

# An Outer Bound for the Gaussian Interference Channel with a Relay

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**Abstract**—A novel sum-rate outer bound for the Gaussian interference channel with a relay is presented. The outer bound is obtained by adapting the genie-aided approach developed for interference channels in [1]. The cut-set bound for this channel is also derived and is shown to be much looser than the new bound. The new bound is also compared to an achievable rate region we introduced in previous work. We show that the inner and outer bounds are close in the regime of strong interference where receivers can decode both messages. The capacity region in strong interference for the discrete memoryless degraded channel is also presented.

## I. INTRODUCTION

Cooperation via relays that forward information in wireless networks improves the performance in terms of rate, coverage, reliability and energy-efficiency. Cooperative strategies for the single-relay channel have been developed in [2], [3], [4], and further generalized to multi-relay channels. Relay channel models consider one communicating pair and hence do not capture cooperation for multiple source-destination pairs. And yet, wireless applications typically involve simultaneous communications from many sources to many destinations. Such scenarios bring in new elements not encountered in the classic relay channel: 1) the presence of interference caused by simultaneous transmissions from multiple sources; 2) the opportunity for joint encoding of messages at a relay; 3) forwarding information to one node in general increases interference to other nodes. These elements impact the optimum ways of relaying. The different aspects of relaying for multiple sources can be captured by considering the smallest such network, which we refer to as *the interference channel with a relay (ICR)* (see Fig. 1). The ICR model contains elements of relay, interference, broadcast and multiaccess channels and thus determining its capacity and associated encoding/decoding schemes is extremely challenging. We previously analyzed ICR communication scenarios [5], [6], [7] by mainly focusing on special cases. We derived inner bounds on the performance, and obtained capacity in the special case of strong interference. We showed that in some communication scenarios, there may be more benefit from increasing interference at the intended destination than in using the classic approach of forwarding desired information.

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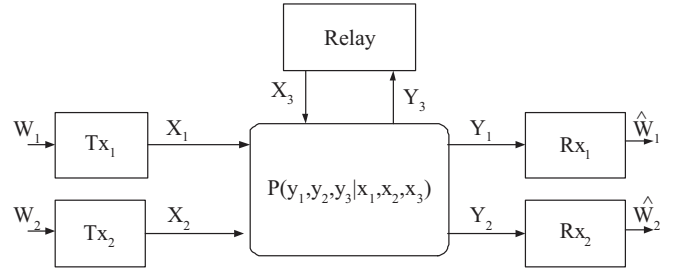


Fig. 1. Interference channel with a relay.

This paper presents a sum-rate outer bound on the capacity of the Gaussian interference channel with a relay. For the Gaussian interference channel, Kramer in [1] introduced the idea of using a genie that provides a receiver with the minimum information necessary to decode both messages. This approach led to a new, improved outer bound for the interference channel. In this work, we apply this idea to the ICR setting. We propose a genie that gives the receiver a noisy observation of the source and the relay channel inputs. Unlike the interference channel in which the channel inputs are independent, in ICRs, channel inputs of the relay and of each encoder are dependent. In our approach, the maximum entropy inequality will guarantee that the bound is maximized by jointly Gaussian inputs. Our bound also applies to the *cognitive* ICR, in which the relay knows a priori the messages sent by the sources. For the cognitive ICR, two outer bounds were developed in [8]. We show that the new bound presented in this paper can be tighter than existing cognitive ICR outer bounds. We also compare the new outer bound to an achievable rate region we presented in [7]. For that encoding scheme, we discuss strong interference conditions under which receivers can decode both messages, in order to compare them to similar conditions in the new outer bound. Generalizing our work in [6], we also determine the capacity region of the discrete memoryless degraded ICR in strong interference.

## Related Work

Inner bounds to the ICR capacity were first presented in [9]. We introduced the idea of interference forwarding and demonstrated its gains in [5], and subsequently in [6], [7]. Works [8], [10] considered a similar channel model under

the assumption that the relay is cognitive, in the sense that it knows a priori the messages to be sent by the two sources. Strong interference conditions for the cognitive Gaussian case were presented in [10]. ICRs with in-band and out-band signaling to/from the relay were considered in [11]. A special case of the ICR channel was considered in the context of cellular networks with relays in [12].

