of probabilistic uncertainty may be more appropriate than numerical values.

This paper therefore investigates the possibility of merging BN and fuzzy logic to provide an alternative means to facilitate offshore risk analysis. The main objective of this research is to propose a method for modelling offshore system safety using an FBN model. The proposed FBN model is particularly based on the work of Furthsirth-Schnater (1993) and Halliwell et al. (2003). The rest of this paper is organised as follows. Section 2 briefly reviews the offshore risk analysis consideration and safety assessment methods. Section 3 gives details of the FBN model and risk analysis flow diagram. A case study of collision risk assessments for FPSO-authorised vessels during operation is described in Section 4. Section 5 provides the conclusions of the paper. Appendices 1 and 2 give brief descriptions of basic fuzzy set theory and linguistic variables, respectively. Appendix 3 provides a calculation example of fuzzy Bayesian inference.

2. Offshore risk consideration and safety assessment methods

The concept of risk is used to assess and evaluate uncertainties associated with an event. Risk can be defined as a combination of the probability and the degree of the possible human injury, damage to property, damage to environment or some combinations of them (Henley and Kumamoto, 1992). Hence, risk can be measured as a pair of the probability of occurrence of an event, and the consequences associated with the event's occurrence. To assess risk associated with an engineering system or a product, the following questions must be answered:

- What can go wrong?
- What are the effects and consequences?
- How often will they happen and what are the causes?

The first two questions can be answered with a HAZID (hazard identification) approach (Pillay and Wang, 2003). Using HAZID, experienced engineers are required to systematically identify all potential failure events with a view of assessing their influences on system safety and performance. The key issue of HAZID is to gather information of hazards, causes and consequences. As an offshore installation is a very complex system, accidents may be caused by multi-factors. It is very important to explore the relationships among those inter-related factors. Human injury, for instance, may be caused by collision between an FPSO unit and a shuttle tanker or a support vessel. The collision may be caused due to faulty position of the shuttle tanker or support vessel. The faulty position of shuttle tanker or support vessel may be caused by human errors or bad weather conditions. Those factors have direct or indirect relationships. It is therefore expected to use probabilistic inference to identify the underlying influence between those relationships. It is necessary to determine, for example, the conditional probabilities of human errors or bad weathers when it is known that there is human injury/death. Therefore, the third question must be answered by establishing the causal relationships among risk elements and estimating the occurrence likelihood of each event. This needs flexible causality modelling techniques to be developed and applied.

In recent years, BN has attracted increasing attention because of the new algorithms such as Message-Passing algorithm, Variable Elimination algorithm and Best-first Search algorithm (Lauritzen and Spiegelhalter, 1988; Pearl, 1988; Zhang et al., 2004). BN has several features:

- It has the ability to incorporate new observations in the network and to predict the influence of possible future observations onto the results obtained (Heckerman, 1996).
- It can not only let users easily observe the relationships among variables, but also give an understandable semantic interpretation to all the parameters in a Bayesian network (Myllymaki, 2005). This allows users to construct a Bayesian network directly using domain expert knowledge. Furthermore, a Bayesian network has both a causal and probabilistic semantics, and thus it provides an ideal representation scheme for combining prior knowledge (which often comes in causal form) and data.
- It can handle missing and/or incomplete data. This is because the model has the ability to learn the relationships among its nodes and encodes dependencies among all variables (Heckerman, 1997).
- It can conduct inference inversely.

Many applications have proven that BN is a powerful technique for reasoning relationships among a number of variables under uncertainty. For example, Hayes (1998) applied BN successfully to ecological risk assessment. Kang and Golay (1999) applied BN successfully to fault diagnosis in complex nuclear power systems. However, when using BN in offshore safety analysis, there are some difficulties e.g. how to deal with incomplete and vague information that largely exists during the early system design stage. In the prior research, approximate reasoning and evidential synthesis approaches have been proposed (Wang et al., 1995; Sii et al., 2001; Ren et al., 2005). In the above research, three fundamental parameters, i.e. *failure rate, consequence severity* and *failure consequence probability*, were used to describe the uncertainties. Due to the high level of uncertainty or the qualitative nature of failure data, safety analysts may often have to use subjective descriptors to describe the above three parameters (Karwowski and Mital, 1986; Wang et al., 1995). Furthermore, even some quantitative variables are often difficult to evaluate with accurate data. For instance, *failure consequence probability* may be more conveniently estimated by experts as a fuzzy number than calculated using BN.

However, conventional BN can only deal with crisp probability and crisp sets. To solve problems with fuzzy input parameters, further research is required to develop novel and flexible risk analysis techniques for dealing with vagueness, ignorance, and randomness properly as well as conducting reasoning under uncertainties on a rational basis.

In order to find a suitable method, extensive literature review was conducted. The literature study was centred on *FBN* and *fuzzy Bayesian inference* and extended to *fuzzy/linguistic probability theory*. Since Zadeh (1984) first proposed the concept of fuzzy probability, many researchers have contributed to this theory and its application. Fuzzy probability measures can be classified into the following three major categories on which uncertainty and information measures are based:

- Fuzzy sets and conventional probability theory.
- Crisp sets and fuzzy number based probability theory.
- Fuzzy sets and fuzzy number based probability theory.

A review of the literature has identified a variety of treatments of fuzzy probability and BN. A piece of work on fuzzy Bayesian inference was conducted in the area of safety project studies in structural reliability research (Chou and Yuan, 1993). Perichi (1991) extended the conventional Bayesian inference mechanism by introducing an interval-based approach, which uses intervals to describe the maximum uncertainty of the observed values. Yang and Cheung (1995) observed two important problems with computation of the fuzzy posterior probability:

- Difficulty in the determination of the likelihood density function, which is complicated and time-consuming for parameter estimation.
- Difficulty in computation of the integral that defines the likelihood probability for the fuzzy valued evidence.

In order to solve the above problems, León-Rojas et al. (2003) proposed a fuzzy Bayesian partnership algorithm to estimate fuzzy likelihood and fuzzy prior probability. Using maximum likelihood solution, they avoided complicated likelihood function estimation and thus provided a way to simplify computation. Although their method was successfully used in population annoyance level assessment caused by noise exposure, the underlying algorithm is similar with likelihood density function estimation. The calculation is still complicated and time-consuming. Darwiche (2003) proposed a differential approach to inference in BN. Their idea is based on evaluating and differentiating arithmetic circuits using a polynomial. Recently Li and Kao (2005) proposed a methodology for solving abductive reasoning problems in BN involving fuzzy parameters and extra constraints. Their method developed a non-linear programming model for dealing with constrained abductive reasoning on BN. The shortcomings of this method are that there is no general solution for the non-linear programming model and the algorithms are case specific.

The idea of using fuzzy numbers to replace crisp numbers in Bayesian inference was proposed by Furthsirth-Schnater (1993) and Halliwell and Shen (2002). To fit fuzzy numbers into the axiomatic basis of probability theory, they defined “Bayesian fuzzy probability” as convex, normal fuzzy set of [0,1]. Using a new terminology “subsumption”, they relaxed Complementation Law to extend a partially defined linguistic probability measure. Their method has been successfully used in forensic statistics (Halliwell et al., 2003).

In summary, on one hand, safety assessment and risk analysis should cover areas where it is difficult to apply traditional safety assessment techniques. Lack of reliable safety data and lack of confidence in safety assessment have been two major challenges in safety analysis of various engineering activities. On the other hand, the existing algorithms capable of dealing with fuzzy and judgemental variables are computationally complicated and time-consuming in parameter estimation. So far few work has been reported in offshore safety analysis in finding a right balance between the above two issues. To solve such problems, this paper proposes a novel and flexible FBN method for dealing with uncertainty including vagueness and randomness in offshore risk analysis. The proposed method extends Halliwell and Shen (2002)’s definition of the fuzzy probability measure and provides four equations to support fuzzy Bayesian inference. The method is also a modular representation of uncertain knowledge and thus can be potentially integrated into the existing offshore safety assessment systems.

3. Fuzzy Bayesian network model and risk analysis diagram

3.1 Fuzzy probability measure and FBN model

A classical BN is a pair N={(V, E), P} where V and E are the nodes and the edges of a Directed Acyclic Graph (DAG), respectively, and *P* is a probability distribution over V. Discrete random variables $V = \{X_1; X_2; ...X_n\}$ are assigned to the nodes while the edges *E* represent the causal probabilistic relationship among the nodes. Each node in the network is annotated with a Conditional Probability Table (CPT) that represents the conditional probability of the variable given the values of its parents in the graph. The CPT contains, for each possible value of the variable associated to a node, all the conditional probabilities with respect to all the combinations of values of the variables associated with the parent nodes. For nodes that have no parents, the corresponding table will simply contain the prior probabilities for that variable. The principles behind BN are Bayesian statistics and concentrate on how probabilities are affected by both prior and posterior knowledge (Buckley, 2002). In order to extend the classic BN into FBN which is capable of dealing with fuzzy variables, fuzzy numbers and their operations must be used (a detailed description is presented in Appendix 1).

