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AN OPTIMAL SYSTEM DESIGN PROCESS
FOR A MARS ROVING VEHICLE
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Analysis and Design of a Capsule Landing System and Surface Vehicle Control System for Mars Exploration

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The problem of determining the optimal design for a Mars roving vehicle is considered.

A system model is generated by consideration of the physical constraints on the design parameters and the requirement that the system be deliverable to the Mars surface.

An expression which evaluates system performance relative to mission goals as a function of the design parameters only is developed.

The use of nonlinear programming techniques to optimize the design is proposed and an example considering only two of the vehicle subsystems is formulated and solved.

Recommendations for future work are presented.

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Special Note: In all cases, $1 \mathrm{~kg}=1 \mathrm{kgwt}$ on earth $=9.806(\mathrm{~kg}$-mass $) \mathrm{m} / \mathrm{sec}^{2}$ System Design Optimization

| NLP | nonlinear programming |
| :---: | :---: |
| $\mathrm{x}_{\mathrm{n}}$ | n -vector of design parameters |
| $f\left(\mathrm{X}_{-\mathrm{n}}\right)$ | system objective function, a scalar function |
| $\mathrm{g}_{\mathrm{i}}(\mathrm{x} \rightarrow$ ) | system inequality constraints; $i=1,2, \ldots, m ;$ all scalar functions |
| $h_{j}\left(x_{n}\right)$ | system equality constraints, the equations of the system model; j=1,2,..,k; all scalar functions |
| $k^{\prime}$ | number of $h_{j}$ functions which are transcendental |
| SUMT | sequential unconstrained minimization technique |

## Communications Subsystem

$\mathrm{R}_{\text {com }}$ data transmission rate, bits/sec
$\mathrm{P}_{\mathrm{t}} \quad$ r.f. power output, watts
$D_{\text {com }} \quad$ rover antenna diameter, meters
$\Delta \theta^{\circ} \quad$ rover antenna pointing error, degrees
$\lambda$ carrier wavelength, $m$
$W_{c} \quad$ earth weight of electronics section, kg
$Q_{c} \quad$ heat dissipation, watts
$\mathrm{L} \quad$ link distance, m
$T_{n}$ noise equivalent temperature, ${ }^{0} K$
$D_{r} \quad$ receiver antenna diameter, $m$
$B_{0} / B \quad$ communication efficiency
$W_{\text {ant }}$ earth weight of antenna and associated steering motors and supports, kg
$P_{r}$ received signal power at earth, watts
$G_{t} \quad$ transmitting antenna gain
$\mathrm{L}_{\mathrm{p}} \quad$. space loss attenuation


| ${ }^{\mathrm{t}} \mathrm{a}$ | time to accelerate to $\mathrm{v}_{\mathrm{f}}$ from stop, sec |
| :---: | :---: |
| ${ }^{P}$ a | power used for acceleration, watts |
| $\mathrm{P}_{\mathrm{v}}$ | power used to maintain constant velocity on a zero slope, watts |
| $F$ | locomotion force, newtons |
| $\mu_{k}$ | coefficient of kinetic friction of vehicle |
| $\mathrm{g}_{\mathrm{m}}$ | acceleration of gravity on Mars, m/sec ${ }^{2}$ |
| $\mathrm{P}_{\text {sl }}$ | power used to rove on a slope, watts |
| $\Psi$ | angle of inclination of slope, degrees |
| $E_{\text {batt }}$ | energy of batteries, watt-hr |
| $\mathrm{W}_{\mathrm{p}}$ | earth weight of power subsystem, kg |

## Thermal Control Subsystem

$a_{1}$ proportionality constant for equipment package length
$a_{2} \quad$ proportionality constant for equipment package width
$a_{12}$ proportionality constant for equipment package surface area
$Q_{h} \quad$ maximum heater output, watts
$A_{r}$
radiator area, $\mathrm{m}^{2}$.
$L_{i}$, insulation thickness, $m$
$\mathrm{K}_{\mathrm{q}}$
$W_{\theta}$
$T_{r n} \quad$ temperature of radiator at night, ${ }^{\circ} \mathrm{K}$
$\mathrm{T}_{\mathrm{rd}}$ temperature of radiator during day, ${ }^{\circ} \mathrm{K}$
A .total equipment package surface area, $\mathrm{m}^{2}$
Qid daytime internal heat dissipation, watts
$Q_{i n} \quad$ nighttime intemal heat dissipation, watts
a. radiometric albedo of Mars

| $\mathrm{A}_{\text {sr }}$ | area of body surface radiating heat, $\mathrm{m}^{2}$ |
| :---: | :---: |
| Q ${ }_{\text {sol }}$ | incident solar energy, watt-hr |
| Qrad | radiated heat, watts |
| $Q_{\text {conv }}$ | convective heat loss, watts |
| $Q_{\text {cond }}$ | conductive heat loss, watts |
| $\mathrm{k}_{\mathrm{i}}$ | insulation conductivity, watts $/ \mathrm{m}-{ }^{\circ} \mathrm{K}$ |
| $\epsilon_{\mathrm{s}}, \epsilon_{\mathrm{r}}$ | emissivity of equipment package surface and radiator |
| $\alpha_{s}, \alpha_{r}$ | absorptivity of equipment package surface and radiator |
| $\mathrm{S}_{\mathrm{c}}$ | solar constant |
| $\mathrm{h}_{\mathrm{c}}$ | average convective transfer coefficient |
| $\sigma$ | Stephan-Boltzman constant |
| $A_{\operatorname{sun}}(\mathrm{s})$ | area of equipment package on which sunlight is incident, $\mathrm{m}^{2}$ |
| Aalb (s) | area of equipment package on which albedo radiation is incident, $\mathrm{m}^{2}$ |

## Navigation Subsystem

$\Delta \beta$ error in detection of local vertical, degrees
$W_{\text {nav }}$ earth weight of navigation subsystem, kg
$P_{\text {nav }} \quad$ average power required by navigation subsystem, watts .

