



Aalto University
School of Electrical
Engineering

Analog and Digital Self-interference Cancellation in Full-Duplex MIMO-OFDM Transceivers with Limited Resolution in A/D Conversion

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Introduction

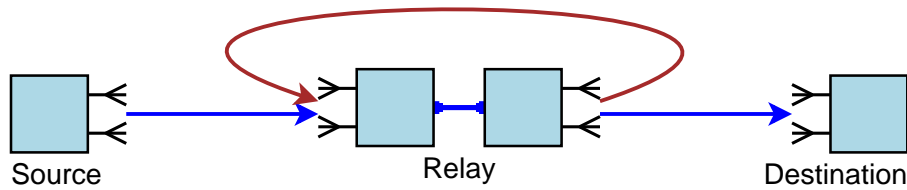
Asilomar — The Cradle of Full-Duplex Wireless

- 2007 T. Riihonen, R. Wichman, and J. Hämäläinen: “Co-phasing full-duplex relay link with non-ideal feedback information” was unsuccessful, presented later at IEEE ISWCS 2008
- 2008 T. Riihonen, S. Werner, J. Cousseau, and R. Wichman: “Design of co-phasing allpass filters for full-duplex OFDM relays”
- 2009 T. Riihonen, S. Werner, and R. Wichman: “Spatial loop interference suppression in full-duplex MIMO relays”
- 2010 M. Duarte and A. Sabharwal: “Full-duplex wireless communications using off-the-shelf radios: Feasibility and first results”
- ++++ P. Lioliou, M. Viberg, M. Coldrey, and F. Athley: “Self-interference suppression in full-duplex MIMO relays”
- ++++ T. Riihonen, S. Werner, and R. Wichman: “Residual self-interference in full-duplex MIMO relays after null-space projection and cancellation”
- 2011 B. P. Day, D. W. Bliss, A. R. Margetts, and P. Schniter: “Full-duplex bidirectional MIMO: Achievable rates under limited dynamic range”
- ++++ E. Everett, M. Duarte, C. Dick, and A. Sabharwal: “Empowering full-duplex wireless communication by exploiting directional diversity”
- ++++ T. Riihonen, S. Werner, and R. Wichman: “Transmit power optimization for multiantenna decode-and-forward relays with loopback self-interference from full-duplex operation”
- 2012 Two special sessions and ten papers! The ultimate breakthrough for this research topic?

Full-Duplex Wireless: What? Why? When?

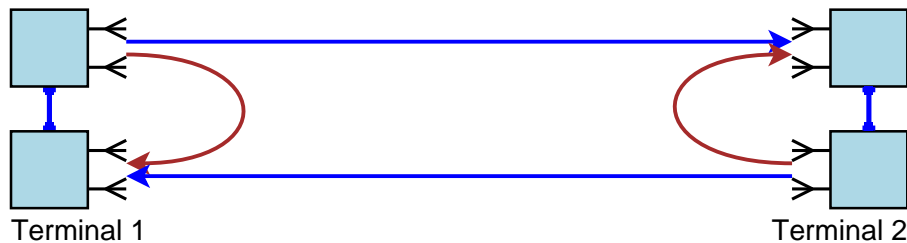
- “*Full-duplex*” wireless communication
 - = systems where some node(s) may transmit (Tx) and receive (Rx) simultaneously on a *single* frequency band
- Progressive physical/link-layer *frequency-reuse* concept
 - = up to double spectral efficiency at system level, if the significant technical problem of *self-interference* is tackled
- *Temporal symmetry* is needed to make the most of full duplex
 - = Tx and Rx should use the band for the same amount of time
 - (a)symmetry of traffic pattern, i.e.,
requested rates in the two simultaneous directions
 - (a)symmetry of channel quality, i.e.,
achieved rates in the two simultaneous directions

Full-Duplex Communication Scenarios



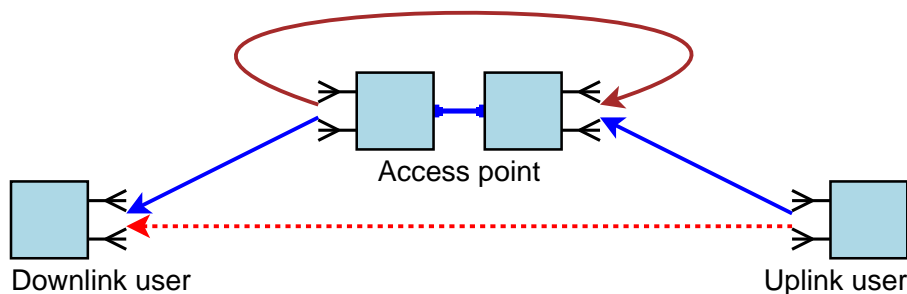
1) Multihop relay link

- Symmetric traffic
- Asymmetric channels
- Direct link may be useful



2) Bidirectional communication link between two terminals

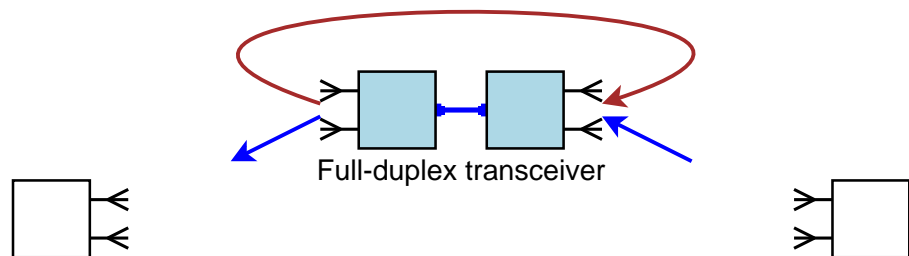
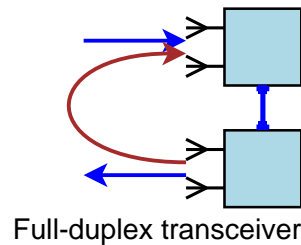
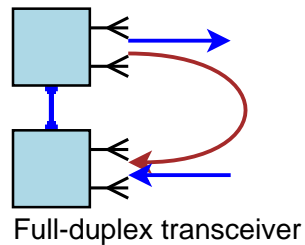
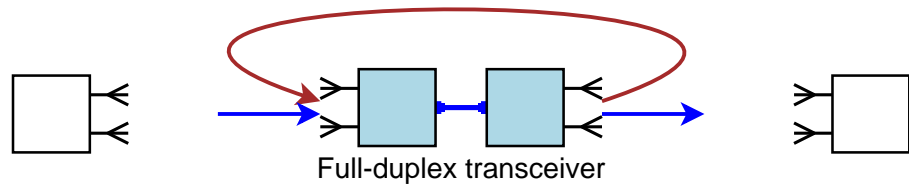
- Asymmetric traffic (maybe)
- Symmetric channels (roughly)



3) Simultaneous down- and uplink for two half-duplex users

- Asymmetric traffic
- Asymmetric channels
- Inter-user interference!

