An optimal continuous time control strategy for active suspensions with preview

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An Optimal Continuous Time Control Strategy for Active Suspensions with Preview

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SUMMARY

A continuous time control strategy for an active suspension with preview, based on optimal control theory, is presented. No approximation is needed to model the time delay between the excitation of the front and the rear wheels. The suspension is applied to a two DOF model of the rear side of the tractor of a tractor-semitrailer. The purpose of the suspension is to reduce either the required suspension working space or the maximum absolute acceleration of the sprung mass, without an increase of the dynamic tire force variation. For a step function as road input, reductions of 65% and 55%, respectively, are possible compared with a passive suspension.

1. INTRODUCTION

In the design of a passive suspension for road vehicles, increasing the comfort of the occupants and decreasing the required suspension working space are conflicting demands [10]. In case of a tractor-semitrailer, the maximum acceleration of the cargo and of the pitch motion of the cabin should be reduced, and the required suspension working space and the dynamic tire force variation should be minimized. Again, these wishes can not be fulfilled simultaneously. To improve the performance quantities nevertheless, semi-active and active suspensions have been developed [1, 10, 11].

If, in the control of the (semi-) active suspensions, the road surface is supposed to be "known" a certain time before it enters the wheels, *preview* is available. In this case, a significant improvement of the performance quantities is possible [3, 4, 5, 8]. In [4, 5], some sensors are used to measure the road surface in front of the vehicle.

A form of preview which seems suitable for use in practice is obtained if the excitation of the rear wheels of the vehicle is assumed to be a time-delayed version of the excitation of the front wheels [3, 5, 8, 12]. In this case, two strategies to incorporate preview in the control of the active suspension are known.

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The first is a *continuous time* strategy in which the time delay is approximated, for instance by a Padé-approximation [3, 5, 9]. An advantage of this approach is that finding the optimal control results in a conventional state-space formulation without time delays. Standard methods can then be used in the controller design. Disadvantages are the increase of the model order and the fact that the Padé-approximation is valid up to about 20 Hz. However, with today's computing power and recognizing that the vehicle models used are only valid up to 20 Hz (note that the frequency area of interest is 0.15 — 20 Hz), these disadvantages may not be an insuperable problem.

The second strategy is a *discrete time* control in which the time-delay is incorporated exactly [8].

In this paper, a *continuous time* control strategy with preview is derived for an active suspension for a tractor-semitrailer, in which the time-delay is incorporated *exactly*. The strategy is based on optimal control theory [7] and no increase of the model order occurs.

Because the relevant motions of the cabin are mainly caused by the road excitation of the rear wheels of the tractor, the idea is to replace the conventional passive suspension at this place by a (semi-) active suspension with preview. In practice, the available *preview time*, i.e. the time-delay between the excitation of the front and the rear wheels of the tractor, is at least 1/8 sec. The excitation of the rear wheels is a time-delayed version of the excitation of the front wheels and the road surface is assumed to be reconstructable from accelerations and relative displacements, measured at the front wheels of the tractor.

Just to show the feasibility of the control strategy, the active suspension is applied to a two DOF model which represents the rear side of the tractor. The suspension has to reduce the required suspension working space and the maximum acceleration of the sprung mass, without increase of the dynamic tire force variation. The road surface is considered as a deterministic signal. In this paper, simulations are only done for a step function as road input.

In Section 2, the model of the tractor-semitrailer and the derivation of the control strategy are presented. Results of the simulations are treated in Section 3. Finally, conclusions are drawn and the current investigation is described briefly in Section 4.

2. MODELLING AND DERIVATION OF CONTROL STRATEGY

The road surface at the rear wheels of the tractor is considered as a deterministic signal. It is supposed to be reconstructable from measurements of accelerations and relative displacements at the front wheels.

As usual [1, 10, 11], in this feasibility study the rear side of the tractor with (semi-) active suspension is modelled by a two DOF model. In the near future, the model will be elaborated to at least a half-vehicle model to include the reconstruction of the road surface at the front side of the tractor.