It is essential to choose proper fuzzy probability measure to conduct fuzzy Bayesian inference. Based on the work of Furthsirth-Schnater (1993) and Halliwell et al. (2003), fuzzy probability measure and fuzzy probability space are defined as follows:

Definition (Fuzzy probability measure): Let Ω be a set of outcomes, ε be a sigma algebra of events of interest (note: a sigma-algebra ε over a set Ω is a family of subsets of Ω which is closed under countable set operations). A fuzzy probability measure is defined over ε that is, a function $P_f : \varepsilon \to F(R)$ is termed as a fuzzy probability measure (Ω, ε) if and only if:

(a) $0_x \prec P_f(A) \prec 1_x$ for all $A \in \varepsilon$.

(b) $P_f(\Omega) = 1_x$ and $P_f(\phi) = 0_x$.

(c) If *A* and *B* are disjoint events in ε (i.e. $A \cap B = \phi$) then $P_f(A) \oplus P_f(B) \supseteq P_f(A \cup B)$.

(d) If *A* and *B* are events in ε then:

$$P_f(A) \circ P_f(B) = \begin{cases} 0_x & if \quad P_f(A) \circ P_f(B) \le 0_x \\ P_f(A) \circ P_f(b) & if \quad 0_x \le P_f(A) \circ P_f(B) \le 1_x \\ 1_x & if \quad P_f(A) \circ P_f(B) \ge 1_x \end{cases}$$

In the above, $(\Omega, \varepsilon, P_f)$ is termed as a fuzzy probability space. "$\circ$" is denoted as one of the fuzzy number arithmetic operations i.e. *addition*, *subtraction*, *multiplication* and *division*. To distinguish fuzzy number arithmetic operations from classical arithmetic operations, "$\oplus$" and "$\otimes$" are used to denote fuzzy *addition* and *multiplication,* respectively. It should be noted that 1_x stands for a fuzzy subset of real number 1 and 0_x stands for a fuzzy subset of real number 0.

Based on the above four axioms, four fuzzy Bayesian rules can be defined to support an FBN model:

Fuzzy conditional independence

$$P_f(X_1, X_2, ..., X_n) \cong \prod_{i=1}^{n} P_f(X_i \mid Parents(X_i)) \tag{1}$$

Fuzzy joint probability

$$P_f(Y = y_j, X = x_i) \cong P_f(X = x_i) \otimes P_f(Y = y_j \mid X = x_i) \tag{2}$$

Fuzzy marginalization rule

$$P_f(Y = y_j) \cong \sum_i P_f(X = x_i) \otimes P_f(Y = y_j \mid X = x_i) \tag{3}$$

Fuzzy Bayesian rule

$$P_f(X = x_i \mid Y = y_j) \cong \frac{P_f(X = x_i) \cdot P_f(Y = y_j \mid X = x_i)}{P_f(Y = y_j)} \tag{4}$$

To distinguish fuzzy probability equations from conventional equations, "$\cong$" is used instead of "$=$". Using the above equations, fuzzy BN inference can be conducted.

3.2. FBN based risk analysis flow diagram

There is a need for a generic FBN based offshore risk analysis flow diagram to be developed. In the flow diagram, offshore system hazards identification and FBN inference should be dealt with in an integrated manner. The flow diagram should include all processes in FBN inference and may provide a basis for the further development of a generic offshore risk analysis framework. The proposed flow diagram is suggested and depicted in Figure 1. It outlines the necessary steps required for risk analysis in a holistic way supported by FBN. It is a guideline in carrying out cause-and-effect inference based on subjective judgements, and thus provides a logical solution as it emulates the human reasoning process through forward/backward propagating within a specific domain of knowledge, codes and standards based on the company's policy using an FBN approach.

The proposed flow diagram consists of two major components. Particularly, the FBN module describes the steps in Bayesian inference and serves as integration interface of information that is mainly caused by randomness and vagueness. This makes the flow diagram more functional and more effective to amalgamate safety of an offshore engineering system. The major steps of the components used in the flow diagram are outlined as follows:

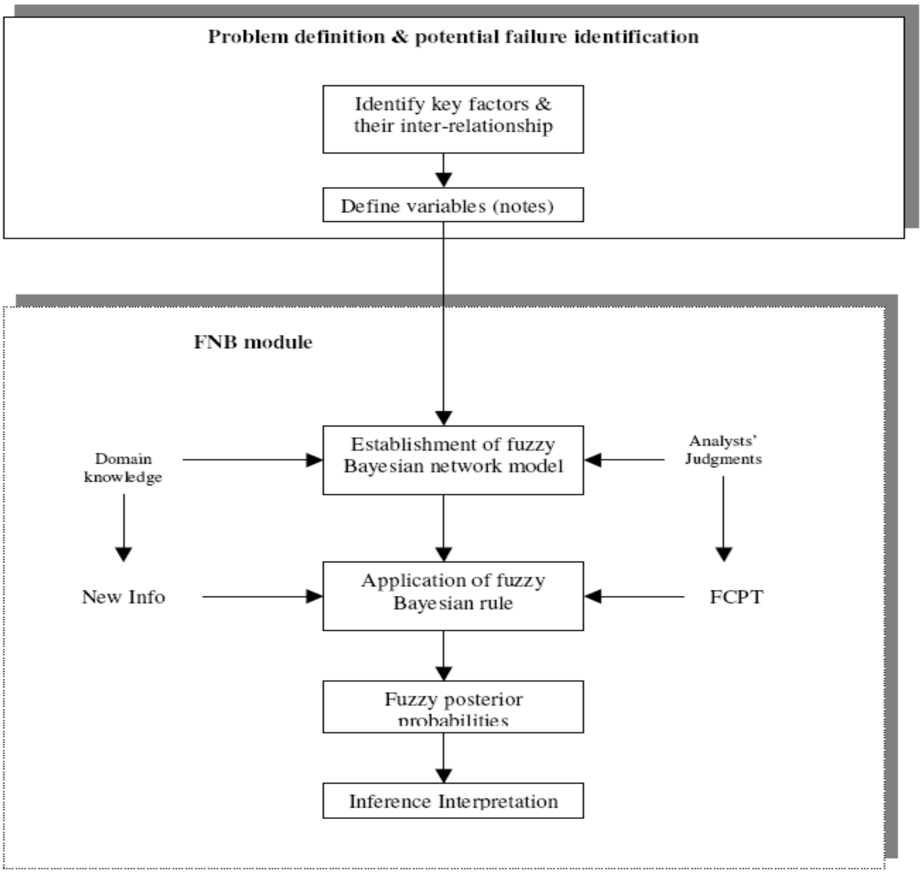

Figure 1: Diagram of FBN based risk analysis

Component 1: Problem definition & potential failure identification

- Identify all anticipated causes/factors to potential failures of an offshore engineering system. Particular attention must be paid to causal relationships among those anticipated causes/factors.
- Define variables (nodes) to represent the identified potential failures.

Component 2: Development of an FBN model

- Construct a BN structure from a generated list of identified hazards specific to the problem under investigation.
- Select the types of fuzzy membership function used to delineate variables (nodes) and provide interpretation for each fuzzy set of each variable (node).
- Integrate domain specific knowledge obtained from available data included in regulatory rules and standards, databases and data networks, and, if necessary, also from simulations and experiments into the network structure.
- Specify states and assign input values for the fuzzy conditional probability table (FCPT) of each variable (node).
- Update the values of all nodes by calculating posterior probabilities via the four Bayesian equations (i.e. Equations (1), (2), (3) and (4)) when new information is available.
- Interpret the causal relationships and provide the generated results for approximate reasoning based safety synthesis.

4. Case study: collision risk of FPSO & authorised vessels

Human errors are the causes of many well-known major incidents such as the loss of Piper Alpha tragedy. According to the P&I Club studies into accidents and claims, approximately 80% of maritime accidents are attributable to human errors (Mitchell and Bright, 1995). Understanding human errors and system failures is particularly important with respect to offshore installations. In this section, a case study of the collision risk between an FPSO unit and the authorised vessels due to human errors during a tandem offloading operation is addressed to illustrate the application of the proposed FBN model. The analysis process follows the diagram proposed in Section 3 (see Figure 1).

4.1. General description and BN model establishment

An FPSO unit is one of the most popular floating systems used by the offshore oil and gas industry. In the UK, crude oil from an FPSO is normally transported to shore using shuttle tankers specially designed for dealing with the harsh weather conditions. Shuttle tankers equipped with a bow-loading system are connected to FPSO or storage facilities by mooring hawser and loading hose through which cargo is offloaded. Tandem loading/offloading is a complex marine operation. It is with high risk due to the close proximity required between the two large vessels. In addition, FPSO units are also routinely serviced by support vessels. During the operation of service, support vessels could collide with FPSO units due to faulty positioning. In a generic scenario, FPSO units can collide with these ships. The consequence of the collision varies from minor contact to incidents that may cause personnel injury/loss, environment pollution and/or damage to the vessel. For demonstration purposes, this case study considers six factors: Human Errors, Adverse Weather, Shuttle Tanker Position, Malfunction of Support Vessel, Collision with FPSO Unit and Personnel Injury/Loss. The causal relationships among those six factors are addressed in a way that human errors or weather conditions may cause the shuttle tanker or support vessel to be in a faulty position. Such a faulty position of the shuttle tanker or support vessel may cause collision with the FPSO, and thus may cause personnel injury/loss. The causal relationships are demonstrated in Figure 2. As can be seen in Figure 2, the six nodes are organised by the acyclic arrows that represent the causal relationships among them. One of the most interesting questions is to find out that if there is a personnel injury/loss observed, then in what possibility it is caused by human errors.