Generic Full-Duplex MIMO Transceivers



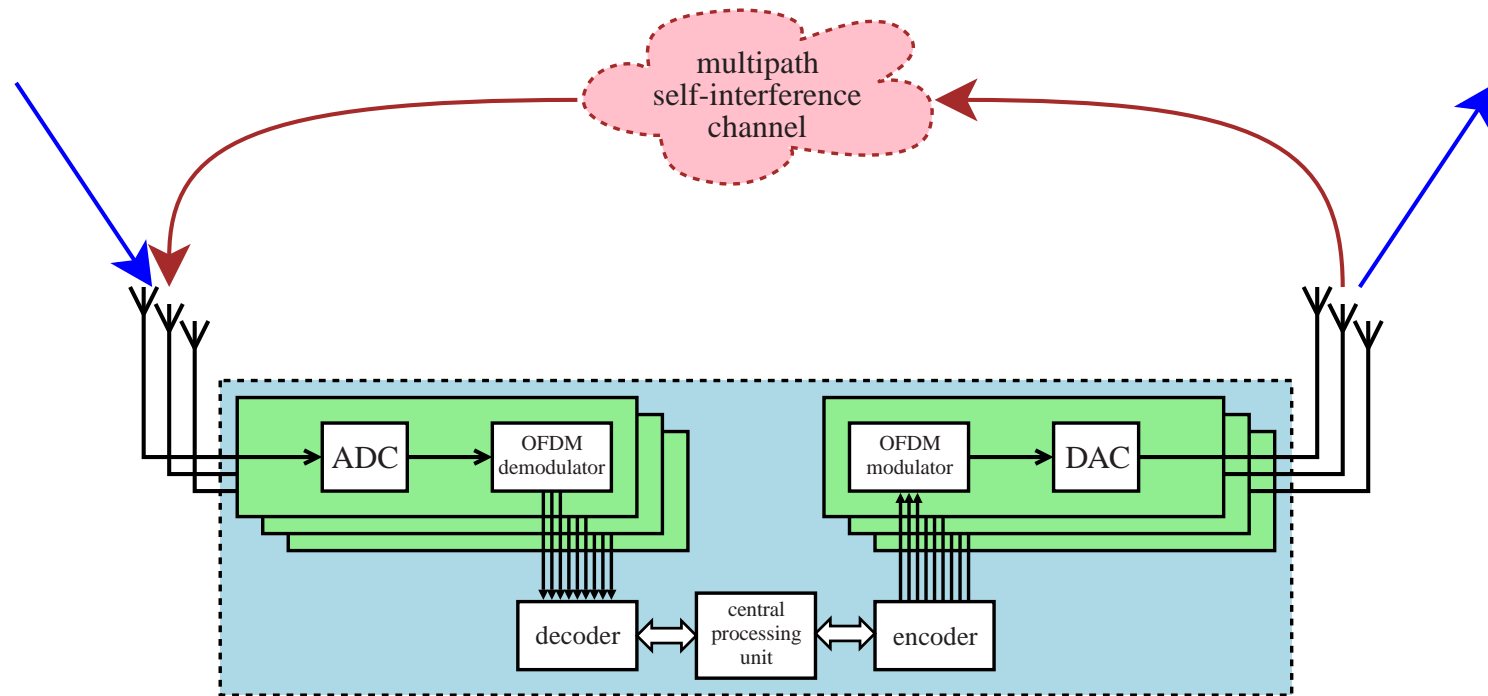
The basic building block for more complex networks

- The benefits go beyond the physical layer!
- Will single-array full-duplex transceivers be viable some day?

In this work: OFDM signal

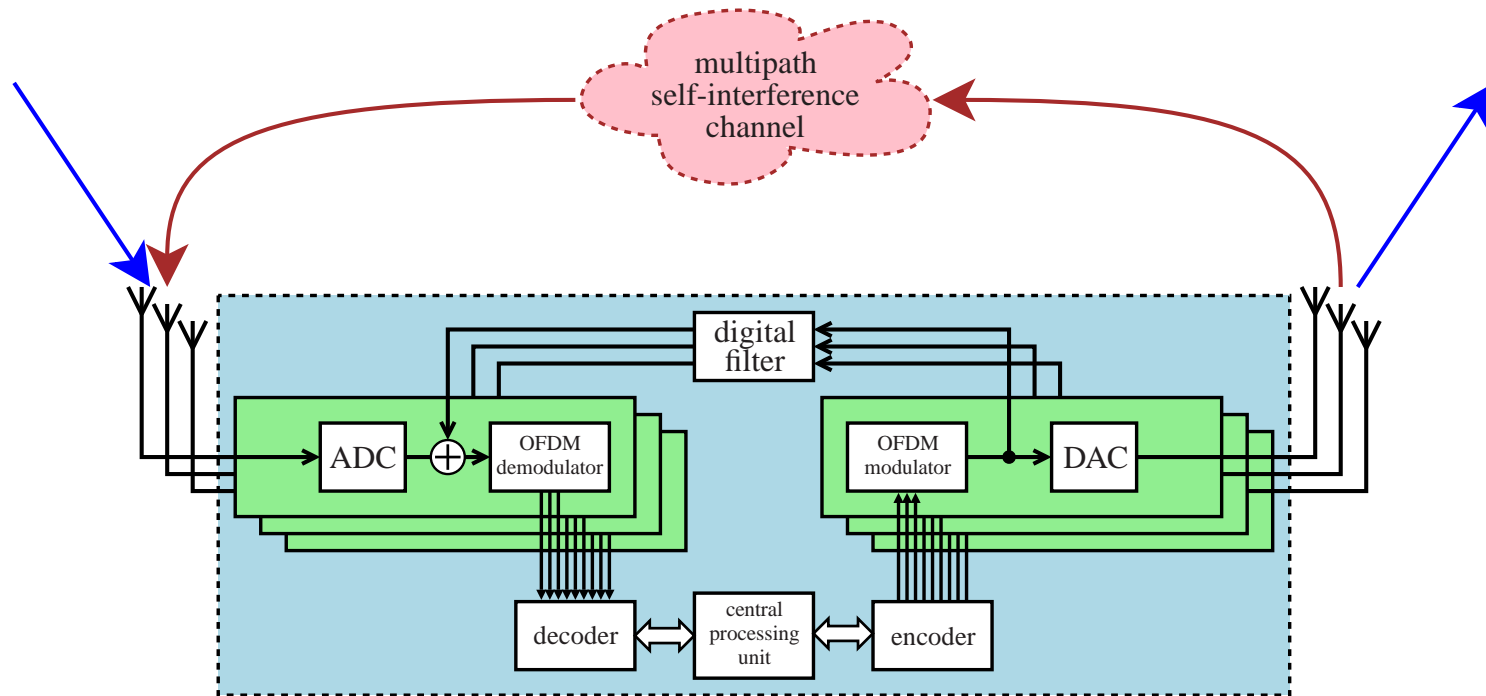
- + limited Rx dynamic range (= realistic A/D conversion)
 - ▷ b -bit quantization
 - ▷ adaptive gain control
- + analog- vs. digital-domain self-interference cancellation

