## An urn model for recall order

An um model based on an associationistic scheme of response ordering in recall is described. The model produces sequences of binary events where each event is identified by membership in one of two verbal classes. Data show that this series of binary events tends to reflect recall sequences of items from two categories.

Of the various urn models of behavior that have been proposed (e.g., Block, 1965; Arnold, 1965), none have dealt with one-trial or immediate memory. The phenomenon with which this report is concerned is the organization of responses (response ordering) during a single recall. A list of recall items must be capable of appearing in two or more different orders if recall organization is to be considered. This presumes the existence of at least two subsets of stimuli that are categorically different. In the simplest of cases, a list of verbal items to be recalled might comprise words taken from two categories such as names of animals and names of musical instruments. In the most complex of cases, no two words would be related. There would be as many categories as there are words in the list with a single word in each category. For simplicity the model will restrict itself to stimulus inputs of two categories ( $C_{1}$ and $C_{2}$ ). Such a restriction does not preclude the analysis of an $n$ category list since such a list can be divided into arbitrarily designated "category" and "non-category" items. A further simplifying assumption will be made that each item in a given category is indistinguishable from any other item in the same category.

Let us assume that the stimulus words have been presented and the $S$ recalls a $C_{1}$ item. (Generally, a stimulus list is presented in multiple random orders to suppress the serial position effect.) This item will act as a stimulus which will elicit either another $\mathrm{C}_{1}$ item or an item from $\mathrm{C}_{2}$. The recall of another $\mathrm{C}_{1}$ item would imply a greater number of "within-category" (e.g., $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}$ ) than "between-category" (e.g., $\mathrm{C}_{1} \rightarrow \mathrm{C}_{2}$ ) connections. In fact, if there were $w$ within-category connections with $\mathrm{C}_{1}$ and $b$ between-category associations with $C_{2}$, a $C_{1}$ item would occur in the second recall position with a probability $\mathrm{w} /(\mathrm{w}+\mathrm{b})$. Thus, the occurrence of a $C_{1}$ or $C_{2}$ item in any recall position is contingent on the balance of associations between that item and the previous recall. The strength of these associations can be assessed from free association data (Bousfield et al, 1964). Items seldom, if ever, appear twice in the same recall. Thus, if a $C_{1}$ item is given it becomes unavailable for recall again, and a change in the balance of a within-category and betweencategory association set is produced. Specifically, if a
$\mathrm{C}_{1}$ word is recalled, the number of $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}$ and $\mathrm{C}_{2} \rightarrow \mathrm{C}_{1}$ connections is decreased. If a $\mathrm{C}_{2}$ word is given the $\mathrm{C}_{2} \rightarrow \mathrm{C}_{2}$ and $\mathrm{C}_{1} \rightarrow \mathrm{C}_{2}$ associations are reduced.

This change in the association structure during recall is completely predictable, and if the states are properly defined, the process can be regarded as a Markov chain. The appropriate mathematical description, as well as a more detailed discussion of the associative process, are given elsewhere (Cowan, 1966). Discrete Markov processes, like other probability phenomena, can be represented by urn models. The urn model which seems appropriate for recall ordering is diagramed in Fig. 1. The recall list depicted comprises six words with an equal number of words in each category. The values of the various associations are given in the figure legend. Two urns are given; one is labeled $\mathrm{C}_{1}$, the other $\mathrm{C}_{2}$. The white balls in each urn represent the mean or expected number of within-category associations ( $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}$ or $\mathrm{C}_{2} \rightarrow \mathrm{C}_{2}$ ) between any item, the category of which is given by the urn, and the other items in the same category. The black balls in each urn represent the expected number of between-category connections ( $\mathrm{C}_{1} \rightarrow \mathrm{C}_{2}$ or $\mathrm{C}_{2} \rightarrow \mathrm{C}_{1}$ ) between any item (the category again given by the urn) and the items in the other category. The selection of an urn at any point in the process implies the recall of an item from the category represented by the chosen urn. Initially an urn is selected according to some rational scheme. A ball is randomly selected from the chosen urn. If a within-category (white) ball is chosen, then the next


Fig. 1. Representation of the urn scheme for the recall sequence $\mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{2} \mathrm{C}_{1} \mathrm{C}_{2} \mathrm{C}_{1}$. The double headed broken arrow indicates that a ball is sampled, noted, then replaced. Balls are then removed from the urns and another ball is sampled. Black balls represent between-category connections, and white balls are within-category associations. If the process moves to an urn on the right, a $\mathbf{C}_{2}$ word has been recalled. The left urn represents $C_{1}$ items. The mean association values used in the example are as follows: $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}=2, \mathrm{C}_{1} \rightarrow \mathrm{C}_{2}=1, \mathrm{C}_{2} \rightarrow \mathrm{C}_{2}=3$, and $\mathrm{C}_{2} \rightarrow \mathrm{C}_{1}=1$.
choice is from the same urn implying that another word from the same category has been recalled. If the black ball is selected, implying that a between-category association was made, the next draw is taken from the other urn. This would indicate the recall of a word from the other category.
Since selected items become unavailable for further recall, a number of white balls corresponding to an appropriate number of within-category connections must be removed from the selection urn. An appropriate number of black balls representing the between-category associations of the recall item must be removed from the other urn. The "appropriate"' number in each case is the expected number of connections between suitable word pairs. The number of black and white balls diminish with each draw and eventually no balls will remain in one of the urns signifying the end of recall. An electronic model can and has been built that will produce a linear display of on ( $\mathrm{C}_{1}$ ) or off ( $\mathrm{C}_{2}$ ) lights in which each display position corresponds to each position in a recall sequence. The machine incorporates a programming feature by which limited parameter values may be introduced.

## Procedure

The data are taken from a previous study (Cowan, 1966). The items to be memorized were 10 words which pertained to food and eating (HUNGRY, KITCHEN, OLIVE, etc.) and 20 miscellaneous words (ACCORDION, PAIL, BLOTTER, etc.). Immediately after a scrambled visual presentation of the items the Ss were asked to write the words in the order they occurred in memory.

## Results

The percent of $\mathrm{C}_{1}$ ("food") words occurring in each recall position were found for three subdivisions of the data. The results are given in Fig. 2. The expected number of associations are the same for all three cases. These are as follows: $\mathrm{C}_{1} \rightarrow \mathrm{C}_{1}=14.80, \mathrm{C}_{1} \rightarrow \mathrm{C}_{2}=$ $1.25, \mathrm{C}_{2} \rightarrow \mathrm{C}_{2}=4.80, \mathrm{C}_{2} \rightarrow \mathrm{C}_{1}=.75$. The mean number of $C_{1}$ and $C_{2}$ words recalled were 6 and 10 respectively. The simulated data were taken from 18 sequences;


Fig. 2. Percent subjects recalling a $C_{1}$ item as a function of recall position when the first recalled word belonged to $A$ ) $C_{1}$, B) $\mathrm{C}_{2}$, and C) $\mathrm{C}_{1}-28 \%, \mathrm{C}_{2}-72 \%$. Open circles represent the simulated data. Since the student's recalls varied, the number of measures per position dropped. These numbers are as follows: A) $22-14, B) 58-35$, and C) $80-49$.
the number of human Ss are given in the figure legend. If the artificial data can be considered as a first order approximation, then there is reasonable agreement between the simulation and the human Ss.

## References

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## Note

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