The remainder of this paper is organized as follows. The channel model is given in Section II. A new sum-rate outer bound is presented in Section III. The cut-set bound for the ICR is presented in Section IV. Numerical comparisons between the new bound, the cut-set bound and an achievable rate region are given in Section V. Section VI discusses the strong interference regime for the ICR, and presents a capacity result in strong interference. Section VII concludes the paper.

## II. CHANNEL MODEL

The discrete interference channel with a relay consists of three finite input alphabets  $\mathcal{X}_1, \mathcal{X}_2, \mathcal{X}_3$ , three finite output alphabets  $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$ , and a probability distribution  $p(y_1, y_2, y_3 | x_1, x_2, x_3)$ . Each encoder  $t$ ,  $t = 1, 2$ , wishes to send a message  $W_t \in \mathcal{W}_t = \{1, \dots, 2^{nR_t}\}$  to decoder  $t$ ,  $t = 1, 2$  (see Fig. 1). The channel is memoryless and time-invariant in the sense that

$$p(y_{1,i}, y_{2,i}, y_{3,i} | x_1^i, x_2^i, x_3^i, y_1^{i-1}, y_2^{i-1}, y_3^{i-1}, w_1, w_2) = p_{Y_1, Y_2, Y_3 | X_1, X_2, X_3}(y_{1,i}, y_{2,i}, y_{3,i} | x_{1,i}, x_{2,i}, x_{3,i}). \quad (1)$$

In most of the paper we will consider the Gaussian channel described by the following input-output relationship:

$$\begin{aligned} Y_1 &= X_1 + h_{12}X_2 + h_{13}X_3 + Z_1 \\ Y_2 &= h_{21}X_1 + X_2 + h_{23}X_3 + Z_2 \\ Y_3 &= h_{31}X_1 + h_{32}X_2 + Z_3 \end{aligned} \quad (2)$$

where  $Z_t \sim \mathcal{N}[0, 1]$ ,  $E[X_t^2] \leq P_t$ ,  $t = 1, 2, 3$ , and  $\mathcal{N}[0, \sigma^2]$  denotes the normal distribution with zero mean and variance  $\sigma^2$ .  $X_1$  and  $X_2$  are channel inputs at the sources and are statistically independent.

An  $(R_1, R_2, n)$  code for the ICR consists of two message sets  $\mathcal{W}_1 = \{1, \dots, 2^{nR_1}\}$ ,  $\mathcal{W}_2 = \{1, \dots, 2^{nR_2}\}$ , an encoding function at each encoder,  $X_1^n = f_1(W_1)$ ,  $X_2^n = f_2(W_2)$ , an encoding function at the relay  $X_{3,i} = f_{3,i}(Y_3^{i-1})$ , and two decoding functions  $\hat{W}_t = g_t(Y_t^n)$ ,  $t = 1, 2$ . The average error probability of the code is given by  $P_e = P[\hat{W}_1 \neq W_1 \cup \hat{W}_2 \neq W_2]$ .

The capacity region of the ICR is the closure of the set of rate pairs  $(R_1, R_2)$  for which receivers can decode their messages with an arbitrarily small positive error probability.

## III. A NEW SUM-RATE OUTER BOUND

We let

$$Y_{1g} = d_1X_1 + d_2X_2 + d_5X_3 + d_3Z_1 + d_4\tilde{Z}_1 \quad (3)$$

where  $d_i$ ,  $i = 1, \dots, 5$  are real numbers, and  $\tilde{Z}_1$  is zero-mean Gaussian with unit variance, independent of other random

variables. The following theorem states the main result of our paper.

*Theorem 1:* The capacity region of the ICR is contained in the set of rate pairs  $(R_1, R_2)$  satisfying

$$R_1 + R_2 \leq \min_{\{d_i\}_{i=1}^5} I(X_1, X_2, X_3; Y_1, Y_{1g}) \quad (4)$$

where the mutual information is evaluated for Gaussian inputs of the form  $p(x_1)p(x_2)p(x_3|x_1, x_2)$  and parameters  $d_i$ ,  $i = 1, \dots, 5$  that satisfy

$$(1/h_{12} + \beta(d_3 - d_2/h_{12}))^2 + (\beta d_4)^2 \leq 1 \quad (5)$$

$$d_5 = (h_{23} - \alpha h_{13})/\beta \quad (6)$$

$$\alpha = (1 - \beta d_2)/h_{12} \quad (7)$$

for some real numbers  $\alpha$  and  $\beta \neq 0$ .

