An Overview of Different Wideband Direction of Arrival(DOA) Estimation methods

SANDEEP SANTOSH[1], O.P.SAHU[2], MONIKA AGGARWAL[3]
Senior Lecturer, Department of Electronics and Communication Engineering,[1]
Astt. Prof., Department of Electronics and Communication Engineering,[2]
National Institute of Technology, Kurukshetra ,[1,2]
Astt. Prof., Centre For Applied Research in Electronics (CARE)[3],
Indian Institute of Technology, New Delhi.[3]
INDIA.
profsandeepkkr@gmail.com http://www.nitkkr.ac.in

Abstract: - The Direction of Arrival (DOA) estimation methods are useful in Sonar, Radar and Advanced Satellite and Cellular Communication systems. In this paper different Direction of Arrival(DOA) methods such as Coherent Signal Subspace Processing (CSSM), the Weighted Average of Signal Subspaces (WAVES) and Test of Orthogonality of Projected Subspaces (TOPS) and Incoherent MUSIC(IMUSIC) is presented and their performance is also compared . The TOPS method performs better than others in mid signal–to-noise ratio (SNR) ranges, while CSSM and WAVES work better in low SNR. Incoherent methods like IMUSIC works best at high SNR.

Key-Words: - Direction of Arrival, CSSM, WAVES, TOPS,IMUSIC, SNR.

1 Introduction

The problem of locating radiating sources with sensor arrays has been a central topic of Signal Processing. Direction–finding algorithms have straight forward applications in Sonar and Radar and are also useful in Advanced Satellite and Cellular Communication systems for adaptive beamforming of smart antennas. In this research paper, we present an overview of some of the already existing wideband DOA estimation methods such as CSSM, WAVES and TOPS. The TOPS method performs better than others in mid signal–to-noise ratio (SNR) ranges, while CSSM and

WAVES work better in low SNR. Incoherent methods like IMUSIC works best at high SNR. We consider an array of M wide-band sensors which receive the wavefield generated by d wide bandpass sources in the presence of an arbitrary noise wave field. The array geometry can be arbitrary but known to the processor The source signal vector is,

$$S(t)=[s_1(t),s_2(t),..,s_d(t)]^T \qquad (1)$$

is assumed to be stationary over the observation interval T_O with zero mean. Superscripts T and H denote transpose and Hermitian transpose, respectively. The source–spectral density matrix is denoted as $P_s(f)$, $f \in F_+ = [f_0 - BW/2, f_0 + BW/2]$ and $F_- = [-f_0 - BW/2, -f_0 + BW/2]$ with BW comparable to f_0. $P_s(f)$ is an arbitrary $d \times d$ nonnegative Hermitian matrix unknown to the processor. The algorithm for CSSM is based on an appropriate frequency- domain averaging of the narrowband spatial covariance matrices. The frequency domain averaging can remove the singularity in $P_s(f)$.
The noise wavefield is assumed to be independent of the source signals with an arbitrary noise spectral density matrix $P_n(f)$, $M \times M$ known to the system designer except for a multiplicative constant σ_n^2. The array

output x(t), M×1, has a spectral density matrix ,

$$P(f) = A(f)P_s(f)A^H(f)+\sigma_n^2 P_n(f_j) \quad (2)$$

where A(f) is M × d transfer matrix of source- sensor array system with respect to some chosen reference point. It is assumed that the sensor number M is larger than the number of sources d and that the rank of A(f) is equal to d for any frequency and angles of arrival. The array output vector x(t) is first decomposed in the temporal domain into non-overlapping narrow-band components by using Discrete Fourier Transform (DFT) over a time segment of ΔT, the decomposed narrow-band components are uncorrelated and the covariance matrix for the component f_j can be expressed as ,

$$cov(X(f_i)) = (1/\Delta T)\, P_x(f_j)$$

$$= (1/\Delta T)A(f_j)P_s(f_j)A^H(f_j) + ((\sigma_n^2)/\Delta T).\, P_n(f_j) ,$$

$$j=1,2….,J. \quad (3)$$

We assume that the array output x(t) observed over T_0 seconds is sectioned into K subintervals of duration ΔTs each. Thus, ΔT is the duration of one snapshot in the usual terminology of narrow-band array processing and K is the total number of snapshots. We denote the jth narrow-band component obtained from the kth snap shot by $X_k(f_j)$,k= 1,2…,K, and j=1,2…,J. Our aim is to determine the number of sources d and estimate the angles θ_i, i=1,2,…d from data $X_k(f_j)$,k=1,2,..K and j=1,2…J. [1]

2 Coherent Signal Subspace Processing

Coherent Signal Subspace Processing (CSSM) presents a method of single signal-subspace for high resolution-estimation of the angles of arrival of multiple wideband plane waves. This technique relies on an approximately coherent combination of the spatial signal spaces of the temporally narrow-band decomposition of the received signal vector from an array of sensors. The algorithm for CSSM is presented. It is possible to combine the signal subspaces at different frequencies in a manner to generate a single signal subspace with algebraic properties indicative of the number of sources and angles of arrival. The following two theorems are considered for CSSM method.[1]

Theorem 1*:* Under the condition that $A(f_j)$, j=1,2…,J, have a rank of d, there exist nonsingular M×M matrices $T(f_j)$, j=1,2,…,J such that

$$T(f_j)A(f_j)=A(f_0), j=1,2,…J. \quad (4)$$

Theorem 2*:* Let λ_i and e_i , i=1,2,…,M be the eigen values and the corresponding eigen vectors of the matrix pencil (R,R_n) with λ_i in the descending order. The following is true.

