# An Optical MEMS-Based Shear Stress Sensor
# for High Reynolds Number Applications

D. Fourguette[1], D. Modarress[1], D. Wilson[2], M. Koochesfahani[3], M. Gharib[4]

[1] VioSense Corporation, Pasadena, CA

[2] Jet Propulsion Laboratory, Pasadena, CA

[3] Michigan State University, East Lansing, MI

[4] California Institute of Technology, Pasadena, CA

## Abstract

In an effort to extend wall shear stress measurements to high Reynolds number flows, a new MEMS-based optical shear stress sensor was fabricated and tested in the 2 feet wind tunnel at the California Institute of Technology for Reynolds numbers of up to $5.6 \times 10^6$. The description of this sensor and the test results are reported in this paper. The sensor, the Dual Velocity sensor, designed using recent developments in diffractive and integrated optics, was small enough to be embeddable in test models. The sensor measured the average flow velocity at two probe volumes located within the first 110 micrometers above the flush-mounted sensor surface. The velocity gradient at the wall was estimated by fitting the Spalding formula to the average velocity measurements, once mapped using the inner-law variables u+ and y+. The results obtained with the Dual Velocity sensor were in excellent agreement with measurements obtained in the same tunnel using other techniques such as the oil film interferometry technique and with another MEMS-based optical shear stress sensor, the Diverging Fringe Doppler sensor. All wall shear stress measurements were also in agreement with those calculated from boundary layer surveys obtained with a miniature LDV.

## Introduction

The wall-shear stress is an essential quantity to compute, measure or infer in turbulent flows. Time-averaged values of this quantity are indicative of the global state of the flow along a surface and can be used to determine body-averaged properties like skin-friction drag. The instantaneous wall-shear stress can be used for control purposes, e.g. drag-reduction or separation delay.

Micro machined wall shear stress sensors (an excellent review of these micromachined sensors is currently in press[1]) calculate the shear stress from measurements performed at the surface, mechanically using a floating element or thermally using heat dissipation, or infer the shear stress at the wall from velocity measurements performed within the viscous sublayer of the boundary layer. These velocity measurements are performed using hot wires positioned within a few microns above the surface[2] or using particle-based velocimetry[3-4].

Presently, no wall shear stress measurement approach is free and clear of significant limitations. Surface mounted thermal sensors suffer from heat transfer problems, thus making an accurate calibration a difficult task while velocity measurements are limited to relatively low Reynolds numbers because of the commonly accepted requirement for the measurement to be located within the linear sublayer, y+<5.

Using optical MEMS technology, a wall shear stress sensor based on a measurement technique demonstrated by Naqwi and Reynolds[5] was developed and described in previous publications[3-4]. This sensor measured the flow velocity at 66 $\mu$m above the sensor surface using a diverging fringe pattern originating at the surface. This method yielded accurate results as long as the measurements were conducted within the linear sub-layer, thus limiting the sensor application to Reynolds number flows less than $10^6$. (The accuracy vs. Reynolds number

for the Diverging Fringe Doppler sensor is reported below in Figure 3).

This paper describes a new optical MEMS sensor, the Dual Velocity sensor, designed to extend the range of shear stress measurements up to Re = $10^8$. This new optical sensor measures the average flow velocity at two different probe locations and an empirical method is used to estimate the velocity gradient at the wall. The sensor was tested in a wind tunnel simultaneously with two other shear stress sensors: an oil film interferometer, and the Diverging Fringe Doppler sensor. The results obtained from all three sensors were in excellent agreement with each other. These results were also in excellent agreement with the wall shear calculated from the measured boundary layer profiles using a traversing MiniLDV™ for all experimental conditions. The design and fabrication of the sensor are described here along with the comparative results obtained with other sensors.

## MEMS-based Optical Shear Stress Sensors

The Diverging Fringe Doppler sensor is a precursor to the Dual velocity sensor and is briefly described here.