*Proof:* We consider signalling at achievable rates  $(R_1, R_2)$ . Each receiver  $t$ ,  $t = 1, 2$  can then reliably decode its desired message  $W_t$ . A genie gives receiver 1 the signal  $Y_{1g}$  given by (3).

Receiver 1 processes its channel output and the information obtained from the genie in the same manner as in [1]: after decoding  $W_1$  it forms:

$$\hat{Y}_1 = \alpha Y_1 + \beta Y_{1g} + (h_{21} - \alpha - \beta d_1)X_1 \quad (8)$$

for some real numbers  $\alpha$  and  $\beta \neq 0$ . This yields

$$\begin{aligned} \hat{Y}_1 &= h_{21}X_1 + (\alpha h_{12} + \beta d_2)X_2 + (\alpha h_{13} + \beta d_5)X_3 \\ &\quad + (\alpha + \beta d_3)Z_1 + \beta d_4\tilde{Z}_1. \end{aligned} \quad (9)$$

We choose:

$$\alpha h_{12} + \beta d_2 = 1 \quad \alpha h_{13} + \beta d_5 = h_{23} \quad (10)$$

which yield conditions (6)-(7). Then, (9) becomes

$$\hat{Y}_1 = h_{21}X_1 + X_2 + h_{23}X_3 + (\alpha + \beta d_3)Z_1 + \beta d_4\tilde{Z}_1. \quad (11)$$

Comparing (11) to the channel output at receiver 2 given by (2), we conclude that when the equivalent noise variance in (11) is smaller than the noise variance at receiver 2, i.e., when

$$(\alpha + \beta d_3)^2 + (\beta d_4)^2 \leq 1 \quad (12)$$

then, since receiver 2 can decode  $W_2$ , receiver 1 can decode  $W_2$  as well. This conclusion holds *regardless* of what the relay channel input is. By substituting the expression for  $\alpha$  from (7) into (12), we obtain (5), which is an equivalent condition to the condition for the interference channel bound in [1].

Because receiver 1 can reliably decode both messages, we can now bound the sum-rate using Fano's inequality as

$$\begin{aligned} &n(R_1 + R_2) \\ &\leq^{(a)} I(W_1, W_2; Y_1^n, Y_{1g}^n) \\ &= \sum_{i=1}^n \left[ H(Y_{1i}, Y_{1gi} | Y_1^{i-1}, Y_{1g}^{i-1}) - \right. \\ &\quad \left. H(Y_{1i}, Y_{1gi} | Y_1^{i-1}, Y_{1g}^{i-1}, W_1, W_2) \right] \\ &\leq \sum_{i=1}^n \left[ H(Y_{1i}, Y_{1gi}) - H(Y_{1i}, Y_{1gi} | Y_1^{i-1}, Y_{1g}^{i-1}, W_1, W_2) \right] \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^n \left[ H(Y_{1i}, Y_{1gi}) - \right. \\
&\quad \left. H(Y_{1i}, Y_{1gi} | Y_1^{i-1}, Y_{1g}^{i-1}, X_1^i, X_2^i, X_3^i, W_1, W_2) \right] \\
&=^{(b)} \sum_{i=1}^n \left[ H(Y_{1i}, Y_{1gi}) - H(Y_{1i}, Y_{1gi} | Y_1^{i-1}, Y_{1g}^{i-1}, X_1^i, X_2^i, X_3^i) \right] \\
&=^{(c)} \sum_{i=1}^n \left[ H(Y_{1i}, Y_{1gi}) - H(Y_{1i}, Y_{1gi} | X_{1i}, X_{2i}, X_{3i}) \right] \\
&= \sum_{i=1}^n I(X_{1i}, X_{2i}, X_{3i}; Y_{1i}, Y_{1gi}) \quad (13)
\end{aligned}$$

where (a) follows because receiver 1 can decode both messages; (b) follows by causality and (c) follows by the memoryless property of the channel.