Main Practical Problem: Limited Dynamic Range



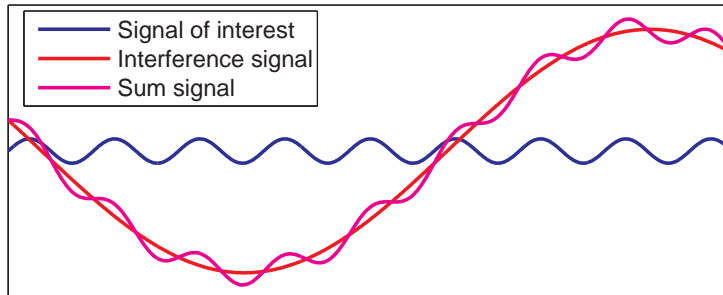
- Self-interference may be much stronger than the signal of interest
- Severe risk of saturating analog-to-digital converters (ADCs)
 - ▷ Quantization noise due to limited resolution
 - ▷ Clipping noise which is pronounced with OFDM
 - ▷ Bias in adaptive gain control (AGC) balancing above effects

Digital Cancellation (DC)

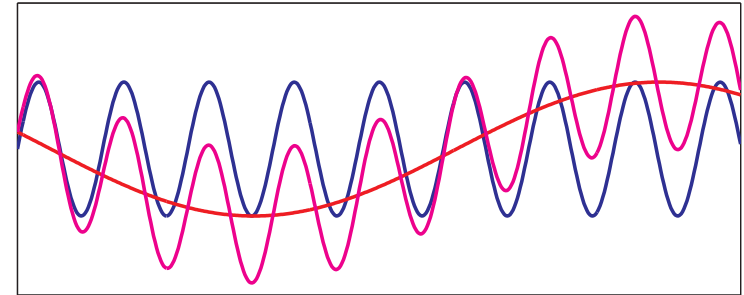


- Interference cancellation is a straightforward task in digital domain
 - ▷ The response of a digital cancellation filter can be adapted to match the frequency-selective self-interference channel
- But nothing can be done at this stage anymore if the signal of interest is already drowned in clipping-plus-quantization noise

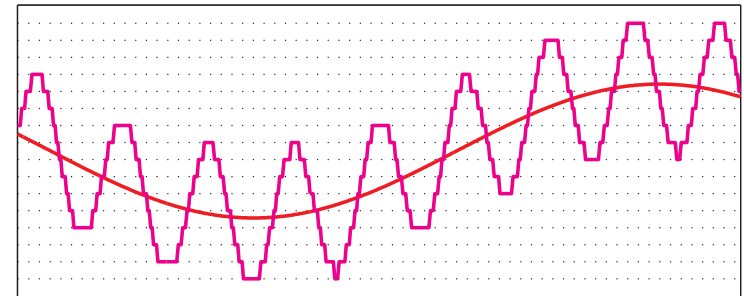
Example on Quantization Noise ($b = 4$)



before ADC



after ADC

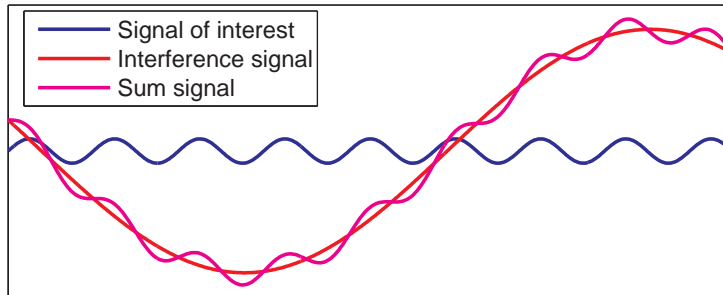


after digital
cancellation
and
scaling

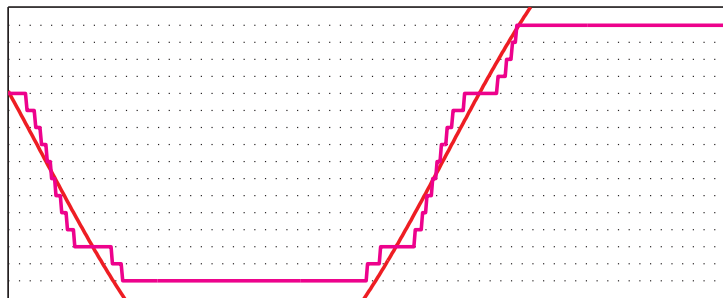
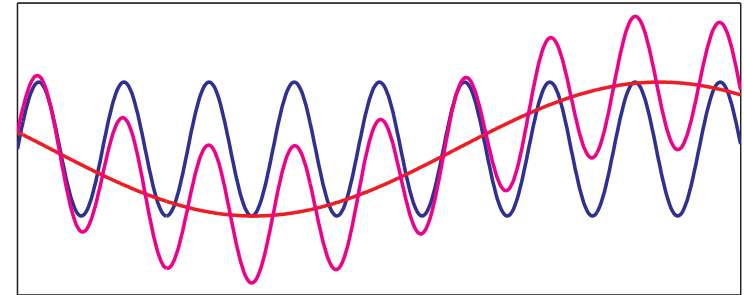
- ~ 1 -bit resolution for the signal of interest