### *The Diverging Fringe Doppler Shear Sensor*

Figure 1 shows a conceptual drawing of the MOEMS wall shear stress sensor principle. The transmitter DOE, illuminated by the output of a single mode fiber, generates diverging interference fringes originating at the surface and extending into the flow. The scattered light from the particle passing through the fringes is collected through a window at the surface of the sensor and focused onto a multimode fiber by the receiver DOE. The probe volume region is defined by the intersection of the transmitter and receiver fields centered at approximately 66 μm above the surface. The key requirement is that the sensor probe volume should be at or near linear region of the boundary layer.
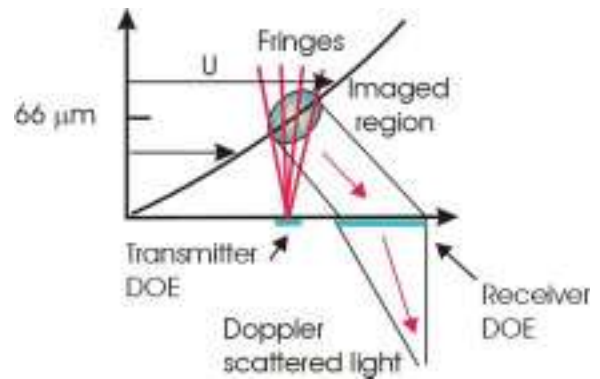


**Figure 1 Schematic of the optical shear stress sensor principle of measurement.**

A schematic drawing of the Diverging Fringe Doppler shear stress sensor is shown in Figure 2. The light output of a single mode optical fiber was allowed to diverge onto a PMGI (Polymethylglutarimide) diffractive lens and was spatially filtered through two parallel slits etched into a chromium layer. The output was a diverging fringe system. The probe volume is located above a window etched into the chromium layer to gather light with a receiver PMGI diffractive lens. The light was imaged onto a multimode fiber coupled to a photodiode. The spatial filter at the surface of the sensor ensures that the fringes originate at the surface of the sensor.
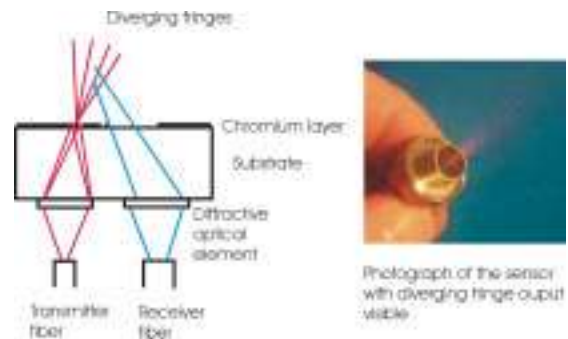


**Figure 2 Diverging Fringe Doppler sensor design**

A detailed description of the sensor is given in a previous paper[3].

### *Shear Stress Sensors for higher Reynolds numbers: The Dual Velocity sensors*

It is clear that the accuracy of estimating the wall velocity gradient can improve if the velocity is known at more than one point.

Using two-point measurements at probe locations $Y_{pv}$ and $2Y_{pv}$, we have used an empirical method to estimate the velocity gradient at the wall. The corresponding wall shear stress accuracy estimate is given in Figure 3 for single and two-point velocity measurements. This plot expresses the downstream location of the measurement normalized by the height of the probe volume above the flat plate vs. the Reynolds number. For example, at 1 m downstream from the leading edge ($x/Y_{pv}=1.51 \times 10^4$), the accuracy of single point measurement will be 90% for a Reynolds number of $3 \times 10^6$. A two-point measurement will be 90% accurate for Reynolds number of $1.5 \times 10^7$, a factor 5 improvement in the sensor's Reynolds number limitation.
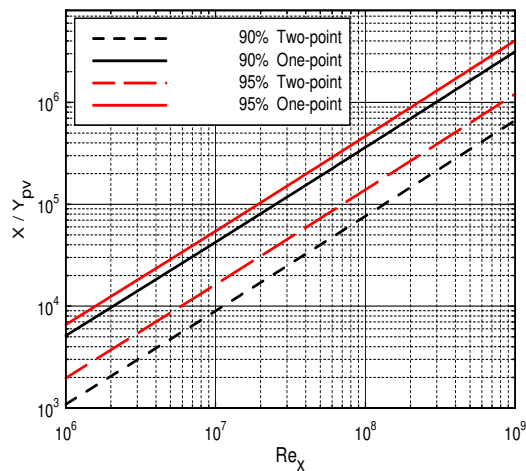