By introducing a time-sharing random variable in (13) as in [13, Thm. 14.10.1], we obtain the sum-rate bound as

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1, Y_{1g}). \quad (14)$$

It follows from the maximum entropy theorem [13, Thm. 9.6.5] that Gaussian inputs maximize the mutual information expression in (14). As the final step, we optimize this bound over parameters  $d_i$ ,  $i = 1, \dots, 5$  subject to (10) and (12).

A corresponding sum-rate bound can be obtained having a genie at the other receiver.

By minimizing the mutual information expression in (4) with respect to  $d_1$ , we obtain the optimum value of  $d_1$  as

$$\begin{aligned}
d_1^* = & \left[ (P_1 + h_{13}\rho_{13}\sqrt{P_1P_3})(h_{12}d_2P_2 + d_5h_{13}P_3 + \right. \\
& (h_{12}d_5 + h_{13}d_2)\rho_{23}\sqrt{P_2P_3} + d_3) \\
& - d_5\rho_{13}\sqrt{P_1P_3}(h_{12}^2P_2 + h_{13}^2P_3 + h_{13}\rho_{13}\sqrt{P_1P_3} \\
& \left. + 2h_{12}h_{13}\rho_{23}\sqrt{P_2P_3} + 1) \right] \\
& \times \frac{1}{P_1(h_{12}^2P_2 + h_{13}^2P_3 + 2h_{12}h_{13}\rho_{23}\sqrt{P_2P_3} - h_{13}^2\rho_{13}^2P_3 + 1)}.
\end{aligned}$$

*Remark 1:* Evaluated for Gaussian inputs, the sum-rate bound (14) will depend on the covariance matrix of source and relay inputs.

*Remark 2:* The bound can be made more general by making the signal given by the genie also dependent on the noise at the relay. This would introduce one more parameter that can be optimized in order to obtain a tighter sum-rate bound.

*Remark 3:* For  $P_3 = 0$ , the bound reduces to the bound in [1].

#### IV. THE CUT-SET BOUND FOR THE INTERFERENCE CHANNEL WITH A RELAY

We next derive the cut-set bound [13, p. 445] for the ICR and compare it to the sum-rate bound presented in Thm. 1.

*Lemma 1:* For the ICR, the cut-set bound is given by

$$\tilde{\mathcal{R}} = \bigcup_{p(x_1)p(x_2)p(x_3|x_1x_2)} \mathcal{R}(p(x_1)p(x_2)p(x_3|x_1x_2)) \quad (15)$$

where  $\mathcal{R}(p(x_1)p(x_2)p(x_3|x_1x_2))$  denotes the set of rate pairs that satisfy

$$\begin{aligned}
R_1 &\leq \min\{I(X_1, X_3; Y_1|X_2), I(X_1; Y_1, Y_3|X_2, X_3)\} \\
R_2 &\leq \min\{I(X_2, X_3; Y_2|X_1), I(X_2; Y_2, Y_3|X_1, X_3)\} \\
R_1 + R_2 &\leq \min\{I(X_1, X_2, X_3; Y_1, Y_2), \\
&\quad I(X_1, X_2; Y_1, Y_2, Y_3|X_3)\}
\end{aligned} \quad (16)$$

evaluated for a specified distribution  $p(x_1)p(x_2)p(x_3|x_1x_2)$ . We observe that all terms in (16) are maximized by Gaussian inputs [13].

Observe that, since the genie gives only the minimum information that receiver 1 needs in order to decode both messages ( $W_1, W_2$ ), this implies that the bound of Thm. 1 is always at least as tight as the first term in the sum-rate of the cut-set bound,  $I(X_1, X_2, X_3; Y_1, Y_2)$ . We next compare the two bounds numerically.

#### V. NUMERICAL RESULTS

Fig. 2 shows an improvement of the sum-rate bound over the sum-rate cut-set bound for a specific choice of channel gains and powers. Fig. 3 shows the comparison with the outer bounds developed for the cognitive ICR in [8, Thm. 2 and Thm. 3], as well as with the cut-set bound (16). In all plots, the sum-rate bound (14) is evaluated together with the individual cut-set bounds on  $R_1$  and  $R_2$  given in (16).