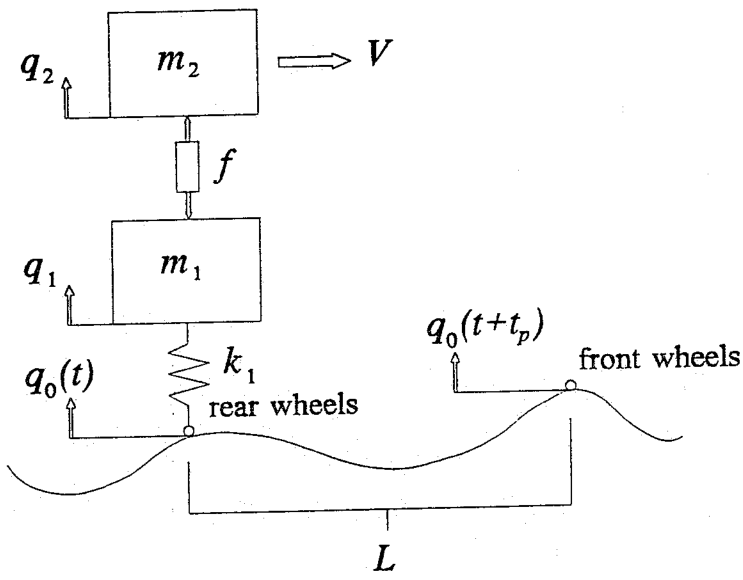

Fig. 1. Two DOF vehicle model with an active suspension with preview.

The model is shown in Fig. 1, where m_1 and m_2 represent the unsprung and sprung mass and k_1 is the tire stiffness. Furthermore, q_0, q_1, and q_2 are the vertical position of the road, of the unsprung mass, and of the sprung mass, respectively. The force, generated by the active suspension, is denoted by f. The preview time t_p is equal to L/V where L is the wheelbase and V is the vehicle speed. The equations of motion are given by

$$m_1\ddot{q}_1 = -f - k_1(q_1 - q_0); \quad m_2\ddot{q}_2 = f, \tag{1}$$

or, in state space notation, by

$$\dot{x}\,(\tau) = \boldsymbol{A}\boldsymbol{x}\,(\tau) + \boldsymbol{B}\boldsymbol{u}\,(\tau) + \boldsymbol{E}\boldsymbol{w}\,(\tau), \quad \boldsymbol{x}\,(t) = x_t, \tag{2}$$

where $\boldsymbol{x}$, $\boldsymbol{u}$, and $\boldsymbol{w}$ are defined by

$$x = \begin{bmatrix} q_1 \\ q_2 - q_1 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}; \quad \boldsymbol{u} = [f]; \quad \boldsymbol{w} = [q_0]. \tag{3}$$

The purpose of the active suspension is to reduce the required suspension working space and the maximum acceleration of the sprung mass, without increase of the dynamic tire force variation. For practical reasons, the input $\boldsymbol{u}$, i.e. the actuator force f, is limited. These demands can be translated in finding the input $\boldsymbol{u}$ which minimizes the quadratic performance index

$$J = \frac{1}{2} \int_{t}^{t+t_p} [y(\tau)^T Q y(\tau) + u(\tau)^T R u(\tau)] d\tau; \quad Q = Q^T, Q \geq 0; \quad R = R^T, R > 0, \tag{4}$$

where the output y is given by the output equation

$$y(\tau) = Cx(\tau) + Fw(\tau); \quad y = \begin{bmatrix} q_1 - q_0 \\ q_2 - q_1 \end{bmatrix}. \tag{5}$$

Since the acceleration $\ddot{q}_2$ of the sprung mass is linear in the actuator force f, it is not necessary to include both $\ddot{q}_2$ and f in J. When a passive spring supports the actuator, both $\ddot{q}_2$ and f can be included in J. The performance index J is defined over the time interval $[t, t + t_p]$ in which the road surface is supposed to be known.

The input u which minimizes the performance index J (see appendix A) is given by

$$u(\tau) = -K_1 x(\tau) + K_2 r(\tau); \quad K_1 = R^{-1} B^T P; \quad K_2 = R^{-1} B^T, \tag{6}$$

where P is the symmetrical, positive definite solution of the algebraic Riccati equation

$$A^T P + PA - PBR^{-1}B^T P + C^T QC = 0 \tag{7}$$

and r is the solution of the differential equation

$$\dot{r}(\tau) = -A_g^T r(\tau) + [PE + C^T QF] w(\tau), \quad r(t + t_p) = Px(t + t_p), \tag{8}$$

where $A_g = A - BK_1$. The state $x(t + t_p)$ at the end of the preview interval is unknown but can be calculated from Eqs. (2), (6), and (8) (see appendix B). To calculate u at time t, only the current state x_t and the solution of Eq. (8) at that time are needed.