- ~ 3 -bit resolution for the signal of interest

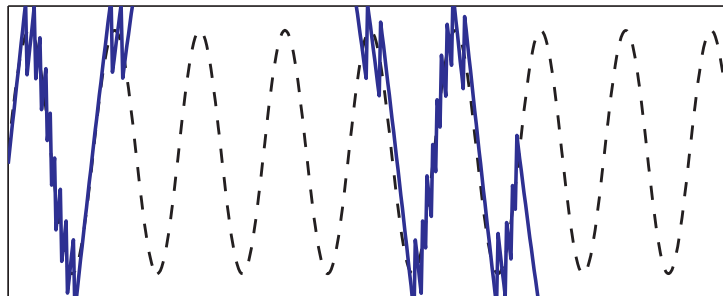
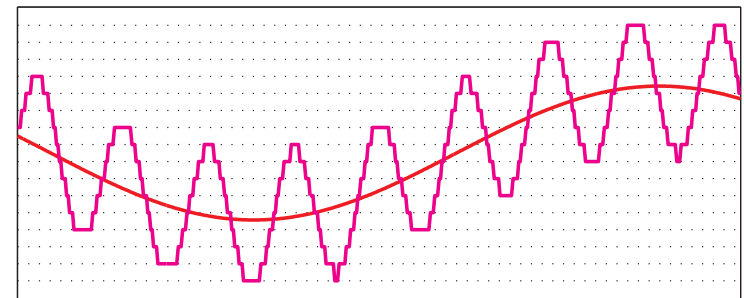
Example on Clipping Noise ($b = 4$)