## Obstacle Avoidance Subsystem

Woa earth weight of obstacle avoidance subsystem, kg
$P_{\text {oa }}$ average power required by subsystem, watts
$\Delta h_{t} \quad$ error in detecting the height of a terrain point, $m$
$r_{a}$. horizontal distance from laser to sensed terrain point, $m$
$\epsilon_{\text {sl }}$ worst case error in slope detection, degrees
$\delta$
horizontal separation between terrain points used in slope calculation, m
s slope of terrain, degrees
Tact . percent terrain impassable for vehicle
0 direction of travel ( }0=0\mathrm{ is desired direction)
p(0) probability of traveling in 0 direction
r step size for path-length ratio simulation
PLR. path-length ratio
RNG
random number generator (uniformly distributed on [0,1] )

```

Computation and Data-handling Subsystem
TOPS thermoelectric outer planet spacecraft
\(P_{c p} \quad\) power required, watts
\(W_{c p}\) earth weight of subsystem, kg
\(v_{c p} \quad\) volume of subsystem, in \({ }^{3}\)

Vehicle Structure Subsystem
\begin{tabular}{|c|c|}
\hline \(\mathrm{w}_{\mathrm{b}}\) & wheelbase of vehicle, m \\
\hline t & Track or width of vehicle, m \\
\hline \(\mathrm{W}_{\mathrm{v}}\) & earth weight of vehicle frame, suspension and motors, kg \\
\hline \[
V_{v}
\] & maximum equipment package volume capability of vehicle, \(\mathrm{m}^{3}\) \\
\hline Other & in \\
\hline \(L_{\text {W }}\) & maximum payload weight of launch vehicle, kg \\
\hline \(\mathrm{A}_{\mathrm{v}}\) & volume of aeroshell, m \({ }^{3}\) \\
\hline \(A_{\text {d }}\) & horizontal diameter of aeroshell, m \\
\hline \(\mathrm{D}_{\text {rov }}\) & straight-line distance traveled, m \\
\hline V & time in one Mars day during which Earth-vehicle communication is possible, hr \\
\hline \[
S_{\text {sci }}
\] & number of science steps per meter of actual distance traveled \\
\hline Cycle & Time between the ends of two consecutive recharge periods \\
\hline \[
\mathrm{N}_{\mathrm{cy}}
\] & number of cycles in one Mars day \\
\hline \(\mathrm{T}_{\text {cy }}\) & time for one cycle, hr \\
\hline \[
\mathrm{D}_{\text {rov }} / \mathrm{C} \cdot \mathrm{y}
\] & \(\mathrm{D}_{\text {rov }}\) in one cycle, m \\
\hline \[
\mathrm{T}_{\mathrm{Sci}} / \mathrm{cy}
\] & \(T_{\text {sci }}\) in one cycle, sec \\
\hline
\end{tabular}

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I. . INTRODUCTION

Design, in the systems: sense, is the process of specifying the information required by the subsystem designers. This information consists of the operating requirements to be met by the subsystem, and all constraints under which the designer must work. For the designer of a communications subsystem for example, such inputs might be that a pulsecode modulated subsystem capable of ' \(x\) ' data rate, not to exceed ' \(y\) ' weight, and drawing ' \(z\) ' watts maximum power is needed.

Systems analysis is the task of determining an accurate system model. Required by this definition is the examination of all design trade-offs in the context of their effect upon the operation of the system as a whole. For a system of non-trivial size, the system design is composed of many parameters and constraints, the interrelationships between the parameters may be complex, and it is necessary to consider all of the parameters and constraints concurrently.

The task of optimization requires that the manner in which the design parameters react is known. It implies the use of a mathematical model of the system. In most applications, the equations of the system model are the result of previous work, usually done by the subsystem designers. Again using the communications example, the system model can include an equation relating communications power input and subsystem weight. This relationship is simply a linear approximation to a curve formed by points corresponding to other communication subsystem designs for already existing units with similar application. Confidence in the model equation is therefore based upon the assumption that it should be possible to design a communications subsystem whose power requirement and weight relate (at least approximately) as the equation predicts.

This example illustrates that system design is really a 'closedloop' process. Information obtained at the subsystem level of design is required to obtain a system model, which will be used eventually to specify parameters that are inputs to the subsystem design procedure. In addition, modifications or innovations in technology which occur on the subsystem level (e.g., a new:material makes it possible to reduce weight) must be used to update the system model. It is important to recognize and utilize this interplay between the two levels, for to inaccurately constrain the design in the modeling stage will most often result in a non-optimal solution.

It is infeasible to expect to be able to force the model to include all possible design variations. Radically different approaches to a design problem will in general have significantly different effects on how the design parameters relate. It becomes necessary then, to make certain assumptions about the system and subsystem configurations. This in turn means that optimization for a single model is not the end product of systems analysis simply because thereare probably other design alternatives not included in that model. To claim that a system design is indeed optimal, it is necessary to first consider the models corresponding to the set of all possible assumptions.

The search for the optimum also implies that there is a standard by which the system quality can be measured. This 'objective': (or objective function) may or may not be unique. Generally, the objective measures how well the system is fulfilling its purpose. If there are alternate: ways of describing how well the system performs, these too are inputs to the optimization process and must be separately considered.

In addition there are assumptions that must be made about external constraints (funding, development of new technology, time schedules,....) which may affect the design and may not be completely deterministic.

The many possible combinations of design assumptions, objectives, and external constraints make system optimization an exhaustive process in the sense that the solution must be obtained for many sets of inputs before confidence in the validity of the optimum is achieved. Schematically, the inputs to a single run of the optimization process can be represented by Figure 1, where now, for a roving vehicle, mission goals are the determining factors in formulating the system objective. The question now is -- how does one go about determining the optimal system design?


FIGURE 1. INPUTS TO THE OPTIMIZATION PROCESS

\section*{II. THE SYSTEM DESIGN OPTIMIZATION PROBLEM}

System design is accomplished by collecting all constraints and attempting to sort out a feasible set of design parameters while keeping in mind the objective of the system. Individual subsystem designers are constrained by the requirements of other subsystem designers. The pointing error of the communications antenna will be affected by its power and weight allocations, which must eventually depend upon how much weight and power are allocated to the other subsystems. Decreasing either antenna pointing power or the weight allocated to the pointing apparatus will probably have the effect of increasing power and/or weight required by the electronics section of the communications package or decreasing the performance level. The problem, then, is to specify a set of design parameters (in this case, power and weight allocations and performance levels) that will best achieve the objective of the system.

There will generally be an infinite number of sets of parameters that will constitute a feasible and acceptable design (i.e., one that will operate to some measure of satisfaction). The system designer is faced with the problem of choosing one of these sets. He obviously wishes to choose that set which will maximize the effectiveness of the system while meeting other constraints he faces such as cost and time limitations.

When the system is complex (which may be a result of having many design parameters to choose and/or complex interrelationships between the parameters), the job of making this choice can be more difficult than the system modeling. Traditionally, the method has been to choose some design parameters to satisfy a system objective to some degree and then to use the model to fix the others. If this set is unacceptable, the designer must change some or all of his original choices and resolve until he is 'satisfied' with the system design. Unfortunately, the nagging question of whether there is a better solution remains. This drawback is inherent
in the method because of its lack of rigor.
If it is possible to describe the effectiveness of the system as a function of the design variables, the optimal solution can most often be identified.

The nonlinear programming (NLP) problem is:
\[
\begin{array}{ll}
\text { extremize }(\max \text { or } \min ): & f\left(x_{n}\right) \\
\text { subject to: } & g_{i}\left(x_{n}\right) \geqslant 0 \quad \\
& h_{j}\left(x_{n}\right)=1,2, \ldots, m
\end{array}
\]
where \(\mathrm{x}_{\mathrm{n}}\) is an n -vector of variables to be chosen by the optimization. The \(f, g_{i}\) 's and \(h_{j}\) 's are all scalar functions (possibly nonlinear) of the components of \(x_{n}\). Unless the solution is unique (implying that there is only one way the system can be designed) or does not exist, it is necessary that the number of equality constraints be less than the number of variables (i.e., \(k<n\) ).

The NLP problem is a natural way to describe the problem of optimal system design. Since the system design is formulated as the determination of \(n\) design parameters, \(f\left(x_{n}\right)\) becomes the objective function previously discussed. The \(g_{i}\) and \(h_{j}\) functions represent the physical and external constraints placed upon the choice of the \(n\) variables. The major advantage of such an approach is that it allows all feasible designs to be identified and considered.

Thus, for a given set of assumptions, the optimization process will consist of three parts:
1) formulation of a mathematical model of the system (identification of constraints),
2) determination of the objective function in terms of the model variables, and
3) imbedding the problem in the nonlinear programming format and locating the optimum.

The modeling process is the determination of what the \(n\) variables
to be evaluated are, and what relationships there must be between them in order for the system design to be physically realizable and to meet any other constraints placed upon it. The model may consist solely of "engineering variables" (weights, powers, data transmission rates, velocities, ...) or it may include "managerial variables" (cost of components, man-hours,...).

Suppose the results of modeling the system yield: in variables, \(m\) inequality constraints, and \(k\) equality relationships. It is then possible (assuming that none of the equalities is transcendental) to use the \(k\) equalities to eliminate \(k\) of the \(n\) variables both. in \(f\left(x_{n}\right)\) and in all the \(g_{i}\left(\ddot{x}_{n}\right), i=1,2, \ldots, m\), yielding a transformed objective \(f_{1}\left(x_{n-k}\right), m\) inequality constraints of \(n-k\) variables and no equality constraints. If \(k^{\prime}\) of the \(k\) equalities are transcendental (not algebraically solvable for any. of the variables), the number of variables can be reduced to \(n-k+k^{\prime}\), and there remain \(k^{\prime}\). equality constraints. In either case, call the reduced set of variables states. Then the order of the system design problem (number of states) is \(n-k+k^{\prime}\). The order represents the number of independent design decisions that must be made by the optimization process, and gives some idea as to the complexity of the problem for a particular system. Note that the set of states is not unique, because the \(k-k^{\prime}\) variables that can be eliminated is likewise not unique.

It is not necessary to eliminate all, or for that matter any, of the \(k-k\), variables. While reducing the order of the problem would seem to simplify the optimization procedure, this is not always the case. In the NLP solution, the partial derivatives of all the scalar functions must be taken with respect to each of the uneliminated variables (Ref. 1). If the form of some of the \(h_{j}\) 's is not sufficiently simple, substitution using these equalities may serve to complicate the situation. (For an example of such a case, see the section on modeling the thermal control subsystem.) Thus, it should remain the designer's option to utilize the substitution for each of the equalities.