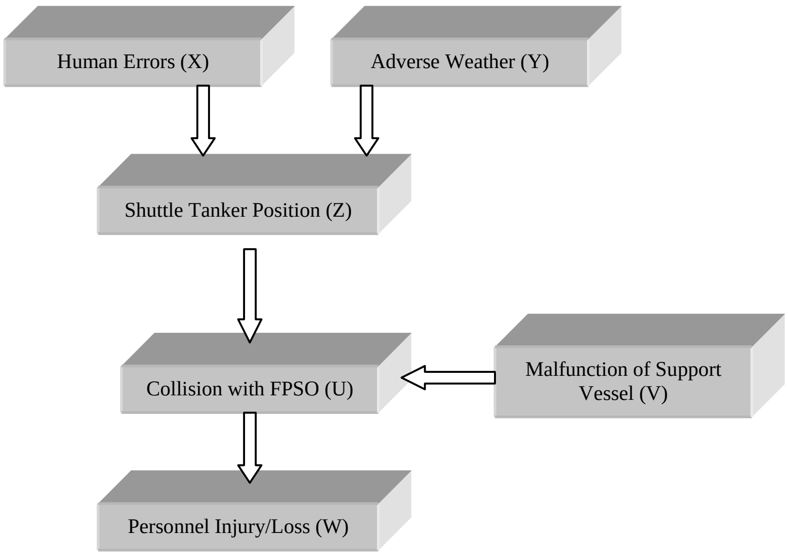

Figure 2: The Bayesian network structure of collision risk of FPSO and authorised vessels

4.2. Fuzzy prior and conditional probabilities

Domain experts were asked to give judgments about the fuzzy probabilities regarding all the nodes. They use linguistic terms to describe the fuzzy probabilities and then refine them with membership functions. The assignment of linguistic terms with associated fuzzy membership functions is case-dependent and expert-dependent (a detailed description of the linguistic variables can be seen in Appendix 2 and demonstrated in Table 8). For example, linguistic term "*Quite unlikely* to happen" was assigned to node "Human Errors" and then defined by fuzzy membership function (0.39,0.40,0.41). Without loss of generality, in this case study triangle (a, b, c) is used although other forms of fuzzy membership function could be employed.

It should be noted that all the membership functions given in this case study are with a narrow interval e.g. (0.39, 0.40, 0.41), this is because wide intervals of fuzzy prior/conditional probabilities may cause the range of inference result too wider to be interpretable. In one instance, the fuzzy posterior probability is calculated out as [0.01, 0.75, 1] given fuzzy prior probability [0.7, 0.8, 0.9]. [0.01, 0.75, 1] almost covers the whole defined universe of discourse [0, 1]; the comparisons between the fuzzy prior and posterior probabilities are therefore meaningless. However, how to deal with such wide range fuzzy probabilities in FBN is obviously important. This is beyond the scope of this paper although it should be studied in further research.

Tables 1, 2 and 4 give the fuzzy prior probabilities of nodes *X*, *Y* and *V*, respectively. Tables 3, 5, and 6 give fuzzy conditional probabilities of nodes *Z*, *U* and *W*, respectively. As shown in Table 1, there are two possible values for "Human Errors" (x_1 or x_2). If *X* is true (x_1), it means that the errors caused by human errors take place. The occurrence likelihood of the event was addressed by domain experts using linguistic term *"Quite unlikely"*. Discussed in Appendix 2 and refereed to Table 8, *"Quite unlikely"* has a value range (0.30, 0.45] and therefore this linguistic term was further defined by domain experts as a triangular fuzzy number $P_f(x_1)$ (0.39, 0.4, 0.41) on the fuzzy scale as shown in Table 1. From Table 1, one can see that the most likely value of $P_f(x_1)$ is 0.4, while 0.39 and 0.41 are the lower and upper least likely values of $P_f(x_1)$, respectively. Similarly in Table 2, y_1 means that "bad weather condition" is observed and the occurrence likelihood of this event is judged as *"Even chance"* defined by a triangular fuzzy number (0.49, 0.50, 0.51).

Table 1 The occurrence probabilities of "Human Errors"

X	$P_f(X)$
x_1	(0.39,**0.4**,0.41)
x_2	(0.59,**0.6**,0.61)

Table 2 The occurrence probabilities of "Adverse Weather"

Y	$P_f(Y)$
y_1	(0.49,**0.5**,0.51)
y_2	(0.49,**0.5**,0.51)

Table 3 gives the conditional fuzzy probabilities of the variable "Shutter Tanker (*Z*)" given the variables "Adverse Weather (*Y*)" and "Human Errors (*X*)". In Table 3, a fuzzy probability is provided for each combination of events (eight in this case). The fuzzy probability value (z_1) under the condition of x_2 and y_2, for example, is shown in the fifth row and third column. The particular value suggests that the faulty position of shuttle tanker is "*Nearly impossible*" to happen with fuzzy probability (0.04,0.05,0.06) if there is no malfunction caused by human errors and bad weather condition. However if there are human operational errors (x_1) that happened under bad weather conditions (y_1), the likelihood occurrence of faulty position of shuttle tanker is increased to "*Quite likely*" with fuzzy probability (0.64,0.65,0.66) (shown in the second row and third column of Table 3).

In Table 4, if *V* is true (v_1), that means "Support Vessel" malfunctions are observed, which may cause collision with the FPSO Unit. The conditional fuzzy probabilities associated with the node "Collision with FPSO (*U*)" given the variables "Shuttle Tanker Position (*Z*)" and "Malfunction of Support Vessel (*V*)" are shown in Table 4. There are eight fuzzy probabilities for each combination of events. The collision accident of the

FPSO is "*Nearly impossible*" (with fuzzy probability (0.04,0.05,0.06)) to happen given no malfunctions of shuttle tanker (z_2) and support vessel (v_2) (shown in the fifth row and third column of Table 5). However once there are position errors of the shuttle tanker (z_1) and support vessel (v_1), the occurrence likelihood of collision with the FPSO is increased to fuzzy probability (0.24,0.25,0.26) (shown in the second row and third column of Table 5).

Table 3 The conditional occurrence probabilities of "Shutter Tanker faulty positioning"

X	Y	$P_f(Z = z_1 \mid X, Y)$	$P_f(Z = z_2 \mid X, Y)$
x_1	y_1	(0.64,**0.65**,0.66)	(0.34,**0.35**,0.36)
	y_2	(0.19,**0.2**,0.21)	(0.79,**0.8**,0.81)
x_2	y_1	(0.19,**0.2**,0.21)	(0.79,**0.8**,0.81)
	y_2	(0.04,**0.05**,0.06)	(0.94,**0.95**,0.96)

Table 4 The occurrence probabilities of "Malfunction of Support Vessel"

V	$P_f(V)$
v_1	(0.34,**0.35**,0.36)
v_2	(0.64,**0.65**,0.66)

Table 5 The conditional occurrence probabilities of "Collision with FPSO"

Z	V	$P_f(U = u_1 \mid Z, V)$	$P_f(U = u_2 \mid Z, V)$
z_1	v_1	(0.24,**0.25**,0.26)	(0.74,**0.75**,0.76)
	v_2	(0.19,**0.2**,0.21)	(0.79,**0.8**,0.81)
z_2	v_1	(0.19,**0.2**,0.21)	(0.79,**0.8**,0.81)
	v_2	(0.04,**0.05**,0.06)	(0.94,**0.95**,0.96)

The two possible values (w_1 or w_2) of variable "Personnel Injury/Loss (*W*)" are shown in Table 6. As can be seen in the second row and second column, the likelihood of occurrence of "Personnel Injury/Loss (*W*)" is "*Very likely*" with fuzzy probability (0.74,0.75,0.76) given the observation of collision of FPSO ($U = u_1$).

Table 6 The conditional occurrence probabilities of "Personnel Injury/Loss"

U	$P_f(W = w_1 \mid U)$	$P_f(W = w_2 \mid U)$
u_1	(0.74,**0.75**,0.76)	(0.24,**0.25**,0.26)
u_2	(0.001,**0.002**,0.003)	(0.997,**0.998**,0.999)

4.3. Fuzzy Bayesian inference and sensitivity analysis

Having obtained the above probabilities, the proposed FBN model can now be used to conduct various types of analysis. The most important use of FBN is in revising probabilities in the light of actual observations of events. It is therefore possible to determine, for example, the posterior probability of human errors when it is known that there is human injury/death observed.