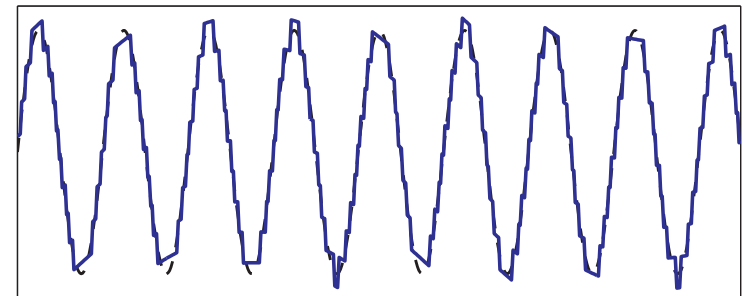
before ADC



after ADC



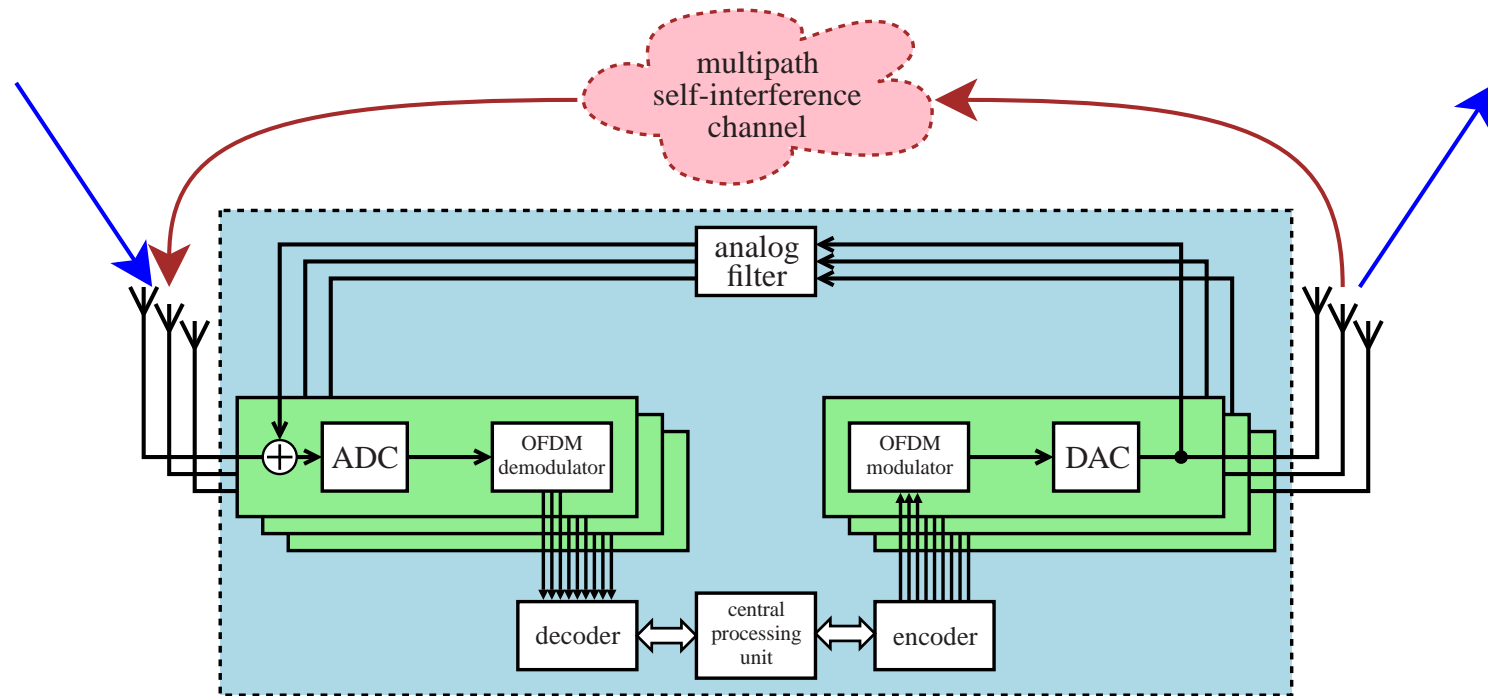
after digital
cancellation
and
scaling



- ~ 2 -bit *clipped* resolution for the signal of interest

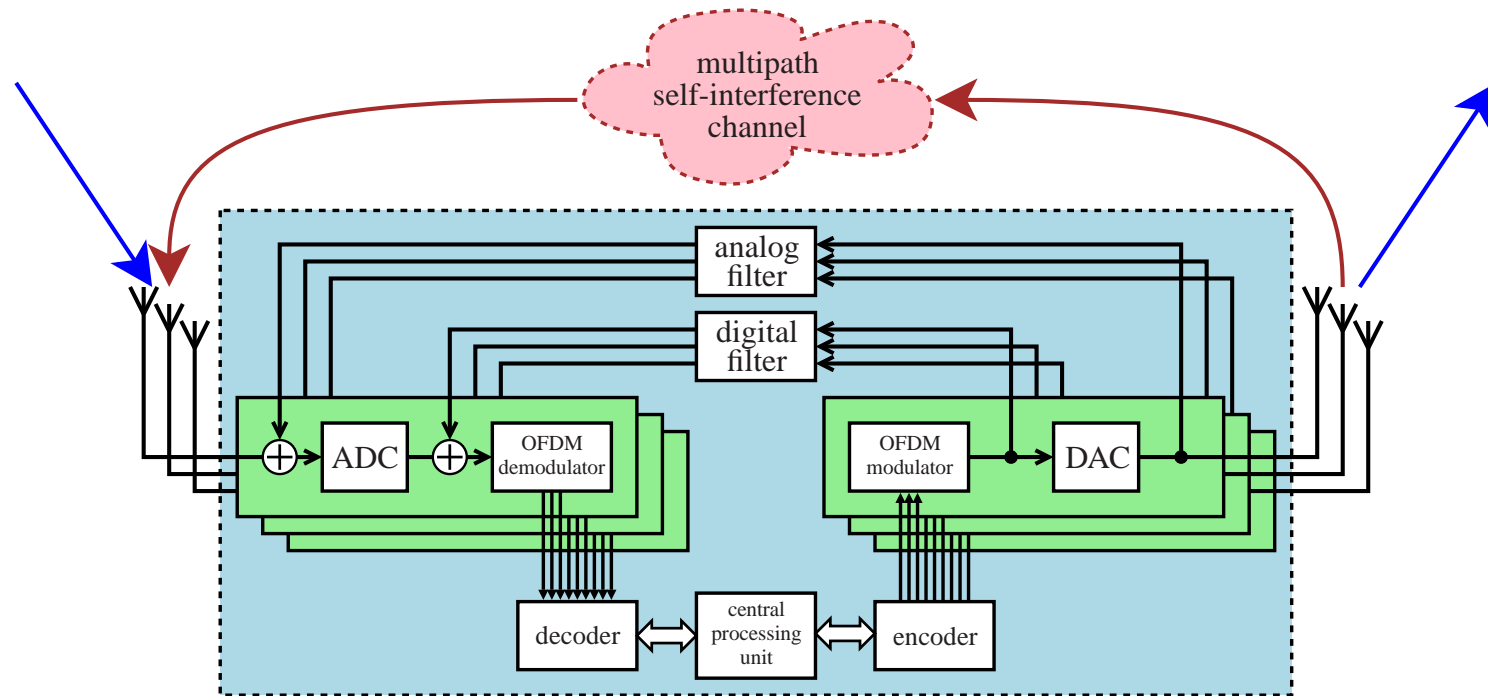
- ~ 3 -bit resolution for the signal of interest

Analog Cancellation (AC)



- It would be desirable to eliminate interference before ADCs
- But it is difficult and expensive to adapt the response of an analog filter to match the time- and frequency-selective MIMO channel
 - ▷ Typical implementation, simple phase shift and amplification in each branch, leaves significant residual interference

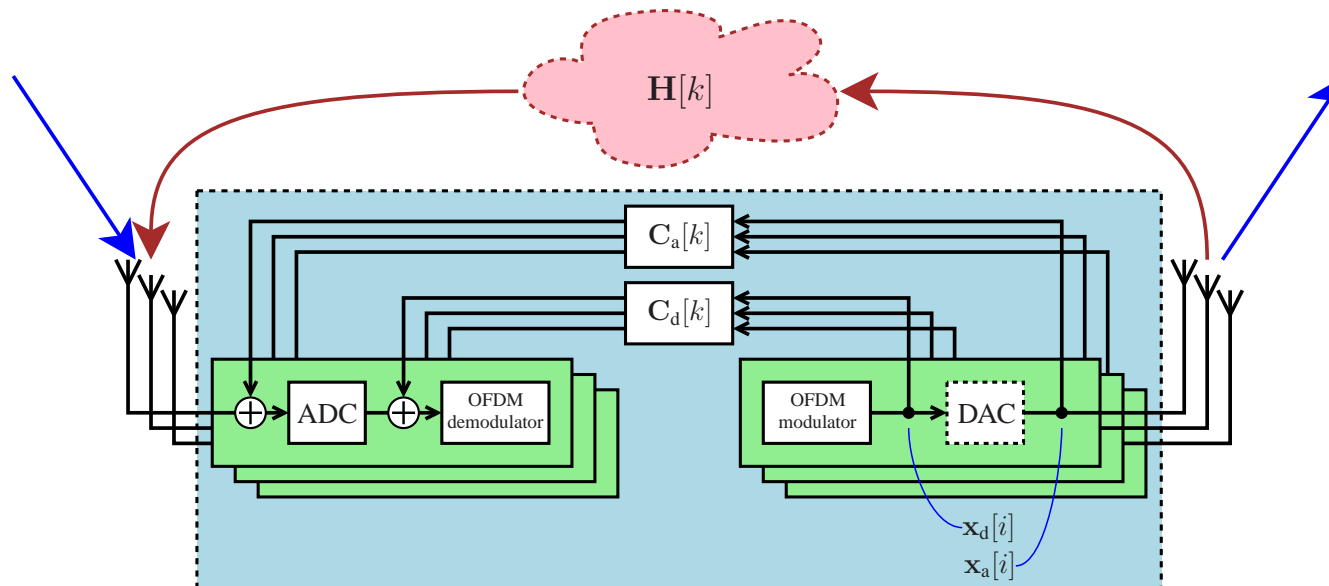
Combined Analog+Digital Cancellation (AC+DC)



- The obvious combination of analog- and digital-domain processing
 - ▷ If analog cancellation could sufficiently suppress the self-interference such that ADC saturation is avoided,
 - ▷ then digital cancellation would be able to efficiently eliminate the remaining self-interference