1) $\lambda_{d+1} = \lambda_{d+2} = …. = \lambda_M = \sigma_n^2$;
2) the column span of $E_n = [\, e_{d+1} ,….,e_m \,]$ is orthogonal to the column span of $\{A(f_0)\}$,i.e., $A^H(f_0)E_n = 0$.

To estimate the coherent signal subspace, we need to estimate the covariance matrices and the transformation matrices using the observed data. The maximum-likelihood estimation of $Cov(X(f_j))$ is, under the normality condition ,simply the snapshot averaged cross-products of $X_k(f_j)$, k=1,2…K, i.e.,

$$C^{\wedge}(X(f_j))= 1/K \sum_{k=1}^{K} X_k(f_j)X_k^H(f_j), j=1,2,…,J \quad (5)$$

It is clear from its definition that construction of $T(f_j)$ requires a knowledge of the unknown angles of arrival. A natural estimator of T, uses preliminary estimates of angles in its formulation. We hypothesize that a knowledge of the neighbourhoods of these angles is sufficient to effect the advantages of coherent processing. . The steps used for coherent signal-subspace processing in the simulations are given below. Variations of these steps especially in obtaining the initial estimates of the angles, are possible.

1) DFT the array output;

2)Form $\hat{C}(X(f_j))$ and perform a preliminary estimation of approximate central angles of arrival $\beta_1, \beta_2, \ldots, \beta_{d1}$ using spatial periodogram as given by equation (5);

3) Form $T(j_j)$ using equation (6) or (7) if $d_1 = 1$;

4) Form $\hat{R}$ or $\hat{R}_n$ using equation (8) or (9);

5) Obtain $\hat{\lambda}_i$ and $\hat{e}_i$;

6) Determine $\hat{d}$ using equation (10),(11)and (12) to determine $\hat{E}_s$ or $\hat{E}_n$;

7) Determine the peak positions in a so-called spatial spectrum , for example ,MUSIC, given by equation (13)

$$\hat{T}(f_j) = [A_\beta(f_0)/B(f_0)][A_\beta(f_j)/B(f_j)]^{-1},\quad j=1,2,\ldots J \tag{6}$$

$$\hat{T}(f_j) = \begin{vmatrix} a_{1\beta}(f_0)/a_{1\beta}(f_j) & \ldots\ldots\ldots & 0 \\ & a_{2\beta}(f_0)/a_{2\beta}(f_j)\ \ldots & \\ 0 & & a_{M\beta}(f_0)/a_{M\beta}(f_j) \end{vmatrix} \tag{7}$$

$$\hat{R} = \Delta T \sum_{j=1}^{J} \hat{T}(f_j)\hat{C}(X(f_j))\hat{T}^H(f_j) \tag{8}$$

$$\hat{R}_n = \sum_{j=1}^{J} \hat{T}(f_j)P_n(f_j)\hat{T}^H(f_j) \tag{9}$$

$$AICE(d) = K\,(M-d)\log(a_0/g_0) + d(2M-d) \tag{10}$$

$$a_0 = (1/M-d)\sum_{i=d+1}^{M} \hat{\lambda}_i \tag{11}$$

$$g_0 = \{\prod_{i=d+1}^{M} \hat{\lambda}_i\}^{1/M-d} \tag{12}$$

$$\hat{P}(\theta) = 1/(A_\theta^H(f_0)\hat{E}_n \hat{E}_n^H A_\theta(f_0))$$

$$= 1/(A_\theta^H(f_0)\hat{R}_n^{-1}A_\theta(f_0) - A_\theta^H(f_0)\hat{E}_s\hat{E}_s^H A_\theta(f_0)) \tag{13}$$

It is pointed out that steps 3) to 7) above can be iterated to improve the estimates. The problem of determining the correct number (detection) and angles of arrival of multiple wide-band plane waves based on measurements by an array of sensors was considered. The signal subspace approach was selected due to its proven high resolution angle estimation properties in the narrow-band case. In this proposed approach, a combined signal subspace is formulated that is a result of aligning and averaging the signal subspaces from constituent narrow-band spaces in the temporospatial received vectors. The coherently constructed signal space results in an appropriately frequency-averaged estimate of the spatial covariance matrix that is statistically more accurate and immune to the degree of correlation between the sources. Therefore, Akaike's information criterion in this space yields accurate determination of number of sources, and the application of methods such as MUSIC give estimates of the angles at a much lower signal-to-noise ratio than corresponding non-coherent methods.[1]

3 Weighted Average of Signal Subspaces

3.1 Introduction

The standard CSSM has several drawbacks. The arrays cannot be generally focused over the entire field of view with small errors by linear transformations. With this constraint, the approximation error increases because fewer degrees of freedom are available for design. Most signal subspace and algorithms are sensitive to model errors. The background noise becomes "colored" in an unpredictable manner in presence of focusing errors. Most algorithms focus the array only over narrow angular sectors that are selected after a preliminary beam forming search. Source cross-spectra and focusing errors vary across the sensor-passband .