The starting point of the inference is to calculate all the marginal probabilities. The marginal probabilities of all the linguistic variables can be computed using Equation (3). For node Z:

$$P_f(Z = z_1) \cong \sum_{X,Y} P_f(X;Y;Z = z_1) = P_f(X = x_1;Y = y_1;Z = z_1) \oplus P_f(X = x_2;Y = y_1;Z = z_1)$$

$$\oplus P_f(X = x_1;Y = y_2;Z = z_1) \oplus P_f(X = x_2;Y = y_2;Z = z_1)$$

It is worth noting that the joint probabilities of the nodes in the BN structure are calculated through the conditional independence rule of probability using Equation (1):

$$P_f(X;Y;Z) \cong P_f(X) \otimes P_f(Y|X) \otimes P_f(Z|X;Y)$$

Using conditional independence relationships, the following is obtained:

$$P_f(X;Y;Z) \cong P_f(X) \otimes P_f(Y) \otimes P_f(Z|X;Y)$$

Hence:

$$P_f(Z = z_1) \cong \sum_{X,Y} P_f(X;Y;Z = z_1) = P_f(X = x_1;Y = y_1;Z = z_1) \oplus P_f(X = x_2;Y = y_1;Z = z_1)$$

$$\oplus P_f(X = x_1;Y = y_2;Z = z_1) \oplus P_f(X = x_2;Y = y_2;Z = z_1)$$

$$= P_f(X = x_1) \otimes P_f(Y = y_1) \otimes P_f(Z = z_1|X = x_1;Y = y_1)$$

$$\oplus P_f(X = x_2) \otimes P_f(Y = y_1) \otimes P_f(Z = z_1|X = x_2;Y = y_1)$$

$$\oplus P_f(X = x_1) \otimes P_f(Y = y_2) \otimes P_f(Z = z_1|X = x_1;Y = y_2)$$

$$\oplus P_f(X = x_2) \otimes P_f(Y = y_2) \otimes P_f(Z = z_1|X = x_2;Y = y_2)$$

For demonstration purposes, the calculation of the first part of the above equation is addressed here in detail. The following fuzzy probabilities can be obtained from Tables 1, 2 and 3.

$P_f(X = x_1)$ = (0.39,0.40,0.41);

$P_f(Y = y_1)$ = (0.49,0.50,0.61);

$P_f(Z = z_1|X = x_1;Y = y_1)$ = (0.64,0.65,0.66).

Therefore:

$P_f(X = x_1) \otimes P_f(Y = y_1) \otimes P_f(Z = z_1 | X = x_1; Y = y_1)$

= (0.39,0.40,0.41) $\otimes$ (0.49,0.50,0.51) $\otimes$ (0.64,0.65,0.66)

Using α -cut based interval multiplication defined in Equation (7):

(0.39,0.40,0.41) $\otimes$ (0.49,0.50,0.51) $\otimes$ (0.64,0.65,0.66)

= (0.1911, 0.2, 0.2091) $\otimes$ (0.64,0.65,0.66)

= (0.1223, 0.1300, 0.1380)

Then using α -cut based interval addition defined in Equation (5), $P_f(Z = z_1)$ can be calculated as:

$P_f(Z = z_1) \cong$ (0.39,0.40,0.41) $\otimes$ (0.49,0.50,0.51) $\otimes$ (0.64,0.65,0.66) $\oplus$ (0.59,0.60,0.61)

$\otimes$ (0.49,0.50,0.51) $\otimes$ (0.19,0.20,0.21) $\oplus$ (0.39,0.40,0.41) $\otimes$ (0.49,0.50,0.51)

$\otimes$ (0.19,0.20,0.21) $\oplus$ (0.59,0.60,0.61) $\otimes$ (0.49,0.50,0.51) $\otimes$ (0.04,0.05,0.06)

= (0.1223, 0.1300, 0.1380) $\oplus$ (0.0549, 0.0600, 0.0653) $\oplus$ (0.0363, 0.0400, 0.0439)

$\oplus$ (0.0116, 0.0150, 0.0187)

= (0.2251, 0.2450, 0.2659)

Similarly, $P_f(Z = z_2)$ can be calculated as:

$$P_f(Z = z_2) \cong \sum_{X,Y} P_f(X; Y; Z = z_2) = P_f(X = x_1; Y = y_1; Z = z_2) \oplus P_f(X = x_2; Y = y_1; Z = z_2)$$

$$\oplus P_f(X = x_1; Y = y_2; Z = z_2) \oplus P_f(X = x_2; Y = y_2; Z = z_2)$$

= (0.7161, 0.7550, 0.7953)

For node U:

$$P_f(U = u_1) \cong \sum_{Z,V} P_f(Z; V; U = u_1) = P_f(Z = z_1; V = v_1; U = u_1) \oplus P_f(Z = z_2; V = v_1; U = u_1)$$

$$\oplus P_f(Z = z_1; V = v_2; U = u_1) \oplus P_f(Z = z_2; V = v_2; U = u_1)$$

= (0.1103, 0.1307, 0.1534)

$$P_f(U = u_2) \cong \sum_{Z,V} P_f(Z;V;U = u_2) = P_f(Z = z_1;V = v_1;U = u_2) \oplus P_f(Z = z_2;V = v_1;U = u_2)$$

$$\oplus P_f(Z = z_1;V = v_2;U = u_2) \oplus P_f(Z = z_2;V = v_2;U = u_2)$$

= (0.7936, 0.8693, 0.9507)

For node W:

$$P_f(W = w_1) \cong \sum_{U} P_f(U;W = w_1) = P_f(U = u_1;W = w_1) \oplus P_f(U = u_2;W = w_1)$$

$$= P_f(U = u_1) \otimes P_f(W = w_1 | U = u_1) \oplus P_f(U = u_2) \otimes P_f(W = w_1 | U = u_2)$$

= (0.0824, 0.0997, 0.1194)

$$P_f(W = w_2) \cong \sum_{U} P_f(U;W = w_2) = P_f(U = u_1;W = w_2) \oplus P_f(U = u_2;W = w_2)$$

$$= P_f(U = u_1) \otimes P_f(W = w_2 | U = u_1) \oplus P_f(U = u_2) \otimes P_f(W = w_2 | U = u_2)$$

= (0.8177, 0.9003, 0.9896)

What is really of interest, however, is how the prior probabilities change when new observations are added into the BN for a particular node. Suppose it is observed that there is human injury, and it is required to inference to what degree this human injury was caused by human errors during operations. This needs to calculate posterior probability $P_f(X = x_1 | W = w_1)$. Using Bayesian rule Equation (4), the relevant calculation is:

$$P_f(X = x_1 | W = w_1) \cong \frac{P_f(X = x_1;W = w_1)}{P_f(W = w_1)}$$

Therefore $P_f(X = x_1;W = w_1)$ is calculated first (the detailed calculation can be seen in Appendix 3):

$$P_f(X = x_1;W = w_1) \cong (0.0386, 0.0461, 0.0546)$$

Thus:

$$P_f(X = x_1 | W = w_1) \cong \frac{P_f(X = x_1;W = w_1)}{P_f(W = w_1)}$$

$$= \frac{(0.0386, 0.0461, 0.0546)}{(0.0824, 0.0997, 0.1194)}$$

= (0.3233, 0.4614, 0.6626)

In fact, the above result is achieved using Equation (8) (see Appendix 1). Note that (0.0386, 0.0461, 0.0546) and (0.0824, 0.0997, 0.1194) are two fuzzy numbers, and (0.0824, 0.0997, 0.1194) does not contain 0. Therefore using Equation (8), the above posterior probability can be obtained. As shown in Figure 3, the posterior probability, i.e. fuzzy number (0.3233, 0.4614, 0.6626) is represented by a solid curve, while the prior probability (fuzzy number (0.39, 0.4, 0.41)) is represented by a dashed curve.

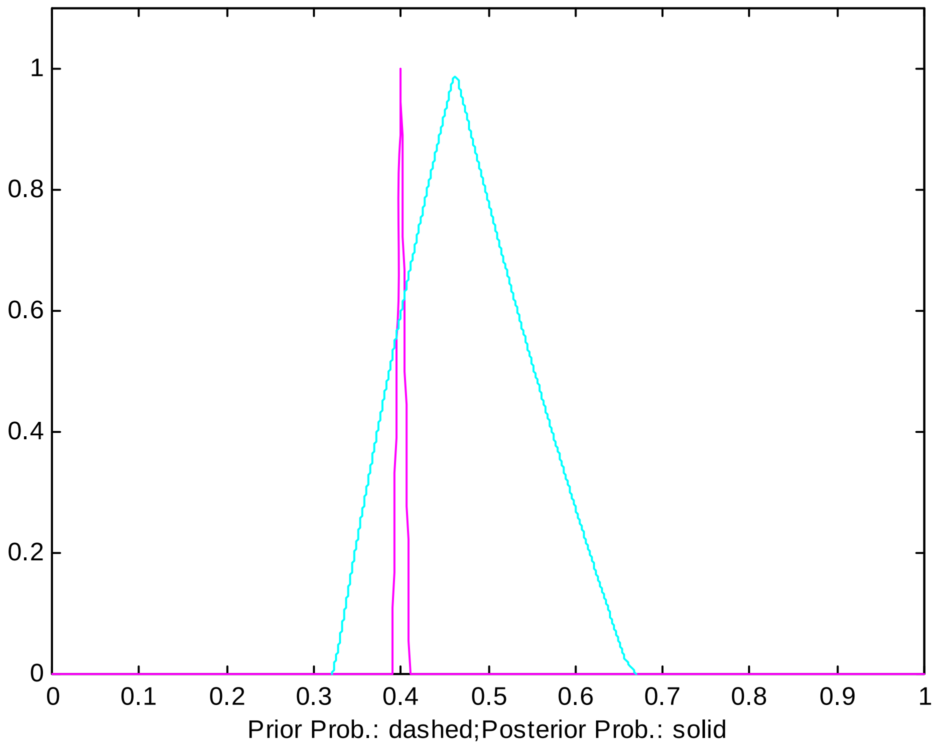

Figure 3: Prior fuzzy probability: $P_f(X = x_1)$ **and Posterior fuzzy probability:** $P_f(X = x_1 | W = w_1)$