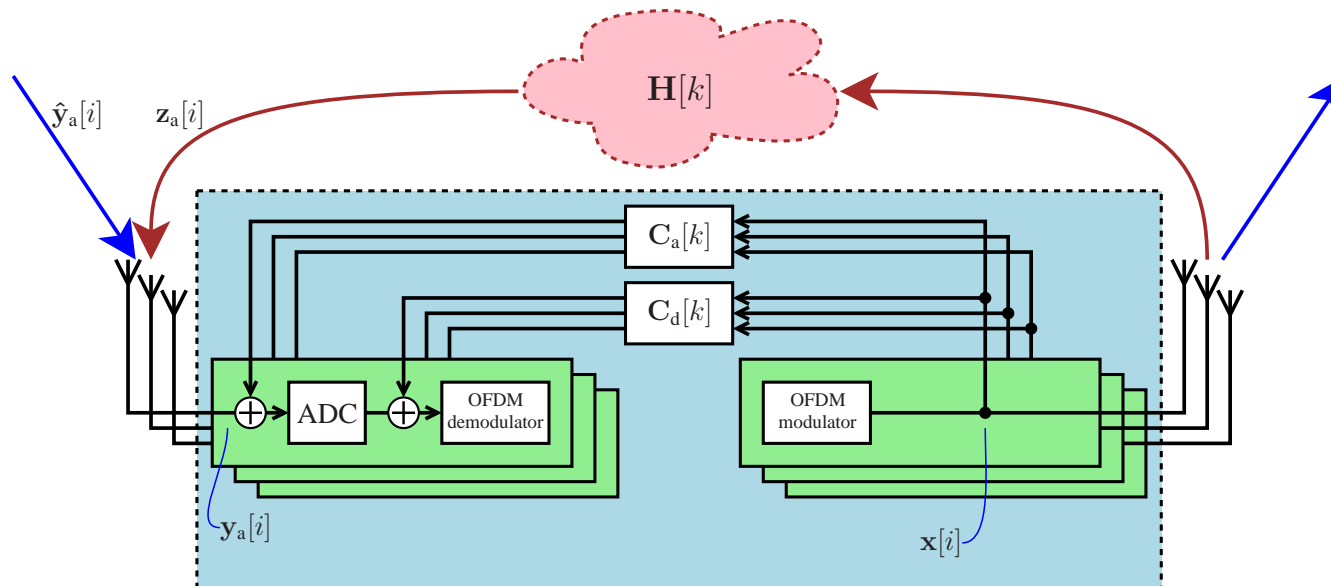
System Model

Transmitted Signals



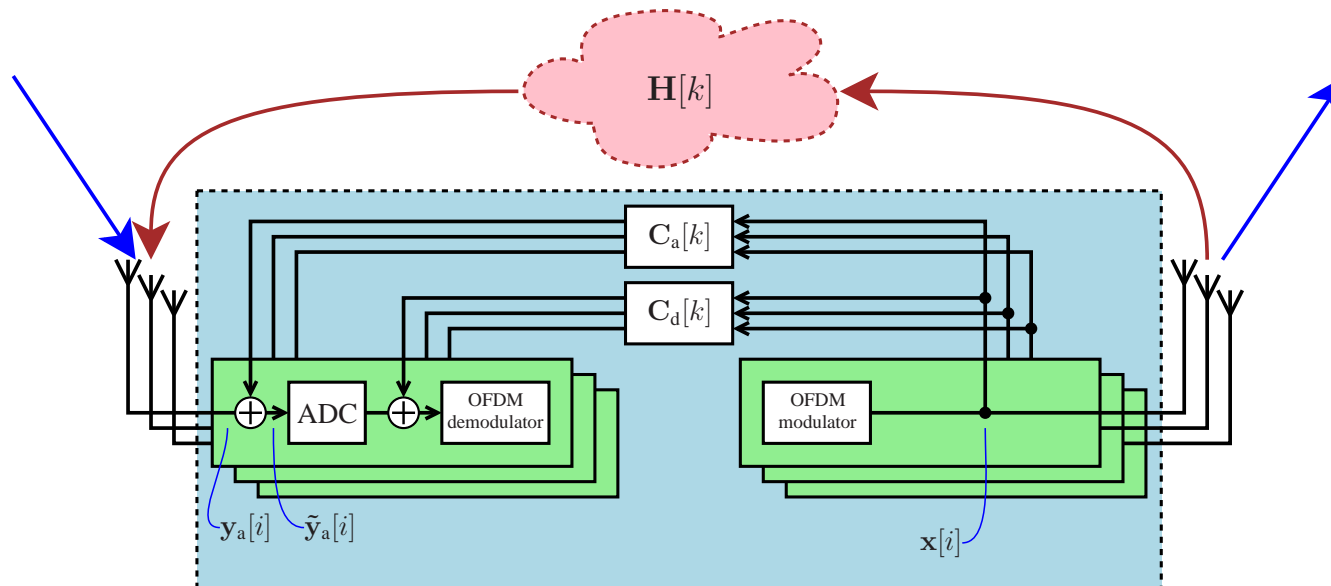
- The full-duplex transceiver tries to receive the signal of interest from a distant transmitter
- while simultaneously transmitting signal $\mathbf{x}[i] \in \mathbb{C}^{N_t \times 1}$ to its own designated destination
 - ▷ Digital-to-analog converters (DACs) are now ideal: $\mathbf{x}_a[i] \simeq \mathbf{x}_d[i]$
- Gaussian-like OFDM signals are assumed throughout this study

Received Signals



- Received analog composite signal: $\mathbf{y}_a[i] = \hat{\mathbf{y}}_a[i] + \mathbf{z}_a[i] \in \mathbb{C}^{N_r \times 1}$
 - ▷ the signal of interest is given by $\hat{\mathbf{y}}_a[i] \in \mathbb{C}^{N_r \times 1}$
and $P_S = \mathcal{E}\{|\{\hat{\mathbf{y}}_a[i]\}_m|^2\}$ denotes its power at the m th antenna
 - ▷ interference signal is given by $\mathbf{z}_a[i] = \sum_{k=0}^{\infty} \mathbf{H}[k] \mathbf{x}[i - k] \in \mathbb{C}^{N_r \times 1}$
and $P_I = \mathcal{E}\{|\{\mathbf{z}_a[i]\}_m|^2\}$ denotes its power at the m th antenna
- Multipath self-interference channel: $\mathbf{H}[k] \in \mathbb{C}^{N_r \times N_t}$, $k = 0, 1, \dots$