But the Weighted Average of Signal Subspaces(WAVES) strategy combines a robust near-optimal data-adaptive statistic, with an enhanced design of focusing matrices to ensure a statistically robust preprocessing of wideband data. The overall sensitivity of WAVES to various error sources ,such as imperfect array focusing, is also reduced with respect to traditional CSSM algorithm. The intrinsic redundancy of wideband array data due to large time-bandwith product of sources is exploited in WAVES., which furnishes a basis for universal focused signal subspace.
Signal Subspace eigenvectors estimated from narrowband decomposition of the array outputs and properly focused generate a pseudodata matrix ,which approximately obeys the narrowband array model. The WAVES estimate is obtained from this matrix through the concept of pseudocovariance which is drawn from robust statistics. The spatial data covariance matrix (SCM) eigenvectors are adaptively weighted to limit the impact of model errors on the final estimate.

The sample WAVES is consistent under the hypothesis of absence of focusing errors when nonunitary focusing matrices are used. It is less sensitive than the Universal spatial covariance matrix (USCM) against actual focusing errors, mistakes in selection of signalsubspace eigen vectors and the presence of weak narrowband sources. Another critical issue for the statistical performance of coherent wideband processing is the synthesis of focusing matrices. Classical least square fitting is not directly related to optimization of statistical performance. Therefore, WAVES is a new angle-dependent error criterion for coherent focusing. It relies on the standard results of the aymptotical analysis of the subspace algorithms , to reduce the bias of beamforming invariance CSSM focusing , which does not require a preliminary search over large angular sectors .[2]

3.2 Signal model

An M-element array receives D signals, radiated by point sources. Each sensor signal is converted to the baseband and is time sampled and frequency decomposed using a filter bank or the DFT into J complex subband components sampled with period T. The bandwidth of each filter is much smaller than its central frequency f_i ($i=1,2,\ldots,J$) so that each subband snapshot $x_i(nT)$($n=0,1\ldots,N-1$) approximately satisfies the classical narrowband equation.

$$x_i(nT)=A(f_i)s_i(nT)+n_i(nT) \qquad (14)$$

Where

$$A(f_i)=[a_1(f_i,\theta_1),\ldots,a_D(f_i,\theta_D)] \qquad (15)$$

is the M×D array transfer matrix at frequency f_i and $s_i(nT)$ is the vector of complex source signals. Columns of $A(f_i)$ are referred to as steering vectors and represent the array response to each wavefront. It is assumed that the generic steering vectors $a(f,\theta)$ has unit L_2 norm and is a known, continous and differentiable function of frequency f and the P location parameters $\{¢_p; p=1,2,\ldots,P\}$contained in vector θ(azimuth, elevation, range,…).For unique identifiability of the model, $A(f_i)$ must have full rank for any set of $D+1<= M$ source locations. $n_i(nT)$ is the M×1 vector of additive noise at frequency f_i and time nT. The background noise is considered white and isotropic at any frequency, with variance $\sigma_n{}^2$ .Coloured noise whose spatio-temporal covariance is known, can be handled by prewhitening. The SCM at frequency f_i , $R_{xx}(f_i)$ obeys the following equation ,

$R_{xx}(f_i)= E[x_i(nT)\, x_i^H(nT)]$

$$=A(f_i)R_{ss}(f_i)A^H(f_i)+\sigma_n^2 I_M . \quad (16)$$

The eigenvectors corresponding to the ή$i \leq D$ largest spatial covariance matrix eigen values $\lambda_1(f_i)\geq\lambda_2(f_i)\ldots,\geq\lambda_{ήi}(f_i)$, collected in the matrix $E_s(f_i)$ constitute a basis for signal subspace at frequency f_i. The signal subspace is contained in the range of $A(f_i)$.The orthogonal complement to the signal subspace is called the noise subspace and is defined by the basis $E_n(f_i)$ of M-ήi eigenvectors that are associated with smallest eigenvalue σ_n^2 of $R_{xx}(f_i)$. In practice , $R_{xx}(f_i)$ is estimated from N≥≥ M snapshots. The SCM estimate $R_{xx}(f_i)$must be consistent and asymptotically unbiased.[2]

3.3 Wideband WSF Criterion and Weighted Average of Signal Subspaces (WAVES)

The classical CSSM spatially transforms both signals and noise .The narrowband Weighted subspace fitting(WSF) algorithm uses only signal subspace eigenvectors to get asymptotically efficient DOA estimates $\tilde{\Theta}_{WSF}$. Extension of WSF criterion to the wideband case is straightforward since the global and concentrated Gaussian log-likelihood is simply the sum of log-likelihoods of narrowband components if snapshots can be considered independent among different frequency bins.