Comparing the posterior probability $P_f(X = x_1 | W = w_1)$ = (0.3233, 0.4614, 0.6626) with prior fuzzy probability $P_f(X = x_1) = (0.39, 0.4, 0.41)$, it can be seen that there is a significant change in the occurrence likelihood of human errors when a personnel injury/loss accident has been observed. In fact, the most likely value of fuzzy number $P_f(X = x_1) = (0.39, 0.4, 0.41)$ is 0.4, whilst the most likely value of fuzzy number $P_f(X = x_1 | W = w_1)$ = (0.3233, 0.4614, 0.6626) is 0.4614. The lower and upper least likely values of fuzzy number $P_f(X = x_1) = (0.39, 0.4, 0.41)$ are 0.39 and 0.41

respectively, whilst the lower and upper least likely values of fuzzy number $P_f(X = x_1 | W = w_1)$ = (0.3233, 0.4614, 0.6626) are 0.3233 and 0.6626 respectively. This suggests that the posterior probability changed significantly. This might imply that node “Personnel Injury/Loss” is quite sensitive to node “Human Errors”, that is, once a personnel injury/loss accident caused by collision of FPSO is observed, it is more likely that human errors during operation are the causes. To further justify the conclusion, sensitivity analysis must be conducted.

Sensitivity refers to how sensitive a model’s performance is to minor changes in the input parameters. Sensitivity analysis is particularly useful in investigating the effects of inaccuracies or incompleteness in the parameters of an FBN model on the model’s output. The most natural way of performing sensitivity analysis is to change the parameters values and then, using an evidence propagation method, monitor the effects of these changes on the posterior probabilities. In this case study, the preliminary conclusion (i.e. node “Personnel Injury/Loss” is quite sensitive to node “Human Errors”) is drawn based on posterior probabilities e.g. $P_f(X = x_1 | W = w_1)$. Thus one of the most important sensitivity analysis aspects is to analysis how they change when prior probabilities take different values. Without loss of generality, $P_f(X = x_1)$ takes eight different values, ranging from (0.14, 0.15, 0.16) to (0.49, 0.50, 0.51) (see Table 7). A Matlab calculation program has been developed to implement the FBN inference. The results in Table 7 were generated using the program and graphically shown in Figure 4. As can be seen in Table 7, the values in the second and third columns indicate that $P_f(X = x_1 | W = w_1)$ clearly changes with $P_f(X = x_1)$. Figure 4 also shows a positively increasing trend in the posterior probability when $P_f(X = x_1)$ steadily increases. Therefore there is a reason to believe that the above conclusion is reliable.

To reduce computational complexity and obtain significant savings in computation time, a Matlab program has been developed. All the FBN inferences including sensitivity analyses in this paper are conducted with the developed Matlab program.

Table 7 Sensitivity analysis results between $P_f(X = x_1 | W = w_1)$ and $P_f(X = x_1)$

No	$P_f(X = x_1)$	$P_f(X = x_1 \| W = w_1)$
1	(0.14,0.15,0.16)	(0.1182,0.1852,0.2847)
2	(0.19,0.20,0.21)	(0.1602,0.2493,0.3669)
3	(0.24,0.25,0.26)	(0.2019,0.3004,0.4468)
4	(0.29,0.30,0.31)	(0.2417,0.3529,0.5245)
5	(0.34,0.35,0.36)	(0.2817,0.4093,0.5959)
6	(0.39,0.40,0.41)	(0.3233,0.4614,0.6626)
7	(0.44,0.45,0.46)	(0.3578,0.5091,0.7400)
8	(0.49,0.50,0.51)	(0.3955,0.5634,0.8092)

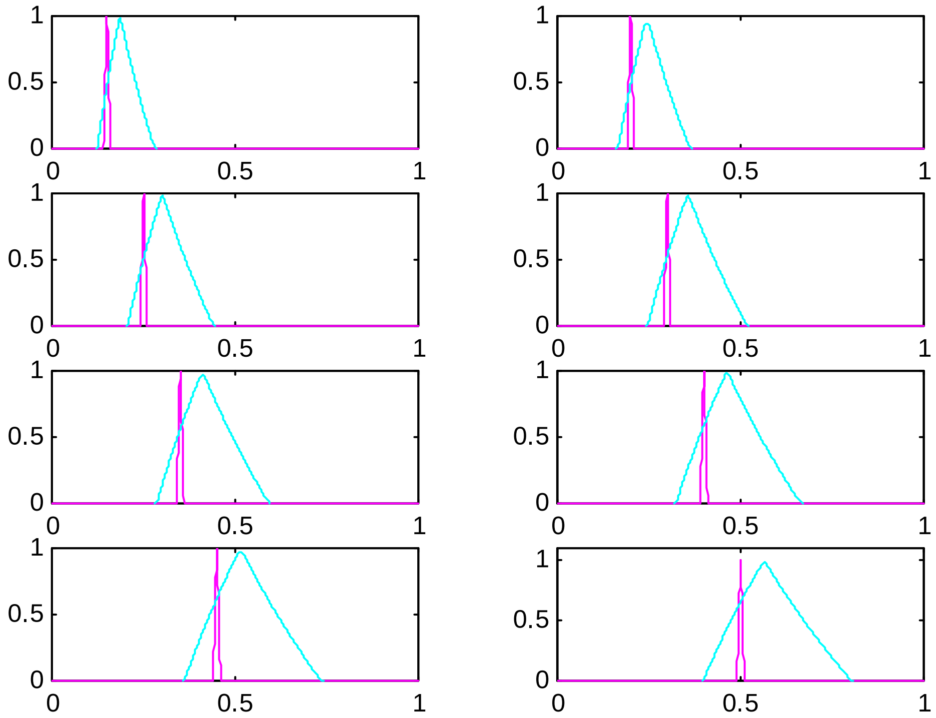

Figure 4: Graphical demonstration of sensitivity analysis between $P_f(X = x_1 | W = w_1)$ **and** $P_f(X = x_1)$

4.4. Comparison between the proposed FBN and conventional BN model

To validate and justify the proposed FBN model, it is essential to compare the results of FBN analysis with those of conventional BN. Using the same structure of BN model and the similar input data, a conventional BN analysis is conducted using software package Hugin. In fact, there are several commercial and research tools designed for BN model establishing and testing. Among the most popular of these tools are Hugin (Jenson, 1993), Netica (Netica 2002) and *MSBNx* (Kadie, et al., 2001). The most significant advantage of Hugin is that it not only serves as "drag and drop" style model construction tool, but also provides libraries of routines for computation of probabilities as well as learning algorithms, facilitating the easy design and authoring of BN models for diagnostics. Using Hugin, it is possible to view the status of any given number of observations, and "run" to obtain posterior probability.

For comparison purposes, fuzzy prior probabilities and CPTs given in Tables 1 - 6 are simplified as crisp numbers, that is, fuzzy numbers are replaced by their most likely values. Fuzzy probability $P_f(X = x_1) = (0.39, 0.4, 0.41)$, for example, is replaced by crisp probability $P(X = x_1) = 0.4$. The bold fonts in the bodies of Tables 1, 2 and 4 are the prior probabilities of nodes *X*, *Y* and *V*, respectively. The bold fonts in the bodies of Tables 3, 5 and 6 are conditional probabilities of nodes *Z*, *U* and *W*, respectively. The meanings of each prior probability and each CPT can be explained in a similar way to those depicted in Section 4.2.

Using the conventional Bayesian rules, posterior probability, $P(X = x_1 \mid W = w_1)$, can be calculated as follows:

$$P(X = x_1 \mid W = w_1) = \frac{P(X = x_1; W = w_1)}{P(W = w_1)} = \frac{\sum_{Y,Z,V,U} P(Y; Z; U; V; X = x_1; W = w_1)}{P(W = w_1)}$$

= 46.21%

To save computation time, this result can be obtained using Hugin software. For example, marginal probabilities of all the nodes and the posterior probabilities can be seen in Figures 5 and 6. Comparing the above result ($P(X = x_1 \mid W = w_1)$ = 46.21%) with prior probability ($P_f(X = x_1) = 40\%$, as shown in Table 1 using bold font in the second row and second column)**,** it is obvious that there is a significant change between the posterior probability and prior probability. This suggests that node "Personnel Injury/Loss" is related to node "Human Errors". This result is consistent with that obtained using FBN. The proposed FBN approach is validated in this way. By comparing the FBN inference and the conventional Bayesian inference, it is noted that although these two methods produce similar results given the same model and similar input data, FBN seems more flexible and interpretable than conventional BN.

