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Journal of King Saud University – Engineering Sciences (2013) xxx, xxx–xxx

King Saud University
Journal of King Saud University – Engineering Sciences
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ORIGINAL ARTICLES

An optimized cosine-modulated nonuniform filter bank design for subband coding of ECG signal

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Received 1 June 2013; accepted 24 October 2013

KEYWORDS
Cosine modulated;
Nonuniform filter bank;
Adjustable window function;
Constrained equiripple FIR technique

Abstract A simple iterative technique for the design of nonuniform cosine modulated filter banks (CMFBS) is presented in this paper. The proposed technique employs a single parameter for optimization. The nonuniform cosine modulated filter banks are derived by merging the adjacent filters of uniform cosine modulated filter banks. The prototype filter is designed with the aid of different adjustable window functions such as Kaiser, Cosh and Exponential, and by using the constrained equiripple finite impulse response (FIR) digital filter design technique. In this method, either cut off frequency or passband edge frequency is varied in order to adjust the filter coefficients so that reconstruction error could be optimized/minimized to zero. Performance and effectiveness of the proposed method in terms of peak reconstruction error (*PRE*), aliasing distortion (AD), computational (CPU) time, and number of iteration (NOI) have been shown through the numerical examples and comparative studies. Finally, the technique is exploited for the subband coding of electrocardiogram (ECG) and speech signals.

1. Introduction

The multirate filter banks have received enormous interest worldwide during the past decades. The ever increasing various applications of filter banks have made it an increasingly important field of research. The research effort was first focused on the design of quadrature mirror filter (QMF) banks Vaidyanathan, 1993, which was later on extended for the design of multichannel uniform filter banks (MUFBS) Jovanovic-Dolecek, 2002. Subsequently, several techniques (Blanco-Velasco et al., 2008; Berger and Antoniou, 2003; Cruz-Roldan et al., 2009; Kumar et al., 2011b) were developed to enhance the performance of filter banks in different fields. Among the different classes of multi-channel uniform filter banks, cosine-modulated filter banks are the most frequently used filter banks due to their simpler design, where analysis and synthesis filter banks are derived by cosine modulation of the low-pass prototype filter. The design of whole filter bank thus reduces to that of a single low-pass prototype filter. Filter banks with such properties have been studied in depth, and various approaches have been successfully developed

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Peer review under responsibility of King Saud University.

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http://dx.doi.org/10.1016/j.jksues.2013.10.001

(Vaidyanathan, 1993; Jovanovic-Dolecek, 2002; Vetterli, 1987).

In numerous applications such as antenna systems, digital audio industry, biomedical signal processing, subband adaptive filtering, and communication (Kumar et al., 2011b; Vetterli, 1987; Manoj and Elizabeth, 2012; Soni et al., 2013a,b; Goodwin, 1996; Parfieniuk and Petrovsky, 2012; Zhang et al., 2008) nonuniform frequency partitioning may be employed to better exploit the signal characteristics. For example in perceptual audio coding, a filter bank with high stopband attenuation, nonuniform frequency sensitivities similar to human auditory system (the Bark scale), and efficient resolution switching are required (Blu, 1996). In ECG signal processing (Jovanovic-Dolecek, 2002; Blanco-Velasco et al., 2008; Berger and Antoniou, 2003; Bergen, 2008; Kumar et al., 2011a), especially for heart beat detection, filter banks with fast switching resolution, adjustable stopband attenuation and nonuniform frequency partitioning is required. Therefore, the nonuniform filter banks have elicited enormous interest among the researchers in recent years due to their ability to differentiate information into different frequency bands based on energy distribution of the signal. In addition, these filter banks are able to provide any sort of rational decimation in each channel, any extent of time–frequency resolution as per the requirement of the application, less quantization error, and low computational complexity. A significant amount of research effort has been devoted toward the theory and design of the nonuniform filter banks (Kovacevic and Vetterli, 1993; Cox, 1986; Xie et al., 2006; Chan et al., 2000; Xie, 2001; Queiroz, 1998; Chen et al., 1998). The most commonly used approach for designing nonuniform filter banks is the direct method, where a direct structure is adopted for designing filter banks (Kovacevic and Vetterli, 1993). However, this method usually involves nonlinear optimization with considerable number of parameters. Another method is the indirect method in which certain channels of a uniform filter bank are merged, giving rise to a near-PR (perfect reconstruction) recombination of nonuniform filter bank (Cox, 1986; Xie et al., 2006). Authors in Chan et al. (2000) have shown that PR can be achieved with indirect method, and have also proposed a new class of nonuniform filter bank based on cosine-modulated FBs (CMFBs). A novel approach based on H_∞ has been presented to determine the synthesis filters from a given set of analysis filters in Chen et al. (1998). While in Nayebi et al. (1993), authors have proposed a time domain approach for designing NUFBs. A detailed review on NUFBs is presented in Akkarakaran and Vaidyanathan (1999), Xie et al. (2005, 2006) and the references therein.

Amongst NUFBs, cosine modulated nonuniform filter bank (CMNUFB) has emerged as an attractive choice due to its simple design and implementation complexities, and good frequency characteristics. Several methods (Niamut and Heusdens, 2003; Xie and Chan, 2006; Princen, 1995; Lee and Lee, 1995a,b; Xie, 2006; Li et al., 1997; Zhang and Yang, 2008) were reported in the literature for designing CMNUFBs with near perfect reconstruction. But, these approaches were not suitable for larger filter due to high degree of nonlinearity. Furthermore, in most of the algorithms, convergence to optimum solution depends on the initial guess value and sometimes, these approaches are unable to find a global optimum solution due to several local minima. Therefore, in Zijing and Yun (2007), authors have proposed a flexible iterative method to overcome the above drawbacks based on the algorithm given in Creusere and Mitra (1995). It has been further modified in Ogale and Ashok (2011), Kumar et al. (2013). Though, almost all the methods developed so far give better performance in terms of reconstruction error, but they converge to an optimum solution in large number of iterations, and more computational time is required. In view of the above, there is a strong motivation to develop a new algorithm that can minimize the reconstruction error, number of iterations as well as computational time in case of larger filter.

In the above context, this paper presents a computationally efficient technique for the design of nonuniform cosine modulated filter bank based on uniform CMFB. The composing filters of NUFB are obtained by merging some relevant uniform filters in the associated uniform CMFB. Therefore, in Section 2, initially the design of uniform CMFB is discussed, followed by NUFB translation. Section 3 describes the proposed method for designing NUFBs. In Section 4, the application of the methodology for designing NUFBs is discussed. Finally, the concluding remarks are included in Section 5.