Because the genie enables receivers to decode both messages, we expect the outer bound to be close to the achievable rates in the regimes in which such decoding is actually possible, i.e., when the receivers experience strong interference. This behavior is illustrated in Fig. 4. The achievable rate region, originally presented in [7, Eq. (14)], is repeated in (29). These rates are achieved by having the relay decode both messages ( $W_1, W_2$ ) and then jointly encode them. Both receiver jointly decode both messages.

The gap between the achievable rates and the outer bound in this regime is due to constraint (6) imposed in the outer

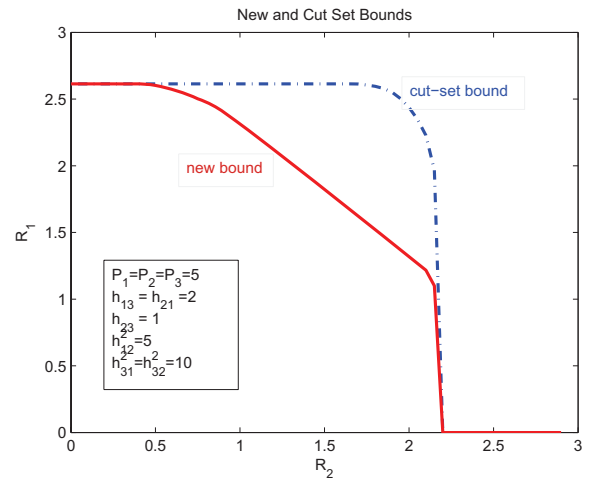


Fig. 2. Outer bound comparison.

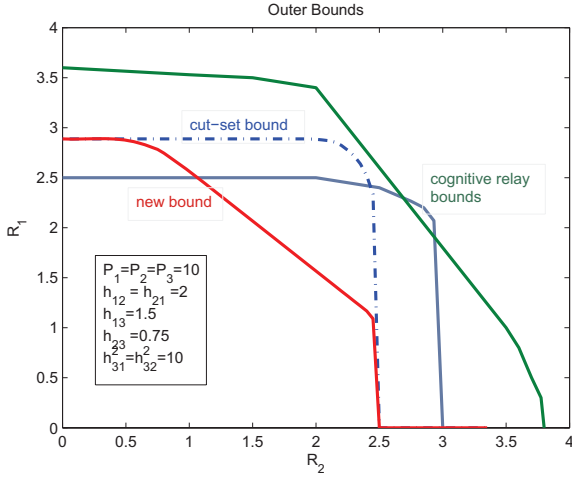


Fig. 3. Outer bound comparison.

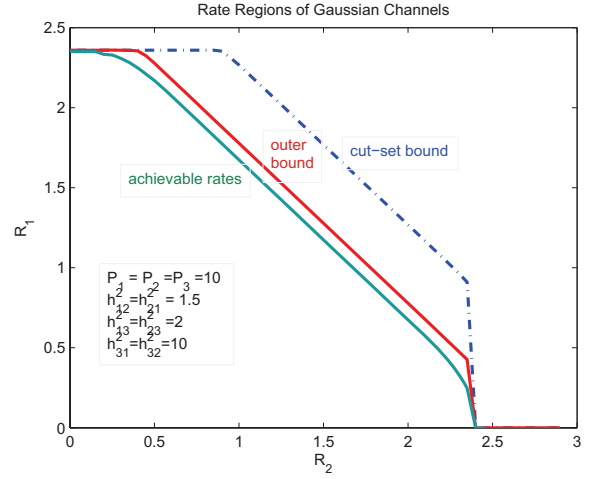


Fig. 4. An achievable rate region and the outer bound.

bound. This constraint may not be always necessary in order to allow receivers to decode both messages. We analyze the strong interference regime in the next section.