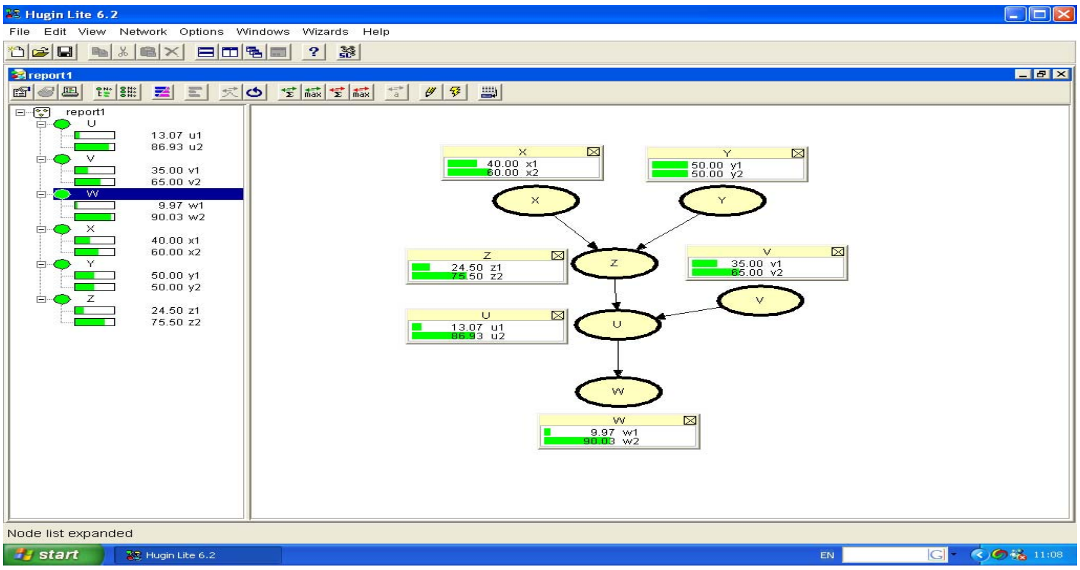

Figure 5: Conventional BN model and marginal probabilities of all the nodes

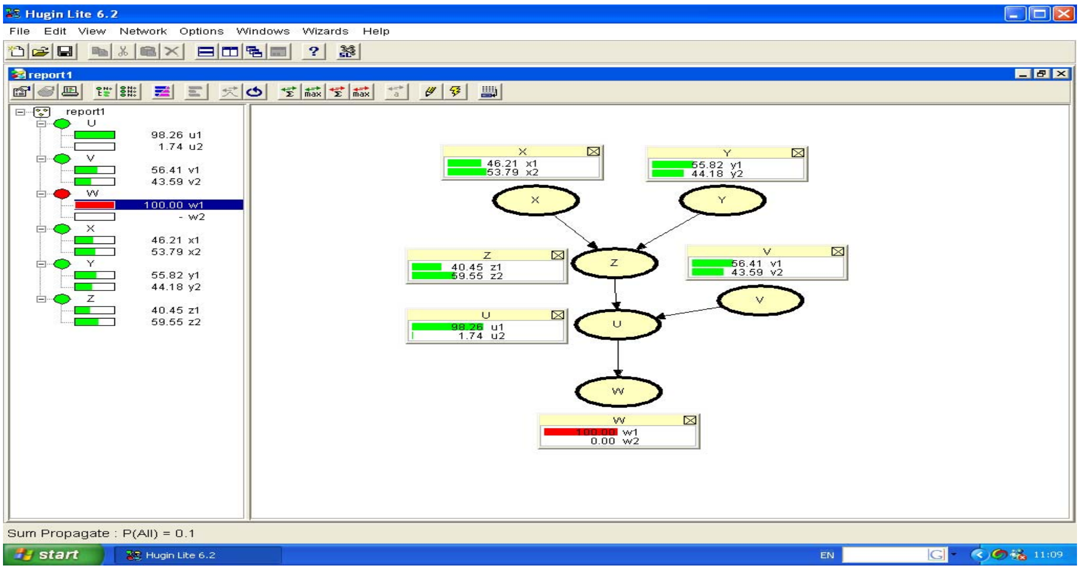

Figure 6: Conventional BN model and posterior probabilities of all the nodes

5. Conclusions

FBN is a technology with huge potential for application across many domains. This paper discusses FBN and its application in offshore risk analysis. The proposed FBN approach uses fuzzy number based probabilities to conduct Bayesian inference, which includes risk factor inter-relationship identification, BN model establishment, fuzzy prior probability and likelihood calculation, and inference and interpretation. The FBN model explicitly represents cause-and-effect assumptions between system variables that may be obscured under other modelling approaches like fuzzy reasoning and Monte Carlo risk analysis. The flexibility of the method allows for multiple forms of information to be used to quantify model relationships, including formally assessed expert opinion when quantitative data are lacking or, only qualitative/vague statements can be made. The proposed FBN model is also a modular representation of uncertainty knowledge caused due to randomness and vagueness, and therefore can be potentially integrated into the existing risk assessment systems. In general, the proposed model ensures that its applications are conducted in a disciplined, well-managed, and consistent manner that promotes the delivery of risk assessment results.

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References

Buckley J.J., “An introduction to fuzzy logic and fuzzy sets”, *Springer-Verlag*, Heidelberg, New York, 2002.

Chou K.G., Yuan J., “Fuzzy-Bayesian approach to reliability of existing structures”, *Journal of Structural Engineering*, Vol.119, No.11, 1993, 3276-3290.

Darwiche A., “A differential approach to inference in Bayesian networks”, *Journal of the ACM*, Vol.50, No.3, 2003, 280-305.

Furthsirth-Schnater S., “On fuzzy Bayesian inferences”, *Fuzzy Sets and Systems,* Vol.60, No.1, 1993, 41-58.

Halliwell J., Keppens J., Shen Q., “Linguistic Bayesian networks for reasoning with subjective probabilities in forensic statistics”, *Proceedings of the 5th International Conference on AI and Law*, Edinburgh, Scotland, UK, June 24-28, 2003, 42-50.

Halliwell J., Shen Q., “Towards a linguistic probability theory”, *Proceedings of the 11th International Conference on Fuzzy Sets and Systems (FUZZ-IEEE '02),* Honolulu (Hawaii), USA, May 16-17, 2002, 596-601.

Hayes K.R., “Bayesian statistical inference in ecological risk assessment”, *Crimp Technical Report,* Australia, Centre for Research on Introduced Marine Pests, CSIRO, Hobart, Australia, No.17, 1998, 100 pages.

Heckerman D., “Bayesian networks for data mining”, *Data Mining and Knowledge Discover*y, Vol.1, No.1, 1997, 79-119.

Heckerman D., Breese J., “Causal independence for probability assessment and inference using Bayesian networks”. *IEEE Transactions on Systems, Man, and Cybernetics,* Vol.26, No.6, 1996, 826-831.

Henley E.J., Kumamoto H., “Probabilistic risk assessment”, *IEEE Press*, New York, 1992.

HSE, “The offshore installations and wells (design and construction, etc.) regulations ”, No. 913, 1996 (ISBN 011054451-X).

Jenson F.V., “Introduction to Bayesian networks: HUGIN”, *Aalborg University Press*, Aalborg, Denmark, 1993.

Kadie C.M., Hovel D., Horvitz E., “MSBNx: a component-centric toolkit for modelling and inference with Bayesian networks”, *Microsoft Research Technical Report MSR-TR-2001-67*, Microsoft Corporation, Redmond, WA 98052, 2001.

Kang C.W., Golay M.W., “A Bayesian belief network – based advisory system for operational availability focused diagnosis of complex nuclear power systems,” *Expert Systems with Applications*, Vol.17, No.1, 1999, 21-32.

Karwowski W., Mital A., “Potential applications of fuzzy sets in industrial safety engineering”, *Fuzzy Sets and Systems*, Vol.19, No.2, 1986, 105-120.

Lauritzen S.L., Spiegelhalter D.J., “Local computations with probabilities on graphical structures and their application to expert systems”. *Journal of the Royal Statistical Society*, Series B, Vol.50, No.2, 1988, 157 - 224.

León-Rojas J. M., Masero V, Morales M, “On the fuzzy Bayesian inference of population annoyance level caused by noise exposure”. *Symposium on Applied Computing*, *Proceedings of the 2003 ACM symposium on Applied computing,* Melbourne, Florida, USA, March 9-12, 2003, 227-234 (ISBN:1581136242).

Li H.L., Kao H.Y., “Constrained abductive reasoning with fuzzy parameters in Bayesian networks”. *Journal of Computers & operations research*, Vol.32, No.1, 2005, 87-105.

Mitchell, K., Bright C.K., “Minimizing the potential for human error in ship operation”. *IMAS 95, Management and Operation of Ships,* Practical Techniques for Today and Tomorrow, 1995.

Moore R. E., “ Interval analysis”, *Prentice Hall*, New Jersey,1966.

Myllymaki P., “Advantages of Bayesian networks in data mining and knowledge discovery”, http://www.bayesit.com/docs/advantages.html, (access date: 11/2/2005).

Netica, “Netica-J reference manual - version 2.21”, *Java Version of Netica API,* Norsys Software Corporation, 2002.

OREDA (Offshore Reliability Data), “OPEDA: offshore reliability data handbook”, OPEDA participants, 535 pages, 1997 (ISBN 8214004381).

Pearl J., “Probabilistic reasoning in intelligent systems”. *Networks of Plausible Inference*, Morgan Kaufmann, Palo Alto, USA, 1988.

Perichi L.R., Walley P., “Robust Bayesian credible interval and prior ignorance”, *International Journal of Statistics*, Vol.58, No.1, 1991, 1-23.

Pillay A., Wang J., "A risk ranking approach incorporating fuzzy set theory and grey theory", *Engineering Reliability & System Safety*, Vol.79, No.1, 2003, 61-67.