2. Outlines of nonuniform cosine modulated filter bank

Consider a general M-channel critically sampled filter bank shown in Fig. 1. Based on the input/output relationship of a filter bank, the reconstructed output is defined as (Vaidyanathan, 1993; Jovanovic-Dolecek, 2002):

$$Y(z) = T_0(z)X(z) + \sum_{i=1}^{M-1} T_i(z)X(ze^{-j2\pi/M}) \quad (1)$$

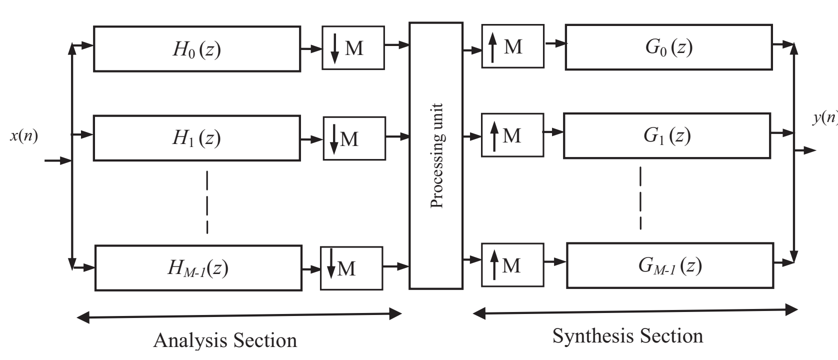

Figure 1 M-band uniform cosine modulated filter bank.

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where, $T_0(z)$ is a distortion transfer function, and determines the overall amplitude distortion to input signal. $T_i(z)$ is an aliasing distortion transfer function. These functions can be expressed, respectively as

$$T_0(z) = \frac{1}{M}\sum_{k=0}^{M-1} G_k(z)H_k(z) \tag{2}$$

and

$$T_i(z) = \frac{1}{M}\sum_{k=0}^{M-1} G_k(z)H_k(ze^{-j2\pi i/M}) \text{ for i} = 1,2,\ldots,M-1 \tag{3}$$

For i = 1, 2, 3, ..., $M-1$, the aliasing term $T_i(z)$ becomes null and the distortion term $T_0(z)$ is equal to z^{-N}, and then it results in perfect reconstruction. Due to their efficient design procedure and implementation, PR cosine modulated filter banks are more attractive. However, PR cosine modulated filter banks having high stopband attenuation are difficult to design. Therefore, there has been some interest in relaxing the PR condition and considering near-perfect reconstruction (NPR) CMFBs. These filter banks can be designed so that the amplitude distortion and phase distortion are canceled, while the aliasing error is kept small. In CMFBs, the entire analysis and synthesis filters are generated from cosine modulation. If $H(z)$ is a prototype filter, then the analysis and synthesis filters of uniform CMFB are, respectively derived using following relationship:

$$h_i(z) = 2h(n)\cos\left(\frac{\pi}{M}\left(k+\frac{1}{2}\right)\left(n-\frac{M}{2}\right)+(-1)^k\frac{\pi}{2}\right) \tag{4}$$

and

$$g_i(z) = 2h(n)\cos\left(\frac{\pi}{M}\left(k+\frac{1}{2}\right)\left(n-\frac{M}{2}\right)-(-1)^k\frac{\pi}{2}\right) \tag{5}$$

The nonuniform CMFBs are characterized by their synthesis equations in terms of uniform CMFBs. After designing the required uniform CMFB, the corresponding NUFB is obtained by merging the relevant band pass filters of analysis and synthesis section of the uniform filter bank (Lee and Lee, 1995b). If $\widetilde{H}_i(z), i = 0,1,\ldots\ldots\widetilde{M}-1$, be the analysis filter of the nonuniform FB obtained after merging $l_0(\geqslant 1)$ adjacent analysis filters of $H_i(z)$ i.e. from $k = n_i$ through $k = n_i + l_i - 1$ of uniform FB, then

$$\text{CM}\widetilde{H}_i(z) = \sum_{k=n_i}^{n_i+l_i-1} H_k(z) i = 0,1,\ldots\ldots\widetilde{M}-1 \tag{6}$$

In similar way, the synthesis filters $(\widetilde{F}_i(z))$ of NUFB, for $i = 0,1,\ldots\ldots\widetilde{M}-1$, then

$$\widetilde{F}_i(z) = \frac{1}{l_i}\sum_{k=n_i}^{n_i+l_i-1} F_k(z) i = 0,1,\ldots\ldots\widetilde{M}-1 \tag{7}$$

These $\widetilde{H}_i(z)$ and $\widetilde{F}_i(z)$ represent the analysis and synthesis filters for the derived M channel-nonuniform CMFB. The minimum value of n_i for $i = 0$ is zero and its maximum value is $n_{\widetilde{M}} = M$ also $l_0 + l_1 + l_2 + \ldots.. + l_{\widetilde{M}-1} = M$. The decimation factor for individual channel of the derived nonuniform CMFB is estimated

$$M_i = \frac{M}{l_i} i = 0,1,\ldots\ldots\ldots\ldots\widetilde{M}-1 \tag{8}$$

where, M_i gives the decimation ratio for the ith channel. The resultant nonuniform CMFB is shown in Fig. 2.

3. Proposed prototype design procedure

As discussed above, the nonuniform cosine modulated filter bank is obtained based on a uniform cosine modulated filter bank by preserving all its indispensable characteristics. Each element of the nonuniform filter bank is derived by merging some of the relevant uniform filters of uniform cosine modulated filter bank. The design of uniform cosine modulated filter bank has, therefore, gained prime importance, and the design procedure of nonuniform cosine modulated filter bank is reduced to the design of uniform cosine modulated filter bank. Hence, the design process of nonuniform CM filter bank reduces to the design of a single prototype filter. In the proposed method, the prototype filter is designed using the method given in Kumar and Singh (2010).

In multichannel cosine modulated filter bank, perfect reconstruction is possible if Eq. (9) is satisfied Zijing and Yun, 2007; Creusere and Mitra, 1995; Ogale and Ashok, 2011; Kumar et al., 2013; Kumar and Singh, 2010:

$$|H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega-\pi/M)})|^2 = 1 \, for \, 0 < \omega \leq \pi/M \tag{9}$$

If Eq. (9) is evaluated at $\omega = \pi/2M$, then it leads to

$$\begin{aligned} |H_0(e^{j\pi/2M})|^2 + |H_0(e^{j(\pi/2M-\pi/M)})|^2 = 1 \\ |H_0(e^{j\pi/2M})| = 0.707 \end{aligned} \tag{10}$$

In this method, either the cut-off frequency (ω_c) or the passband edge frequency (ω_p), based on the type of digital filter design technique employed for designing prototype filter, is linearly optimized so that Eq. (10) is approximately satisfied. In each iteration, only filter coefficients are computed at $\omega = \pi/2M$, which reduces the computational complexity and converges to optimal solution in less number of iterations.