From Eq. (6) it is seen that the input u is composed of a feedback term ($-K_1 x$) and a feedforward term ($-K_2 r$, which contains the road information). This is illustrated in Fig. 2. If no preview is available, only the feedback term remains and an often used optimal control strategy for active suspensions without preview (e.g. [13]) is obtained.

The control strategy with preview, as derived here, is a continuous time strategy and no Padé-approximation, to model the time-delay between the excitation of the front and the rear wheels, is necessary.

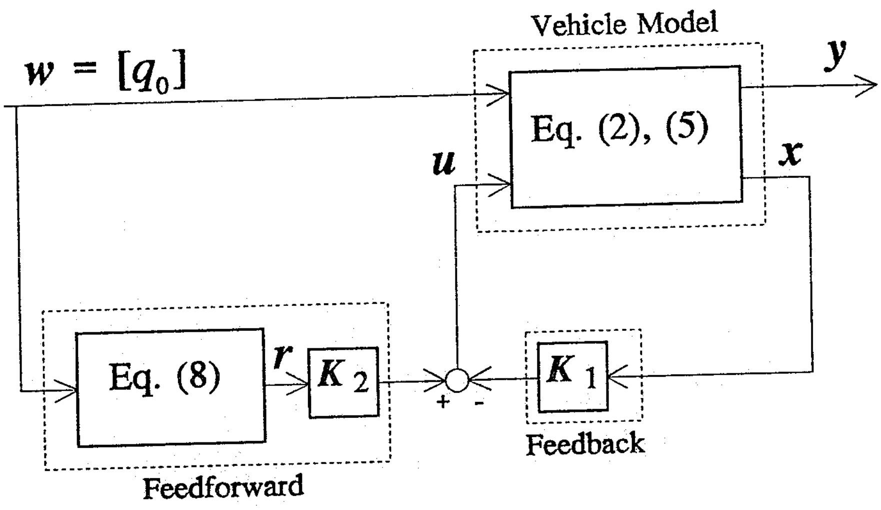

Fig. 2. Calculation of the input vector $\boldsymbol{u}$ as the sum of a feedback and a feedforward part.

3. RESULTS

The control strategy, derived in the previous Section, is applied to the model of Fig. 1.

The performance of the active suspension with preview is tested for a step function as road input and is compared with that of a passive suspension. The passive suspension, shown in Fig. 3, is a model of the suspension used in tractor-semitrailers. The height of the step (7.1 cm) is chosen in such a way that the passive suspension uses all the available suspension working space. The parameter values chosen for m_1, m_2, k_1, k_2, and b_2 are representative for the rear side of

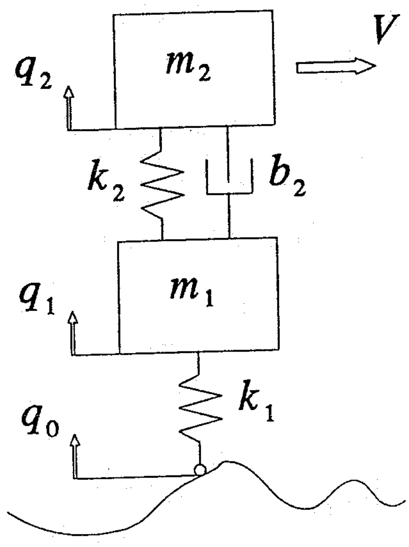

Fig. 3. Two DOF vehicle model with a passive suspension.

