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An optimization procedure to design a Minibus feeder service: an application to the Sintra rail line

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Abstract

This paper presents the formulation of a heuristic encompassing a mixed integer linear program to model the synchronisation between a minibus feeder service and a mass transit system. Our formulation is divided into three main steps: the establishment of stops, scheduling and the fleet dimensioning.
This model was tested in a dense suburban area around a communting rail line in the Lisbon Metropolitan Area. The results show a high potential of the tested service to feed the suburban rail service with gains for the passengers and the establishment of an economically self-sustainable service.

Keywords: mixed integer linear program, rail feeder service, optimisation.

1. Introduction

Major urban areas face, nowadays, considerable mobility problems derived from massive urbanisation processes, also linked with urban sprawl phenomenon. This fact generates increasing externalities in urban areas, especially in highly motorised and car dependent regions.

The large volume of daily commuting trips in a sparsely occupied territory, linked with employment concentration in the city centre produces severe congestion problems during traffic peak periods. This situation leads to low efficiency mobility outputs and substantial space consumption by parked private cars in the streets.

The Lisbon Metropolitan Area (LMA), as other capitals in Europe, faces daily these problems, almost doubling the present population in the city centre during the conventional working period.

One of the main residential areas along the Northwest area of the LMA that generates an intense commuting traffic is commonly designated as the Sintra corridor, composed by two large municipalities: Sintra and Amadora.

This area is served by the busiest suburban railway line in Europe, transporting daily, about, 200,000 passengers. This rail line is linked to several terminal stations at Lisbon, providing connections in the inner city to all subway lines (Entre Campos station – yellow line; Rossio Station – blue and green line; Areeiro – green line; Sete Rios – blue line and Oriente – red line), and to large residential and employment areas (i.e. Benfica and Campolide).

The natural catchment area of this rail line does also present an untolled direct freeway to Lisbon, which makes the private vehicle the main commuting mode. The abusive use of the private vehicle relies on the biased peoples' perception towards this mode travel time, always regarded as faster. Yet, situations where the use of a combination of train and subway outperform the private car can be easily found.

Furthermore, other barriers to a more general use of this rail line as main commuting alternative are related to the low accessibility of the train stations from their catchment area. Although some of the stations present park & ride services, the parking capacity, usually, is not enough to accommodate all the potential demand, limiting the attractiveness of this combination. Moreover, the majority of the train stations do not have a fast, direct and reliable public transport connection, which leaves walking as the main alternative to reach the stations, diverting many people to consider the rail service as their main commuting option.

The implementation of a fast feeder service to the train stations along the Sintra line may be perceived by private vehicle commuters as a significant improvement on the overall public transport alternative performance, and lead a significant share to shift to this new alternative. This would reduce the number of private vehicles circulating every day, during peak hour, towards Lisbon.

The service that we propose is envisaged as the first leg of an intermodal chain that also includes rail and subway. It will make use of low capacity buses (minibuses), picking-up users at pre-established stops and drop them at a train station in the Sintra Line.

The design of the feeder's routes and schedules will take into consideration three different aspects:

- it will guarantee that the customer will arrive to the final destination in a time window defined by his current time of arrival and a maximum of 15 minutes before,
- the compatibility with the train's timetables, at each station, where a customer should arrive in a specific time-window (3 to 10 minutes prior to the train's departure);
- and a non-universal demand coverage.

This paper will be divided in 6 main topics: after this small introduction, we will make a brief literature review, focusing on the research that has been done when modelling feeder services; afterwards, a description of the model framework will be presented with the specification of each intermediate step and the presentation of the obtained results; the paper ends with some conclusions and further developments.

2. Literature review

Aiming at reducing the share of private vehicles in large urban areas around the world, public transport authorities have been promoting for decades integration of public transport system to promote public transport accessibility and affordability, as well as creating a quality alternative to private car trips . One of the key elements of this integration is the definition of a public transport services hierarchy that may allow identifying feeder and trunk services for the different origin destination connections (May, Kelly, & Shepherd, 2006).

The coordination of feeder services with mass transit alternatives (e.g. commuter rail, Bus Rapid Transit) has been studied thoroughly in the public transport literature, especially using small capacity vehicles as the Minibus (Bellini, Dellepiane, & Quaglierini, 2003; Brake, Mulley, Nelson, & Wright, 2007; Brake, Nelson, & Wright, 2004; Hidalgo & Graftieaux, 2008).Some authors have been introducing optimisation solutions to design these feeder bus services using Mixed Integer Linear Programming (MILP) approaches.