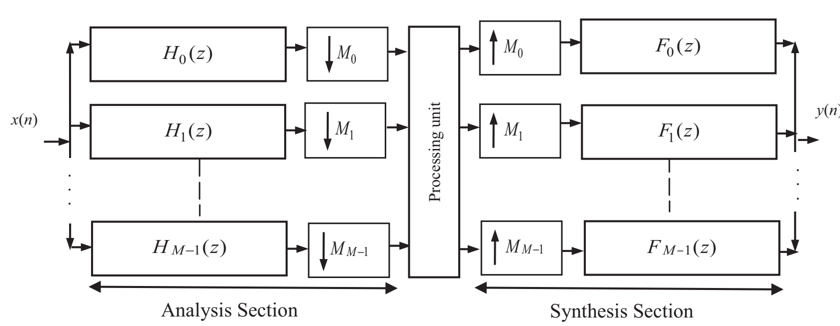

Figure 2 $\widetilde{M}$-band derived nonuniform cosine modulated filter bank.

For this work, the prototype filter is designed by using different window functions and Constrained Equiripple FIR Technique as well. The filter design using window technique is specified by three parameters such as order of the filter (N), window shape parameter and ω_c. In order to have the desired stopband attenuation (A_s), the order of prototype filter (N) is computed by

$$N = \frac{(A_s - 7.95)}{14.95\Delta f} \tag{11}$$

where, Δf is the transition band given by Eq. (18)

$$\Delta f = (\omega_s - \omega_p)/2. \tag{12}$$

In Eqs. 11 and 12, ω_p is the passband edge frequency, and ω_s is the stopband edge frequency. For achieving the desired frequency specifications, cut-off frequency is estimated using Eq. (13), defined by

$$\omega_c = \frac{1}{2}(\omega_p + \omega_s) \tag{13}$$

The window shape parameter is estimated using the given stopband attenuation for the window functions used.

In this work, adjustable window functions such as Kaiser, Cosh and Exponential are used due to their improved spectral characteristics and the low computational complexity caused by the closed form expressions.

3.1. Kaiser

$$w(n) = \frac{I_0\left[\beta\sqrt{1-\left(1-\frac{2n}{N-1}\right)^2}\right]}{I_0[\beta]}, \text{ for } 0 < n < N-1 \tag{14}$$

where, $I_0[.]$ is the modified zero$^{\text{th}}$ order Bessel function, and β, the window shape parameter, given by

$$\beta = \begin{cases} 0, \text{for } A_s \le 21 \\ 0.5842(A_s - 21)^{0.4} + 0.07886(A_s - 21), \text{for } 21 \le A_s < 50 \\ 0.1102(A_s - 8.7), \text{for } A_s > 50 \end{cases} \tag{15}$$

3.2. Cosh

$$w(n) = \frac{\cosh\left[\alpha\sqrt{1-\left(1-\frac{2n}{N-1}\right)^2}\right]}{\cosh(\alpha)}, \text{ for } 0 < n < N-1 \tag{16}$$

3.3. Modified exponential

$$w(n) = \frac{e^{\left[\alpha\sqrt{1-\left(1-\frac{2n}{N-1}\right)^2}\right]}}{e^{\alpha}}, \text{for } 0 < n < N-1 \tag{17}$$

In Eqs. 16 and 17, α is the window shape parameter, which controls the ripple ratio. The detailed discussion on these window functions is given in Avci and Nacaroglu (2008a,b, 2009), Kumar and Kuldeep (2012) and the references there in. The sequential steps required for designing a prototype filter for nonuniform cosine modulated filter banks using window technique are:

- Step 1: Specify the design specifications such as A_s,

ω_s

and ω_p,

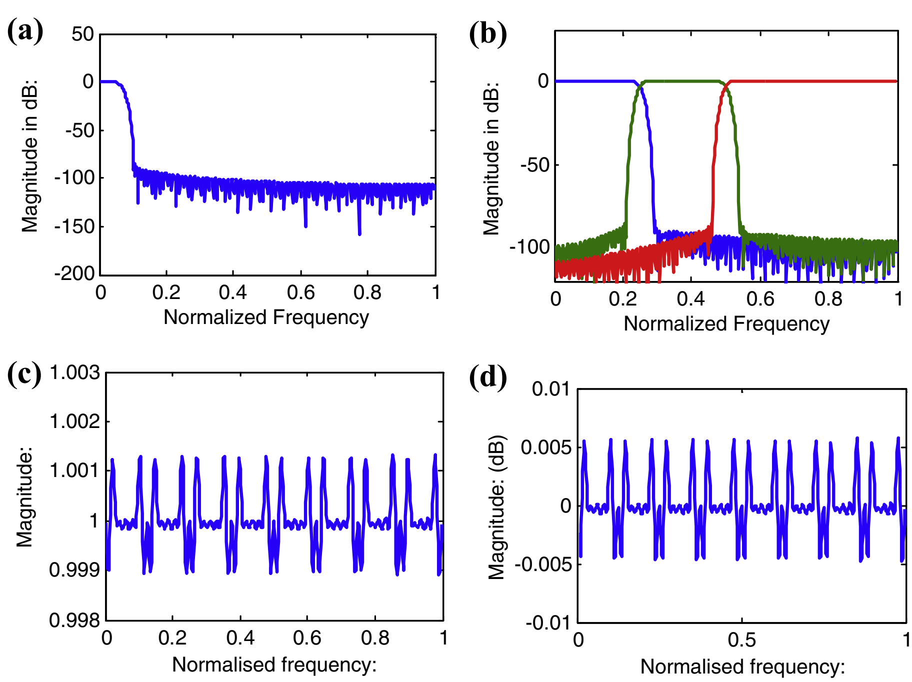

Figure 3 Performance of 3-channel nonuniform cosine modulated filter bank using the proposed method with exponential window. (a) Amplitude response of the prototype filter in dB. (b) Amplitude responses of the analysis filters of NUFB in dB. (c) Variation of reconstruction error. (d) Variation of reconstruction error in dB.