**Figure 3 Accuracy of wall shear stress in a flat plate turbulent boundary layer based on two-point velocity data from probe volumes located at Ypv and 2Ypv. An empirical method is used to estimate the wall velocity gradient. To achieve the indicated level of accuracy, the conditions must lie above the corresponding line.**

The accuracy of wall shear stress estimation from two-point data can be significantly improved using a fit through the velocity measurements[1]. The method works well over a broad range of pressure gradients. The idea behind the method is as follows: Typical turbulent boundary layer profiles are shown in Figure 4 for different external pressure

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[1] M. Koochesfahani, Private communication.

gradients. The important observation is that, once normalized using inner variables, the profiles in the logarithmic overlap layer and the linear sublayer collapse onto one single curve.

The Spalding formula representing $u^+(y^+)$ and shown in Figure 5 presents a good fit to the velocity profiles. Note that Spalding's formula provides a reasonable representation of the mean velocity profile in a turbulent boundary layer all the way from the wall to the end of the log region, to several hundred $y^+$ units, thus including both the linear sub-layer and the logarithmic law.
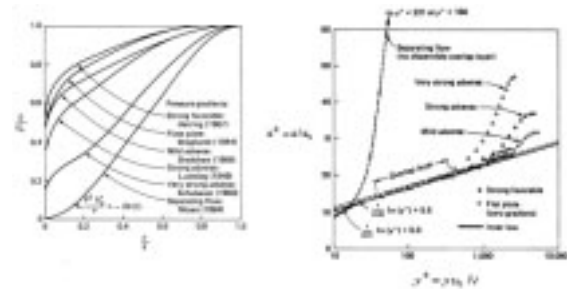


**Figure 4 Left: Experimental turbulent boundary-layer velocity profiles for various pressure gradients (Data from Coles and Hirst (1968), and Right: Replot of velocity profiles shown on the left using inner-law variables $y^+$ and $u^+$. Reprinted from "Viscous Fluid Flow" by F. A. White.**

Estimating the wall shear stress is performed by doing a fit, the mean velocity profile $u(y)$ is transformed into $u^+(y^+)$ using the mapping $u^+ = u/u_\tau$ and $\nu$ is the kinematic viscosity of the fluid. Using an iterative procedure, the friction velocity $u_\tau = \sqrt{\dfrac{\tau_w}{\rho}}$ is determined from the best fit to the Spalding formula

$$y^+ = \frac{y\,u_\tau}{\nu} = u^+ + e^{-\kappa_o B}\left[e^{\kappa_o u^+} - 1 - \kappa_o u^+ - \frac{(\kappa_o u^+)^2}{2} - \frac{(\kappa_o u^+)^3}{6} - \frac{(\kappa_o u^+)^4}{24}\right]$$

using the data within the first several hundred wall units. In principle, if the velocity data are accurate, this method can give an extremely accurate estimate of the wall shear stress (to the extent that the Spalding formula, or another equivalent, is an accurate representation of the mean profile).
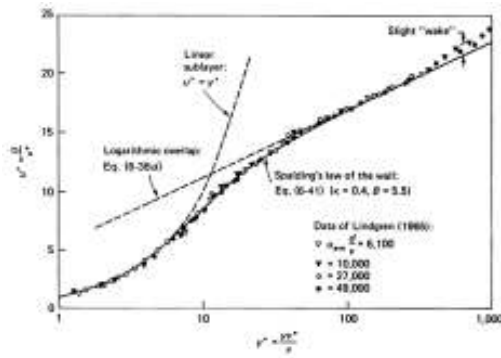
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**Figure 5 Mean velocity profile of a turbulent boundary layer from "Viscous Fluid Flow" by F. A. White.**

Note that Spalding's formula represents well the mean velocity profile in a turbulent boundary layer (for all pressure gradients except for the case of a separating flow as shown in Figure 4) to about $y+=200$. Typical turbulent boundary layer profiles for different external pressure gradients are shown in Figure 4. The important observation is that, once normalized using inner-law variables, the velocity profiles in the overlap layer and the linear sublayer collapse onto one single curve.