There are several different formulations present in the literature, some focusing more in the coordination, optimising simultaneously the bus routing and scheduling and the train timetables at a specific station (Chien & Schonfeld, 1998; Shrivastava & O'Mahony, 2006; Verma & Dhingra, 2006), while others considered the timetable of the high capacity mode as a constraint to which the minibus feeder service has to adjust (Mohaymany & Gholami, 2010; Wirasinghe, 1980).

Chowdury & Chien (2002) focused on the estimation of the optimal headways for transit routes and slack times for coordinated routes of an intermodal transit system (bus and rail). In their formulation, the supplier, the user, the wait, the transfer and the in-vehicle costs are all taken into consideration. The model consists on a four stage procedure that starts by determining the optimal headways without considering coordination and ends with the rail-bus coordination for multiple transfer stations.

The inspection of the literature allowed a more comprehensive formulation of the optimisation problem that this research aims to tackle. The developed methodology, building on previous research, introduces some innovative components discussed in the next section.

3. Methodology

The formulation of the proposed feeder service includes several aspects that should be jointly addressed for a correct and efficient definition of the service, namely: stops, services, schedules and fleet dimensioning.

To achieve this goal, the model was formulated in three different stages: a first stage, which analyses the potential demand of the system, focusing on current private car commuters as the main target; a second stage that will define the best configuration of services to accommodate the potential demand; and a last phase that estimates the best combination of services, minimising the total fleet needed to operate the system.

In the demand estimation phase, a potential clients database was generated through a synthetic travel simulation model developed and calibrated for the LMA, presenting all the trip extremes discretised both in space (at the census block level) and in time (different trip departure and arrival times) (Viegas & Martínez, 2010). This dataset of approximately five million trips was then filtered to reduce it to a subset of suitable trips to the proposed service, bearing in mind that the service was not only designed mainly to divert current private car drivers, but also to improve the quality of the service to current transit users. The filters used in this original dataset were:

- Trips performed during the morning peak period (6:00-10:00 am);
- Trips towards Lisbon and with origins outside Lisbon;
- Trip origins located less than 5,000 m away from a train station of the Sintra line and not closer than 15 minutes walking to any train station (using a 4.0 km/h walking speed);
- Due to the possible interactions with another rail line (Cascais rail line) and the subway system, all trips with origins closer to these stations were removed;
- Total walking time of the trip (access to Minibus stop and from station to work) less than 15 minutes (using a 4.0 km/h walking speed).

After this initial filter, we also estimated the expected total travel time of the proposed service, considering all the legs of the intermodal chain, in order to remove the potential clients that had a lower reported travel time. Due to the diversity of possible train stations, each potential client was associated with the one that had the minimum travel time until its final destination, considering two possible travel options: either walking or using the subway to reach its destination. This resulted in an initial potential demand of the service of 22,061 trips. This demand was used as input for the next stages of the model.

The second phase intends to define the most favourable configuration of services (stops, routes and schedules), constrained to the spatial-temporal attributes of each potential client. This problem falls within the framework of the location-distribution problem, which presents a high combinatorial nature (NP-Complete), usually split into a two steps approach (Nagy & Salhi, 2007).

The formulation of the first step was based in the classical p-median problem, widely used for discrete facility location problems (Teixeira & Antunes, 2008). The used formulation also established that all the potential clients should be located, at most, 500 meters away from a stop (approximately 8 minutes walking). In this step, we wanted to guarantee a minimum number of stops that led to a feasible solution where the distance between stops and the origin of the trip was the shortest possible.

For the definition of the feeder services, we used a traditional Vehicle Routing Problem (VRP) formulation encompassed into a greedy heuristic that intended to maximise the profit of each minibus service respecting all the operational parameters specified above. Moreover, due to the flatness of the objective function, further simplifications had to be introduced to reduce the computational time:

1. **Aggregation of train services** – the Sintra rail line presents two possible services that have different terminal stations at Lisbon. To reduce the problem complexity, the train schedules of the two services were aggregated into a virtual average arrival time at the Benfica station (the train station where the services divert) for trains circulating within the same 5.25 minutes time-window. From this station on, we considered an average travel time between each pair of stations to estimate the arrival time at the stations inside Lisbon. This procedure resulted in the establishment of several "families" of trains that had an average departure time from each station and that were connected to all the train stations of the Sintra network, allowing each user to virtually arrive at each possible station within Lisbon;
2. **Adoption of a heuristic to establish the services** – the large number of possible initial train stations, at each model iteration, results in similar objective function values that increase the convergence complexity of the model, leading to significant large gaps after 1,500 seconds of computation. This problem was bypassed by pre-estimating the potential of each train station to generate minibus services, based on the number of clients within each station influence area. This potential was then used, on each iteration, to randomly generate a destination station and evaluate the creation of a minibus service.