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- Step 2: Assume initial values of error (ε), counter (*Count*), and step size (Δ),
- Step 3: Compute ω_c and N by using respective Eqs. 19 and 17, from the given design specifications,
- Step 4: Evaluate the prototype filter coefficients using different window functions with N and ω_c. Also compute the magnitude response of designed filter (MRD) at $\tilde{M}$ and error or deviation from ideal condition given by Eq. (10)

$$error = 0.707 - MRD \tag{18}$$

- Step 5: Check whether '*error*' is comparable to error (ε). If yes, then derive other analysis and synthesis filters of uniform CM filter bank using this prototype filter, and finally derive their nonuniform counterparts. If no, follow the next step,
- Step 6: Check whether $MRD > 0.707$. If yes, decrease the cut-off frequency by step size and follow the next step. If no, increase ω_c by the step size, and also follow next step,
- Step 7: Increase counter by one and $\Delta = \Delta/2$,
- Step 8: Re-evaluate the prototype filter coefficients using window functions with the same N but with a new cut-off frequency. Also, compute the error or deviation from ideal condition. Then, go to Step 5.

Like window technique, constrained equiripple FIR technique is also exploited for designing prototype filter for nonuniform cosine modulated filter bank. In constrained equiripple technique, the resulting filter minimizes the maximum error between desired frequency response and actual frequency response by spreading the approximation error uniformly over each band. Such filters exhibit equiripple behavior in both the passband and stopband region, and hence, are called as equiripple filters. Here, all the local extremes of the approximation error in the passband are equal. The same is true for the stopband. For designing prototype filter with constrained equiripple FIR technique, same steps can be followed except instead of varying the cut-off frequency, the passband edge frequency is varied in each iteration.

This method has been implemented on a Genuine Intel (R) CPU T2300 @ 1.66 GHz, 1 GB RAM. For computing computational time, the command 'tic' and 'toc' are used.

4. Design examples

In this section, implementation of the proposed method has been carried out in MATLAB 2010, and the simulation results obtained have been discussed. Four attributes defined below are referred for the evaluation of performance of the designed filter bank.

- Peak reconstruction error (PRE):

$$PRE = \max\left\{\sum_{k=0}^{M-1}|H_k(e^{j\omega})|^2\right\} - \min\left\{\sum_{k=0}^{M-1}|H_k(e^{j\omega})|^2\right\} \tag{19}$$

- Number of Iterations (NOI)
- Computational time (CPU time).

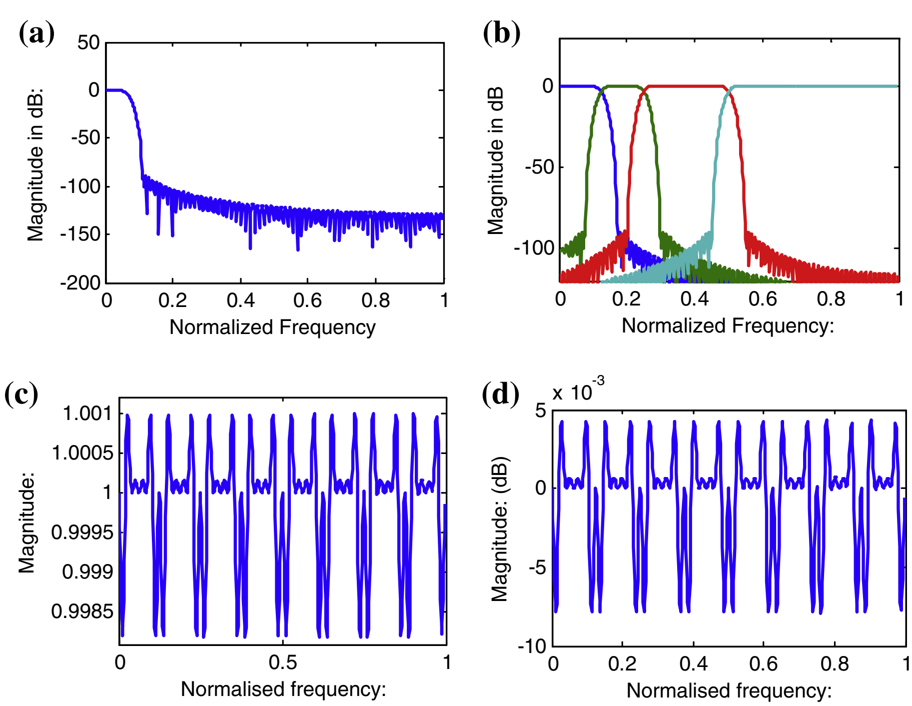

Figure 4 Performance of 4-channel nonuniform cosine modulated filter bank using the proposed method with Exponential window. (a) Amplitude response of the prototype filter in dB. (b) Amplitude responses of the analysis filters of NUFB in dB. (c) Variation of reconstruction error. (d) Variation of reconstruction error in dB.

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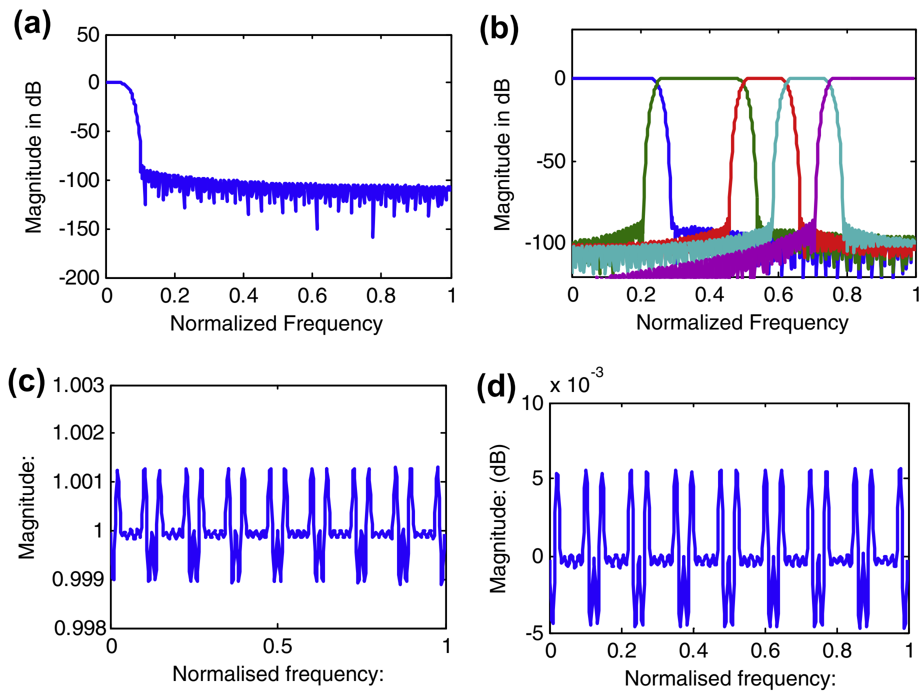

Figure 5 Performance of 5-channel nonuniform cosine modulated filter bank using the proposed method with Exponential window. (a) Amplitude response of the prototype filter in dB. (b) Amplitude responses of the analysis filters of NUFB in dB. (c) Variation of reconstruction error. (d) Variation of reconstruction error in dB.