$$\tilde{\Theta}_{WSF}=\arg_{\Theta} \min \{\sum_{i=1}^{J}|A(f_i,\Theta)\tilde{C}_i - \tilde{E}_s(f_i)P(f_i)|^2_F\}. \quad (17)$$

$\tilde{E}_S$ is the estimated signal subspace at frequency f_i . For Gaussian signals and noise perfectly calibrated array, optimal ήi × ήi weighting matrices $P(f_i)$ are diagonal with elements proportional to ,

$$P(f_i)_{[k,k]} =(\lambda_k(f_i)-\sigma_n^2)/(\lambda_k(f_i)\sigma_n^2)^{1/2} \quad (18)$$

Each D × $ή_i$ matrix $\tilde{C}_i$ in (17) satisfies the relation $\tilde{C}_i = P(f_i)\, \tilde{E}_s(f_i)A^{\dagger}(f_i, \Theta)$. Optimality is preserved even if consistent estimates of weights are substituted in place of true values. In case of Weighted Average of Signal Subspaces(WAVES) , the diagonal shape of the optimal $P(f_i)$ in (18) means that sample signal subspace eigenvectors work like independent observations, at least for Gaussian signals. This property can be exploited to develop a new global statistic, which can replace USCM. Solution of (17) does not change if single terms of summation are premultiplied by unitary M× M matrices $T(f_i)$ that satisfy the perfect focusing condition ,hence equation (17) becomes

$$\tilde{\Theta}_{WSF} = \arg_{\Theta} \min \{\sum_{i=1}^{J} |B(f_0,\Theta)\tilde{C}_i - T(f_i)\tilde{E}_s(f_i)P(f_i)|^2_F\}. \quad (19)$$

Equation (19) suggests the introduction of the matrix

$$\tilde{Z} =(\sum_{i=1}^{J}\eta_i)^{-1/2}.[\, T(f_1)\tilde{E}_s(f_1)P(f_1),\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, T(f_J)\tilde{E}_s(f_J)P(f_J)]$$

$$=\eta^{-1/2}[\tilde{Z}_1,\ldots,\tilde{Z}_J]. \quad (20)$$

The next two theorems state that the $d \leq D$ principal left singular vectors of $\tilde{Z}$ asymptotically furnish an estimate of the universal signal subspace.

Theorem 1 : The sample matrix $\tilde{Z}$ converges with probability 1 to a fixed matrix Z under the following assumptions.

1) Each sample subspace $\tilde{E}_s(f_i)$ converges w.p. 1 to true $E_s(f_i)$ as $N\rightarrow\infty$.
2) Matrices $T(f_i)$ and $P(f_i)$ have full rank and finite L_2 norm.
3) SCM eigenvectors are made unique by a proper scaling.

Theorem 2 : If ,in addition to assumptions of *Theorem 1,*focusing matrices satisfy equation (21), all columns of Z lie in the range of $B(f_0)$. The only tenable conclusion is equation (22).

$$B(f_0)=T(f_i)A(f_i)=[b(f_0,\theta_1)\ldots,b(f_0,\theta_D)] \quad (21)$$

$$Z = \eta^{-1/2} B(f_0)[C_1,\ldots, C_J] =B(f_0)C \quad (22)$$

Theorem 2 indicates that the d-dimensional subspace spanned by columns of Z defines a basis for the universal signal sub-space. Since this subspace can be interpreted as the average of weighted $E_s(f_i)$s, therefore it is referred to as WAVES.

3.4 Direction finding using WAVES.

The sample matrix $\tilde{Z}$ is always full rank because of presence of finite focusing and sample errors. $\tilde{Z}$ can be considered to be *pseudodata* matrix, built with sample SCM eigenvectors, instead of raw array snapshots. Therefore, from *Theorem1* and *Theorem2,* it is quiet natural to estimate a basis for the WAVES from the d principal left singular vectors $\tilde{U}_S$ of the following reduced-size SVD (R-SVD) of $\tilde{Z}$

$$\tilde{Z} = [\tilde{U}_S \ \tilde{U}_N] \begin{bmatrix} \tilde{\Sigma}_S & 0 \\ 0 & \tilde{\Sigma}_N \end{bmatrix} [\tilde{W}_S \ \ \tilde{W}_N]^H. \quad (23)$$

This algorithm can be regarded as a total least squares (TLS) approximation step attempting to "restore" the original rank-d property of Z. U_S asymptotically converges to WAVES. The basis $\tilde{U}_S$ and its orthogonal complement $\tilde{U}_N$ can replace the corresponding CSSM universal subspaces $\tilde{E}_S$ and $\tilde{E}_N$ in any subspace based algorithm for direction finding. A very basic algorithm combining WAVES and MUSIC is given below.