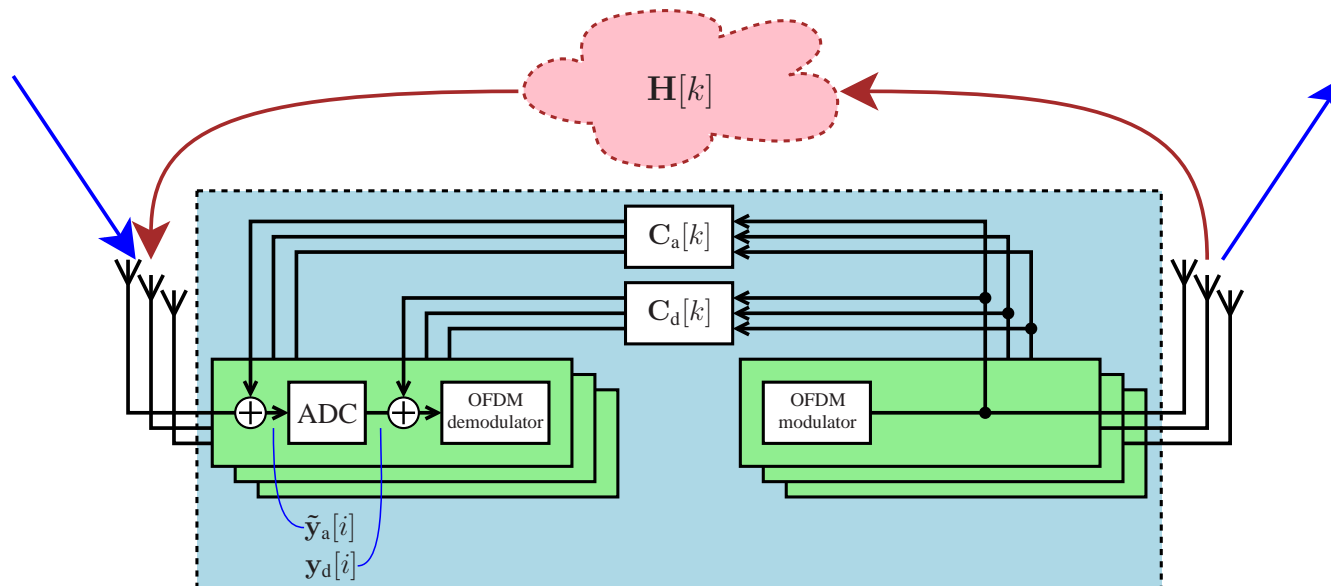
Analog Cancellation (AC)



- After analog cancellation: $\tilde{\mathbf{y}}_a[i] = \hat{\mathbf{y}}_a[i] + \tilde{\mathbf{z}}_a[i]$
 - ▷ the signal of interest $\hat{\mathbf{y}}_a[i]$ is not affected
 - ▷ residual interference signal becomes

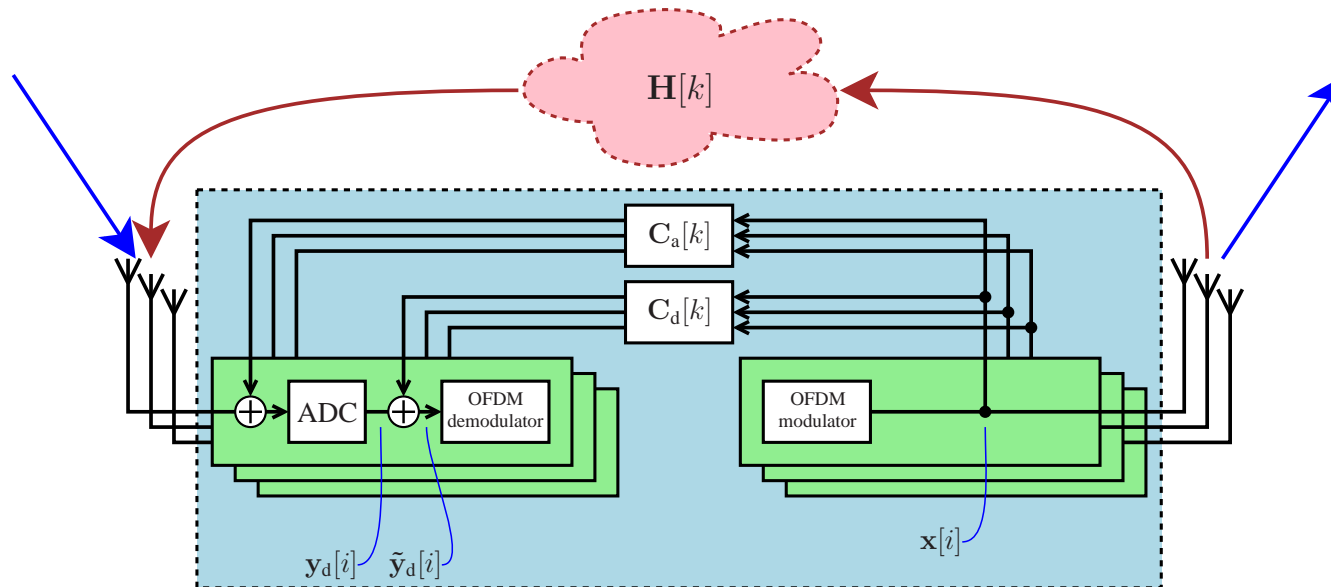
$$\tilde{\mathbf{z}}_a[i] = \sum_{k=0}^{\infty} (\mathbf{H}[k] + \mathbf{C}_a[k]) \mathbf{x}[i - k]$$
- Analog cancellation filter: $\mathbf{C}_a[k] \in \mathbb{C}^{N_r \times N_t}$, $k = 0, 1, \dots$
 - ▷ for example $\{\mathbf{C}_a[k]\}_{m,n} = \begin{cases} -\{\mathbf{H}[k]\}_{m,n}, & \text{if } k = \arg \max_{k'} |\{\mathbf{H}[k']\}_{m,n}|^2 \\ 0, & \text{otherwise} \end{cases}$

Analog-to-Digital Conversion (ADC)