With the help of following examples, performance evaluation of the proposed method is carried out using window technique and constrained equiripple FIR technique. In the last, results obtained with this method are compared with the results derived from other known methods, followed by the application of designed filter bank for subband coding.

4.1. Design examples for window technique

Example-I: A 3-channel nonuniform CMFB has been designed from its uniform counterpart with $n_0 = 0$, $n_1 = 1$, $n_2 = 2$ and $l_0 = 2$, $l_0 = 2$, $l_0 = 4$ i.e. adjacent 2, 2, and 4 channel filters in uniform filter bank have been merged together in order to have a 3-channel nonuniform filter bank. The band edge frequencies included are $\omega_1 = \pi/4$ and $\omega_2 = \pi/2$. The stop band attenuation A_s is 85 dB, and $N = 160$. Simulation results obtained with Exponential window function are graphically illustrated in Fig. 3.

Example-II: In this example, a 4-channel nonuniform CMFB has been derived with $n_0 = 0$, $n_1 = 1$, $n_2 = 2$, $n_3 = 4$ and $l_0 = 1$, $l_0 = 1$, $l_0 = 2$, $l_0 = 4$ from an 8-channel uniform CMFB. Here, the adjacent channel filters 1, 1, 2, 4 are merged. The order of filter and A_s are kept same as previous example and band edge frequencies are $\omega_1 = \pi/8$, $\omega_2 = 2\pi/8$, and $\omega_2 = 4\pi/8$. The design results obtained in case of Exponential window are displayed in Fig. 4.

Example-III: Similar to the previous examples, by relevant merging of the adjacent channel filters of uniform filer bank, a 5-channel nonuniform CMFB has been designed with $n_0 = 0$, $n_1 = 2$, $n_2 = 4$, $n_3 = 5$, $n_3 = 6$ and $l_0 = 2$, $l_0 = 2$, $l_0 = 1$, $l_0 = 1$, $l_0 = 2$. Other specifications are kept as $N = 144$, $A_s = 85$ and band edges at $\omega_1 = \pi/4$, $\omega_2 = \pi/2$, $\omega_2 = 5\pi/8$, and $\omega_2 = 3\pi/4$. In this case, simulation results obtained with Exponential window function are depicted in Fig. 5.

Similarly, the proposed methodology has been also tested with other window functions, and with different design specifications. The performance parameters obtained in each case are tabulated in Table 1. As it can be observed from the simulation results given in Table 1, the proposed method gives satisfactory performance with all the window functions discussed above. The reconstruction error is drastically reduced and the method converges in less number of iterations. The computation time (CPU time) is also effectively reduced even when the order of filter is high. The average computation time required is less than a fraction of second, and is converged to optimal solution within twenty iterations. Therefore, this methodology can be effectively used for designing the nonuniform filter bank with larger filter taps.

4.2. Design examples for constrained equiripple FIR technique

Example I: An 8-channel linear-phase uniform CMFB with $N + 1 = 144$, $A_s = 100$ dB, passband ripple $(A_p) = 0.00001$, and $\omega_p = 0.0319\ \pi$ is designed with the proposed method. From this, a 3-channel nonuniform filter bank with the specifications $n_0 = 0$, $n_1 = 2$, $n_2 = 4$ and $l_0 = 2$, $l_0 = 2$, $l_0 = 4$ is derived. In this case, the peak reconstruction error (*PRE*), computational time (CPU time) and number of iterations (*NOI*) obtained are 1.9×10^{-3}, $1.201s$ and 19, respectively. The design results are shown in Fig. 6.

Example II: A 4-channel linear-phase nonuniform CMFB is designed with the proposed method. The design specification for the prototype filter for uniform FB are $N + 1 = 112$, $A_s = 80$ dB, $A_p = 0.00002$, and $\omega_p = 0.0312\pi$. In this case, nonuniform FB has $n_0 = 0$, $n_1 = 1$, $n_2 = 2$, $n_3 = 4$ and $l_0 = 1$, $l_0 = 1$, $l_0 = 2$, $l_0 = 4$ and the band edge frequencies are $\omega_1 = \pi/8$, $\omega_2 = 2\pi/8$, and $\omega_2 = 4\pi/8$. The performance

Table 1 Performance parameters obtained with the proposed method in case of different window functions.

Type of window	A_S (dB)	Channel	Filter taps (N)	*RE*	*PRE* (dB)	*NOI*	CPU time (*s*)
		Three band (4, 4, 2)	80	2.50×10^{-3}	5.60×10^{-3}	20	0.064
			96	$2.\ 40 \times 10^{-3}$	6.70×10^{-3}	19	0.093
			112	2.60×10^{-3}	6.70×10^{-3}	17	0.102
			144	2.24×10^{-3}	5.80×10^{-3}	22	0.186
Kaiser	85	Four band (8, 8, 4, 2)	80	2.50×10^{-3}	9.50×10^{-3}	20	0.086
			96	2.50×10^{-3}	1.05×10^{-2}	19	0.093
			112	2.70×10^{-3}	1.10×10^{-2}	21	0.124
			144	2.80×10^{-3}	1.12×10^{-3}	20	0.093
		Five band (16, 16, 8, 4, 2)	80	2.50×100^{-3}	1.10×10^{-2}	17	0.078
			96	2.50×10^{-3}	1.09×10^{-2}	13	0.042
			112	2.50×10^{-3}	1.10×10^{-2}	17	0.124
			144	2.60×10^{-3}	1.14×10^{-3}	17	0.093
Exponential	85	Three band (4, 4, 2)	80	1.77×10^{-2}	6.64×10^{-2}	21	1.203
			96	7.5×10^{-3}	2.29×10^{-2}	20	1.218
			112	6.2×10^{-3}	2.63×10^{-2}	18	1.172
			144	6.10×10^{-3}	2.65×10^{-2}	21	1.234
		Four band (8, 8, 4, 2)	80	1.77×10^{-2}	7.62×10^{-2}	21	1.172
			96	7.5×10^{-3}	3.27×10^{-2}	20	1.234
			112	6.2×10^{-3}	2.72×10^{-2}	18	1.219
			144	6.10×10^{-3}	2.65×10^{-2}	21	1.268
		Five band (16, 16, 8, 4, 2)	80	1.77×10^{-2}	7.62×10^{-2}	21	1.222
			96	7.5×10^{-3}	3.27×10^{-2}	20	1.172
			112	6.2×10^{-3}	2.72×10^{-2}	18	1.188
			144	6.10×10^{-3}	2.65×10^{-2}	21	1.298
Cosh	85	Three band (4, 4, 2)	80	1.50×10^{-2}	5.43×10^{-2}	20	1.203
			96	6.8×10^{-3}	2.25×10^{-2}	19	1.172
			112	5.8×10^{-3}	2.47×10^{-2}	21	1.234
			144	5.70×10^{-3}	2.47×10^{-2}	20	1.243
		Four band (8, 8, 4, 2)	80	1.50×10^{-2}	6.50×10^{-2}	20	1.172
			96	6.8×10^{-3}	2.95×10^{-2}	19	1.234
			112	5.8×10^{-3}	2.51×10^{-2}	21	1.281
			144	5.70×10^{-3}	2.48×10^{-2}	20	1.312
		Five band (16, 16, 8, 4, 2)	80	1.50×10^{-2}	6.50×10^{-2}	20	1.187
			96	6.8×10^{-3}	2.95×10^{-2}	19	1.187
			112	5.8×10^{-3}	2.51×10^{-2}	21	1.219
			144	5.70×10^{-3}	2.48×10^{-2}	20	1.257