Step 1) For i=1,2,…,J, estimate $\tilde{E}_s(f_i)$ and η_i from eigen value decomposition (EVD)of the sample SCM,

$$\tilde{R}_{xx}(f_j) = 1/N \sum_{n=0}^{N-1} X_i(nT)\, X_i(nT)^H .$$

*Step2)*For i =1,2…,J,Compute $T(f_i)$and $P(f_i)$ according to some available criterion.

Step 3) Form the matrix $\tilde{Z}$ according to (20).

Step 4) Estimate the number of sources $\tilde{D}$, the WAVES $\tilde{U}_S$ and $\tilde{U}_N$ from R-SVD of $\tilde{Z}$ from (23)

Step 5) Estimate angles using the MUSIC criterion with $\tilde{U}_N$ in place of E_N.

An important issue for the consistency of DOA estimators based on $\tilde{Z}$ is that the number of free parameters of matrix C remains finite ,regardless of the number of snapshots used to form each SCM. In this case , WAVES spans exactly the range of $B(f_0)$. Fast iterative algorithms to compute the EVD of $\tilde{R}_{xx}(f_j)$ and the R-SVD as given by equation (23) can be calculated.[2]

3.5 Error Analysis of WAVES.

A rigourous error analysis of WAVES appears mathematically intractable. An approach based on standard first-order perturbative expansions of sample signal subspaces and WAVES worked satisfactorily under the hypothesis of sufficiently small estimation errors .This analysis shed light on the asymptotical performance of WAVES and allowed the synthesis of algorithms resistant to large errors . This analysis is divided into three parts .[2]

1) Sensitivity of the sample signal subspaces.
2) TLS estimation of WAVES.
3) Asymptotical properties of DOA estimates.

The various Sample Subspace Errors are Finite Sample Errors, Calibration and Focusing Errors and other Error Sources.

3.6 Robust Estimation of WAVES.

Signal subspace misalignment essentially produces a leakage in error subspace, which can mask weak signal components and generate “ghost” sources .These effects resemble those induced by the presence of the outliers in multichannel data matrices. Most of the theory developed for robust covariance estimation cannot be directly extended to the problem at hand since fundamental probabilistic concepts, like “distribution,” “contamination”, and “robustness” itself become intrinsically vague when applied to matrix $\tilde{Z}$. A robust WAVES estimator should combat mismodeling.

The mathematical tools developed in robust statistics can be extremely effective as heuristic optimization procedures if linked to WAVES concept. It is likely that some corrupted eigenvectors exist and “blow up” the WAVES estimate obtained by the SVD. Since transformed eigenvectors can be considered to be nearly independent observations of the array response in frequency and space ,hence their impact on WAVES can be effectively controlled by adaptively scaling each column of Z . This operation is equivalent to optimizing diagonal matrices $P(f_i)$ and ensures that the WAVES matrices is invariant with respect to unitary column transformations of Z just as the sample Gaussian ML SCM and the USCM. If most eigenvector weights remain nonzero, Theorem 2 still guarantees the asymptotical convergence of WAVES. The robust estimation of WAVES has following steps i.e, Algorithm Initialization , Weight Optimization , Choice of s(x), Eigenvector Selection ,Eigenvector Whitening and Enhanced Synthesis of focusing matrices.[2]

4 Test of Orthogonality of Projected Subspaces

4.1 Introduction

This new technique estimates DOAs by measuring the orthogonal relation between signal and noise subspaces of multiple frequency components of the sources. TOPS can be used with arbitrary shaped one-dimensional or two-dimensional arrays. Unlike other coherent wideband methods such as CSSM and WAVES, the new method does not require any preprocessing for initial values. This new technique performs better than others in mid signal-to-noise ratio (SNR) ranges ,while coherent methods work best in low SNR and incoherent methods work best in high SNR.
Unlike coherent methods that must align the signal and noise subspaces to form a viable general covariance matrix, TOPS determines whether or not a DOA dependent transformation is able to achieve the alignment. Although, TOPS does not cohere the subspaces over frequency to achieve the processing gain , the multiple alignment test over frequency bins leads to a more robust estimator at lower SNR than incoherent methods.
The advantages of TOPS are 1) It does not require focusing angles or beamforming matrix, 2) It does not suffer from bias at large SNR and 3) at low SNR , it better integrates frequency bins than incoherent methods. Thus, TOPS fills a gap between coherent and incoherent methods. Similar to previous wideband methods, TOPS uses the DFT of sensor outputs given by equation (24)

$$X(\omega_i)=A(\omega_i,\theta)S(\omega_i)+N(\omega_i), i= 0,1,..,K-1. \quad (24)$$

We consider linear arrays with arbitrary sensor locations along with additional constraint that the array manifolds of different DOAs should be independent. Like CSSM and WAVES, TOPS also uses a transformation matrix to exploit multiple frequency components. TOPS does not use the transformation matrix to generate the general correlation matrix $R_{gen.}$ TOPS can provide perfect measurements for infinite SNR. Like Steered Covariance method (STCM), TOPS uses the Rotational Signal Subspace (RSS) focusing matrix designed for a single DOA. But, TOPS uses the transformation matrix at each hypothesized DOA to perform an orthogonality test between the transformed signal subspace and the noise subspace. If the hypothesized DOA corresponds to a true DOA , then orthogonality is preserved otherwise not.