Figure 2 shows the modeling and optimization process for a set of assumptions. The "optimum" value is in quotes because it is optimal only with respect to the validity of the input assumptions. The iteration is with respect to changes in these inputs. For a collection of examples of optimal system design by NLP, the reader is referred to Reference 2.


FIGURE 2. MODELING \& OPTIMIZATION PROCESS
A. SUBSYSTEM MODELING
1. COMMUNICATION SUBSYSTEM

The Earth/Mars communication subsystem is modeled as a direct two way link in the microwave spectrum between a Mars Roving Vehicle and an Earth communication station. A number of such models, with appropriate fixed parameters, would be required to describe all of the possible relay configurations which might be used.

The communication link is divided into an uplink to Mars and a downlink back to Earth. Uplink parameters associated with the rover are found to be negligible in comparison to similar downlink parameters, and were thus not considered directly.

The downlink is composed of the spacecraft transmitter, a high gain parabolic dish antenna, a standby low gain omnidirectional antenna, a free space propagation path, a high gain parabolic dish receiving antenna, and an ultra low noise receiver, as shown in Figure 3.
- The first step in the modeling task is to describe the subsystem mathematically in terms of link farameters. The list of parameters chosen to model the link is given in Table 1 . The parameters can be divided into two classes: those which are fixed by nature, state of the art, or constraints; and those which are design dependent, and therefore a function of the design decisions made (e.g. link distance is fixed by nature, transmitter efficiency is fixed by the state of the art; however, data rate is free to vary over. some range, as a function of the design chosen to implement the link).

Before proceeding further, it is necessary to make assumptions to specify the fixed parameters and constrain the model sufficiently to allow analysis:
1. The carrier is \(X\)-band microwaves of wavelength \(3.3 \times 10^{-2}\) meters, which have been shown to be especially well suited for high speed communications at Mars distances. (Ref. 3)


FIGURE 3.
COMMUNICATION SUBSYSTEM:
DOWNLINK FUNCTIONAL DIAGRAM


TABLE 1. COMMUNICATION SUBSYSTEM: DOWNLINK PARAMETER LIST
2. The ground station antenna is a 64 meter parabolic dish. (Ref. 5)
3. The rover antenna is a parabolic dish with a pointing error of \(1^{\circ}\left(\Delta \theta=1^{\circ}\right)\).
4. Uplink parameters are negitigible:
5. The overall r.f. efficiency of the transmitter is \(20 \%\). This. figure is obtained from a \(25 \%\) TWT efficiency and a very low exciter efficiency. (ref. 4)
6. The worst case link distance of \(5.7 \times 10^{11}\) meters is used.
7. Total equivalent noise temperature for the receiving system on Earth is the sum of the galactic and receiver noise temperatures, and was assumed to be \(30^{\circ} \mathrm{K}\). (ref. 6)
8. The communication efficiency, a measure of the ability of a given modulation scheme to overcome additive channel noise, is \(5 \%\). This corresponds to a \(20: 1\) signal to noise ratio in a typical PCM system. (Ref. 7)

The above assumptions specify many of the entries in the list of parameters. To further reduce the number of unspecified parameters, equations relating the various parameters can be found.
1. Conservation of energy allows two equations to be written:
\[
\begin{aligned}
& P_{t}=e P_{i} \\
& Q_{c}=(1-e) P_{i} .
\end{aligned}
\]
2. Electronics weight is obtained as a function of power input alone from data associated with various prediction efforts in Mars communication, as shown in Figure 4. (Ref. 8)
\[
W_{c}=0.59 \mathrm{~kg} / \text { watt } \mathrm{P}_{\mathrm{i}}+34.0 \mathrm{~kg}
\]
is found to approximate the functionality for \(P_{i}\) expressed in watts and \(W_{c}\) expressed in kg .

3. Similarly, volume can be found to be related to power input in a like manner (Ref. 8). The functionality is found to be approximately linear.
\[
\mathrm{V}_{\mathrm{c}}=8.3 \times 10^{-4} \mathrm{P}_{\mathrm{i}}+4.8 \times 10^{-2} \text { meter }{ }^{3}
\]
4. The weight of the antenna and its associated steering motors can be approximated as a function of antenna diameter, \(\mathrm{D}_{\text {comin }}\), in meters:
\(\therefore \mathrm{W}_{\mathrm{ant}}=2.0 \mathrm{D}_{\mathrm{com}}^{2}+5.0 \quad \mathrm{~kg} .(\operatorname{Ref} .3,9,39)\)
At this point, note that there remain only three of the original parameters in Table 1 which have not been either specified by assumption or related to another specified parameter by the simplifying equations identified above: \(R, P_{i}\) and \(D\). In otheriwords, a knowledge of these three parameters alone will., in the light of the basic assumptions listed, completely specify all of the parameters identified at the begining of the modeling task as being necessary to uniquely describe the entire subsystem, Given these three parameters, a subsystem could be built. However, not every subsystem would satisfy the requirements which this subsystem is being asked to satisfy. In other words, not any random choice of these parameters will produce a satisfactory subsystem. There must exist another equation which will provide a relationship which the defining parameters must satisfy. The equation sought is the classic range equation for a noisy channel.

For a "successful" subsystem, the signal power received on Earth must be sufficiently large to overcome the noise. The received power is given by
\[
\because P_{r}=P_{t} G_{t} L_{p} G_{r},
\]
where
\(P_{r}\) is received signal power,
\(P_{t}\) is transmitted signal power,
\(G_{t}\) is transmitting antenna gain,
\(L_{p}\) is the space loss attenuation,
and
\(G_{r}\) is the receiving antenna gain.
Substituting known parameters, for \(P_{r}\) and \(P_{i}\) in watts, \(D_{\text {com }}\) in meters, the received power is found to be
\[
P_{r}=5 \times 10^{-19} D_{c o m}^{2} P_{i}
\]

For the signal to overcome the noise, the following relation must be satisfied for a PCM subsystem. (Ref. 7)
\[
P_{r} \geq 10^{-23}\left(B / B_{o}\right) \cdot T_{n} \cdot R_{c o m},
\]
where
\(B / B_{O}\) is the inverse of the communication efficiency,
\(T_{n}\) is the system noise equivalent temperature, \({ }^{o_{K}}\), and
\[
R_{\text {com }} \text { is the data rate in bits/sec. }
\]

Substituting known parameters and combining the above two relations yields the desired state variable relationship,
\[
\mathrm{R}_{\mathrm{com}} \leq 42 \mathrm{D}_{\mathrm{com}}^{2} \mathrm{P}_{\mathrm{i}}
\]

Only choices of the three variables satisfying the above relationship will specify subsystems capable of communicating successfully with Earth. Because it will obviously be advantageous to have the upper limit of the equality satisfied, the equation becomes:
\[
R_{\text {com }}=42.0 \mathrm{D}_{\mathrm{com}}{ }^{2} \mathrm{P}_{\mathrm{i}}
\]

In summary, the communication subsystem can be modeled on the basis of only two chosen parameters, as the third is determined by the range equation. If any of the assumptions made at the beginning of the analysis were to be relaxed, then additional variables would be included to uniquely specify the subsystem.

For the present model, if the total weight of the communications subsystem (this includes antenna and electronics section) is denoted by
\(W_{\text {com }}\), the weight equation is:
\[
\begin{equation*}
W_{\text {com }}=0.59 \mathrm{P}_{\mathrm{i}}+2.0 \mathrm{D}_{\text {com }}^{2}+39.0 \mathrm{~kg} . \tag{1}
\end{equation*}
\]

Volume of the electronics section is specified by:
\[
\begin{equation*}
v_{c}=\left(8.3 \times 10^{-4}\right) P_{i}+4.8 \times 10^{-2} \mathrm{~m}^{3} \tag{2}
\end{equation*}
\]
and the data transmission rate \(\left(R_{\text {com }}\right)\) is:
\[
\begin{equation*}
R_{\text {com }}=42.0 \mathrm{D}_{\mathrm{com}}{ }^{2} \mathrm{P}_{\mathrm{i}} \text { bits } / \mathrm{sec} \tag{3}
\end{equation*}
\]
2. SCIENCE SUBSYSTEM

The purpose of an unmanned Martian roving vehicle (MRV) mission would be to gather information about the planet, as well as to develop the technology relevant to autonomous roving vehicles. A roving capability makes it possible to conduct similar tests at many different locations, or to modify tests according to present location and past experimental results. The design of the science subsystem must be based, upon knowing first what information is sought, and second, how to endow the subsystem with the ability to gather this information.

The major thrust of a roving vehicle mission will be to determine the probability of life on the planet (Ref. 10; 24). Knowing whether life is more or less probable than was estimated before the mission would be an acceptable result. In addition, a comprehensive data-gathering program tracking Martian surface parameters (temperatures, atmospheric composition, surface gravity, seismological activity,...) will greatly increase the total knowledge of the planet's surface. Because several stationary landers will proceed an MRV mission, surface parameters will be known at some locations, and this second requirement takes on a slightly lesser priority.

Modeling the payload must result in relationships between the major parameters of the subsystem, especially those which will affect the design of other subsystems. These parameters include: 1) weight, 2) power requirement, 3) stationary science time required, and 4) data processing requirements. It seems that the only way to obtain empirical relationships
between these variables is to know what equipment will be onboard. However, until the parameters of the science subsystem have been chosen, which is the result of the analysis, this information would normally not be known. What can be done is to establish a priority list for the equipment, i.e., a list of which equipment will be added to the payload as weight and power allotted to science are increased. The priority list is set by defining what tests are needed to acquire the information desired, and then ordering these tests according to which information is deemed most useful. A heavy reliance was therefore placed upon the results of an extensive literature search concerning planetary scientific exploration and exobiology (Refs. 3, 10-24).

Science priorities (descending order) were determined to be:
1. test for qualities (properties) associated with life,
2. determine Mars surface parameters at diverse locations and times, and
3. have a "general chemical laboratory" with the ability to perform varied analyses and tests under earth command.

The assumption that the Martian bio-chemistry (if any) is earth-modeled. is not warranted. Free water appears to be in short supply on the surface, ultraviolet radiation (1700-3000 \&) fatal to most earth organisms is incident throughout what would be considered the biosphere, and temperatures are low ( \(180-300^{\circ} \mathrm{K}\) ). Some earth micro-organisms could survive on the planet, but none have been found which could grow in the Martian environment at the week rates of seasonal activity on Mars (the "wave-of-darkening," which mạy be biological in nature).

Life evolution normally progresses through and must exist first on molecular, microbial, and the macroorganismic stages, Therfore, life-search will be most efficient if tests are made for the qualities associated with
life (attempting not to assume a specific bio-chemistry) at the lower levels.

Indications of the presence of life may be functional (dynamical and thermodynamical), morphological, and/or chemical. Testing for functional qualities can be accomplished by certain biological activity tests (radio-isotope, turbidity, pH , calorimetric) which have been shown to be adaptable to space science requirements. Morphological properties can be observed in the large (television and television microscope) or on the molecular level (optical assymmetry tests). Finally, the knowledge of what chemical constituents are present on Mars will be of importance for practically all studies, but specifically for determining the possible bio-chemistries.

Determination of certain Mars surface parameters can be accomplished by Viking-1976-type meteorology and seismology packages (Ref. 3,22). In addition, tests for magnetic properties, surface gravity, and soil moisture should be considered.

Chemical analysis will be accomplished by the use of a gas chromatom graph-mass spectrometer (GC-MS) device. The device must be capable of pyrolyzing samples prior to analysis. Ref. 15 gives details.