The final stage, as already mentioned, consisted in the combination of services that optimised the fleet required to operate the system. Having defined all the profitable services, accounting for fixed and variable vehicle costs, with the corresponding departure and arrival points, as well as the schedules, it was possible to define the optimal combination of services that respected all the spatial and temporal constraints.

The costs of the service were estimated having as reference data from Minibus' operators in the LMA (EasyBus, Barraqueiro), while the system revenue, was based on the taxi fare system, where a user is charged by a fixed amount plus a variable fare dependent on the distance travelled, and the time lost in congestion during the travel (travel time at speed under 30 km/h).

The next sections of the paper present the mathematical formulation of each sub-model and the discussion of the results from their application to the case study.

4. Routing design

4.1. Location of stops

The stop location and user assignment problem formulation was based on a traditional capacitated p-median problem, aiming to minimise the total walking distance of users to stops, given a maximum walking time until that stop. The mathematical formulation of the problem is the following:

Sets: $P = \{1,\dots,C\}$ set of all the residential location of users , where C stands for the total number of users; $N = \{1,\dots,K\}$ set of all the possible stops corresponding to the census block centroids where there is at least one customer, where K is the maximum number of nodes.

Decision variables: Y_j: binary variable for the existence of a stop in node $j \in N$; X_{ij}: binary variable that sets the assignment of user $i \in P$ to the stop created in node $j \in N$;

Data: D_{ij}: matrix that represents the Euclidean distance between user $i \in P$ and the potential stop $j \in N$, considering a sinuosity index.

Constants: d_{max}: maximum distance that is acceptable for a user to walk from their home to the stop; p_{max}: maximum number of stops that the system can generate.

With this notation, the objective function is described by the following expression:

$$Min(WD) = \sum_{i \in \mathrm{P}, j \in \mathrm{N}} D_{\mathrm{ij}} X_{\mathrm{ij}} \tag{1}$$

where the goal is to minimise the total distance between the users and their stop.

This solution space is subject to the following constraints:

$$\sum_{j \in \mathrm{N}} X_{ij} = 1 \quad \forall i \in \mathrm{P} \qquad X_{ij} \le Y_j \quad \forall i \in \mathrm{P}, \forall j \in \mathrm{N} \qquad \sum_{j \in \mathrm{N}} Y_j \le p_{max} \tag{2}$$

Ensures that each user is assigned only to an existent stop, and that the total number of stops generated is not greater than the established maximum number of stops;

$$X_{ij} \times D_{ij} \le d_{max} \quad \forall i \in \mathrm{P} \tag{3}$$

Ensures that the distance of each user to the bus stop does not exceed the maximum established value.

The model was computed for the available dataset of potential clients, using as parameters: $\mathrm{p_{max}} = 147$ stations and $\mathrm{d_{max}} = 500$ m.

The results obtained in this phase are summarised in Table 1, where we can denote that, in average, each passenger will walk 3.5 minutes to the closest minibus stop.

Table 1 Results from the computation of stops

Indicator	Average	Maximum
Number of people per stop	150.07	600
Distance until the stop [meters]	231.09	499.38

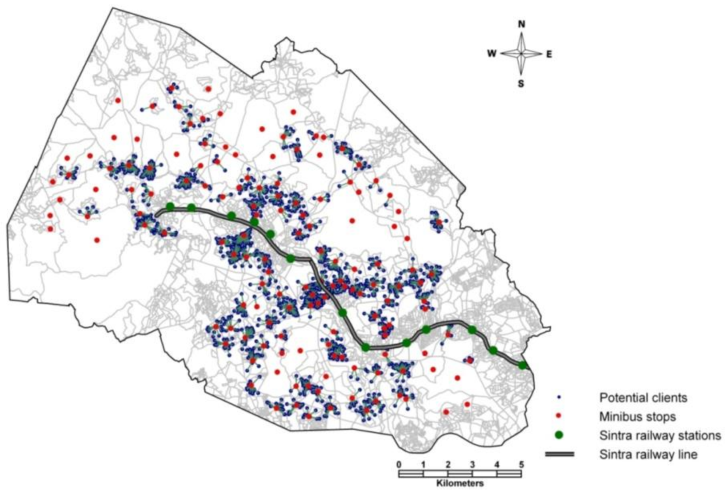


Figure 1 Minibus stops distribution

Analysing the spatial distribution of the demand and the location of minibus stops along the Sintra commuter rail line (see Figure 1), we can identify a rather good coverage. Nevertheless, the created stops with more clients aggregated are located not too far from the rail line. This fact might have an impact on the estimated services in the following stages of the model, where short services will tend to be more profitable.