Ren J., Jenkinson I., Sii H.S., Xu D.L., Wang J., Yang J.B., "An offshore safety assessment framework using fuzzy reasoning and evidential synthesis approaches", *Journal of Marine Engineering and Technology*, Part A, No. A6, 2005, 3-16.

Sii H.S., Ruxton T., Wang J., "A synthesis using fuzzy set theory and Dempster-Shafer based approach to compromise decision making with multiple attribute applied to risk control options selection", *Proceedings of the Institution of Mechanical Engineers Part E, IMechE Journal of Process Mechanical Engineering*, Vol.212, 2001, 251-261.

UK Department of Energy, "The public inquiry into the Piper Alpha disaster (Cullen Report)", London, UK, 1990 (ISBN 010113102).

UKOOA, "Industry guidelines on a framework for risk related decision making", *UK Offshore Operators Association*, Aberdeen, UK, April, 1999.

Wang J., "A review of marine and offshore safety assessment", *Quality and Reliability Engineering International*, Vol.22, No.1, 2006, 3-19.

Wang J., Kieran O., "Offshore safety assessment and safety based decision making – the current status and future aspects", *Journal of Offshore Mechanics and Arctic Engineering – Transactions of the ASME,* Vol.122, No.2, 2000, 93-99.

Wang J., Ruxton T., Wang J., Sii H. S., Yang J.B., Pillay A., Maistralis E., Saajedi A., "Use of advanced in technology in marine risk assessment", *Risk Analysis*, Vol.24, No.4, 2004, 1011-1033.

Wang J., Yang J.B., Sen P., "Safety analysis and synthesis using fuzzy set modelling and evidential reasoning", *Reliability Engineering and System Safety*, Vol.47, No.2, 1995, 103-118.

Wang J., Yang J.B., Sen P., "Multi-person and multi-attribute design evaluations using evidential reasoning based on subjective safety and cost analyses", *Reliability Engineering and System Safety*, Vol.52, No.2, 1996, 113-129.

Yang C.C., Cheung K.M., "Fuzzy-Bayesian analysis with continuous-valued evidence", *In proceedings of IEEE International Conference on Systems, Man, and Cybernetics*, Vancouver, Canada, October 22-25, 1995, 441-446.

Yang J.B., Xu D.L., "On the evidential reasoning algorithm for multiattribute decision analysis under uncertainty", *IEEE Transactions on Systems, Man, and Cybernetics, Part A: Systems and Humans*, Vol.32, No.3, 2002, 289-304.

Yang J.B., Liu J., Wang J., Sii H.S., Wang H.W., "A belief rule-base inference methodology using the evidential reasoning approach - RIMER", *IEEE Transactions on Systems, Man, and Cybernetics, Part A*, Vol.36, No.2, 2006, 266- 285.

Zadeh L.A., "Fuzzy sets", *Information and Control*, Vol.8, 1965, 338-353.

Zadeh L.A., "Fuzzy probabilities", *Information Proceeding Management*, Vol.20, 1984, 363-372.

Zhang G., Bai C., Lu J. and Zhang C., "Bayesian network based cost benefit factor inference in e-services", *Proceedings of the 2nd International Conference on Information Technology for Application(ICITA),* Harbin, P. R. China, January 7-10, 2004, 464-469.

Appendix 1 Fuzzy set, fuzzy numbers and their operations

Fuzzy set theory was first introduced by Zadeh (1965). Fuzzy sets were derived from generalizing the concept of set theory. Fuzzy sets can be thought of as an extension of classical sets. In a classical set or crisp set, the objects in a set are called elements or members of the set. An element x belonging to a set A is defined as $x \in A$, an element that is not a member in A is noted as $x \notin A$. A characteristic function or membership function $\mu_A(x)$ is defined as an element in the universe U having a crisp value of 1 or 0. For every $x \in$ U,

$$\mu_A(x) = \begin{cases} 1 & \text{for } x \in \mathrm{A}, \\ 0 & \text{for } x \notin \mathrm{A}. \end{cases}$$

this can also be expressed as $\mu_A(x) \in \{0,1\}$. For the classical set or crisp set, membership functions take a value of 1 or 0. However, for fuzzy sets, a membership function can take values in the interval [0,1]. The range between 0 and 1 is referred to as the membership grade or degree of membership. A fuzzy set *A* is defined below:

$$A = \{(x, \mu_A(x)) \mid x \in A, \mu_A(x) \in [0,1]\}$$

where $\mu_A(x)$ is a membership function belonging to the interval [0,1].

There are different kinds of membership function. A triangular membership function (Figure 7), for example, is one of the most popular ones. It is normally defined by parameters (*a, b, c)*, where *a* is the membership function's left intercept with the grade equal to *0*, *b* is the centre point where the grade is *1* and *c* is the right intercepts at grade equal to *0*. The function *y = triangle (x, (a, b, c))* is written to return the membership values corresponding to the defined universe of discourse *x*. The parameters that define the closed interval membership function (*a, b, c)* must be in the discretely defined universe of discourse.

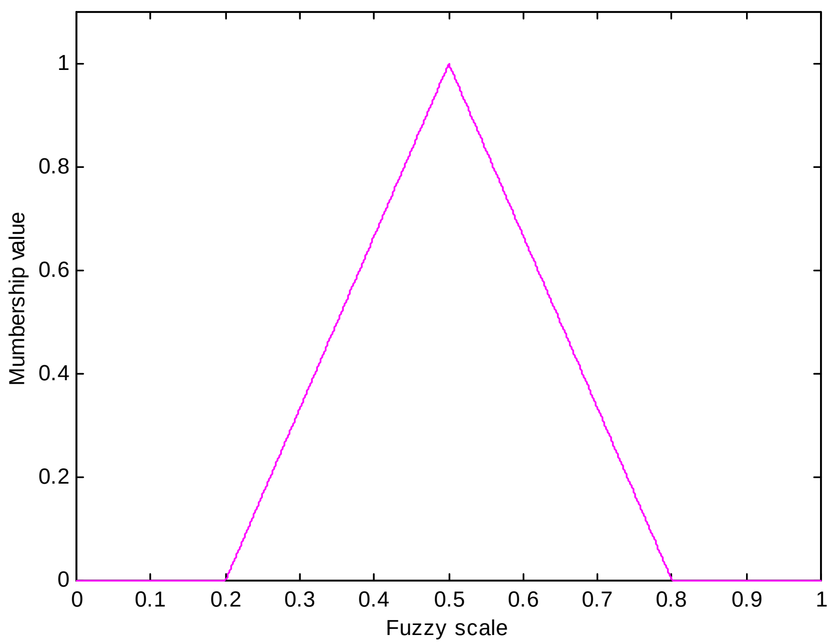

Figure 7: A triangular distribution defined by a most likely value of 0.5, with a lower least likely value of 0.2 and upper least likely value of 0.8

Fuzzy numbers are very special fuzzy subsets of the real numbers. The general definition of a fuzzy number *X* is a fuzzy subset of *R*. If the membership function of *X* is denoted as $\mu_X(x)$, *X* must meet the following conditions:

(a) The core of *X* is non-empty, i.e. $\exists x \in R$ such that $\mu_X(x) = 1$.

(b) α-cuts of *X* are all closed, bounded intervals.

(c) It has a bounded support, i.e. $\exists N \in R$ such that $\forall x \in R$, if $|x| \geq N$ then $\mu_X(x) = 0$.

Note that an α-cut of a fuzzy number *X* is an interval number X_α that contains all the values of real numbers that have a membership grade in *X* greater than or equal to the specified value of α. This can be written as

$$X_\alpha = [a, b] = \{x \in X | \ \mu_X(x) \geq \alpha\}.$$

To implement fuzzy number operations using α-cut and interval arithmetic method, it is essential to perform interval-based calculations at each α-cut. Given two fuzzy numbers *X* and *Y*, their α-cuts are X_α = [a, b] and Y_α =[c, d] for every $\alpha \in [0,1]$, respectively. The interval operations over those two α-cuts [*a*, *b*] and [c, d] are therefore calculated by the following formulae (Moore, 1966):

Sum:

$$X_\alpha + Y_\alpha = [a, b] + [c, d] = [a + c, b + d] \quad (5)$$

where inverse addiction is defined as: $-X_\alpha = [-b, -a]$

Subtraction:

$$X_\alpha - Y_\alpha = X_\alpha + (-Y_\alpha) = [a, b] + [-d, -c] = [a - d, b - c] \quad (6)$$

Multiplication:

$$X_\alpha \times Y_\alpha = [a, b] \times [c, d] = [\min\{ac, bc, ad, bd\}, \max\{ac, ad, bc, bd\}] \quad (7)$$

Inverse multiplication is defined as: $X_\alpha^{-1} = 1/X_\alpha = [1/b, 1/a]$, $0 \notin X_\alpha$.

Division:

$$X_\alpha / Y_\alpha = X_\alpha \times 1/Y_\alpha = [a, b] \times [1/d, 1/c]$$

$$= [\min\{a/d, b/d, a/c, b/c\}, \max\{a/d, b/d, a/c, b/c\}], \text{ if } 0 \notin [c, d] \quad (8)$$

When α takes value from 0 to 1, the above formulae give the results of *addition, subtraction, multiplication and division* of two fuzzy number *X* and *Y*, respectively.