## VI. STRONG INTERFERENCE

The following rate region was shown to be achievable in [7, Thm. 1]. Rate pairs  $(R_1, R_2)$  that satisfy

$$R_1 \leq I(X_1, X_3; Y_1 | U_2, X_2) \quad (17)$$

$$R_2 \leq I(X_2, X_3; Y_2 | U_1, X_1) \quad (18)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_1) \quad (19)$$

$$R_1 + R_2 \leq I(X_1, X_2, X_3; Y_2) \quad (20)$$

$$R_1 \leq I(X_1; Y_3 | X_2, X_3) \quad (21)$$

$$R_2 \leq I(X_2; Y_3 | X_1, X_3) \quad (22)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_3 | X_3) \quad (23)$$

for any distribution

$$p(u_1, x_1)p(u_2, x_2)p(x_3|u_1, u_2) \quad (24)$$

are achievable.

We next show the following capacity result that generalizes the result in [6]:

**Theorem 2:** Under strong interference conditions

$$I(X_1, X_3; Y_1 | X_2) \leq I(X_1, X_3; Y_2 | X_2) \quad (25)$$

$$I(X_2, X_3; Y_2 | X_1) \leq I(X_2, X_3; Y_1 | X_1) \quad (26)$$

satisfied for all distributions  $p(x_1)p(x_2)p(x_3|x_1, x_2)$  and the following degradedness condition:

$$p(y_1, y_2 | y_3, x_3, x_1, x_2) = p(y_1, y_2 | y_3, x_3) \quad (27)$$

the achievable rate region (17)-(23) is the ICR capacity region.

*Proof: (Outline).* Bounds (17) and (18) can be shown by using the same approach as in [3, Sec. 3]. This approach will also imply the chain (24). Bounds (21)-(23) can be shown using similar steps as in [2, Lemma 4] and the degradedness condition (27). Sum-rate bounds (19)-(20) can be shown using

the same approach as in [6, Thm.2]. Details of the proof are omitted. ■

We next evaluate (17)-(23) for Gaussian inputs chosen as:

$$\begin{aligned} U_1 &\sim \mathcal{N}[0, \alpha P_1], \quad X_{10} \sim \mathcal{N}[0, \bar{\alpha} P_1], \quad X_1 = X_{10} + U_1 \\ U_2 &\sim \mathcal{N}[0, \beta P_2], \quad X_{20} \sim \mathcal{N}[0, \bar{\beta} P_2], \quad X_2 = X_{20} + U_2. \end{aligned} \quad (28)$$

Thus, the encoders 1 and 2 split their power between sending new information (respectively with  $\bar{\alpha} P_1$  and  $\bar{\beta} P_2$ ) and between cooperating with the relay. The power at the relay is split between forwarding messages  $W_1, W_2$  as:

$$X_3 = \sqrt{\frac{\gamma P_3}{\alpha P_1}} U_1 + \sqrt{\frac{\bar{\gamma} P_3}{\beta P_2}} U_2$$

where  $0 \leq \alpha, \beta, \gamma \leq 1$ . Parameter  $\gamma$  determines how the relay splits its power for forwarding  $W_1, W_2$ . A higher  $\gamma$  results in more power dedicated for forwarding  $W_1$ .

The region (17)-(23) evaluates to [7, Eqn. (14)]:

$$\begin{aligned} R_1 &\leq C(P_1 + h_{13}^2 \gamma P_3 + 2h_{13} \sqrt{\alpha P_1 \gamma P_3}) \\ R_2 &\leq C(P_2 + h_{23}^2 \bar{\gamma} P_3 + 2h_{23} \sqrt{\beta P_2 \bar{\gamma} P_3}) \\ R_1 + R_2 &\leq C(P_1 + h_{12}^2 P_2 + h_{13}^2 P_3 + 2h_{13} \sqrt{\alpha P_1 \gamma P_3} \\ &\quad + 2h_{12} h_{13} \sqrt{\beta P_2 \bar{\gamma} P_3}) \\ R_1 + R_2 &\leq C(h_{21}^2 P_1 + P_2 + h_{23}^2 P_3 + 2h_{21} h_{23} \sqrt{\alpha P_1 \gamma P_3} \\ &\quad + 2h_{23} \sqrt{\beta P_2 \bar{\gamma} P_3}) \\ R_1 &\leq C(h_{31}^2 \bar{\alpha} P_1) \\ R_2 &\leq C(h_{32}^2 \bar{\beta} P_2) \\ R_1 + R_2 &\leq C(h_{31}^2 \bar{\alpha} P_1 + h_{32}^2 \bar{\beta} P_2) \end{aligned} \quad (29)$$

where  $C(x) = 0.5 \log(1 + x)$ .