4.2. Definition of services

As discussed above, this step of the second phase used a multi-vehicle routing problem formulation including the stops' scheduling, a key component to warrant the standards of the service. The algorithm intends to maximise the operational profit of the system.

The developed model, due to its NP-Completeness, was formulated using a greedy approach, where each service is generated independently, although, constrained by the reduction of clients already served on previous iterations.

The mathematical formulation of the problem is the following:

Sets: S = {1,…,C} set of all the bus stops where users are picked up, where C is the total number of stops; T = {'Benfica', 'Santa Cruz/Damaia',…, 'Sintra'} set of all the train stations that are possible final destinations (a total of 11 train stations); N = {1,…,C, 'Depot', {T}} set of all the bus stops where users are picked up, plus the initial and final stops (depot and train stations); P = {1,…,C, {T}} set of all the bus stops where users are picked up, plus the final stops (train stations); Np = {1,…,J} set of all the location of clients, where J is the total number of clients; O = {1,…,C, 'Depot'} set of all the possible origins of trips; V = {1,…,K} set of the available vehicles to use in the routing problem; HT = {1,...,Ft} set of all the possible train time schedules were Ft represents the maximum number of trains for station t ϵ T.

Decision variables: B_{ijk}: binary variable that identifies if i ϵ N precedes j ϵ N in the route of vehicle k ϵ V; Z_{ik}: binary variable that represents if customer i ϵ NP is allocated to vehicle k ϵ V; H_{ik}: real variable that defines the pick-up/drop-off time in stop i ϵ N by vehicle k ϵ V, including the depart time from the depot; St_{ijk}: binary variable that sets whether a station i ϵ T at time j ϵ HT was used by vehicle k ϵ V or not.

Data: D_{ij}: matrix that represents the travel time estimates during the peak hour to travel from stop i to stop j, i,j ϵ N; WT_i: vector that represents the time that a user i ϵ Np takes to reach his/her assigned stop; $tmin_i$: vector that represents the minimum acceptable time for a customer i ϵ Np to arrive at Benfica; $tmax_i$: vector that represents the maximum acceptable time for a customer i ϵ Np to arrive at Benfica; $Speed_{ij}$: matrix that represents the average speed during the peak traffic hour between stop i and stop j, i,j ϵ N; $TBenf_i$: vector that represents the total travel time by train from station i ϵ T to Benfica; $TRAIN_{ij}$: train departure time of schedule j ϵ HT at station i ϵ T; U_i: vector that represents the stop that is associated to the client i ϵ Np.

Constants:$Tariff_{fixed}$: value to be charged in the fixed part of the service fare; $Tariff_v$: value to be charged to the user per kilometre travelled; cost: value of the cost per kilometre travelled; cap: capacity of the Minibus.

With this notation, the objective function is described by the following expression:

$$Max(Balance) = \sum_{i \in \mathrm{Np},\ k \in \mathrm{V}} \mathrm{Z_{ik}} \times Tariff_{fixed} + \sum_{i \in \mathrm{Np},\ k \in \mathrm{V}} Z_{ik} \times Tariff_v \times D_{U_i j} \times Speed_{U_i j} - \sum_{i,j \in \mathrm{P},\ k \in \mathrm{V}} B_{ijk} \times D_{ij} \times cost \times Speed_{ij} \tag{4}$$

The objective function computes the estimated profit of each service, as stated above.

This solution space is subject to the following constraints:

$$\sum_{i \in \mathrm{N},\ j \in \mathrm{T}} B_{ijk} = 1 \ \ \forall k \in \mathrm{V} \quad \sum_{j \in \mathrm{N}} B_{Depot,j,k} = 1 \ \ \forall k \in \mathrm{V} \tag{5}$$

Ensures that all the vehicles start at the depot and end at one station;

$$\sum_{i \in \mathrm{O}} B_{ijk} \leq 1 \ \ \forall j \in \mathrm{P}, k \in \mathrm{V}, \mathrm{i} \neq \mathrm{j} \qquad \sum_{j \in \mathrm{P}} B_{ijk} \leq 1 \ \ \forall i \in \mathrm{O}, k \in \mathrm{V}, \mathrm{i} \neq \mathrm{j} \tag{6}$$