Certain portions of the TV microscope and chemical laboratory, seen in the literature as the automated biological laboratory (ABL, Ref. 13), may be used to give the science package flexibility. The ABL is a general reagent laboratory, which when equipped with a minimal number of motor functions (moving samples, mixing, heating,...) will enable scientists on earth to request certain tests based upon what the MRV has observed up to that time.

Table 2 lists science equipment in order of priority as chosen by the authors along with other data important to the operation of the package. Data processing requirements were not considered in this.finst analysis.

TABLE 2. SCIENCE SUBSYSTEM: EQUIPMENT PRIORITY LIST AND SOME DEVICE CHARACTERISTICS

* all entries assume that time required to sample from the atmosphere is 15 sec ., and soil samples require 60 sec . Time req'd. includes the time necessary to transmit the outcome of the activity.
** portions of the total package may be used

Based upon the information in Table 2, two approximate relationships between subsystem parameters can be derived by plotting cumulative time and power vs. cunulative weight (i.e., total weight as equipment is added to the payload). The data points are plotted in Figures 5 and 6 along with linear approximations which are:
\[
\begin{align*}
& P_{\text {sci }}=3.44 \mathrm{~W}_{\text {sci }} \\
& T_{\text {esci }}=35.75 \mathrm{~W}_{\text {sci }}-135.0  \tag{4}\\
& T_{\text {sci }} \doteq\left(T_{\text {esci }}+\frac{3 \times 10^{6}}{R_{\text {com }}}\right)=35.75 W_{\text {sci }}+\frac{3 \times 10^{6}}{R_{\text {com }}}-135.0 \tag{5}
\end{align*}
\]
where
\[
\begin{aligned}
\mathrm{W}_{\text {sci }}= & \text { weight of science payload, } \mathrm{kg} \\
\mathrm{P}_{\text {sci }}= & \text { power required, watts } \\
\mathrm{T}_{\text {sci }}= & \text { time required to obtain and transmit science data per } \\
& \text { stop, sec } \\
\mathrm{T}_{\text {esci }}= & \text { time required to obtain science data per stop, sec } \\
R_{\text {com }}= & \text { data transmission rate for science data, bits } / \mathrm{sec}
\end{aligned}
\]

Note the first instance of coupling between subsystems. A communications subsystem parameter can be seen to directly affect the relationship between two of the science parameters.

A more accurate indication of the power requirement for science might be the average power expended overtime as a function of total weight. In other words, the average power ( \(\mathrm{P}_{\text {scia }}\) ) for any weight is the sum of all the products of experiment power times experiment time, divided by the sum of all the times. Suprisingly, this number is nearly constant with total weight, and to a good approximation:
\[
\begin{equation*}
P_{\text {scia }}=26 \text { watts. } \tag{6}
\end{equation*}
\]

\section*{3. POWER GENERATION AND STORAGE SUBSYSTEM}

In order to meet any mission requirements, the Martian roving vehicle must contain a suitable power source. Suchica power generation subsystem must be capable of sustained operation in a hostile environment and under


FIGURE 6. SCIENCE SUBSYSTEM: TIME REQUIRED FOR EXPERIMENTATION VS. TOTAL
adverse loading conditions. . References 25 and 26 develop a power system comprised of radioisotope thermoelectric generators (RTG's) and hermetically sealed batteries: the work in this section follows substantially along the same lines.

The power subsystem is assumed to operate in a dual-mode fashion: the batteries supply the energy to operate the craft and are subsequently recharged by the RTG's during 'rest' periods (no locomotion). The relationships governing the functioning of this subsystem are based upon the expression
\[
\frac{\text { Energy }}{\text { Power }}=\text { Time. }
\]

The product \(\left(C_{a v}\right)\left(D_{a v}\right)\left(M_{b}\right)\) represents the consolidated energy output of all the batteries (numerator term in equation above). \(C_{\text {av }}\) represents the average capacity of the cells in watt \(=\mathrm{hr} / \mathrm{kg}-\) mass; \(\mathrm{D}_{\mathrm{av}}\) is the mean depth of discharge; and \(M\) is the mass of the batteries ( \(k g\) mass). The subscript av is used to obtain a weighted value of \(C\) and \(D\) since more than one type of battery can be used simultaneously. The product ( \(M_{R T G}\) ) ( \(O_{\text {RTG }}\) ) represents the power output in watts of the RTG's ( \(O_{\text {RTG }}\) being the output in watts \(/ \mathrm{kg}\)-mass). An expression for the time necessary to recharge the onboard batteries ( \(T_{r}\) ) can be obtained:
\[
T_{r}=\frac{E_{f f}\left(C_{a v}\right)\left(D_{a v}\right)\left(M_{b}\right)}{\left(M_{R T G}\right)\left(O_{R T G}\right)-P_{\text {Ster }}}
\]
where \(P_{\text {str }}\) is the power consumed by onboard vehicular subsystems while the vehicle is in the recharging state, and \(E_{f}\) is the depth of discharge divided by the efficiency of the recharging process.

A similar equation results for the case of the vehicle in the roving state
\[
T_{\text {rov }}=\frac{E_{f d}\left(C_{a v}\right)\left(D_{a v}\right)\left(M_{b}-E_{\mathrm{st}}\right.}{\mathrm{P}_{\text {prop }}+P_{\mathrm{mv}}-\left(M_{\mathrm{RTG}}\right)\left(0_{\mathrm{RTG}}\right)}
\]
where \(T_{\text {rov }}\) is the maximum time allowable for locomotion, \(P_{\text {prop }}\) is the power used to propel the vehicle, and \(P_{m v}\) is the power used by onboard subsystems while the vehicle is moving (exclusive of \(P_{\text {prop }}\) ). \(E_{f d}\) is the depth of
discharge. \(E_{s t}\) is the total energy used by all equipment during stops in the roving period.

A dominant term in the last equation is \(P_{\text {prop }}\), the power required to propel the rover. This term is not a constant, in fact it is highly dependent upon the roving velocity, and the vehicle's mass - to mention only two parameters. The power used to drive the vehicle is the result of three factors: \(P_{a}\), the power used by the rover to accelerate from a stationary position to the roving velocity; \(P_{v}\), the power required to maintain the velocity of the rover (the velocity was assumed to be constant); and \(P_{\text {sl }}\), the power needed for slope traversal.

The first term, \(P_{a}\), is obtained by applying the energy-power equation in the form of
\[
\frac{\text { Energy }}{\text { Time }}=\text { Power. }
\]

Therefore,
\[
P_{a}=\frac{M_{r} \dddot{v}_{f}^{2}}{2 t}
\]
where \(M_{r}\) is the total mass of the rover, and \(t_{a}\) is the time necessary to accelerate the rover to the constant velocity ( \(\mathrm{v}_{\mathrm{f}}\) ).
\(P_{v}\) is the power used by the rover to overcome the force of friction while traversing the planet at a constant velocity. Since on a flat plane
\[
P_{v}=F v_{f},
\]
then
\[
P_{v}=\mu_{k} M_{r} g_{m} v_{f},
\]
where \(\mu_{k}\) is the coefficient of kinetic friction and \(g_{m}\) is the acceleration of gravity on Mars.

The final term, \(P_{s 1}\), in the power equation is found to be
\[
P_{s l}=M_{r} g_{m} v_{f} \sin , \psi,
\]
where \(\psi_{\text {is the }}\) the angle of inclination of the slope being traversed. Combining the last 2 equations yields
\[
P_{v}+P_{s l}=M_{r} g_{m}{ }_{f}\left(\mu_{k}+\sin \psi\right)
\]

This last equation can be modified to take into account wheel slippage: a two degree additive slope factor approximates the effect of any slippage (Ref. 27) so:
\[
P_{v}+P_{s l}=M_{r} g_{m} v_{f}\left(\mu_{k}+\sin \left(\psi+2^{\circ}\right)\right)
\]

Because the \(P_{a}\) term applies only to the case where the vehicle is accelerating to \(\mathrm{V}_{\mathrm{f}}\), and because in that case power assigned to \(\mathrm{P}_{\mathrm{v}}\) can be utilized, an approximation to \(P_{\text {prop }}\) might be:
\[
\begin{equation*}
P_{\text {prop }}=M_{r} g_{m} v_{f}\left(\mu_{k}+\sin \left(\psi+2^{0}\right)\right) \tag{7}
\end{equation*}
\]

If: the substitutions
\[
\begin{aligned}
P_{\text {RTG }} & =M_{R T G} 0_{R T G} \\
\text { and } E_{\text {batt }} & =C_{a v} D_{a v} M_{b}
\end{aligned}
\]
are made, the time equations become:
\[
\begin{align*}
& T_{r}=\frac{E_{f} E_{\text {batt }}}{P_{\text {RTG }}-P_{\text {str }}}  \tag{8}\\
& T_{\text {rov }}=\frac{E_{\text {fd }} E_{\text {batt }}-E_{\text {st }}}{P_{\text {prop }}+P_{\text {mv }}-P_{\text {RTG }}} \tag{9}
\end{align*}
\]
"The terms \(P_{m v}\) and \(P_{\text {str }}\) must be determined by the operating characteristics of the subsystems. They will consist of the power usages of the subsystems for the roving and recharging states. This work is reported in section IIIB.

The weight of the power subsystem must be found as a function of subsystem variables. Ref. 3 estimates the weight of relays, converters and shunts required for an RTG-battery configuration to be 14 kg , which should be fairly constant within the working range of the subsystem parameters. The projection of RTG technology circa 1975 is for a 5.94 watts \(/ \mathrm{kg}\) capability with practically infinite lifetime when compared to the duration of the mission (Ref. 25).

Table 3 presents data on battery types considered dependable enough for space applications (Ref. 28). Silver-zinc batteries have too high a degradation rate for use on a 6-18 month mission. The choice between silver-cadmium and nickel-cadmium batteries might best be made by running the optimization problem with each of them (this is a good example of a simplifying assumption that must be investigated by later allowing it to change). For the first run, a conservative (more cycles, lower degradation rate) choice of NiCd was made. NiCd batteries have a 27.0 watt-hrs/kg ratio. The weight of the power subsystem \(\left(W_{p}\right)\) can be described by:
\[
\begin{equation*}
\mathrm{W}_{\mathrm{P}}=.168 \mathrm{P}_{\mathrm{RTG}}+.037 \mathrm{E}_{\mathrm{batt}}+14.0 \ldots \mathrm{~kg} . \tag{10}
\end{equation*}
\]
4. THERMAL CONTROL SUBSYSTEM

The function of the thermal control subsystem is to maintain a satisfactory environment in which critical equipment can be operated. The basic assumption made in the modeling effort is that a compartment shall be temperature controlled to remain at \(300^{\circ} \mathrm{K}\) despite Martian environmentr. ; variations.
, Variations of the Martian environment are vital inputs. Maximum temperatures occur at Martian noon and are estimated to be about \(300^{\circ} \mathrm{K}\), while minimum temperatures of \(200^{\circ} \mathrm{K}\) are expected at night. However, in the event of prolonged absence of sunlight, temperatures could be expected to drop to as low as \(150^{\circ} \mathrm{K}\). (Ref. 12, 29) Such dark periods could result from a dust storm, or from the Rover's stopping in a shaded position. Therefore, the extreme temperatures for which the subsystem must perform satisfactorily are \(300^{\circ} \mathrm{K}\) and \(150^{\circ} \mathrm{K}\), respectively.

Other constraints affecting the subsystem design are low atmospheric pressure, low thermal conductivity of the atmosphere, day/night cyclical incident.energy variations, abrasive dust storms, and limited power and weight available.

Prior to the modeling effort, it was concluded that the configuration of the subsystem would have to be specified to some extent, or the modeling

TABLE 3. POWER GENERATION AND STORAGE SUBSYSTEM: STATISTICS ON BATTERIES FOR SPACE APPLICATIONS
\begin{tabular}{|r|c|c|c|}
\hline Type & Energy capacity (watt-hr/gm) & useful life (cycles) & \begin{tabular}{c} 
degradation \\
(\%/cycle) \\
\(\%\)
\end{tabular} \\
\hline Nicd & 0.027 & 10,000 & 0.003 \\
AgZn & 0.053 & 2,000 & 0.015
\end{tabular}
task would be insurmountable. Therefore, from previous work done on choosing a thermal control configuration for a Martian laboratory, (Ref. 30) a preferred scheme was selected from a list of the many feasible alternatives. The choice was made on the basis of a list of desired features, such as simplicity, reliability, range of control, proven performance, insensitivity to Martian atmospheric parameters, ability to survive sterilization procedures, ease of development, resistance to dust storm damage, and required weight. The configuration chosen is an electrically heated, heat pipe-cooled insulated compartment, as shown in Figure 7 (note that this figure defines the variables \(a_{1}\) and \(a_{2}\) ).

Having selected the configuration, a list of describing parameters can be compiled. These parameters are given in Table 4. A number of heat balance equations can be written by noting that the assumption of isothermal compartments implies that for each isothermal volume, the heat input equals the heat output. Furthermore, the heat balance is satisfied both at night and in the day. This allows six equations to be written. Also, an equation for subsystem weight can be derived.

A sample heat equation and the weight equation are shown here. For the outer skin during the day, let:
```