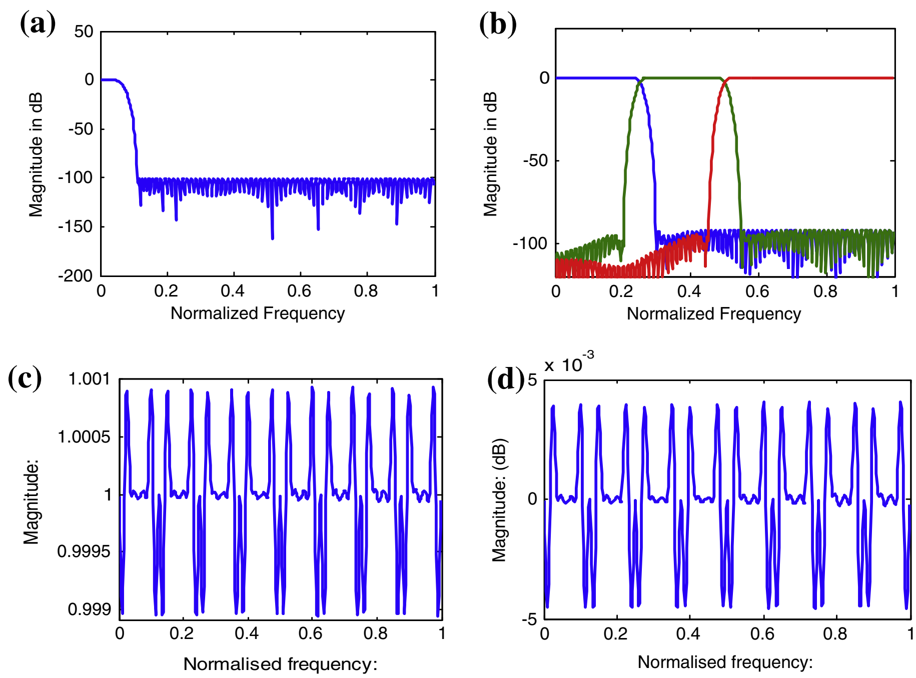

Figure 6 Performance of 3-channel non-uniform CMFB (4, 4, and 2) designed using the proposed method. (a) Magnitude responses of the prototype filter in dB. (b) Magnitude responses of the analysis filters in dB. (c) Variation of peak reconstruction error. (d) Variation of peak reconstruction error in dB.

indices obtained in this example are: $PRE = 2.2 \times 10^{-3}$, CPU time = 0.920s, $NOI = 14$ and the simulation results are graphically depicted in Fig. 7.

Example III: An 8-channel uniform FB having the same design specifications as used earlier in example II is exploited for deriving a 5-channel nonuniform CMFB with decimation factors 4, 4, 8, 8, 4, and $n_0 = 0$, $n_1 = 2$, $n_2 = 4$, $n_3 = 5$, $n_3 = 6$ and $l_0 = 2$, $l_0 = 2$, $l_0 = 1$, $l_0 = 1$, $l_0 = 2$ along with band edges at $\omega_1 = \pi/4$, $\omega_2 = \pi/2$, $\omega_2 = 5\pi/8$, and $\omega_2 = 3\pi/4$. The design results obtained are shown in Fig. 8. Similarly, the proposed method is also tested with other design specifications, and the results are summarized in Table 2. It is evident from the design results listed in Table 2 that the proposed method gives better performance in terms of reconstruction error, *NOI*, and *CPU* time. The important parameter, peak reconstruction error is reduced appreciably. The average peak reconstruction error in dB obtained in case of 3-channel and 4-channel nonuniform CMFBs is 3.32×10^{-3}, and 3.35×10^{-3}, respectively, while in case of 5-channel nonuniform CMFBs, it is 3.77×10^{-3}. The other parameters such as computation time and number of iterations required are extremely low even if the number of filter taps is more. Hence, nonuniform filter bank with larger filter taps can also be designed efficiently using this technique. It is also evident from Tables 1 and 2 that the proposed technique gives better performance irrespective of the filter design technique used for the prototype filter.

4.3. Comparison with other methods

Results of the comparative studies made are given in Table 3, which clearly show the superiority of the proposed methodology over other exiting algorithms (Xie et al., 2006; Li et al., 1997; Lee and Lee, 1995b; Zijing and Yun, 2007; Ogale and Ashok, 2011). For this, nonuniform filter banks with similar design specifications as published earlier have been designed and compared. As, it can be seen that the performances of the proposed method are significantly improved as compared to other known techniques in terms of peak reconstruction error, computational time (CPU time) and number of iterations. The average peak reconstruction error obtained by the proposed method is 2.57×10^{-3} with constrained equiripple FIR technique, 2.57×10^{-3} with Kaiser window, 2.57×10^{-3} with Cosh window, and 2.57×10^{-3} with Exponential window. The average percentage reduction in *PRE* provided by the proposed method compared to others (Xie et al., 2006; Chen et al., 1998; Lee and Lee, 1995b; Zijing and Yun, 2007; Ogale and Ashok, 2011) is 50.39%, 50.39%, 85.68%, 43.12%, and 18.14%, respectively. Since, in the proposed method, only filter coefficients are computed at $\omega = \pi/2M$, so it is more computationally efficient as compared to earlier published results. The number of iterations required for these algorithms is 66 in Zijing and Yun (2007) and 56 in Ogale and Ashok (2011). The computation time required for designing nonuniform filter bank with algorithms (Zijing and Yun, 2007; Ogale and Ashok, 2011) is 1.475s and 1.409s, respectively. While, the average number of iterations required with the proposed method is 20. Therefore, in real or quasi real time applications where processing speed is a matter of concern, this technique becomes very efficient and employable.