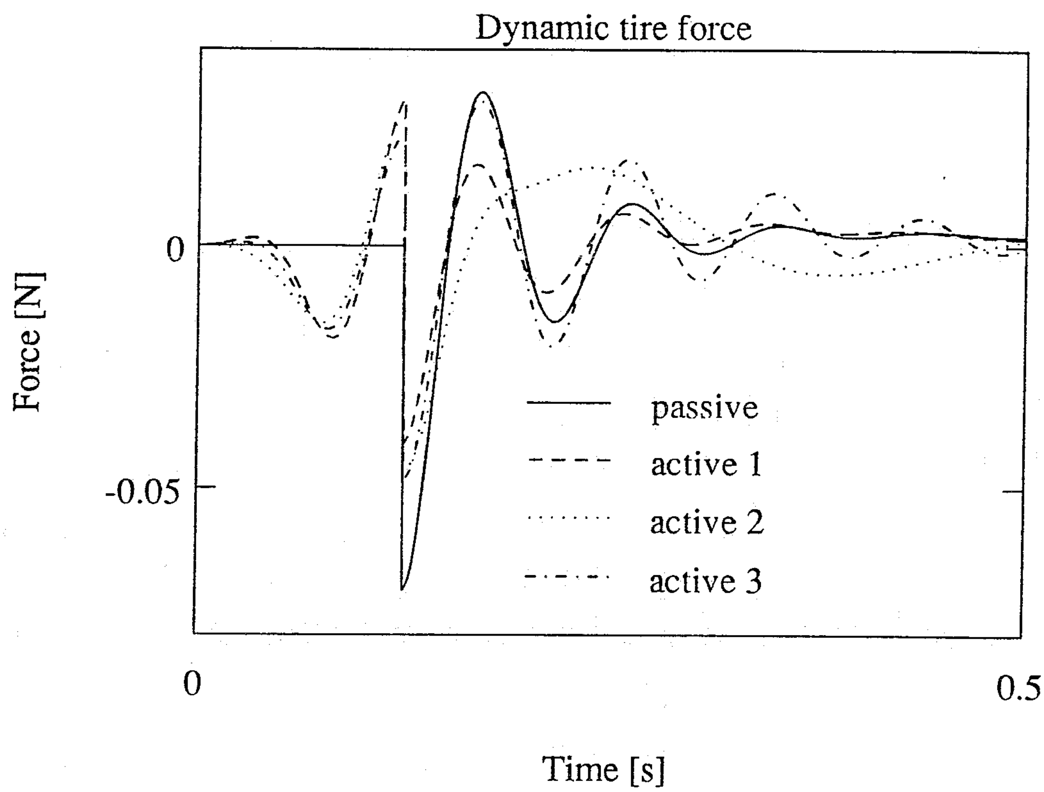

Fig. 4.

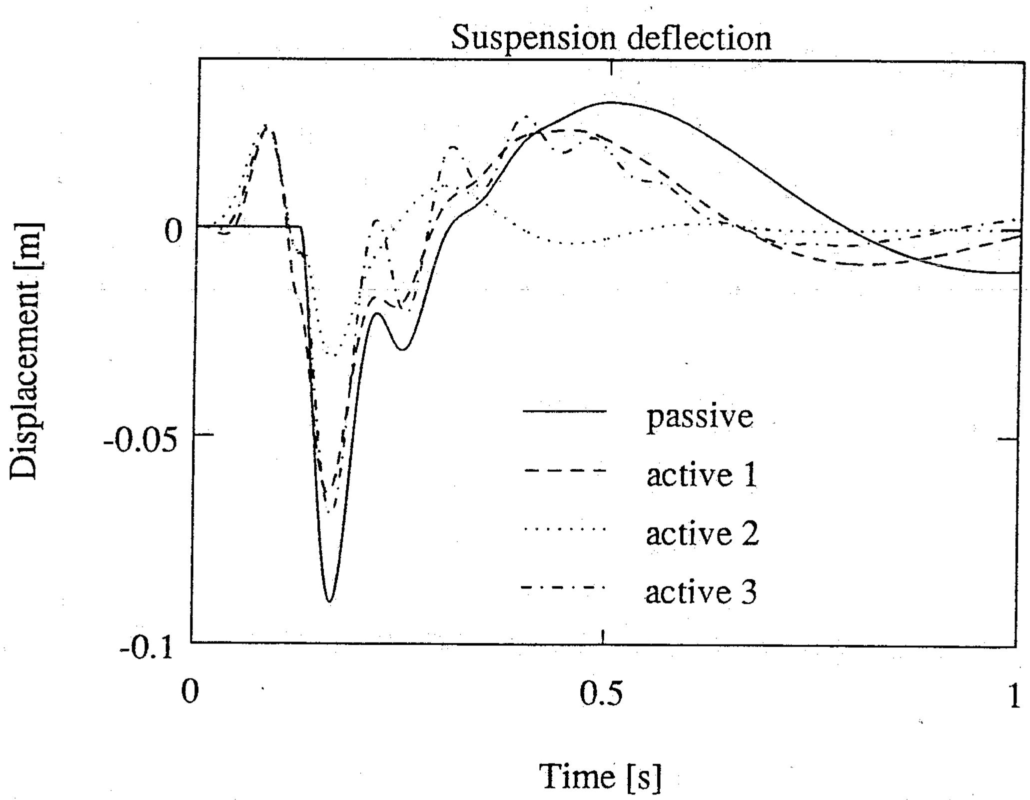

Fig. 4 and 5. Dynamic tire force and suspension deflection of the active suspension compared with that of a representative passive suspension for a step function as road input. The preview time is $t_p = 1/8$ sec. Active 1 = best overall performance, active 2 = required suspension working space minimized, and active 3 = maximum absolute acceleration of sprung mass minimized.