Area of surface which radiates heat $=A_{\text {sr }}$
Radiometric Albedo = . $295=\mathrm{a}$
Incident solar energy $=Q_{\text {sol }}$
Radiated heat $=Q_{\text {rad }}$
Convective heat loss $=Q_{\text {conv }}$
Conductive heat loss $=Q_{\text {cond }}$
Insulation conductivity $=\mathrm{k}_{\mathrm{i}}=0.0216 \frac{\text { watts }}{\dot{\mathrm{m}}^{\mathrm{O} \mathrm{K}}}$
Surface emissivity $=\epsilon_{s}=0.8=\epsilon_{r}$
Surface absorptivity $=\alpha_{S}=0.5=\alpha_{r}$
Incident solar energy (Solar Constant) $=S_{c}=235 \frac{\mathrm{BTU}}{\mathrm{hr}} \mathrm{ft}^{2}=750 \frac{\text { Watts }}{\mathrm{m}^{2}}$

```


\section*{CROSS SECTION}

Environmentally controlled


FIGURE 7.
THERMAL CONTROL SUBSYSTEM:
BASIC CONEIGURATION
\begin{tabular}{|c|c|c|}
\hline PARAMETER & SYMBOL & UNITS \\
\hline Maximum Heater output & \(Q_{h}\) & watts \\
\hline Radiator area . & \[
A_{r}
\] & \(m^{2}\) \\
\hline Insulation thickness & \(L_{i}\) & m \\
\hline Heat pipe cooling capacity & \(\mathrm{K}_{\mathrm{q}}\) & watts/ok \\
\hline Weight (Mass) & \(W_{\theta}\) & kg \\
\hline Night skin temp & \(\mathrm{T}_{\mathrm{bn}}\) & \({ }^{\circ} \mathrm{K}\) \\
\hline Day skin temp & T bd & \({ }^{\circ} \mathrm{K}\) \\
\hline Night radiator temp & \(\mathrm{T}_{\mathrm{rn}}\) & \({ }^{\circ} \mathrm{K}\) \\
\hline Day radiator temp & \(\mathrm{T}_{\mathrm{rd}}\) & \({ }^{\circ} \mathrm{K}\) \\
\hline
\end{tabular}

INPUT PARAMETERS
(FUNCTIONS OF OTHER SUBSYSTEM STATE VARIABLES)
\begin{tabular}{|l|c|c|}
\hline PARAMETER & SYMBOL & UNITS \\
\hline Total package surface area & A & \(\mathrm{m}^{2}\) \\
Daytime internal dissipation & Qid & watts \\
Night internal dissipation & Qin & watts \\
\hline
\end{tabular}

TABLE 4.
THERMAL CONTROL SUBSYSTEM:

PARAMETER LIST
\[
\begin{aligned}
& \text { Average convective transfer coefficient }=h_{c} \\
& \text { Stephan - Boltzman Constant }=\sigma=5.67 \times 10^{-8} \frac{\text { watts }}{m^{2}\left({ }^{0} K\right)^{4}}
\end{aligned}
\]

The heat transfer equations are (Ref. 31, 32):
\[
\begin{align*}
& Q_{\text {cond }}=\left(A-A_{r}\right)\left(k_{i} / L_{i}\right)\left(T_{a}-T_{b}\right)  \tag{A1}\\
& Q_{\text {conv }}=\left(A-A_{r}\right) h_{c}\left(T_{b}-T_{m}\right),  \tag{A2}\\
& Q_{\text {rad }}=\left(\varepsilon_{s} \sigma A_{S r} T_{b}^{4}\right), \tag{A3}
\end{align*}
\]
and
\[
\begin{equation*}
Q_{S O l}=\alpha_{s}\left(A_{\operatorname{sun}(s)}+A_{a l b(s)}\right) S_{c} \tag{A4.}
\end{equation*}
\]

For equilibrium the heat input must equal the heat output (i.e. zero heat build-up), therefore,
\[
\begin{equation*}
Q_{\text {cond }}+Q_{\text {sol }}=Q_{\text {conv }}+Q_{\text {rad }} \tag{A5}
\end{equation*}
\]

Substituting values for the variables in equation (A5) yields the final heat balance equation:
\[
\begin{align*}
& \left(A-A_{r}\right)\left(k_{i} / L_{i}\right)\left(T_{a}-T_{b}\right)+\alpha_{s}\left(A_{\operatorname{sun}(s)}+a A_{a l b}(s) S_{c}\right. \\
& =\left(A-A_{r}\right) h_{c}\left(T_{b}-T_{m}\right)+\varepsilon_{s} \sigma A_{s r^{T}} T_{b}^{4} \tag{A6}
\end{align*}
\]

If \(W_{\hat{\theta}}\) is the weight of the thermal control subsystem in kg :
\[
\therefore \quad W_{\theta}=55.5 \mathrm{AL}_{i}+7.28 \dot{A}_{\dot{r}}+3.64 \frac{\left(\mathrm{a}_{1} \mathrm{a}_{2}\right)}{\mathrm{a}_{12}} \mathrm{~A}+0.1 \mathrm{~A} \underset{\mathrm{r}}{\mathrm{~K}_{1}} \cdot
\]

The result of the above analysis is that the \({ }^{12}\) list of nine unknowns in in Table 4 are related by seven equations. In theory the model could then be reduced to only two parameters, or state variables, which would uniquely specify the subsystem. However, as the sample heat balance shows, "insolvable" transcendental equations result from the fourth power radiation law terms; and in practice, the system is best reduced only to four state variables and two state variable relationships. The four state variables chosen in this model are \(T_{b n}, T_{b d}, T_{r n}\), and \(T_{r d}\), the day and night design temperatures of the outer skin and radiator, respectively. Admittedly, four more physically obvious parameters could be chosen as the state variables, but this choice results in the simplést set of state variable relationships, and thus a more easily usable model. The two state variable relationships, even so,
are very complicated, and are shown here:
\[
\begin{align*}
& \left(A-A_{r}\right) \frac{k_{i}}{L_{i}} T_{b n}+\varepsilon_{s} \sigma\left[A\left(1-\frac{a_{1} a_{2}}{2 a_{12}}\right)-A_{r}\right] T_{b n}^{4}=  \tag{11}\\
& \text { ( } A-A_{r} \text { ) } \frac{k_{i}}{L_{i}}(300) \text {, } \\
& \left(A-A_{r}\right) \frac{k_{i}}{L_{i}} T_{b d}+\varepsilon_{-} \sigma\left[A\left(1-\frac{L_{i} a_{2}}{2 a_{12}}\right)-A_{r}\right] T_{b d}^{4}=  \tag{12}\\
& \left(A-A_{r}\right) \frac{k_{i}}{L_{i}}(300)+\alpha_{s} S_{c}\left(A_{\operatorname{sun}(s)}+a A_{a l b}(s) 0\right) \text {, }
\end{align*}
\]
where
\[
\begin{align*}
& L_{i}=\frac{\sum_{i}\left(300-T_{r n}\right)}{\varepsilon_{r} \sigma\left(T_{r n}^{4}-100^{4}\right)},  \tag{13}\\
& A_{r}=\frac{Q_{i d}-\frac{A k_{i}}{L_{i}}\left(300-T_{b d}\right)+\alpha_{r}\left(A_{\operatorname{sun}(r)}+A_{a l b}(r)\right) S_{c}}{\varepsilon_{r} \sigma\left(T_{r d}^{4}-100^{4}\right)+\frac{k_{i}}{L_{i}}\left(300-T_{b d}\right)}  \tag{14}\\
& Q_{h}=\left(A-A_{r}\right) \frac{k_{i}}{L_{i}}\left(300-T_{b n}\right)+\frac{A_{r} k_{i}}{L_{i}}\left(300-T_{m}\right)-Q_{i n},  \tag{15}\\
& K_{q}=Q_{i d}-\left(A-A_{r}\right) \frac{k_{i}}{L_{i}}\left(300-T_{b d}\right)+A_{r} \frac{k_{i}}{L_{i}}\left(300-T_{r d}\right)  \tag{16}\\
& \left(300-\mathrm{T}_{\mathrm{rd}}\right)
\end{align*}
\]
and
\[
\begin{equation*}
W_{\theta}=55.5 \mathrm{AL}_{i}+3.64\left(\frac{a_{1} a_{2}}{a_{12}}\right) A+.1 A_{r} K_{q}:+7.28 A_{r} \tag{17}
\end{equation*}
\]

Some rather important sensitivities can be examined at this point by setting up a nominal example design. If a cubic compartment, one meter on a side, with radiator area of two square meters and emissivity of 0.8 is examined, it is easy to calculate the relation between insulation thickness and heater power required at night, as shown in Figure 8. It is interesting to note that increases in insulation thickness above about 0.1 meters do not reduce heater requirements. very much, but will add to the weight, and make it much more difficult to dissipate excess heat during the day hours. On the other hand, insufficient insulation forces heater power to be ridiculously high, causing the power


FIGURE 8.
THERMAL CONTROL SUBSYSTEM:
GRAPH OF NIGHT HEATER POWER VS.
INSULATION THICKNESS
subsystem to be larger. The various trade-offs are plainly evident from the mathematical model, and optimization could be performed at this point, as is usually done in most design efforts; but by using the state variable approach to leave the subsystem underspecified, optimization on the systems level is allowed, and a truly optimal system is obtained.

Another interesting sensitivity evident from the model is that of daytime internal heat production and heat pipe cooling capacity required to maintain the target temperature of \(300^{\circ} \mathrm{K}\). This relation is given in Figure 9. It is very important to note that cooling requirements go up rather rapidly for increases in heat dissipated inside the compartment during the day. For the sake of efficiency, it would be highly desirable to perform functions associated with large amounts of internal dissipation during the night, when the heat given off as a byproduct could be used to maintain night temperature, rather than overtaxing the cooling function.
5. NAVIGATION SUBSYSTEM

Navigation is taken to mean the location of vehicle position with respect to a şet of coordinates centered in Mars. The scheme considered for first analysis is one devised by a group of the RPI-MRV project at Rensselaer Polytechnic Institute,

The coordinate system is established by instruments which locate what would be the position of a true pole star of Mars (Ref. 33) and the direction of local vertical (Ref. 34). The initial estimate of position is obtained by tracking an orbiter with known orbital parameters (Refs. 35, 36). A direct velocity sensor (Ref. 37) measures vehicle velocity relative to the surface in a body-bound frame. A system for updating the estimate of position with vehicle movement (Ref. 38) has been devised.

Ideally, modeling of the navigation subsystem would include equations describing how power and weight allocations to the equipment affect the accuracy of the subsystem. In addition, the error in detecting local vertical ( \(\Delta \beta\) ) has


FIGURE 9.
THERMAL CONTROL SUBSYSTEM:
GRAPH OF REQUIRED HEAT PIPE COOLING CAPACITY VS. INTERNAE HEAT DISSIPATION
a direct effect on the obstacle avoidance (terrain sensing and path selection) subsystem. However, because:
1. the form of these equations appears to be complex, and the time required to derive them considerable,
2. an error in position location does not directly:affect the operation of any other subsystem, and
3. the error in local vertical is fairly invariant for forseeable values of the design parameters, it was decided to allocate certain constant values of power and weight to the navigation subsystem, and make a worst case estimate of the local vertical detection error. Weight and power allocations appear in Table 5. The local vertical error is assumed to be \(0.25^{\circ}\) (Ref. 34). Thus, the navigation subsystem does not appear in any of the system model equations.

\section*{6. OBSTACLE AVOIDANCE SUBSYSTEM}

The obstacle avoidance subsystem is responsible for identifying terrain hazards and choosing a safe path for travel by the vehicle. The system considered utilizes a laser rangefinder which scans the terrain in front of the moving vheicle in repeating arcs and determines the height of the terrain at the sensed points (Ref. 3). This method can be modified to estimate slopes by assuming the terrain is linear between sensed points. This information is : utilized by a dual-mode routing algorithm (Ref. 40). The algorithm assumes that previous fly-by and orbiter missions have sufficiently mapped the surface so that a coarse path (segments on the order of kilometers) can be pre-programmed. Local deviations in the coarse path are achieved by following the outer contour of all obstacles encountered.