Guarantees that each bus does not enter or leave the same stop more than once;

$$\sum_{j \,\epsilon\, \mathrm{P}} B_{ijk} = \sum_{\mathrm{j} \,\epsilon\, \mathrm{O}} B_{jik} \quad \forall i \,\epsilon\, \mathrm{S}, k \,\epsilon\, \mathrm{V} \qquad B_{ijk} + B_{jik} \leq 1 \; \forall i, j \,\epsilon\, \mathrm{N}, k \,\epsilon\, \mathrm{V} \tag{7}$$

Ensures the continuity of flow in intermediate stops avoiding the formation of cycles;

$$\sum_{i \,\epsilon\, \mathrm{T},\; \mathrm{j} \,\epsilon\, \mathrm{HT}} St_{ijk} = 1 \; \forall \mathrm{k} \,\epsilon\, \mathrm{V} \qquad \mathrm{B_{ijk}} \times \mathrm{D_{ij}} + \mathrm{M} \times (1 - \mathrm{B_{ijk}}) \geq 500 \; \forall \mathrm{i} \,\epsilon\, \mathrm{S}, \mathrm{j} \,\epsilon\, \mathrm{T}, \mathrm{k} \,\epsilon\, \mathrm{V} \tag{8}$$

Assures that each vehicle is linked to one train schedule at one of the possible stations, and the closest stop of the Minibus is further than 500 meters. This constraint is only active if there is a direct path between i and j, otherwise a relaxation to this constraint is introduced by parameter M set with a high value;

$$\sum_{i \,\epsilon\, \mathrm{S}} B_{ijk} \geq \sum_{h \,\epsilon\, \mathrm{HT}} St_{jhk} \quad \forall j \,\epsilon\, \mathrm{T}, k \,\epsilon\, \mathrm{V} \tag{9}$$

Ensures that a time schedule can only be selected if the Minibus arrives to that station.

$$H_{jk} \geq H_{ik} + B_{ijk} \times D_{ij} + \left(\frac{19 + 2.6 \times \sum_{p \,\epsilon\, \mathrm{NP}} Z_{pk}}{3600}\right) - M \times \left(1 - B_{ijk}\right) \qquad \forall i \,\epsilon\, \mathrm{S}, j \,\epsilon\, \mathrm{O}, k \,\epsilon\, \mathrm{V}, i \neq j, U_p = j \tag{10}$$

$$H_{jk} \leq H_{ik} + B_{ijk} \times D_{ij} + \left(\frac{19 + 2.6 \times \sum_{p \,\epsilon\, \mathrm{NP}} Z_{pk}}{3600}\right) + M \times \left(1 - B_{ijk}\right) \qquad \forall i \,\epsilon\, \mathrm{S}, j \,\epsilon\, \mathrm{O}, k \,\epsilon\, \mathrm{V}, i \neq j, U_p = j \tag{11}$$

Guarantees that the pick-up time of one stop is equal to the pick-up of the previous stop plus the travel time between the two stops, deceleration, acceleration and boarding of the customers in the current stop. Where the values of time losses are discussed in (Braca, Bramel, Posner, & SimchiLevi, 1997); and where M guarantees the linearity of the constraint, as explained above;

$$H_{jk} \geq H_{ik} + B_{ijk} \times D_{ij} + 0.02 - M \times \left(1 - B_{ijk}\right) \forall i \,\epsilon\, \mathrm{S}, j \,\epsilon\, \mathrm{T}, k \,\epsilon\, \mathrm{V}, i \neq j \tag{12}$$

$$H_{jk} \leq H_{ik} + B_{ijk} \times D_{ij} + 0.02 + M \times \left(1 - B_{ijk}\right) \forall i \,\epsilon\, \mathrm{S}, j \,\epsilon\, \mathrm{T}, k \,\epsilon\, \mathrm{V}, i \neq j \tag{13}$$

Ensures that, at the final destination, the drop-off time is equal to the previous pick-up time plus the travel time from the previous stop, deceleration, acceleration and the exiting of the clients considering, as a simplification, that the Minibus is full;

$$H_{jk} - (H_{U_i k} - WT_i) \leq t_{max} + M \times \left(1 - \sum_{d \,\epsilon\, P} B_{U_i dk}\right) \forall i \,\epsilon\, N, j \,\epsilon\, T, k \,\epsilon\, V \tag{14}$$

Assures that the total service time, including the walking time from the user's home until the stop, is less than the maximum service time. Where M is a big number that guarantees the linearization of the constraint as explained above;

$$H_{Depot,k} \geq H_{ik} + B_{Depot,i,k} \times D_{Depot,i} - M \times \left(1 - B_{Depot,i,k}\right) \forall i \,\epsilon\, \mathrm{S}, k \,\epsilon\, \mathrm{V} \tag{15}$$