Appendix 2 Linguistic variables

Offshore risk analysis is usually a multi-criteria and multi-expert analysis process. The measurement of the probability of an unwanted/unsuccessful event may conveniently be assessed by linguistic terms on a subjective basis. Linguistic variables provide appropriate elements in human knowledge representation. When human experts are asked to evaluate the occurrence likelihood of a possible event with limited historical data, it is more likely that they use words like "very unlikely to happen" or "quite likely to happen". By introducing the concept of linguistic variables, it is possible to formulate vague descriptions in natural languages in precise mathematical terms. In fact, a variable that is imprecisely defined using linguistic descriptors is called a "linguistic variable." A linguistic variable takes on linguistic value whereas a numerical variable takes on numerical value. Uncertainty about a numerical value can be described using probability distributions. Uncertainty about a linguistic value can be expressed using fuzzy measures.

Linguistic variables can be described with membership functions. It is commonly accepted that the type of linguistic variable is case-dependent and expert-dependent, that is, same linguistic term may be assigned with different membership functions in different cases and different domain experts may assign different membership functions to the same linguistic term. Judgement "very unlikely to happen", for example, could be defined by expert *A* with a triangularly shaped membership function (0.19, 0.2, 0.21) while being defined by expert *B* with (0.24, 0.25, 0.26). This is the nature of human recognition. In addition, membership functions may reflect experts' belief degree and uncertainty in the judgement. (0.1, 0.2, 0.3), for example, has a higher uncertainty than (0.19, 0.2, 0.21) while they have the same core value (0.2). The domain expert can provide complete or partial information about the linguistic variables. In some cases, precise values could be found from relevant databases, like the OREDA database (OPEDA, 1997) in offshore risk analysis. Precise number is a special case of fuzzy number e.g. value 0.2 could be represented by triangularly shaped fuzzy number (0.2, 0.2, 0.2).

One of the most complex steps in defining linguistic variables is the generation of fuzzy partitions, that is, the definition of the fuzzy sets that define the meaning of the linguistic terms considered in the offshore risk analysis. Brainstorming within a panel of domain experts is the most popular method to establish the relationship between linguistic terms and membership functions. For demonstration purposes in this research, nine levels of linguistic variables are used for linguistic probabilities in safety analysis of engineering systems, such as *Impossible, Nearly impossible, Very unlikely, Quite unlikely, Even chance, Quite likely, Very likely, Nearly certain and Certain.*

Table 8 describes the linguistic labels and associated meanings of the linguistic variables. As can be seen in the table, the first column gives the linguistic label and the second column gives meanings according each linguistic label whilst third column defines possible value range for each label.

Table 8 Example linguistic terms and meanings with value ranges

Linguistic label	Meaning	Value range
Impossible	Events never happen.	0.00
Nearly impossible	The occurrence likelihood of possible events is *nearly impossible* (extremely unlikely to exist in the system or during operations).	(0.00, 0.05]
Very unlikely	The occurrence likelihood of possible *events* is highly unlikely (highly unlikely to exist in the system or during operations).	(0.05, 0.30]
Quite unlikely	The occurrence likelihood of possible *event*s is *quite* unlikely but possible (improbable to exist even in rare occasions on the system or during operations).	(0.30, 0.45]
Even chance	The occurrence likelihood of possible *events* is *even chance* (likely to exist on rare occasions in the system or during operations).	(0.45, 0.55]
Quite likely	It is quite likely that *events* occur (i.e. exist from time to time on the system or during operations, possibly caused by a potential design fault or malfunction during operations).	(0.55, 0.70]
Very likely	It is highly likely that *events* occur (i.e. often exist somewhere on the system or during operations due to a highly likely potential hazardous situation or design and/or operations procedural drawback).	(0.70, 0.95]
Nearly certain	*Events always* happen (i.e. likely to exist repeatedly during operations due to an anticipated potential procedural drawback in design and operation).	(0.95, 1.00)
Certain	*Events definitely* happen.	1.00

Two issues must be given more attention during the assignment of linguistic terms with fuzzy membership functions. First, a single linguistic term may have more than one membership function e.g. "*Very unlikely*" may have membership functions (0.19, 0.20, 0.21) and (0.24, 0.25, 0.26). This is because when domain experts assess the occurrence likelihood of two possible events, they may use single term to describe them e.g. "*Very unlikely"* to happen. However, if they can tell a difference between the occurrence likelihood of the two possible events, then different membership functions should be used. This provides accuracy to linguistic variables and flexibility to use fuzzy membership functions.

Second, an assigned fuzzy membership function may lie across the boundary between two value ranges. (0.04, 0.05, 0.06), for instance, does not lie completely within one single value range. It has intersection with both *"Nearly impossible"* and *"Very unlikely"*. This means that experts made judgement of this fuzzy probability as *"Very unlikely* and very close to *Nearly impossible"*.

Appendix 3 A detailed calculation of $P_f(X = x_1; W = w_1)$

Using extended Bayesian Equations (2) and (3), $P_f(X = x_1; W = w_1)$ is obtained as follows:

$$P_f(X = x_1; W = w_1) \cong \sum_{Y,Z,V,U} P_f(w_1|U) \otimes P_f(U|Z;V) \otimes P_f(V) \otimes P_f(Z|x_1;Y) \otimes P_f(x_1) \otimes P_f(Y)$$

$$= P_f(x_1) \otimes \{P_f(y_1) \otimes \sum_{Z;V;U} P_f(w_1|U) \otimes P_f(U|Z;V) \otimes P_f(V) \otimes P_f(Z|x_1;y_1)$$

$$+ P_f(y_2) \otimes \sum_{Z;V;U} P_f(w_1|U) \otimes P_f(U|Z;V) \otimes P_f(V) \otimes P_f(Z|x_1;y_2)\}$$

$$= P_f(x_1) \otimes \{P_f(y_1) \otimes \{P_f(v_1) \otimes \{\sum_U P_f(w_1|U) \otimes P_f(U|z_1;v_1) \otimes P_f(z_1|x_1;y_1) \oplus \sum_U P_f(w_1|U) \otimes P_f(U|z_2;v_1) \otimes P_f(z_2|x_1;y_1)\}$$

$$\oplus P_f(v_2) \otimes \{\sum_U P_f(w_1|U) \otimes P_f(U|z_1;v_2) \otimes P_f(z_1|x_1;y_1) \oplus \sum_U P_f(w_1|U) \otimes P_f(U|z_2;v_2) \otimes P_f(z_2|x_1;y_2)\}\}$$

$$\oplus P_f(y_2) \otimes \{P_f(v_1) \otimes \{\sum_U P_f(w_1|U) \otimes P_f(U|z_1;v_1) \otimes P_f(z_1|x_1;y_2) \oplus \sum_U P_f(w_1|U) \otimes P_f(U|z_2;v_1) \otimes P_f(z_2|x_1;y_2)\}$$

$$\oplus P_f(v_2) \otimes \{\sum_U P_f(w_1|U) \otimes P_f(U|z_1;v_2) \otimes P_f(z_1|x_1;y_2) \oplus \sum_U P_f(w_1|U) \otimes P_f(U|z_2;v_2) \otimes P_f(z_2|x_1;y_2)\}\}$$

$$= P_f(x_1) \otimes \{P_f(y_1) \otimes \{P_f(v_1) \otimes \{P_f(w_1|u_1) \otimes P_f(u_1|z_1;v_1) \otimes P_f(z_1|x_1;y_1) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_1;v_1) \otimes P_f(z_1|x_1;y_1)$$

$$\oplus P_f(w_1|u_1) \otimes P_f(u_1|z_2;v_1) \otimes P_f(z_2|x_1;y_1) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_2;v_1) \otimes P_f(z_2|x_1;y_1)\}$$

$$\oplus P_f(v_2) \otimes \{P_f(w_1|u_1) \otimes P_f(u_1|z_1;v_2) \otimes P_f(z_1|x_1;y_1) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_1;v_2) \otimes P_f(z_1|x_1;y_1)$$

$$\oplus P_f(w_1|u_1) \otimes P_f(u_1|z_2;v_2) \otimes P_f(z_2|x_1;y_1) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_2;v_2) \otimes P_f(z_2|x_1;y_1)\}\}$$

$$\oplus P_f(y_2) \otimes \{P_f(v_1) \otimes \{P_f(w_1|u_1) \otimes P_f(u_1|z_1;v_1) \otimes P_f(z_1|x_1;y_2) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_1;v_1) \otimes P_f(z_1|x_1;y_2)$$

$$\oplus P_f(w_1|u_1) \otimes P_f(u_1|z_2;v_1) \otimes P_f(z_2|x_1;y_2) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_2;v_1) \otimes P_f(z_2|x_1;y_2)\}$$

$$\oplus P_f(v_2) \otimes \{P_f(w_1|u_1) \otimes P_f(u_1|z_1;v_2) \otimes P_f(z_1|x_1;y_2) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_1;v_2) \otimes P_f(z_1|x_1;y_2)$$

$$\oplus P_f(w_1|u_1) \otimes P_f(u_1|z_2;v_2) \otimes P_f(z_2|x_1;y_2) \oplus P_f(w_1|u_2) \otimes P_f(u_2|z_2;v_2) \otimes P_f(z_2|x_1;y_2)\}\}\}$$

= (0.0386, 0.0461, 0.0546)