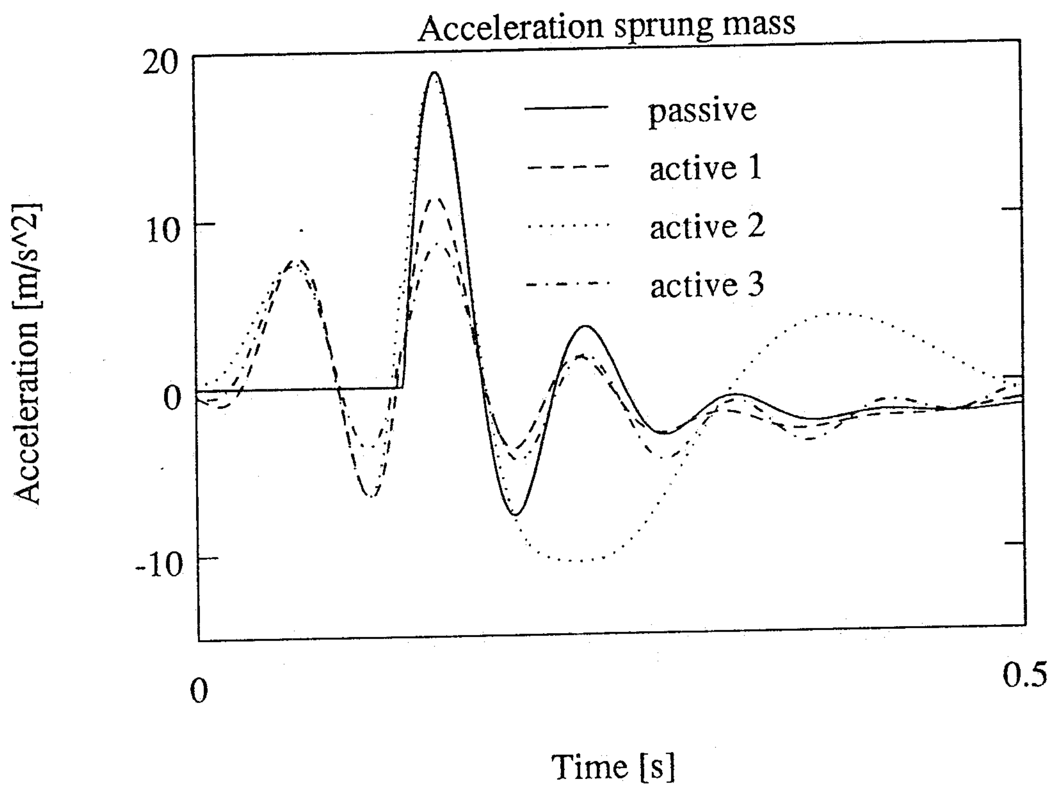

Fig. 6. Acceleration of the sprung mass of the active suspension compared with that of a representative passive suspension for a step function as road input. The preview time is $t_p = 1/8$ sec. Active 1 = best overall performance, active 2 = required suspension working space minimized, and active 3 = maximum absolute acceleration of sprung mass minimized.

the tractor of a current tractor-semitrailer combination. For a tractor wheelbase $L = 3.5$ m and a vehicle speed $V = 28$ m/sec ($\approx$ 100 km/h), the available preview time t_p is 1/8 sec. This preview time is used in the simulations.

Three combinations of the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are chosen. One combination is chosen in such a way that for the step function as road surface the required suspension working space is minimal. The second is chosen in such a way that the maximum acceleration of the sprung mass is minimal. A third combination is chosen in such a way that both the required suspension working space and the maximum acceleration of the sprung mass are minimized. In this case the "best overall performance" is achieved. Note that for the three combinations of $\boldsymbol{Q}$ and $\boldsymbol{R}$, all performance quantities (i.e., the required suspension working space, the maximum absolute acceleration of the sprung mass, and the dynamic tire load variation) are less than or equal to that of the passive suspension. Moreover, because a deterministic road surface is used, peak-values are minimized in stead of, for example, RMS-values.