4.4. Application of NUFBs

Quantization noise prevails in speech coding, and hence it degrades its quality. In order to reduce the effect of quantization noise in speech coding, the concept of subband coding was introduced in 1976 (Vaidyanathan, 1993). It is a frequency domain coding analogous to transform coding. Due to one of its disadvantage of dividing the source output artificially into

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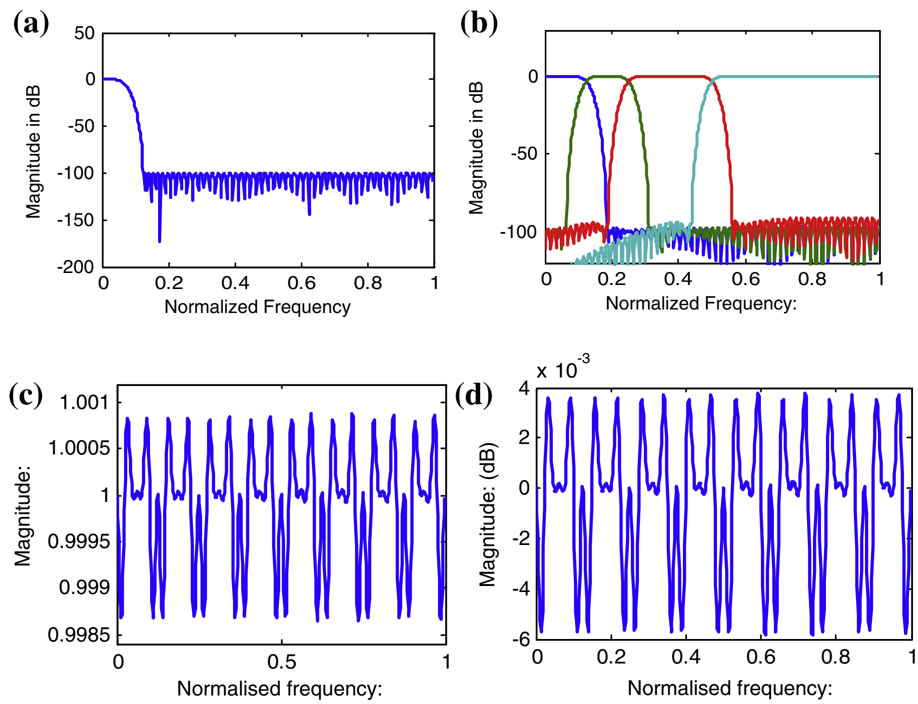

Figure 7 Performance of 4-channel nonuniform CMFB (8, 8, 4, 2) designed using the proposed method. (a) Magnitude responses of the prototype filter in dB. (b) Magnitude responses of the analysis filters in dB. (c) Variation of peak reconstruction error. (d) Variation of peak reconstruction error in dB.

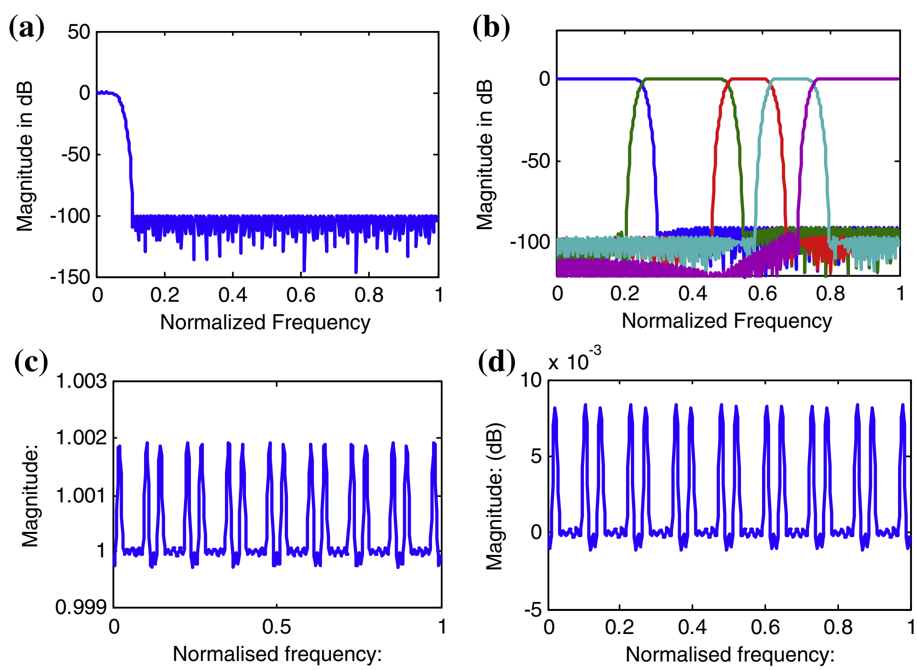

Figure 8 Performance of 5-channel Non-uniform CMFB (4, 4, 8, 8, 4) designed using the proposed method. (a) Magnitude responses of the prototype filter in dB. (b) Magnitude responses of the analysis filters in dB. (c) Variation of peak reconstruction error. (d) Variation of peak reconstruction error in dB.

blocks which ultimately renders coding artifacts at the block edges, transform coding is less preferable over subband coding. Advantageous part of subband coding lies in its ability to decompose the input signal into various frequency bands without imposing any artificial block structure. Generally, properties of analysis filter bank of CMFB have their matching pair with the characteristics of input signals fed to filter bank. Because of this, CMFBs are generally more preferred over

Table 2 Performance of the proposed method for designing NUFB using constrained Equiripple FIR Technique.

Band (M)	Filter taps (N)	RE	PRE(dB)	NOI	CPU time (s)
Three band (4, 4, 2)	80	3.50×10^{-3}	1.33×10^{-2}	17	0.780
	96	5.30×10^{-3}	1.95×10^{-2}	18	1.072
	112	2.50×10^{-3}	8.60×10^{-3}	18	1.762
	144	2.00×10^{-3}	4.60×10^{-3}	18	2.542
Four band(8, 8, 4, 2)	80	3.72×10^{-3}	1.45×10^{-2}	17	0.842
	96	5.50×10^{-3}	1.07×10^{-2}	15	1.092
	112	2.20×10^{-3}	9.70×10^{-3}	14	1.348
	144	2.00×10^{-3}	8.40×10^{-3}	18	1.524
Five band (4, 4, 8, 8, 4)	80	3.30×10^{-3}	1.41×10^{-2}	17	0.842
	96	7.80×10^{-3}	1.87×10^{-2}	16	1.248
	112	1.81×10^{-3}	8.46×10^{-3}	16	1.567
	144	2.20×10^{-3}	9.50×10^{-3}	17	2.976

Table 3 Comparison of the proposed methodology with earlier published results.