Preliminary analysis demonstrated that the errors caused by changes in power and weight allocations to the subsystem would be small compared to errors inherent in the method which are due primarily to errors in the detection of local vertical (Refs. \(41,42,43\) ). A weight allocation ( \(W_{o a}\) ) of 5 kg , and a

TABLE 5. NAVIGATION SUBSYSTEM: POWER AND WEIGHT ALLOCATIONS FOR SUBSYSTEM COMPONENTS
\begin{tabular}{|l|c|c|}
\hline device & weight, kg & power, watts \\
\hline pole star detector \\
local vertical sensor \\
laser (ranging to orbiter) \\
\begin{tabular}{l} 
position update (gyrocompass, \\
velocity sensor)
\end{tabular} \\
platform, motors (torquors)
\end{tabular}\(\quad\)\begin{tabular}{l}
1 \\
\hline total
\end{tabular}
* not applicable, the laser is in operation only a few seconds per day (present estimate is 3 seconds every 2.5 hours). Let \(P_{\text {nav }}=6\) watts.
continuous power draw ( \(\mathrm{P}_{\mathrm{oa}}\) ) of 15 watts were chosen.
An error in estimating the height of a portion of the terrain ( \(\Delta h_{t}\) ) can be written:
\[
\Delta h_{t}=r_{a} \sin \Delta \beta \doteq r_{a} \Delta \beta
\]
where \(\quad r_{a}=\) horizontal distance to sensed terrain point. When calculating estimates of terrain slopes, the worst case error ( \(\varepsilon_{\text {sl }}\) ) can be shown to be:
\[
\varepsilon_{s l}=\frac{2 r_{\dot{a}} \Delta \beta}{\delta} \quad \text { degrees }
\]
where \(\delta=\) horizontal separation between the terrain points used in slope approximations, meters.

Slope segments become a real concern when their span approaches the wheelbase of the vehicle. Because the nominal separation between sensed terrain points (in the direction of vehicle travel) will be much smaller than the wheelbase, it is possible to consider only sets of points such that:
\[
\varepsilon_{s 1}=\frac{2 r_{a} \Delta \beta}{w_{b}} ; \text { i.e., } \delta=\cdot w_{b}
\]
where \(\mathrm{w}_{\mathrm{b}}=\) wheelbase of the vehicle. For all succeeding work, it was assumed: (as per Ref. 3) that \(r_{a}=30\) meters.

To find the effect of the error on vehicle travel, a model of the Mars terrain is required. Ref. 44 establishes that the probability that a terrain segment of 61 m interval will have an average slope less than or equal to \(s\) (in degrees) is:
\[
P(s \leq s)=\int_{0}^{s} 0.17 e^{-.17 \lambda} \mathrm{~d} \lambda
\]

The distribution for slopes with smailer span can be assumed equivalent (Ref. 45).

The percentage of terrain impassable for the vehicle on the same scale as the vehicle wheelbase ( \(\mathrm{T}_{\text {act }}\) ) is a function of the maximum slope the vehicle will be allowed to traverse (s\%):
\[
\mathrm{T}_{\mathrm{act}}=\int_{\mathrm{S}^{*}}^{\infty} .17 \mathrm{e}^{-.17 \mathrm{~s}} \mathrm{ds},
\]
but considering the error the vehicle will make in interpreting slopes,
the percentage terrain considered impassable by the vehicle ( \(T\) ) will be:
\[
\begin{equation*}
\mathrm{T}=\int_{\mathrm{s} *-\varepsilon}^{\infty}: .17 \mathrm{e}^{-.17 \mathrm{~s}} \mathrm{ds} \tag{18}
\end{equation*}
\]
where, again in this case, the error is assumed to have a worst case effect. Note that \(T\) is a function of \(s^{*}, r_{a}, \Delta \beta ; \dot{w}_{b}\) and the Martian terrain model.

The dual-mode routing algorithm. requires that a coarse path be chosen prior to the mission. This large-scale path will be determined basically by the crater distribution on Mars. To a good approximation, it will not - be a function of vehicle capability, but will simply be a path chosen to detour around craters. Ref. 45 shows that the percentage of terrain area encompassed by craters is approximately \(50 \%\). Because the average crater wall is too steep for safe vehicle tṛavel, that portion of the terrain will be considered impassable in the large-scale case. For small-scale deviations from the largescale path, \(T\) will be determined by slope distributions and the maximum slope, the vehicle will be allowed to traverse.

Considering both these cases jointly, the modeling procedure requires a measure of how efficient the obstacle avoidance subsystem is as a function of the parameters discussed above. A useful descriptor is the path-length ratio (PLR), defined as the ratio of actual path length to straight-line (great circle) distance.

Simulation was employed to determine PLR for both cases. Given that the vehicle is at a point on the terrain and wishes to travel in the \(\theta=0^{\circ}\) direction, the probability that it will travel in the \(\theta\) direction, \(p(\theta)\), would have the form of Figure 10. (Theta is dimensionless; there are only a finite number of possible directions.) Briefly, thisais due to the fact that the vehicle looks for a free path by considering directions in the following order: \(0,1,-1,2,-2,3\), ..... The probability of \(\theta=0\) (i.e., the probability of traveling in the desired direction) can be assumed 1-T if the step size is not too much greater than the obstacle size. Given that \(\theta=0\) is not a free path, the probability of


FIGURE 10. OBSTACLE AVOIDANCE SUBSYSTEM: \(\begin{aligned} & \text { TYPICAL CHOICE-OF DIRECTION } \\ & \text { PROBABILITY FUNCTION }\end{aligned}\)

1 or -1 being free is small (obstacles have size). As the scan gets further away from the known obstacle, the probability of the path being free should increase. Finally, as \(|\theta|\) gets large, \(p(\theta)\) should decrease because a large \(|\theta|\) will only be chosen if all smaller ( in \(|\theta|\) ) paths are blocked. The problem with assuming this type of distribution is that the statistics of the "humps" are functions of statistics of the obstacles, which are unknown for Mars.

For purposes of simplification, the simulation used a distribution with \(p(0)=1-T\) and all other probabilities equal. If \(\delta \theta\) is defined as the angular deviation between possible paths, let \(\delta \theta=5^{\circ}\) be assumed (this gives a separation of approximately 3 m at the maximum laser range for \(\left.r_{a}=2030 \mathrm{~m}\right)\). Then,
\[
p\left(i 5^{\circ}\right)=\left\{\begin{array}{cl}
1-T, & i=0 \\
\cdots, 70, & i= \pm 1, \pm 2, \ldots, \pm 35 .
\end{array}\right.
\]

A computer program simulated traveling from ( 0,0 ) to ( 1,0 ) in Cartesian coordinates. The variables in the simulation were \(T\) and \(r\) (the step size, analagous to \(r_{a}\) ). For the large-scale path, \(T=0.50\) as previously established, and since the simulation required that the step size approached the average obstacle size, \(r=25 \mathrm{~km} / 1000 \mathrm{~km}=0.025\) (the average Mars crater is 16.3 km , with heavy debris outside the edge; the mission range will hopefully approach 1000 km ). For small-scale path deviations, \(T\) is a running variable. The value of \(\underset{\sim}{r}=\mathrm{w}_{\mathrm{b}} / 25 \mathrm{~km}\), or 0.00012 if \(\mathrm{w}_{\mathrm{b}} \doteq 3 \mathrm{~m}\). Table 6 reports the results (averages) of many simulations at varying \(T\) and \(r\).

The next step was fitting the data of Table 6 with a continuous function for use in the model. The total PLR is the product of the large-scale.and small-scale PLRs. Therefore, at \(T=0\), PLR should be 2.0. At \(T=1\), PLR

TABLE 6. OBSTACLE AVOIDANCE SUBSYSTEM: RESULTS OF PATH-LENGTH RATIO SIMULATION

must approach infinity. The function
\[
\operatorname{PLR}=\frac{2}{1-T}
\]
fits this form, but was not sufficiently accurate for intermediate values. The function
\[
\begin{equation*}
\text { PLR }=\frac{2\left(1+0.05 T+0.167 T^{2}\right)}{1-T} \tag{19}
\end{equation*}
\]
fits all data points within \(7 \%\). Figure 11 compares thessimulation results with the functional approximation.

A PLR simulation of a different approach by Eisenhardt and Murtaugh (Ref. 46) originally applied to a Surveyor mission, gives small-scale PLRs which vary from deviations \(4 \%\) lower at low \(T\) (.20)monotonically increasing to \(15 \%\) lower at high \(T(.60)\) as compared with results presented here.

\section*{7. COMPUTATION AND DATA-HANDLING SUBSYSTEM}

The onboard computational and data-handling requirements for a semiautonomous MRV are succinctly stated and explained in Ref. 47. Briefly stated, they are:
1. conditioning of onboard sensor data
2. fiavigation, guidance and special sensor (antenna, celestial) pointing computations
3. terrain modeling, path selection and motion control commands
4. energy bookkeeping and management functions regarding the vehicular state
5. logic for event sequencing and synchronization sequencing of the total vehicle system.

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The datarhandling subsystem for the Thermoelectric Outer Planet Spacecraft (TOPS) meets the MRV requirements, and exceeds the lifetime requirement by a factor of ten (Ref. 3). Table 7 presents power, weight and volume data for the TOPS subsystem. These numbers will be considered constant inputs to the MRV model. Data from Refs. 48 and 49 indicates the validity of this approach.

\section*{8. VEHICLE STRUCTURE SUBSYSTEM}

There are a number of candidate vehicles for a roving exploratory Mars mission. Both 4 -and 5 -wheeled vehicles have been proposed. The AC Electronics Division of the General Motors Corp. (Ref. 50) and McDonnell Astronautics (Ref. 51) have studied 6-wheeled mobility subsystems, Work at Rensselaer Polytechnic Institute under the RPI-MRV project has led to the proposal of a 4 -wheeled vehicle with an optional 3 -wheeled mode (Ref. 52). It is this latter version that was considered toward formulating the system model. Figure 12 shows a simplified sketch of the concept.

Because the RPI-MRV is dynamically scaled, all major dimensions are dependent; defining
\[
\begin{aligned}
& \mathrm{w}_{\mathrm{b}}=\text { wheelbase or front-to-rear distance between wheels } \\
& \mathrm{t}=\text { track or side-to-side distance between wheels } \\
& \mathrm{w}_{\mathrm{v}}=\text { weight (frame, suspension, motors) } \\
& \mathrm{v}_{\mathrm{v}}=\text { equipment package volume, }
\end{aligned}
\]
the following relationships hold:
\[
\begin{align*}
& w_{b}=t  \tag{20}\\
& {\left[\frac{t}{t_{0}}\right]^{3}=\frac{w_{v}}{w_{v o}}}  \tag{21}\\
& {\left[\frac{t}{t_{0}}\right]^{3}=\frac{v_{v}}{v_{v o}}} \tag{22}
\end{align*}
\]
where the subscript zero indicates the nominal design values, which are

TABLE 7. COMPUTATION AND DATA-HANDLING SUBSYSTEM: POWER AND WEIGHT ALLOCATIONS FOR SUBSYSTEM COMPONENTS
\begin{tabular}{|c||c|c|c|}
\hline component & weight, kg & power, watts & volume, in \({ }^{3}\) \\
\hline flight data subsystem \\