## Description of the Dual Velocity sensor

Based on the findings described in the previous section, a MEMS-based optical sensor was designed to measure flow velocity at two locations above the sensor surface. This sensor was composed of one transmitter DOE (diffractive optic element) and two receiver DOEs, as shown in Figure 6. The transmitter DOE was illuminated by the output of a singe mode fiber pigtailed to a diode laser. The transmitter generated two probe volumes, one located approximately at 65 μm and a second located approximately at 110 μm above the sensor surface. These probe locations were measured in air. Each probe volume was composed of two elongated light spots, 75 μm long and 13.7 μm apart. The scattered light generated by particles intersecting the probe volume, generating two intensity spikes, was focused onto a fiber by the receiver DOE and brought to an avalanche photodetector. An autocorrelation performed on the digitized output of each photodetector yielded the time elapsed between the spikes. The distance

between the light spots divided by the elapsed time yielded the instantaneous flow velocity.
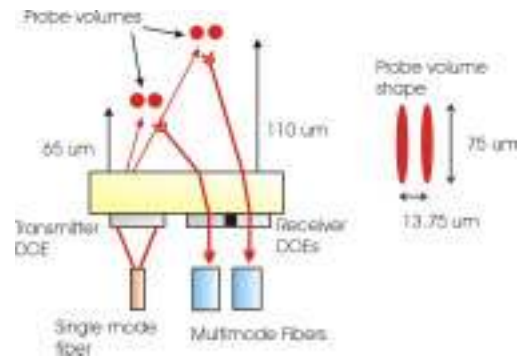


**Figure 6 Design of the Dual Microvelocimeter with two probe volumes, one probe volume located at 65 μm and a second probe volume located at 110 μm above the surface of the sensor.**

A photograph of the transmitter DOE and the resulting probe volumes generated by the transmitter is shown in Figure 7. The DOE was fabricated by direct-write electron beam lithography[8] using 0.25 mm square pixels and 64 depth levels. As a result, the two peaks in the signal are well defined and the processing yields a data rate 4 times higher than in previous velocity sensors.
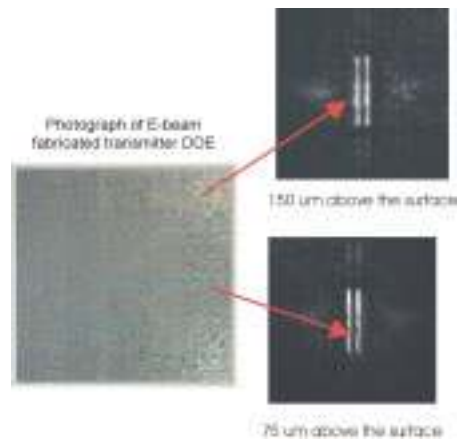


**Figure 7 Photographs of the transmitter DOE and of the two probe volumes. The locations of the probe volumes are given for use in water.**

## Wall shear stress measurements

Tests were carried out at the California Institute of Technology 2' wind tunnel. For the particle-based measurements, the tunnel was seeded with oil droplets using a pressurized nebulizer (the average droplet

diameter was estimated at 3 μm). The shear stress sensor results were compared with the results from the oil interferometry and MiniLDV™ results. The oil interferometry technique[3] measured the thinning rate of an oil film dropped on a surface (the tunnel bottom wall in this case) as it is being sheared by the flow. A monochromatic light source (sodium light) was used to generate an interferometric pattern in the oil film and a camera was used to record the pattern. The oil viscosity was calibrated and the tunnel temperature recorded. Prof. Nagib from the Illinois Institute of Technology conducted the oil film interferometry.

In addition, VioSense's miniature laser Doppler velocimeter, the MiniLDV™, was mounted on a traverse attached below the flat plate and was used to characterize the boundary layer profile through a window installed in the flat plate. The Reynolds number for the experimental conditions varied from $0.3 \times 10^6$ to $5.7 \times 10^6$. This corresponded to a free stream velocity of 2.75 m/s to 40 m/s. The measurement was carried out on the tunnel bottom wall at 2.08 meters beyond the converging section. The boundary layer was tripped using a 2″ wide sand paper strip placed at the entrance of the test section.

A photograph of the experimental setup is shown in Figure 8. The sensors, the MiniLDV and the Diverging Fringe Doppler sensor, are already mounted on the bottom wall of the test section. To the left of these sensors is a MEMS skin friction sensor designed and fabricated by Ho and Tai[6] as a joint Caltech/UCLA program. Testing the MEMS skin friction sensor is currently in process.