Algorithms	Technique	No. of channels	Filter taps (N)	A_s (dB)	PRE	NOI	CPU time (s)
Li et al. (1997)	Cosine modulation	Three channels (4, 4, 2)	64	60	7.80×10^{-3}		
Xie et al. (2006)	Recombination	Three channels (4, 4, 2)	64	110	7.80×10^{-3}		
Lee and Lee (1995b)	Cosine modulation	Five Channels (4, 4, 8, 8, 4)	40	46.3	2.70×10^{-2}		
Zijing and Yun (2007)	Cosine modulation	Five Channels (4, 4, 8, 8, 4)	164	110	6.80×10^{-3}	66	1.475
Ogale and Ashok, 2011	Cosine Modulation	Three channels (4, 4, 2)	64	110	2.99×10^{-3}	55	1.245
		Five Channels (4, 4, 8, 8, 4)	164	110	6.50×10^{-3}	55	1.573
	Cosine modulation Kaiser	Three channels (4, 4, 2)	64	70	3.80×10^{-3}	20	0.1092
		Five Channels (4, 4, 8, 8, 4)	144	110	2.60×10^{-3}	19	0.1404
Proposed method (windowing)	Cosine modulation Cosh	Three channels (4, 4, 2)	64	70	3.80×10^{-3}	20	0.1092
		Five Channels (4, 4, 8, 8, 4)	144	110	5.70×10^{-3}	19	0.1404
	Cosine modulation exponential	Three channels (4, 4, 2)	64	70	3.80×10^{-3}	20	0.1092
		Five Channels (4, 4, 8, 8, 4)	144	110	6.10×10^{-3}	19	0.1404
Proposed method (constrained equiripple)	Cosine modulation	Three channels (4, 4, 2)	144	100	2.00×10^{-3}	18	2.542
			160	110	3.10×10^{-3}	17	1.226
		Five Channels (4, 4, 8, 8, 4)	144	110	2.20×10^{-3}	19	1.201
			160	100	3.30×10^{-3}	17	2.976

Table 4 Fidelity assessment parameters in the proposed algorithm with Kaiser window.

Signal	PRD	MSE	ME	SNR
MIT-BIH Rec. 800	6.14×10^{-2}	1.77×10^{-8}	3.86×10^{-4}	64.23
MIT-BIH Rec. 810	5.92×10^{-2}	2.94×10^{-8}	6.76×10^{-4}	64.55
MIT-BIH Rec. 825	5.58×10^{-2}	5.32×10^{-9}	2.31×10^{-4}	65.07
MIT-BIH Rec. 829	3.01×10^{-2}	6.77×10^{-9}	3.94×10^{-4}	70.43
MIT-BIH Rec. 840	5.33×10^{-2}	4.36×10^{-9}	2.60×10^{-4}	65.47
MIT-BIH Rec. 855	5.78×10^{-2}	5.49×10^{-9}	2.61×10^{-4}	64.75
Speech, 'L', Eng.: M 45	9.87×10^{-2}	9.64×10^{-9}	1.50×10^{-4}	60.11
Speech, 'O', Eng.: M 45	7.65×10^{-2}	4.67×10^{-8}	1.40×10^{-3}	62.32
Speech, 'U', Eng.: M 45	8.39×10^{-2}	3.42×10^{-9}	1.33×10^{-4}	61.52

all other filter banks for audio coding and ECG signal processing (Blanco-Velasco et al., 2008; Berger and Antoniou, 2003; Cruz-Roldan et al., 2009; Kumar et al., 2011a,b; Bergen, 2008). Signal enhancement or compression, beat detection, and beat classification are the major areas, where CMFBs are employed. In all these applications, signal is split into various uniform spectrums for analysis. However, this uniform band division does not work efficiently, especially when there is an uneven energy distribution. Here, the proposed method has been used for subband coding of ECG and speech signals. For this, 3-channel nonuniform CMFB is designed with the same design specifications as in example I, and is exploited for subband coding. Several fidelity parameters such as mean square error (MSE), maximum error (ME), and percent root mean square difference (PRD) given in Blanco-Velasco et al. (2008), Bergen (2008), Kumar et al. (2011) are computed and

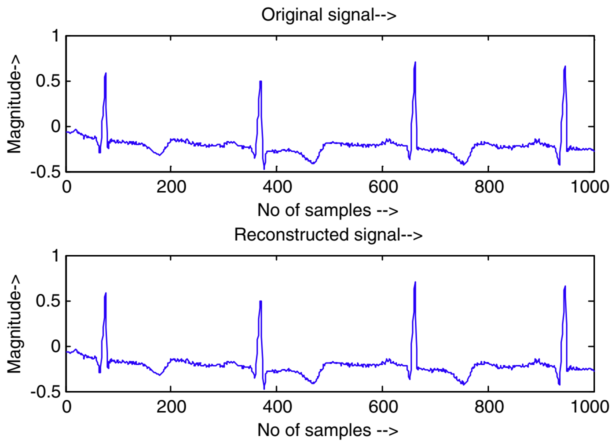

Figure 9 Sub-band coding of ECG signal MIT-BIH Rec.820.

tabulated in Table 4. Fig. 9 shows the original ECG signals (MIT/BIH-820) and its reconstructed version.

As it can be observed from Table 4, all the fidelity parameters obtained with this methodology are much better than the acceptable range in practice. PRDs in the range of 2–10% are considered as acceptable in practice (Berger and Antoniou, 2003; Jalaleddine et al., 1990). Thus, it is apparent that the proposed method can be effectively used for subband coding of the real time signals.

5. Conclusions

A simple and efficient algorithm for the design of cosine modulated nonuniform filter bank is presented in this paper. Results of comparative studies given in Tables 2 and 4 clearly show the key advantageous features of the proposed algorithm over others. It is found that not only the amplitude distortion is reduced considerably, computational time as well as number of iterations required are also reduced. It is simple and easy to implement, attributed to its single parameter optimization technique. Also, the acceptable range of calculated fidelity parameters makes it useful for subband coding of real time signals. Therefore, it can be concluded that the proposed algorithm can be used for high quality reconstruction of the speech signals; ECG signals etc., and can also be used for the filters with larger taps.

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