centralized computer subsystem \\
data storage subsystem & 12.7 & 25 & 1000 \\
\hline Total & 11.4 & \(\ddots\) & \\
\hline
\end{tabular}


FIGURE 12: VEHICLE STRUCTURE SUBSYSTEM: SIMPLIFIED SKETCH OF MARTIAN ROVING VEHICLE CONCEPT
\[
\begin{aligned}
t_{\mathrm{o}} & =10 \mathrm{ft}=3.04 \mathrm{~m} \\
\mathrm{~W}_{\mathrm{Vo}} & =400 \mathrm{lbs}=182 \mathrm{~kg} \\
\mathrm{~V}_{\mathrm{vo}} & =130 \mathrm{ft}^{3}=3.653 \mathrm{~m}^{3} .
\end{aligned}
\]

The slope climbing and other obstacle capabilities of the vehicle are such that they should not be the limiting factors in choosing the optimal design. The power requirements for slope climbing will probably be the limiting factor. This is a supposition which may require refinement or change during the actual optimization process.

\section*{III B. SYSTEM CONSTRAINTS}

This section regarding system constraints completes the identification and formulation of all the equality and inequality constraints between the design parameters. That is, the end result of this work is the system model.

To this point, there are 23 equations (the 22 numbered equations in section IIIA plus the identity \(a_{12}=a_{1}+a_{2}+a_{1} a_{2}\), eqn. 23) in 70 parameters. Table 8 lists the design parameters. All of the 70 parameters must fall into one of three categories:
1. Those that are considered constant,
2. Those that are expressable in terms of other design parameters by appropriate assumptions, and
3. Those that are true variables, related only by the 23 equations already given.

The 26 constants are circled in Table 8 and their values are given in Table 9. The model is now reduced to 23 equalities in 44 unknowns. Other variables can be shown to be dependent upon some "type 3 " parameters.

Using the equipment package dimensions of the MRV (remember that the vehicle dimensions are interrelated, i.e. \(a_{1}\) and \(a_{2}\) are constants) and assuming worst case solar radiation effects:
\begin{tabular}{|c|c|c|c|}
\hline subsystem & \# equations & \# parameters & parameter list \\
\hline communications & 3 & 5 & \(W_{\text {com }}, \mathrm{D}_{\text {com }}, \mathrm{V}_{\mathrm{c}}, \mathrm{P}_{\mathrm{i}}, \mathrm{R}_{\text {com }}\) \\
\hline science & 3 & 4 & \(\mathrm{P}_{\text {scia }}, \mathrm{W}_{\text {sci }}, \mathrm{T}_{\text {sci }}, \mathrm{T}_{\text {esci }}\) \\
\hline power & 4 & . 15 &  \\
\hline thermal control & 8 & 27 & \[
\begin{aligned}
& Q_{h}, A_{r}, L_{i}, K_{q}, W_{\theta}, T_{b n}, T_{b d}, T_{r n}, \\
& \left.T_{r d}, A, Q_{i d}, Q_{i n} K_{i}, \varepsilon_{s}, \varepsilon_{r}, \alpha_{s}, \alpha_{n}\right)
\end{aligned}
\] \\
\hline & & & \[
\left.\left(a_{1}, a_{2}\right), a_{12}\right) A_{c} \operatorname{sun}(s), A \operatorname{alb}(s),
\] \\
\hline navigation & 0 & 3 &  \\
\hline obstacle avoidance & 2 & 6 & \[
\mathrm{P}_{\mathrm{a}} \mathrm{~s}^{*}, \mathrm{~T}, \mathrm{PLR}, \mathrm{~W}_{\mathrm{Oa}} \mathrm{PO}
\] \\
\hline computation and data-handling & 0 & 3 & \[
W_{C D}, P_{C D} V_{C D}
\] \\
\hline vehicle & 3 & 7 & \[
w_{b}, t t_{0} w_{v} \mathrm{wv}_{\mathrm{vo}} \mathrm{v}_{\mathrm{vo}} \mathrm{v}_{\mathrm{v}}
\] \\
\hline & 23 & 70 & \\
\hline
\end{tabular}

TÁBLE 9. CONSTANT PARAMETERS AND THEIR VALUES

Number of constants \(=26\)
\[
\begin{aligned}
\mu_{\mathrm{k}} & =0.10 \\
E_{\mathrm{fd}} & =0.40 \\
E_{\mathrm{f}} & =0.57 \\
\mathrm{k}_{\mathrm{i}} & =0.0216 \text { watts } / \mathrm{m}^{\circ} \mathrm{K} \\
\varepsilon_{\mathrm{s}} & =0.8 \\
\varepsilon_{\mathrm{r}} & =0.8 \\
\alpha_{\mathrm{s}} & =0.5 \\
\alpha_{\mathrm{r}} & =0.5 \\
\sigma & =5.67 \times 10^{-8} \text { watts } / \mathrm{m}^{2} o_{\mathrm{K}}{ }^{4} \\
\mathrm{~s}_{\mathrm{c}} & =750 \mathrm{watts} / \mathrm{m}^{2} \\
\mathrm{a} & =0.295 \\
W_{\mathrm{nav}} & =15 \mathrm{~kg} \\
\dot{P}_{\mathrm{nav}} & =6 \mathrm{watts} \\
\Delta B & =0.25^{\circ} \\
\mathrm{r}_{\mathrm{a}} & =30 \mathrm{~m}
\end{aligned}
\]
\[
\begin{align*}
& A_{\operatorname{sun}(s)}=0.50 \mathrm{~A}  \tag{24}\\
& \mathrm{~A}_{\mathrm{alb}(s)}=0.843 \mathrm{~A} \\
& \mathrm{~A}_{\operatorname{sun}(r)}=0.50 \mathrm{~A} \\
& \mathrm{~A}_{\mathrm{alb}(r)}=\mathrm{A}_{r} \tag{27}
\end{align*}
\]

These relations can be used to simplify several of the terms in the thermal control subsystem equations. Specifically:
\[
\begin{aligned}
& A_{\operatorname{sun}(s)}+a A_{a l b}(s)=0.749 \mathrm{~A} \\
& A_{\operatorname{sun}(r)}+a A_{a l b}(r)=0.795 A_{r} .
\end{aligned}
\]

The equation for \(P_{\text {prop }}\), eqn. (6), contains 2 terms which can be described by other parameters. First, the mass of the total system ( \(M_{r}\) ) can be written:
\[
\begin{align*}
M_{r} & =\frac{\text { weights of all subsystems in } \mathrm{kg}}{9.806 \mathrm{~m} / \mathrm{sec}^{2}} \\
& =0.1020\left[\mathrm{~W}_{\mathrm{com}}+\mathrm{W}_{\mathrm{sci}}+W_{\mathrm{p}}+W_{\theta}+W_{\mathrm{nav}}+\mathrm{W}_{\mathrm{oa}}+\mathrm{W}_{\mathrm{cp}}+\mathrm{W}_{\mathrm{v}}\right] \mathrm{kg}-\text { mass }  \tag{28}\\
& =0.1020\left[\mathrm{~W}_{\mathrm{com}}+\mathrm{W}_{\mathrm{sci}}+\mathrm{W}_{\mathrm{p}}+\mathrm{W}_{\theta}+\mathrm{W}_{\mathrm{v}}+66.8\right] \mathrm{kg} \text {-mass }
\end{align*}
\]
because \(W_{\text {nav }}\), \(W_{\text {oa }}\), and \(W_{c p}\) are constants.
, Psi \((\psi)\) is the angle of the slope being traversed by the vehicle. The equations are not meant to apply in a time-varying sense, but rather try to describe the "average" performance of the vehicle. The average slope angle ( \(\psi_{\text {avg }}\) ) over a large distance will satisfy:
\[
\int_{0}^{\psi} \operatorname{avg} 0.17 \mathrm{e}^{-0.17 \mathrm{~s}} \mathrm{ds}=0.5 \int_{0}^{\mathrm{s}^{*}} 0.17 \mathrm{e}^{-0.17 \mathrm{~s}} \mathrm{ds}
\]
which implies \(\psi_{\text {avg }}=4.075-5.88 \ln \left(1+e^{-0.17 s *}\right)\) degrees. (29)

To describe internal heat dissipation and power uses at various times, the power-use profile must be established. The vehicle system will normally operate in one of four modes:
2. recharge
3. science and communication
4. minimal operation.

Minimal operation occurs during the period when communication between Earth and the vehicle is impossible due to the Mars-Earth configuration. At this time, only necessary functions (thermal control, navigation checks, computer control of system functions) and recharging are permissible. At all times, \(20 \%\) of total science power will be allotted toward maintaining ongoing science functions (sample treatment, monitoring experiments, ...). Thus, minimal operation power consumption, which also will be the internal heat dissipation for this period, is:
\[
\begin{equation*}
P_{c p}+0.5 P_{\text {nav }}+0.2 P_{\text {scia }}=Q_{i n} \tag{30}
\end{equation*}
\]

Likewise, the power profile for other modes is:
\[
\begin{equation*}
\mathrm{P}_{\mathrm{str}}=\mathrm{P}_{\mathrm{nav}}+\mathrm{P}_{\mathrm{cp}}+0.2 \mathrm{P}_{\mathrm{scia}}+0.1 \mathrm{P}_{\mathrm{com}} \tag{31}
\end{equation*}
\]
and
\[
\begin{equation*}
P_{\text {mv }}=P_{\text {nav }}+P_{c p}+P_{\text {oa }}+0.20 P_{\text {scia }}+0.1 P_{\text {com }} \tag{32}
\end{equation*}
\]
where the \(10 \%\) communication power allotment is for the continuous transmission of engineering data to Earth.

Internal heat dissipation during modes 1 thru 3 will approximately average:
\[
\begin{equation*}
\mathrm{P}_{\mathrm{cp}}+0.2 \mathrm{P}_{\mathrm{scia}}+\mathrm{P}_{\mathrm{nav}}+0.05 \mathrm{P}_{\mathrm{com}}+0.25 \mathrm{P}_{\mathrm{oa}}=\mathrm{Q}_{\mathrm{id}} \tag{33}
\end{equation*}
\]

Now, the 10 new equalities make the system model 33 equalities in 44 parameters.

It is necessary that the sume of all the subsystem volumes (excluding the vehicle) be less than the volume of the equipment package itself:
\[
\begin{equation*}
\sum \text { volumes of subsystems } \leq 2 \mathrm{v}_{\mathrm{v}} \tag{34}
\end{equation*}
\]

The requirement that the vehicle be able to support the weight of the other subsystems can be written:


The constraints identified so far represent real physical limitations upon the interrelationships of the parameters. These constraints are inherent to the system itself. External constraints, those placed upon the system by influences other than those which guarantee that the system will be physically realizable, have not yet been considered.

The cost of research, development, and construction of the system is a major factor, but it is outside the scope of this study. Another factor is the requirement that the system be deliverable to the surface of Mars. This imposes weight, volume, and size limitations on the vehicle system. They are:
\[
\begin{align*}
\sum \text { weights of all subsystems } & \leq L_{w}  \tag{36}\\
\sum \text { volumes of all subsystems } & \leq A_{v}  \tag{37}\\
1.7 \mathrm{w}_{\mathrm{b}} & \leq \mathrm{A}_{\mathrm{d}}
\end{align*}
\]
where \(L_{W}=\) maximum payload weight of launch vehicle
\[
A_{v}=\text { volume of aeroshell }
\]
and
\[
A_{d}=\text { horizontal diameter of aeroshell. }
\]

Any optimization process requires that the system performance be measureable with respect to some standard. When the expression of measure (hereafter called the objective, or objective function) is written as a function of the design parameters, the optimal design problem becomes one of choosing the design parameters to extremize (maximize or minimize) the value of the objective function while assuring that the parameters meet all the equality and inequality constraints of the system model.

The expression for system evaluation, i.e. the objective function, is generally not unique. There may be many different factors one would like to make large or small, each of which describes a different aspect of the system operation. Generally, it is good practice to attempt to incorporate all of the basic system functions into the objective.

An MRV has two basic functions:
1. rove the surface of the planet, and
2. obtain and transmit science data.

Note, that the second function is actually a combination of two functions, but that the system model groups these two together by considering science time as the time required to experiment and communicate the results.

The objective must express the ability of the vehicle to perform both of these functions concurrently. In formulating the objective function, one must be careful not to allow either of these measures to go to "zero." Allogical form, then, is to measure the system performance by the product of experimental science time and straight-line distance roved ( \(\mathrm{D}_{\text {rov }}\) ). That is, denoting the objective function as " f ":