TOPS uses a diagonal unitary transformation matrix, similar to STCM given by equation (25).

$$T_i = \mathrm{diag}\{e^{j(\omega_i - \omega_0) v_0 \sin(\theta_f)}, .., e^{j(\omega_i - \omega_0) v_{M-1} \sin(\theta_f)}\} \quad (25)$$

The kth term on the diagonal of the transformation matrix $\Phi(\omega_i, \theta_i)$ is given as,

$$[\Phi(\omega_i,\theta_i)]_{(k,k)}=\exp(-j\omega_i v_k \sin\theta_i). \quad (26)$$

The transformation matrix has been able to preserve the array manifold as shown in the lemma below.

Lemma 1 : Given a linear array manifold $a_i(\theta_i)$, and a matrix $\Phi(\omega_j, \theta_j)$,the product is a new array manifold

$$a_k(\theta_k)=\Phi(\omega_j,\theta_j)a_i(\theta_i) \quad (27)$$

where the relations between frequencies and DOAs are,

$$\omega_k=\omega_i+\omega_j \quad (28)$$

$$\sin\theta_k =(\omega_i / \omega_k) \sin\theta_i +(\omega_j/\omega_k) \sin\theta_j \quad (29)$$

Lemma 2 : Let $\Delta\omega = \omega_j - \omega_i$. Then the following two range spaces are identical :

$$R\{\mathbf{\Phi}(\Delta\omega, \Phi)F_i\}=R\{A(\omega_j,\theta^{\wedge})\} \quad (30)$$

Where the new angles $\theta^{\wedge}$ depend on Φ in the following manner,

$$[\theta^{\wedge}]_i = \arcsin\{(\omega_i / \omega_j) \sin\theta_i +(\Delta\omega / \omega_j) \sin\Phi\} \quad (31)$$

Theorem : Assume that $2P \leq M$ and $K \geq P+1$. .Let's define the M * P matrices $U_i(\Phi)$ as,

$$U_i(\Phi) =\mathbf{\Phi}(\Delta\omega_i,\Phi)F_0, i = 1,2,\ldots, K-1. \quad (32)$$

Where $\Delta\omega_i = \omega_i - \omega_0$, and Φ is a hypothesized azimuth angle. Define a P * (K−1)(M −P) matrix $D(\Phi)$ as,

$$D(\Phi)=[U_1^H W_1| U_2^H W_2|\ldots|U_{K-1}^H W_{K-1}]. \quad (33)$$

Then the following applies:
a) If $\Phi = \theta_l$ for some l , $D(\Phi)$ loses its rank and becomes rank deficient.
b) If $D(\Phi)$ is full rank matrix $\Phi \neq \theta_l$ for all l.

This theorem holds as long as the source signals are not fully correlated. TOPS works when the dimension of F_i is same as that of signal subspace for all i. However, when some of the sources are fully correlated, the dimensionality of signal subspace decreases and like the performance of most DOA estimators, the performance of TOPS degrades. In order to derive TOPS to work with arbitrary arrays, the following two conditions should hold.

a) There always exists a transformation matrix such as $\mathbf{\Phi}(\omega, \Phi)$ in (27).
b) The $D(\Phi)$ matrix should be full rank unless Φ is equal to one of the DOAs.[3]

4.2 Signal Subspace Projection

In practise, the correlation matrices are unavailable so estimated correlation matrices are used in place of true correlation. In order to estimate the correlation matrix, the sensor outputs are divided into j blocks ,with the number of samples in one block being equal to the number of DFT points. If we let $x_{j,I}$ be the sensor DFT output at $\omega = \omega_i$ for the jth block ,then the estimated correlation matrix is,

$$\hat{R}_i = 1/J \sum_{j=0}^{J-1} x_{j,I} x_{j,I}^H \quad (34)$$

From $\hat{R}_i$, we can find the signal and noise subspaces $\hat{F}_i$ and $\hat{W}_i$. The DOA estimation performance depends on the quality of estimated correlation matrix which in turn is determined by number of snapshots and the SNR, which are not usually under the control of the processor. By subspace projection it is possible to reduce some error terms in the TOPS matrix $\hat{D}(\Phi)$. If we define a projection matrix $P_i(\theta)$ on to the null space of $a_i(\Phi)$,then we have,

$$P_i(\Phi) = I - (a_i^H(\Phi) a_i(\Phi))^{-1} a_i(\Phi) a_i^H(\Phi) \quad (35)$$

Hence , we can form a modified $D(\Phi)$ matrix where $U_i(\Phi)$ is replaced by ,

$$U_i'(\Phi) = P_i(\Phi) U_i(\Phi). \quad (36)$$

This projection reduces the signal subspace component leakage in the estimated noise subspace. Estimates from this modified $D(\Phi)$ matrix exhibit less mean square error. This projection method is used only when the distance between sensors is less than half the wavelength of the highest frequency used in processing and not the center frequency otherwise aliasing occurs.[3]

4.3 Algorithm

Since both the signal subspace and noise subspace are estimated, it is unlikely that D would ever become rank deficient. We can find how close a matrix is to being rank-deficient by looking at the condition number or the minimum singular value of the matrix. The smallest singular value is a better choice because we are looking for the case where D becomes rank deficient when one of its row vectors become zero vector. The following steps summarize the TOPS method of finding DOAs of wideband sources for 1-D arrays.