The performance of the active suspension for the three combinations of $\boldsymbol{Q}$ and $\boldsymbol{R}$ is shown in Figs. 4 to 6. These figures show that either a 65% reduction of the required suspension working space or a 55% reduction of the maximum absolute acceleration of the sprung mass is possible without an increase of the maximum dynamic tire load variation. The active suspension with the best overall performance reduces the required suspension working space and the maximum absolute acceleration of the sprung mass with 30% and 40% respectively.

To investigate the influence of the preview time t_p , the step response of the active suspension with the best overall performance is determined for $t_p = 0$ sec and $t_p = 1/8$ sec, and for $t_p \rightarrow \infty$. The response of the acceleration of the sprung mass is shown in Fig. 7. From this figure it is seen that the performance for $t_p = 1/8$ sec is not as good as for infinite preview time. However, the performance improvement compared with the passive suspension is still significant. It is also clear that the performance of the active suspension without preview ($t_p = 0$ sec) is disappointing; the performance is hardly better than that of the passive suspension. Variation of the weighting matrix Q over a wide range does not improve the performance significantly.

The performance of the presented control strategy is not compared with the other strategies mentioned in Section 1. However, preliminary studies using a two DOF vehicle model show that there are no big differences in performance.

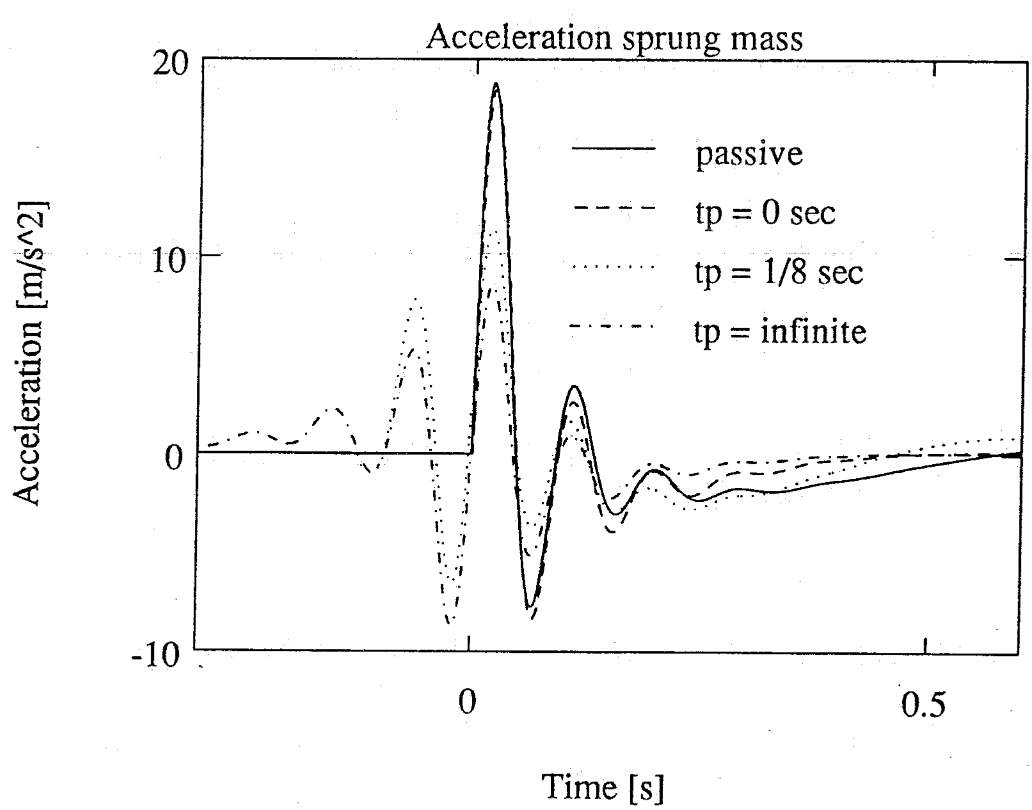

Fig. 7. Effect of the preview time on the performance of the active suspension.

A step function has been used to show the performance of the active suspension. In literature, however, often frequency responses are used to illustrate the performance of a suspension (e.g. [1, 6]). Moreover, it is also worthwhile to test the suspension for other deterministic road surfaces, like rounded pulses (e.g. [2]). The active suspension tested here has already been tested for these pulses. It appeared that the same performance improvement is possible compared with the passive suspension. However, for "low-frequent pulses", the performance improvement decreases because then a preview time $t_p = 1/8$ sec is not sufficient anymore. In the near future, especially frequency responses will be investigated.