\[
f=T_{\text {esci }} D_{\text {rov }}
\]

Define a cycle as comprising the activities between the ends of two recharges. The time in a cycle will then be the sum of the time spent roving, the time to recharge, and the total time spent on science and communication between recharges. The time spent on science and communication
in a cycle can be expressed as:
\[
T_{s c i / c y}=T_{s c i} S_{s c i} v_{f} T_{\text {rov }} h r,
\]
where \(S_{\text {sci }}=\) number of science stops per meter of actual distance traveled.

Now, \(E_{s t}\) can be written as:
\[
\begin{equation*}
E_{s t}=\left\{P_{n a v}+P_{c p}+\left[\frac{3 \times 10^{6}}{R_{c o m} T_{s c i}}\right] P_{i}+P_{s c i a}\right]\left[T_{s c i} S_{s c i} v_{f} T_{r o v}\right] \tag{39}
\end{equation*}
\]
which can be substituted into eqn. (9) to solve for \(\mathrm{T}_{\text {rov }}\) and eliminate \(E_{s t}\) from the model, if desired. The total time for a cycle ( \(T_{c y}\) ) will be:
\[
T_{c y}=T_{s c i} S_{s c i} v_{f} T_{r o v}+T_{r o v}+T_{r} h r .
\]

If \(V\) is the number of hours in a Martian day during which communication between the vehicle and Earth is possible, the number of cycles in a Martian day ( \(\mathrm{N}_{\mathrm{cy}}\) ) is:
\[
N_{c y}=\frac{V}{T_{c y}}
\]

Because the time spent communicating the science information back to Earth is non-productive in the sense that other vehicle activities must cease, it is reasonable to wish to deal with scientific experimentation time ( \(\mathrm{T}_{\text {esci }}\) ) instead of total science time. This time per cycle is:
\[
\mathrm{T}_{\text {esci/cy }}=\mathrm{T}_{\text {esci }} \mathrm{S}_{\text {Sci }} \mathrm{V}_{\mathrm{f}} \mathrm{~T}_{\text {rov }} .
\]

The straight-line distance roved in a cycle is:
\[
\mathrm{D}_{\mathrm{rov} / \mathrm{cy}}=\frac{\mathrm{v}_{\mathrm{f}} \mathrm{~T}_{\mathrm{rov}}}{\mathrm{PLR}}
\]
on an average "daily" basis then,
\[
f=T_{\text {esci/cy }} \quad D_{\text {rov/cy }}\left[\frac{V^{2}}{T_{c y}}\right]
\]
or in terms of the parameters of the system model:

The value of \(\mathrm{S}_{\mathrm{sci}}\) in the solution to the optimization problem will be part of the optimal operating policy for the vehicle. It will be the optimal manner of determining when the vehicle should stop for science
investigation. This number can be pre-programmed and will have the effect of maximizing the product of distance roved and experimentation time for a vehicle designed with parameters equal to those in the optimized solution.

Note that since \(V\) is not a variable in the problem (i.e., it may take on many values according to the Earth-Mars configuration, but for any run of the problem it is considered a constant, perhaps the average over, the mission lifetime) it has no effect upon the determination of the optimal design. Maximizing \(f\) is equivalent to maximizing \(f / v^{2}\). But also note that this is trus solely because of the form of the objective function, and it is possible that a different formulation for the system objective would result in the optimal design being dependent upon \(V\).

Consider a system composed of solely the communications and science subsystems as modeled in Section III. Suppose that this new system will arbitrarily be limited to a maximum weight of 100 kg and maximum power requirement of 75 watts. Let it be desired to maximize the quotient of experimental science time with total science time.

Maximizing the objective function requires letting experimental science time approach total science time. This is accomplished by making the data transmission rate high. However, increasing the data transmission rate forces the power and weight of the communication subsystem higher, and since both power and weight are limited, decreases the weight and power allocations to the science package. A decreased weight allocation to science decreases both of the science times, and the effect on their ratio depends upon the nominal values.

The preceeding paragraph demonstrates the difficulties involved: in attempting to optimize the system design without the use of some formal method. Methematically, the optimal system design problem becomes:
\[
\begin{aligned}
& \text { maximize } \frac{\mathrm{T}_{\text {esci }}}{\mathrm{T}_{\text {sci }}} \\
& \text { subject to: } T_{\text {esci }}=35.75 \mathrm{~W}_{\text {sci }}-135 \\
& \mathrm{P}_{\mathrm{SCi}}=3.44 \mathrm{~W}_{\text {sci }} \\
& T_{s c i}=35.75 \mathrm{~W}_{\mathrm{sci}}+\frac{3 \times 10^{6}}{R_{\text {com }}}-135 \\
& W_{\text {com }}=0.59 \mathrm{P}_{\mathrm{i}}+2.0 \mathrm{D}_{\text {com }}^{2 \mathrm{com}}+39.0 \\
& R_{\text {com }}=42.0 \mathrm{D}_{\text {com }}^{2} \mathrm{P}_{\mathrm{i}} \text {, }
\end{aligned}
\]
and: \(\quad W_{\text {sci }}+W_{\text {com }} \leq 100\)
\[
P_{s c i}+P_{i} \leq 75
\]
which is an NLP problem in 8 variables subject to 5 equality and 2 inequality constraints. In addition, there are the trivial constraints that all the variables be non-negative.

Using the equality substitution technique of Section II (see page 7), the problem can be rewritten in three variables as:
\[
\begin{aligned}
& \operatorname{minimize} \quad-f\left(x_{3}\right)=\frac{-\left(35.75 x_{1}-135\right)}{35.75 x_{1}+\frac{3 \times 10^{6}}{42 x_{2} x_{3}}-135} \\
& \text { subject to } g_{1}\left(\underline{x}_{3}\right)=61-x_{1}=2 x_{2}=0.59 x_{3} \geq 0 \\
& g_{2}\left(\underline{x}_{3}\right)=75-3.44 x_{1}-x_{3} \geq 0 \\
& \text { where } \\
& x_{1}=W_{\text {sci }} \\
& x_{2}=D_{\text {com }}^{2} \\
& x_{3}=P_{i}
\end{aligned}
\]
which is in the NLP format and is ready for computer solution.
Solution of this problem was accomplished by utilizing the sequential unconstrained minimization technique (SUMT) of Fiacco and McCormick (Ref. 53). The optimal design is characterized by:
\[
\begin{aligned}
& W_{\text {com }}=85.965 \mathrm{~kg} \quad W_{\text {sci }}=14.035 \mathrm{~kg} \\
& P_{i}=26.720 \text { watts } \quad P_{\text {sci }}=48.263 \text { watts } \\
& D_{\text {com }}=13.950 \text { misiono } \quad T_{\text {sci }}=537.490 \mathrm{sec} \\
& R_{\text {com }}=17.494 \mathrm{Kbits} / \mathrm{sec} \quad \mathrm{~T}_{\text {esci }}=366.573 \mathrm{sec}
\end{aligned}
\]
which maximizes the time ratio at 0.68155 . Note that both constraints are "active" (the equality is taken on at the solution point). That is, the optimization process makes full use of all available resources.

It is proposed that optimization of the MRV system be carried ou't in a manner similar to the above example. For the MRV, the problem willbee formulated as maximizing \(f\) (eqn. 40) subject to the 31 equalities of equations (1), (3)-(5),(7)-(21),(23), and (39) and the inequalities of expressions (35),(36), and (38). All relations concerning volumes will be discarded for the first run. The reasons for this are that the model is incomplete in terms of description of subsystem volumes, and that the weight constraints appear to be the critical ones. If, however, the values
of the design parameters at the optimal solution violate the volume inequalities, they will be reinserted along with the complete model and the problem will be resolved.

The number of design parameters to be determined by the process is 42 (the 43 variables of Table 8 , minus the two volumes, plus the operating factor \(S_{\text {sci }}\); i.e., now \(P_{\text {scia }}\) is considered a constant and equation (6) is dropped). The problem is of order \(11(42-31=11)\). The only remaining: decision is to choose the launch vehicle, which will establish values for the right side of inequalities (36) thru (38). Current belief is that some sort of Titan III-C configuration will be utilized for the mission (Refs. 3, 13, 24), but the choice of launch vehicle may be one of the inftial assumptions that must be varied in succeeding runs.

The feasibility of the proposed method of optimal system design is contingent upon obtaining a solution to the NLP problem resulting from system modeling and the determination of the system objective function. The NLP problem has no known closed-form solution and the iterative techniques to date do not guarantee locating the optimum. The approach considered "best" (SUMT) has been examinied and tested. The tests show that attaining a solution is a function of the specific problem, and while the approach works most often, there are some problems for which the iterative technique does not converge. The approach generally has more difficulties with equality than inequality constraints.

The system model generated by the work reported here is:not restricted to use by an optimal system design process. The model itself is a useful tool for the system design problem with the optimality consideration discarded. Being able to describe the parameter interrelationships before one attempts to pick values for some or all parameters is an obvious aid foward designing' a physically reảizable system which will meet any external constraints placed upon it.

While the optimal design process may give a solution to the problem as it is presently formulated, changing any of the initial assumptions made will invalidate the optimal property of the obtained solution. For each set of assumptions there will probably be changes in the system model, and there will almost definitely be a new and different optimal solution. If solutions can be generated corresponding to all major sets of assumptions, they can be compared (and perhaps weighted by cost and time constraints not directly included in the model) so as to locate a solution considered optimal independent of assumptions.

Future work will be directed toward the following areas:
1. continuation of the present effort; locating the optimal design for the present model; changing assumptions, the
system model and objective, and finding the optimal solution for all cases.
2. in parallel: with the above, study the problem of sensitivity of the optimal design to perturbations in the model and in the design parameters themselves.
3. a comprehensive analysis of onboard computation and datahandling requirements as functions of the capabilities and activities of the vehicle system; such matters as the effect of the autonomous vs. earth-control trade-off upon the computer, and the corresponding effects on other subsystems, will be studied.
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