Guarantees that the Minibus leaves the depot on time to reach the first stop;

$$Z_{ik} \leq \sum_{j \,\epsilon\, \mathrm{P}} B_{U_i jk} \; \forall i \,\epsilon\, \mathrm{Np}, k \,\epsilon\, \mathrm{V} \tag{16}$$

Ensures that a customer only enters the Minibus if the service includes the corresponding stop;

$$-\frac{10}{60} - M \times \left(1 - St_{ijk}\right) \leq H_{ik} - TRAIN_{ij} \leq -\frac{3}{60} + M \times \left(1 - St_{ijk}\right) \qquad \forall i \,\epsilon\, \mathrm{T}, j \,\epsilon\, \mathrm{HT}, k \,\epsilon\, \mathrm{V} \tag{17}$$

Guarantees that the Minibus arrives at the train station between 3 and 10 minutes before the train's departure. M is set as a big value and warrants that this constraint is only active when the train at hour j at station i is the destination of that Minibus;

$$H_{jk} \geq \left(tmin_i - TBenf_j\right) - M \times (1 - Z_{ik}) \forall i \in \mathrm{Np}, j \in \mathrm{T}, k \in \mathrm{V} \quad (18)$$

$$H_{jk} \leq \left(tmax_i - TBenf_j\right) + M \times (1 - Z_{ik}) \forall i \in \mathrm{Np}, j \in \mathrm{T}, k \in \mathrm{V} \quad (19)$$

Assures that the individual time of arrival constraints at Benfica are satisfied;

$$\sum_{i \in \mathrm{Np}} Z_{ik} \leq cap \ \ \forall k \in \mathrm{V} \quad (20)$$

Guarantees that the clients transported by the Minibus do not exceed its capacity;

$$\sum_{j \in \mathrm{S},\ j \in \mathrm{N}} B_{ijk} \geq 1 \ \ \forall k \in \mathrm{V} \qquad \sum_{j \in \mathrm{S},\ j \in \mathrm{N}} B_{ijk} \leq 5 \ \ \forall k \in \mathrm{V} \quad (21)$$

Ensures that a service must have at least one intermediate stop and a maximum of five.

The model was computed using the results from the previous phase and the following parameters: $\mathrm{Tariff_{fixed}} = 0.602$ €/pax ; $\mathrm{Tariff_v} = 0.71$ €/km ; $\mathrm{cost} = 0.201$ €/km ; $\mathrm{cap} = 24$; $t_{max} = 30\ minutes$.

The generated services were able to capture 19,864 potential clients (90.04 percent of the initial clients' dataset), from which 6,036 were private car users (30.39 percent).

The characteristics of the obtained services are summarised in Table 2, where services averagely present approximately 3 boarding stops, although 28.64 percent of them are direct connections to train stations with reduced service times. The positive average travel time savings also confirms our initial hypothesis that this new service could be faster than the private car commuting, which might attract a significant share of drivers to this service.

Table 2 Results from the computation of services

Number of generated services	993
Average number of stops	2.64
Average service time [min]	15.31
Average number of transported clients	20.00
Average percentage of stop time	13.47%
Average travel time savings [min]	17.98

5. System fleet dimensioning

After the definition of the best configuration of services for the existing travel demand in the Sintra rail line during the morning peak period, we developed an algorithm to aggregate services into routes, in order to perform the fleet dimensioning of the system for the set operation time (6:00-10:00 am). The objective of the algorithm is to merge into the same route, services that demand similar levels of vehicle capacity, whose trip extremes, to be connected, are not located far from each other, and that the time lost between the operations of both services is as small as possible. This complex objective was defined as a global profit of the system, which accounts for the revenues from the fares of the services, the costs of the vehicle acquisition and use, the time lost between services, and the distance travelled between the points where the service takes place.

The mathematical formulation of the problem is the following:

Sets: N = {1,…,C} set of all the available nodes that are origins or destinations of services, where C is the maximum number of nodes; A = {1,…,K} set of all the available arcs that represent a services between nodes where K is the maximum number of arcs; B={ 8, 16, 24} set of the capacity of vehicles considered in the operation of routes of the service.

Decision variables: M_{ij}: binary variable that identifies if service j ϵ A is preceded by service i ϵ A.