1) Divide the sensor output into J identical sized blocks .
2) Compute the temporal DFT of the J blocks.
3) For the jth block, select $x_{j,k}$ at preselected ω_k, where k= 0,1,….,K–1 and j=0,1,…J–1.
4) Compute the signal subspace $\hat{F}_1$ and the noise subspace $\hat{W}_K$ for k = 1, …,K–1 by SVD of estimated covariance matrices $\hat{R}_k$.
5) Generate $\hat{D}(\Phi)$ using (36) and (33) for each hypothesized DOA Φ.
6) Estimate $\hat{\theta}$ by,
   $\hat{\theta} = \arg\max_{\Phi} 1/\sigma_{min}(\Phi)$ where $\sigma_{min}(\Phi)$ is the smallest singular value of $\hat{D}(\Phi)$.The estimation is now to find P local maxima by doing a one-dimensional search.[3]

4.4 Computational Complexity

It is not easy to calculate the exact computational cost for TOPS. The number of computations for an M* M SVD is $O(M^3)$.The minimum nonzero singular values of the D matrix can be found via an SVD of a P*P matrix ,so $O(P^3)$ computations have to be done for each hypothesized Φ. For CSSM and WAVES , once they have formed the coherent correlation matrix, only a single SVD is required to use a typical narrowband signal subspace method. These methods require fewer computations than TOPS . But the process of finding the RSS focusing matrices requires an SVD of a M*M(M › 2P) matrix for each frequency bin. Thus, if we consider the computational cost for preprocessing and the performance results, TOPS is still a viable alternative choice for wideband DOA estimation.[3]

4.5 Concept of Focusing Angles

For the coherent methods, the focusing angles are one of the most important factors that impact the estimator's performance . Two different kinds of focusing are used in the simulation. First, we use perturbations of true DOAs by adding Gaussian random noise, so we have

$$\Theta_F \sim N(\theta, \sigma^2_f I) \qquad (37)$$

as focusing angles .Various values of σ_f were used in the simulations. The errors in focusing angles are usually coupled, so it is little bit unrealistic to use independent errors between focusing angles. The simulations with those perturbed focusing angles can give some idea of how much error is tolerable. The second type of focusing angles is much more realistic. i.e 1) Find approximate DOAs μ_i using low-resolution algorithms and 2) use μ_i , μ_i + 0.25BW and μ_i - 0.25BW as focusing angles where BW denotes the beamwidth. Unlike the focusing angles which are fixed earlier, we use variable μ_i's that are estimated during each run. This seems more natural since the μ_i's are also random variables. Capon's method is employed as low-resolution algorithm to obtain the estimates μ_i . DOA estimates of multiple frequency bins by Capon are averaged and used in coherent methods .[3]

5 Incoherent Methods

If Multiple Signal classification (MUSIC)is used as a narrowband method , the wideband DOA estimate is

$$\hat{\theta} = \arg\min_\theta \sum_{i=0}^{K-1} a_i^H(\theta)\, W_i W_i^H\, a_i(\theta) \qquad (38)$$

where W_i is the noise subspace at frequency ω_i . The noise subspace matrix W_i is estimated from spatial correlation matrix R_i. Since the final estimates are averages of magnitude squared functions from different signal subspaces, ISSM is called an incoherent subspace method.[3]

6 Applications in parameter estimation and detection

Some of the main areas of parameter estimation and detection in array signal processing are 1) Personal Communications 2) Radar and Sonar Communications 3)Industrial Applications and 4)Future Directions.

1)Personal Communications: Receiving arrays and related estimation/detection techniques have long been used in High Frequency communications . These applications have emerged recently and received significant attention of the researchers for numerous problems in personal communications .One of the most important problems in multiuser asynchronous environment is the inter-user interference which can degrade the performance quite severely. This is also the case in practical Code-Division Multiple Access(CDMA)systems because the varying delays of different users induce non-orthogonal codes. The base stations in mobile communication systems are using spatial diversity for combating fading due to severe multipath . But using an antenna array of several elements introduces additional degrees of freedom which is used to obtain higher selectivity . An adaptive receiving array is steered in the direction of one user at a time while simultaneously nulling interference from other users in the same manner as Beamforming technique.[4],[5]

2)Radar and Sonar Communications: The classical application of array signal processing is in Radar and Sonar , and modern model based techniques have also found their ways in these areas. The antenna array is used for source localization , interference cancellation and ground clutter suppression . In Radar ,the mode of operation is active. This is because of the role of the antenna array based system which radiates pulses of electro-magnetic energy and listens for return .The parameter space of interest may vary according to the geometry and sophistication of the antenna array. The Radar returns enables estimation of parameters such as velocity(Doppler frequency), range and Direction of Arrivals (DOA) of Targets of interest.. Using passive far-field listening arrays only the DOAs can be estimated.