We next derive sufficient conditions that allow decoders to decode both messages, when signalling with the above inputs. When achievable rates  $(R_1, R_2)$  are used for signaling, receiver 1 can decode  $W_1$ , form  $U_1^n(W_1)$ , and with one block

delay (due to block Markov encoding) also form  $X_1^n(W_1)$  in order to evaluate:

$$\hat{Y}_1 = Y_1 - X_1(1 - h_{21}) - \sqrt{\frac{\gamma P_3}{\alpha P_1}} U_1(h_{13} - h_{23}). \quad (30)$$

From (2) and (30) we obtain:

$$\begin{aligned} \hat{Y}_1 &= h_{21}X_1 + h_{12}X_2 + h_{23}\sqrt{\frac{\gamma P_3}{\alpha P_1}}U_1 + h_{13}\sqrt{\frac{\bar{\gamma} P_3}{\beta P_2}}U_2 + Z_1 \\ Y_2 &= h_{21}X_1 + X_2 + h_{23}\sqrt{\frac{\gamma P_3}{\alpha P_1}}U_1 + h_{23}\sqrt{\frac{\bar{\gamma} P_3}{\beta P_2}}U_2 + Z_2. \end{aligned} \quad (31)$$

By comparing  $\hat{Y}_1$  and  $Y_2$  in (31) we conclude that receiver 1 obtains a less noisy signal carrying  $W_2$  than receiver 2 if

$$\begin{aligned} P_2 + h_{23}^2 \bar{\gamma} P_3 + 2h_{23} \sqrt{\beta \bar{\gamma} P_2 P_3} \\ \leq h_{12}^2 P_2 + h_{13}^2 \bar{\gamma} P_3 + 2h_{12}h_{13} \sqrt{\beta \bar{\gamma} P_2 P_3}. \end{aligned} \quad (32)$$

Therefore, since decoder 2 can decode  $W_2$ , so can receiver 1. Similarly, receiver 2 can decode  $W_1$  when

$$\begin{aligned} P_1 + h_{13}^2 \gamma P_3 + 2h_{13} \sqrt{\alpha \gamma P_1 P_3} \\ \leq h_{21}^2 P_1 + h_{23}^2 \gamma P_3 + 2h_{21}h_{23} \sqrt{\alpha \gamma P_1 P_3}. \end{aligned} \quad (33)$$

Conditions (32)-(33) have to be satisfied for all  $\alpha, \beta, \gamma \in [0, 1]$ . Evaluating the maximum of the left-hand side terms and the minimum of the right hand side terms we obtain sufficient conditions for (32)-(33) to hold as:

$$\begin{aligned} |h_{12}| &\geq |1 + h_{23} \sqrt{P_3/P_2}| \\ |h_{21}| &\geq |1 + h_{13} \sqrt{P_3/P_1}|. \end{aligned} \quad (34)$$

Therefore, in this scenario receivers can decode each other's message under conditions (32)-(33) without the help of a genie. In general, in the regime of strong interference there will be a gap between the achievable rate region and our outer bound because, in Thm. 1, constraint (6) does not allow for  $\beta = 0$ , (i.e., to turn off the genie.)

## VII. CONCLUSIONS

We present a new outer bound for the interference channel with a relay. The capacity region in strong interference for the discrete memoryless degraded ICR is also presented. The outer bound is obtained by adapting the approach developed for the interference channels in [1]. The bound is significantly tighter than the cut-set bound. One limitation of the bound is that it requires a receiver to decode *both* messages. This requirement could be relaxed by using the genie technique of [14] that led to the sum-capacity of the interference channel in the low interference regime [15], [16], [17]. The biggest difficulty of this approach when applied in our scenario is in showing the optimality of Gaussian inputs. This is one direction of our future work. The other one is to apply the genie-approach employed in this bound to larger networks.

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