**Figure 8 Photograph of the wind tunnel test section equipped with the Miniature LDV and the Doppler sensor**

## *Boundary layer profiles obtained with the Miniature LDV*

The MiniLDV™ [7], successfully used to characterize shear stress sensors[3], was mounted on a traverse attached to the wall of the wind tunnel. Figure 9 shows a plot of the turbulent velocity profiles obtained in the boundary layer using the MiniLDV probe for Reynolds number between $0.38 \times 10^6$ and $1.89 \times 10^6$.
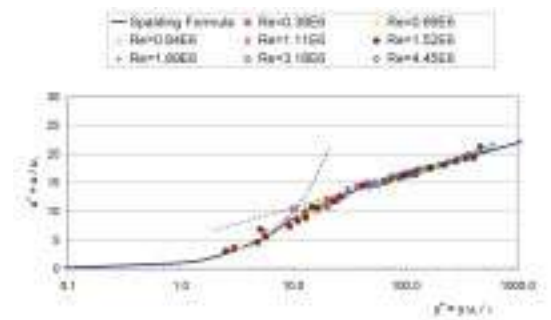


**Figure 9 Turbulent velocity profiles measured in the 2′ wind tunnel**

The slope at the wall $\left(\dfrac{\partial u}{\partial y}\right)_0$ was obtained from each boundary layer survey using a fit of the measured mean velocity to the Spalding Universal Law.

The wall shear stress
$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_0 = 0.0576\left(\frac{1}{2}\rho U_\infty^{\;2}\right)\left(\frac{U_\infty x}{\nu}\right)^{-1/5}$$
was calculated as a reference for the results obtained with the Diverging Doppler sensor and the Dual velocity sensor.

## *Wall shear stress measurements obtained with the Diverging Doppler sensor*

Wall shear stress measurements were obtained using the Diverging Doppler shear stress sensors. Particle passing through the sensor probe volume generate a signal burst similar to that of an LDV. The frequency of the Doppler burst was calculated using a Fast Fourier Transform. The velocity gradient at the wall was calculated from the product of the Doppler burst frequency and the sensor

fringe divergence $\dfrac{\mathrm{d}d_f}{\mathrm{d}y}$ measured during the fabrication of the sensor.

The wall shear stress measurements were compared to the results obtained with the Miniature LDV and the oil film interferometry technique. The results obtained with the three measurement techniques are displayed in Figure 10. The graph shows that the results agree extremely well both among the measurements techniques and with the theoretical formula for the wall shear stress given in the previous paragraph.
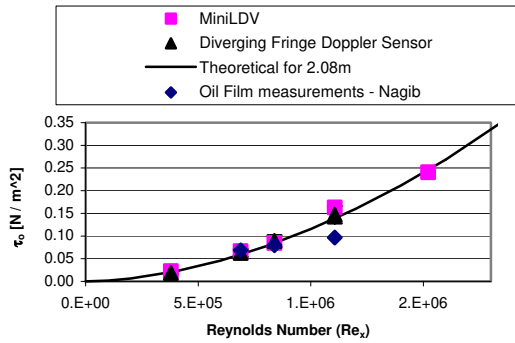


**Figure 10 - Wall shear stress measurements obtained with the diverging fringe Doppler sensor compared to that obtained with the MiniLDV.**

### _Wall shear stress measurements obtained with the Dual Velocity sensor_

The Diverging Fringe Doppler sensor was replaced in the tunnel with the Dual Velocity shear sensor and data were acquired for the same flow conditions as that used for the Doppler shear sensor. The average velocity was calculated by averaging 100 instantaneous measurements. The sensor provided velocity measurements at two locations 65 μm and 110 μm above the sensor.