But in case of Sonar ,the signal energy is usually acoustic, and measured only using an array of hydrophones .The Sonar can operate in active as well as passive mode .In a passive mode the receiving array has the capability of detecting and locating distant sources. Deformable array models are often used in Sonar as the receiving antenna is typically towed under water . In an active mode, a sonar system emits acoustic (electro-magnetic arrays are also used under water)energy and monitors and retrieves any existing echo .This can again be used for parameter estimation such as bearings and velocity using the delay of the echo. Inspite of its limitations due to bending speed-of-propagation profiles and the high propagation losses, Sonar remains a reliable tool for range, bearing estimation and other imaging tasks in underwater applications The difficult propagation conditions underwater may call for more complex signal modeling such as in matched field processing .

3) Industrial Applications: Sensor array signal processing techniques have dsrawn much interest from industrial application such as manufacturing and medical applications. In medical imaging and hyperthermia treatment , circular arrays are commonly used as a means to focus energy in both an injection mode as well as reception mode .It has also being used in treatment of tumors. In Electrocardiograms, planar arrays are used to track the evolution of wavefronts which in turn provide information about condition of a patient's. Array Processing methods are also used to localize brain activity using bio-magnetic sensor arrays .Signals emanating from a womb is of are of more interest than those of the mother. Other applications in industry are automatic monitoring and faul detection/localization. In engines, sensors are placed in a judicious and convenient manner to detect and potentially localize faults such as knocks and boken gears. It is also used in object shape-characracter ization in Tomography.[6]

4) Future Directions: We will be witnessing an explosive development of array processing algorithms. We look forward for more model based array signal processing in various imaging problems . The examples are Synthetic Aperture Radar(SAR) and underwater acoustic imaging .The interest in remote sensing and imaging is expected to grow due to its applications in the environmental studies. Incoherent and Coherent Wideband array processing techniques are used for aeroacoustic detection and tracking of ground vehicles. Target identification using battlefield acoustic sensor arrays is an important for the army. In applications such as medical imaging or nondestructive evaluation active sensor array are used. Active sensor arrays not only receive but also transmit a signal. Seismic sensor arrays are used for buried landmine detection .In seismic array system , multiple transmitters and multiple receivers are used. Each transmitter sends an acoustic pulse into the medium and all the receivers record the reflection at the same time . The time-domain duration of the transmitted pulse should be as short as possible in order to increase the range resolution so its frequency bandwidth is wide .TOPS can be tested with seismic sensor data.[7]

6 Conclusion

IMUSIC works best at high SNR. Coherent methods work best in low SNR. The performance of TOPS lies between coherent and incoherent methods in the whole SNR range. This method is different from coherent methods that form a general coherent correlation matrix using focusing angles. It is different from usual incoherent methods since it takes advantage of subspaces from multiple frequencies simultaneously. TOPS represents a new way of processing multiple subspaces and would be able to improve DOA estimates not only for wideband sources but also for acoustic sources having multiple harmonics.

References:

[1] H. Wang and M. Kaveh, " Coherent signal -subspace processing for the detection and estimation of angles of arrival of multiple wideband sources" , IEEE Transaction on Acoustic, Speech, Signal processing vol ASSP-33,no.4,pp 8233-8330,Aug. 1985.

[2] E. Di Claudio and R. Parisi, "WAVES: Weighted average of signal subspaces for robust wideband direction finding", IEEE Trans. On Signal processing, vol 49, no. 10, pp 2179-2190,Oct. 2001.

[3] Y.S Yoon, L.Kaplan and J.McClellan , "TOPS: New DOA estimator for wideband signals", IEEE Transactions on Signal processing, vol 54,no. 6, June 2006.

[4] H. Krim and M.Viberg, "Two decades of array signal processing research: The parametric approach" , IEEE Signal processing magazine, vol 13, no.4, pp 67-94,July, 1996.

[5] D.W.Bliss, K.W.Forsythe, 'Angle of Arrival estimation in the presence of Multiple sources', WSEAS Transactions on Computers, v.7 p.990-999 July 2008.

[6]Kourzaev, Guennadi, R.Eckhoff, " Direction of Arrival determination using 3-axis crossed array" , WSEAS Transactions on Computers 2006;5(10) ,2221-2224

[7] G.E.Athanasiadon, I.J.Wassell,, 'Comparisons on Ray Tracing Predictions and Field Trial results for Broadband fixed Wireless Access Scenarios' , WSEAS Transactions on Communications', ISSN 1109-2742,Issue 8,vol. 4,August 2005, p.p 717-721.

About Authors:

1.) Prof. Sandeep Santosh is Sr. lecturer in ECE deptt. of NIT Kurukshetra.

2.) Dr. O.P.Sahu is Astt. Prof . in ECE deptt of NIT Kurukshetra.

3.) Dr Monika Agrawal is Astt. Prof. in CARE, IIT Delhi.