Some remarks have to be made on the realizability of the active suspension presented here. The limited bandwidth of the actuator (e.g., a hydraulic cylinder) has not been taken into account. The road surface is assumed to be perfectly reconstructable and full state feedback is used. In practice, both the road surface and the state have to be determined from a limited number of measurements. The robustness of the control strategy to modelling errors and parameter variations has not been investigated yet. To calculate the actuator force at time t, the computational effort is high compared with a control strategy in which the time delay is modelled by a Padé-approximation. Finally, the power consumption of the actuator could be too high. However, in practice, the actuator will be supported by a spring and a damper and will be used only incidentally, e.g. in case of a railway crossing or a pot-hole.

Despite these remarks and the fact that only a two DOF vehicle model is used, the improvement of the performance compared with the passive suspension is significantly enough to make further investigation useful.

4. CONCLUSIONS

A continuous time control strategy for an active suspension with preview is presented. In this strategy, the time-delay between the excitation of the front and the rear wheels can be taken into account exactly.

The active suspension with preview is applied to a two DOF model of the rear side of a tractor-semitrailer. In case of a step function as road input and a preview time $t_p = 1/8$ sec, either a 65% reduction of the required suspension working space or a 55% reduction of the maximum absolute acceleration of the sprung mass is possible, compared with a representative passive suspension, without increase of the dynamic tire force variation.

The minimum available preview time for a tractor-semitrailer is 1/8 sec. For the step function as road input, the performance of the active suspension with $t_p = 1/8$ sec is not as good as for infinite preview time but still significantly better than that of the passive suspension. The performance of the active suspension without preview is disappointing compared with that of the suspension with preview.

Further research on this topic is being carried out and includes (1) the reconstruction of the road surface from a limited number of measurements (relative displacements and accelerations) at the front wheels of the truck and (2) the derivation of a control strategy for a semi-active suspension with preview. The robustness of the control strategy to modelling errors and parameter variations, the dynamic behaviour and the power consumption of the actuator, the computational effort, and the frequency response of the active suspension presented here will be investigated too.

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APPENDIX A

The input $\boldsymbol{u}$ has to minimize the performance index

$$J = \frac{1}{2} \int_{t}^{t+t_p} [y(\tau)^T Q y(\tau) + u(\tau)^T R u(\tau)] d\tau, \tag{A.1}$$

under the constraints (more general than those in Section 2)

$$\dot{x}(\tau) = Ax(\tau) + Bu(\tau) + Ev(\tau), \quad x(t) = x_t, \tag{A.2}$$

$$y(\tau) = Cx(\tau) + Du(\tau) + Fw(\tau), \tag{A.3}$$

In the minimization, the Lagrange multiplier λ is used to incorporate Eq. (A.2). Now, minimization of J results in the following equations:

$$u(\tau) = R_d^{-1} B^T \lambda(\tau) - R_d^{-1} D^T QCx(\tau) - R_d^{-1} D^T QFw(\tau). \tag{A.4}$$

$$\dot{\lambda}(\tau) = -A_d^T \lambda(\tau) + C_d x(\tau) + f_\lambda(\tau), \quad \lambda(t+t_p) = 0, \tag{A.5}$$

$$\dot{x}(\tau) = A_d x(\tau) + B_d \lambda(\tau) + f_x(\tau), \quad x(t) = x_t, \tag{A.6}$$

where A_d, B_d, C_d, f_λ, and f_x are defined by

$$\begin{aligned} R_d &= R + D^T QD, \\ A_d &= A - BR_d^{-1} D^T QC, \\ B_d &= BR_d^{-1} B^T, \\ Q_d &= Q - QDR_d^{-1} D^T Q, \\ C_d &= C^T Q_d C, \\ f_\lambda &= C^T Q_d Fw, \\ f_x &= Ev - BR_d^{-1} D^T QFw. \end{aligned} \tag{A.7}$$