The wall shear stress was calculated from the dual velocity measurements using a Spalding fit through the linear sublayer and the logarithmic overlap. Figure 11 shows the Spalding fit to five sets of dual velocity measurements in the sublayer.
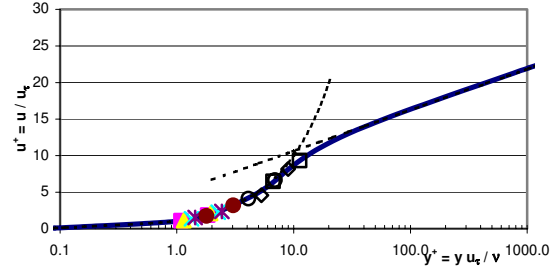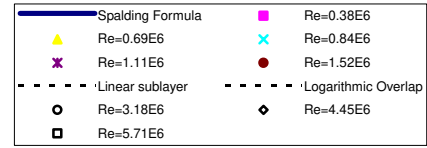


**Figure 11 Spalding fit to dual velocity measurements using the Dual Velocity sensor**

The wall shear stress measurements obtained using the Dual Velocity sensor are shown in Figure 12, combined with the results from the boundary layer survey obtained with the Mini LDV. These results show that the dual velocity approach to measure wall shear stress performs very well at Reynolds number in excess of 5.5 million and is a viable extension to the Diverging Doppler sensor.
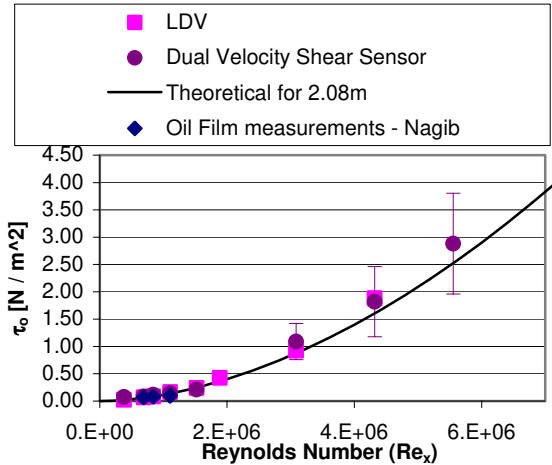


**Figure 12 Wall shear stress measurements with the Mini LDV, the Diverging Doppler sensor and oil film interferometry.**

The local skin friction $c_f^{'} = \dfrac{\tau_0}{\dfrac{1}{2}\rho U_{\infty}^2}$, was calculated for all sensors used in the tests and the results are plotted in Figure 13. The Diverging Fringe Doppler sensor performs well for Reynolds numbers up to $10^6$ and the

Dual Velocity sensor is in reasonable agreement with the theoretical curve.
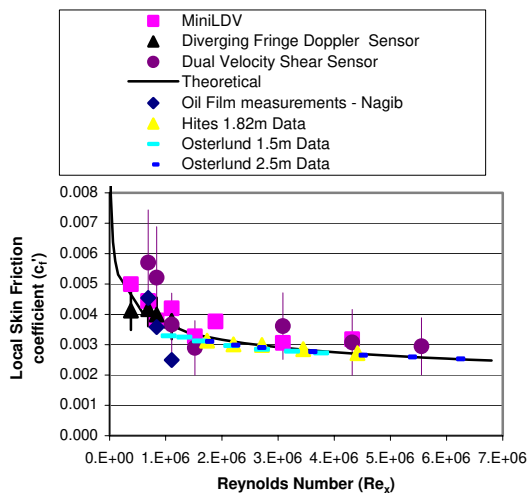


**Figure 13 Comparison of local skin friction measurements obtained with the diverging Doppler sensor, the dual Velocity sensor, LDV, oil film, and additional results obtained by Österlund[9] and Hites[10].**

## Conclusions

An optical MEMS shear stress sensor, the Dual Velocity sensor, was fabricated and tested in a wind tunnel at Reynolds numbers up to $5.6 \times 10^6$. This Dual Velocity sensor yielded velocity measurements at two locations close to the wall and the wall shear is obtained by fitting a Spalding curve through the data. The results obtained with this sensor were in good agreement with measurements obtained in the same tunnel using other techniques such as the oil film interferometry technique and with another MEMS-based optical shear stress sensor, the Diverging Fringe Doppler sensor. The results obtained with this sensor demonstrate that wall shear stress can be accurately estimated by measuring the velocity in the boundary layer at two points located within the first hundred $y^+$. Experiments are currently underway to demonstrate this two-point approach for accurate wall shear measurements for higher Reynolds number flows.

## Acknowledgements

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