Data: D_i: vector that defines the numbers of customer assigned to service i ϵ A; $Time_{ik}$: vector that defines the departure and arrival time of service i ϵ A, where k=1 for the departure time and k=2 for arrival time; TT_{ij}: matrix that represents the travel time estimates during the peak traffic hour to travel from node i to node j, i, j ϵ N; $DIST_{ij}$: matrix that represents the road network distance from node i to node j (Euclidean distance × sinousity index), where i, j ϵ N; $Costf_b$: fixed cost of operation of the Minibus of capacity type b ϵ B; $Costv_b$: variable cost of each kilometre travelled by each Minibus of capacity type b ϵ B.

Constants: t_{max}: maximum waiting time allowed between two consecutive services performed by the same vehicle; cap: maximum capacity difference between two services aggregated into the same route; $Tariff_{fixed}$: fixed component of the ticket price charged to the passengers for a single trip; $Tariff_{km}$: variable fee charged to the user by kilometre travelled; $Tariff_{time}$: variable fee charged to the user by travel time.

With this notation, the objective function is described by the following expression:

$$\begin{aligned} &Max(Operational\ Profit) = \\ &\left[D_i \times \sum_{i\,\epsilon\,A}\left(Tariff_{fixed} + \left(Tariff_{km} \times DIST_{A_{i1}A_{i2}}\right) + Tariff_{time} \times \left(Time_{i2} - Time_{i1}\right)\right)\right] \\ &\qquad - \left[\sum_{i\,\epsilon\,A}\left(1 - \sum_{j\,\epsilon\,A} M_{ij}\right) \times Costf\left[ceiling\left({}^{D_i}/_{8}\right)\right]\right. \\ &\qquad \left. + \sum_{i\,\epsilon\,A}\left(DIST_{A_{i1}A_{i2}} + \sum_{j\,\epsilon\,A} DIST_{ij} \times M_{ij}\right) \times Costv\left[ceiling\left({}^{D_i}/_{8}\right)\right]\right] \end{aligned} \tag{22}$$

The objective function computes the total operational profit of the aggregated services, considering all the costs and profits described above.

This solution space is subject to the following constraints:

$$\sum_{j\,\epsilon\,A} M_{ij} = 1 \quad \forall i\,\epsilon\,A \qquad \sum_{j\,\epsilon\,A} M_{ji} \leq 1 \quad \forall i\,\epsilon\,A \tag{23}$$

Ensures that all the services are connected with another service, at least with itself (single service route).

$$Time_{j1} \geq \left(Time_{i2} + TT_{ij}\right) \times M_{ij} - L \times \left(1 - M_{ij}\right) \forall i,j\,\epsilon\,A \tag{24}$$

Ensures that the departure time for service *j* is greater than, the arrival time of service *i* plus the travel time between these location, when they are aggregated in the same route as consecutive services. The constant L, which is a large number, guarantees that this constraint is only active when M_{ij} is greater than zero.

$$\left(Time_{j1} - Time_{i2} - TT_{ij}\right) \times M_{ij} \ \leq\ t_{max}\ \forall i,j\,\epsilon\,A \tag{25}$$

Ensures that the linkage time (time waiting to perform the next service) between two consecutive services of the same route is smaller than a maximum established time.

$$\left|D_j - D_i\right| \times M_{ij} \leq\ cap\ \forall i,j\,\epsilon\,A \tag{26}$$

Guarantees that the difference of demand of two services grouped into the same route is smaller than a maximum difference for services of the same route.

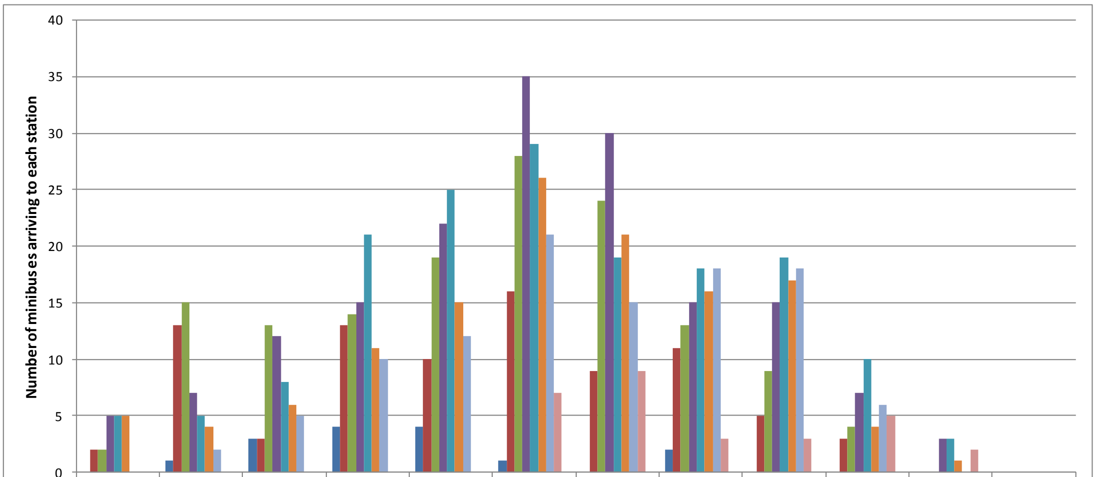
The model was computed using the results from the previous phase and the following parameters: $t_{max} =$ 1.0 h; cap = 8; $Tariff_{fixed} = 0.75$ €/pax; $Tariff_{km} = 0.168$ €/km; $Tariff_{time} = 0.077$ €/h.