To solve Eqs. (A.5) and (A.6), a new vector is introduced, defined by

$$r(\tau) = \lambda(\tau) + Px(\tau). \tag{A.8}$$

If P is chosen as the symmetric positive definite solution of the algebraic Riccati equation

$$A_d^T P + PA_d - PB_d P + C_d = 0, \tag{A.9}$$

then Eqs. (A.5) and (A.6) can be written as

$$\dot{r}(\tau) = -A_g^T r(\tau) + Pf_x(\tau) + f_\lambda(\tau), \quad r(t+t_p) = Px(t+t_p), \tag{A.10}$$

$$\dot{\boldsymbol{x}}(\tau) = \boldsymbol{A}_g \boldsymbol{x}(\tau) + \boldsymbol{B}_d \boldsymbol{r}(\tau) + \boldsymbol{f}_x(\tau), \quad \boldsymbol{x}(t) = \boldsymbol{x}_t, \tag{A.11}$$

where $\boldsymbol{A}_g = \boldsymbol{A}_d - \boldsymbol{B}_d \boldsymbol{P}$. Eq. (A.4) becomes

$$\boldsymbol{u}(\tau) = -\boldsymbol{R}_d^{-1} [\boldsymbol{B}^T \boldsymbol{P} + \boldsymbol{D}^T \boldsymbol{QC}]\, \boldsymbol{x}(\tau) + \boldsymbol{R}_d^{-1} \boldsymbol{B}^T \boldsymbol{r}(\tau) - \boldsymbol{R}_d^{-1} \boldsymbol{D}^T \boldsymbol{QFw}(\tau). \tag{A.12}$$

To calculate the actuator force at time t, the current state $\boldsymbol{x}_t$ and the solution $\boldsymbol{r}(t)$ of Eq. (A.10) is needed. To solve $\boldsymbol{r}(t)$, the state $\boldsymbol{x}$ at time $t + t_p$ is needed. However, this state is not known a priori. This problem is dealt with in Appendix B.

APPENDIX B

From Eq. (A.11) it is seen that

$$x(t + t_p) = \Phi(t_p)\, x_t + \int_t^{t+t_p} [\Phi(t + t_p - \tau)\, [\boldsymbol{B}_d \boldsymbol{r}(\tau) + \boldsymbol{f}_x(\tau)]] d\tau, \tag{B.1}$$

were Φ is defined by

$$\Phi(\tau) = e^{A_g \tau}. \tag{B.2}$$

The solution of Eq. (A.10) is

$$\boldsymbol{r}(\tau) = \Phi^T (t + t_p - \tau)\, \boldsymbol{Px}(t + t_p) + \boldsymbol{r}_0(\tau), \tag{B.3}$$

where $\boldsymbol{r}_0$ is defined as the solution of the differential equation

$$\dot{\boldsymbol{r}}_0(\tau) = -\boldsymbol{A}_g^T \boldsymbol{r}_0(\tau) + \boldsymbol{Pf}_x(\tau) + \boldsymbol{f}_\lambda(\tau), \quad \boldsymbol{r}_0(t + t_p) = \boldsymbol{0}. \tag{B.4}$$

With Eqs. (B.1) and (B.3), $\boldsymbol{x}(t + t_p)$ is written as

$$\boldsymbol{x}(t + t_p) = \Phi(t_p)\, \boldsymbol{x}_t + \boldsymbol{GPx}(t + t_p) + \boldsymbol{q}_0(t + t_p), \tag{B.5}$$

in which $\boldsymbol{G}$ and $\boldsymbol{q}_0(t + t_p)$ are defined by

$$\boldsymbol{G} = \int_t^{t+t_p} [\Phi(t + t_p - \tau)\, \boldsymbol{B}_d \Phi^T (t + t_p - \tau)\, d\tau, \tag{B.6}$$

$$\dot{\boldsymbol{q}}_0(\tau) = \boldsymbol{A}_g \boldsymbol{q}_0(\tau) + \boldsymbol{B}_d \boldsymbol{r}_0(\tau) + \boldsymbol{f}_x(\tau), \quad \boldsymbol{q}_0(t) = \boldsymbol{0}. \tag{B.7}$$

It can be shown that matrix $\boldsymbol{G}$ satisfies

$$\boldsymbol{G} = \boldsymbol{Z} - \Phi(t_p)\, \boldsymbol{Z} \Phi^T (t_p), \tag{B.8}$$

where $\boldsymbol{Z}$ is the solution of the Lyapunov equation

$$A_g Z + Z A_g^T = -B_d; \quad Z = Z^T \tag{B.9}$$

With Eq. (B.5), an explicit expression of the boundary value $\boldsymbol{x}(t + t_p)$ is calculated:

$$x(t + t_p) = (I - GP)^{-1} [\Phi(t_p) x_t + q_0(t + t_p)] \tag{B.10}$$