Filtering the resulting profitable vehicles, we obtain 142 routes that generate a total profit of 10,257.09 €. The average estimated profit is 72.23 €/minibus with a standard deviation of 52.03 €, which indicates that there is a high uncertainty on the expected profit by vehicle. Considering only the profitable routes, the system would transport 17,195 passengers, approximately 90 percent of the original demand.

The average operational indicators by minibus of the system show a profitable operational time of 1.37 hours, a total loading of 127.32 passengers, and operating 5.85 services during the morning peak period. The estimated fleet composition of the system is presented in Table 3, where we can denote a prevalence of 24 seats' vehicles, which do also tend to be more profitable derived from the high ridership obtained (21.76 passengers), resulting in load factors above 55 percent, close to 100 percent for the 24 seats vehicles.

Table 3 Summary of the operation indicators for the different vehicle types in estimated fleet

Type of minibus	No. of routes generated	No. of passengers	Pax/(veic.MPP)	Services	Load factor	Profit/veic. [€/veic.]
8	4	84	21.00	15	70.00%	6.58
16	16	709	44.31	76	58.31%	10.87
24	122	17,286	141.69	740	97.33%	82.43



	Sintra	Portela de Sintra	Algueirão-Mem Martins	Mercês	Rio de Mouro	Agualva-Cacém	Massamá-Barcarena	Monte Abraão	Queluz-Belas	Amadora	Reboleira	Santa Cruz/Damaia
06:00	0	1	3	4	4	1	0	2	0	0	0	0
06:30	2	13	3	13	10	16	9	11	5	3	0	0
07:00	2	15	13	14	19	28	24	13	9	4	0	0
07:30	5	7	12	15	22	35	30	15	15	7	3	0
08:00	5	5	8	21	25	29	19	18	19	10	3	0
08:30	5	4	6	11	15	26	21	16	17	4	1	0
09:00	0	2	5	10	12	21	15	18	18	6	0	0
09:30	0	0	0	0	0	7	9	3	3	5	2	0
10:00	0	0	0	0	0	0	0	0	0	0	0	0

Figure 2 Summary of the services operated by station and time period

A summary of the system operation profile during the morning peak, feeding the rail station until reaching the Lisbon border is presented in Figure 2. The defined operation intervals of 30 minutes show that the main destinations of the feeder services are Agualva-Cacém and Massamá-Barcarena stations (290 services), located inside the most dense residential area of the Sintra corridor. Other relevant stations are Rio de Mouro (107 services) and Monte Abraão (96 services). As expected, the busiest time periods are between 7:30-8:00 am and 8:00-8:30 am, with more than 162 services operated in each 30 minutes period, resulting in, approximately, 12 minibuses per train service. We should also acknowledge the existence of some cases where the number of buses in the same station, at the same time period, is very high (more than 30 services), which would require a detailed design of the parking areas of buses outside the stations, in order to avoid bunching effects of buses loading and unloading passengers.

It is worth pointing out that, in average, the minibuses have 38.16 percent of unprofitable time, where they are either travelling from the end of one service to the other or waiting for its starting time, owing to the chaining of the high number of short estimated services.

6. Conclusions

The modelling framework presented in this paper showed that it is possible to model, efficiently and in detail, the several phases of the design process of a dedicated rail feeder service.

The large dimension of the study area required several simplifications in the modelling stages to allow its computation in an acceptable time period.

The results of the model revealed a high potential for an organised feeder service to reduce the traffic volume in the most congested freeway of the LMA, by improving the accessibility to a mass transit service.

This formulation intends to be adapted and applied, in the future, to the afternoon peak period and to other railway lines in the LMA, in order to estimate a full impact that this type of services might produce to the overall traffic volume of the LMA.

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