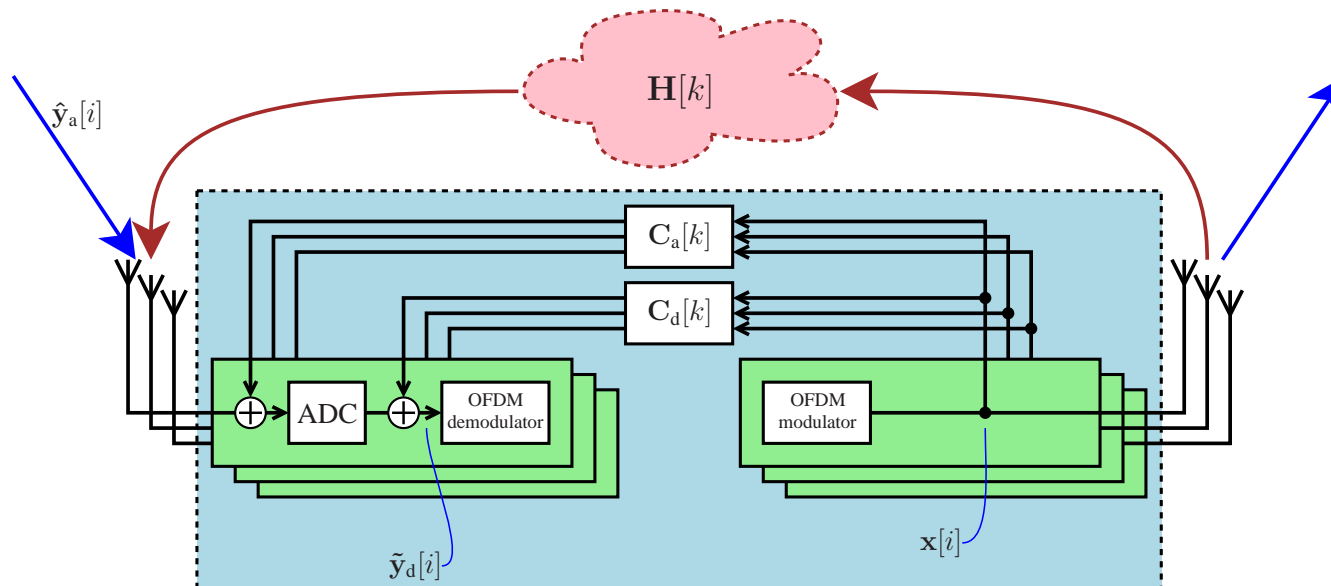
- $2 \times N_r$ ADCs: $\text{Re}(\{\mathbf{y}_d[i]\}_m) = \mathcal{Q}(\sqrt{g_m} \text{Re}(\{\tilde{\mathbf{y}}_a[i]\}_m))$
 $\text{Im}(\{\mathbf{y}_d[i]\}_m) = \mathcal{Q}(\sqrt{g_m} \text{Im}(\{\tilde{\mathbf{y}}_a[i]\}_m))$
 - ▷ AGC tunes variable gain amplifier (VGA) setting g_m to keep signal level within the fixed range of quantization block $\mathcal{Q}(\cdot)$
- The theory of non-linear memoryless devices: $\mathbf{y}_d[i] = \mathbf{A}\tilde{\mathbf{y}}_a[i] + \mathbf{n}[i]$
 - ▷ clipping-plus-quantization noise power is $P_N = \mathcal{E}\{|\{\mathbf{n}[i]\}_m|^2\}$

Digital Cancellation (DC)



- After digital cancellation: $\tilde{\mathbf{y}}_d[i] = \mathbf{A}\hat{\mathbf{y}}_a[i] + \tilde{\mathbf{z}}_d[i] + \mathbf{n}[i]$
 - ▷ interference signal is transformed from $\mathbf{z}_d[i] = \mathbf{A}\tilde{\mathbf{z}}_a[i]$ to $\tilde{\mathbf{z}}_d[i] = \sum_{k=0}^{\infty} (\mathbf{A}(\mathbf{H}[k] + \mathbf{C}_a[k]) + \mathbf{C}_d[k]) \mathbf{x}[i - k]$
 - ▷ clipping-plus-quantization noise term $\mathbf{n}[i]$ is not suppressed!
- Digital cancellation filter: $\mathbf{C}_d[k] \in \mathbb{C}^{N_r \times N_t}$, $k = 0, 1, \dots$
 - ▷ ideally $\mathbf{C}_d[k] = -\mathbf{A}(\mathbf{H}[k] + \mathbf{C}_a[k])$ if there is no estimation error

Complete Signal Model



- After putting everything together:

$$\tilde{\mathbf{y}}_d[i] = \mathbf{A}\hat{\mathbf{y}}_a[i] + \sum_{k=0}^{\infty} (\mathbf{A}(\mathbf{H}[k] + \mathbf{C}_a[k]) + \mathbf{C}_d[k]) \mathbf{x}[i - k] + \mathbf{n}[i]$$

- Powers of signal components at the m th antenna:

$$\mathcal{E}\{|\{\tilde{\mathbf{y}}_d[i]\}_m|^2\} = \alpha^2 P_S + \mathcal{E}\{|\{\tilde{\mathbf{z}}_d[i]\}_m|^2\} + P_N$$

$$\text{where } \alpha = \{\mathbf{A}\}_{m,m}$$

- SINR can be formulated after calculating $\mathcal{E}\{|\{\tilde{\mathbf{z}}_d[i]\}_m|^2\}$

Analytical Results

Signal to Interference and Noise Ratio (SINR)

- The ratio of *desired* signal power to *residual* interference and *clipping-plus-quantization* noise power becomes

$$\gamma = \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a} \quad \text{where} \quad \rho = \frac{\alpha^2(P_S + P_I/\Delta_a)}{P_N}$$

- ▷ interference suppression due to cancellation:

$$\Delta_a = \frac{\mathcal{E}\{|\{\mathbf{z}_a[i]\}_m|^2\}}{\mathcal{E}\{|\{\tilde{\mathbf{z}}_a[i]\}_m|^2\}} \quad \text{from AC}$$

$$\Delta_d = \frac{\mathcal{E}\{|\{\mathbf{z}_d[i]\}_m|^2\}}{\mathcal{E}\{|\{\tilde{\mathbf{z}}_d[i]\}_m|^2\}} \quad \text{from DC}$$

- ▷ signal-to-interference ratio (SIR):

$$\begin{aligned} & \frac{P_S}{P_I} && \text{without cancellation} \\ & \Delta_a \frac{P_S}{P_I} && \text{after AC} \\ & \Delta_a \Delta_d \frac{P_S}{P_I} && \text{after DC} \end{aligned}$$

- A/D conversion affects SINR only through $\rho = \rho(\alpha, P_N, P_S + P_I/\Delta_a)$
 - ▷ $\gamma \rightarrow \Delta_a \Delta_d \frac{P_S}{P_I}$ if dynamic range is *not* the limiting factor ($\rho \rightarrow \infty$)

SNR with Limited ADC Resolution

- The ratio of signal power to clipping-plus-quantization noise power after ADC, i.e., *dynamic range*: $\rho = \frac{\alpha^2(P_S + P_I/\Delta_a)}{P_N} = \frac{\alpha^2 p}{g P_N}$
 - ▷ AGC tunes VGA setting g such that normalized input power to the quantization block is constantly $p = g(P_S + P_I/\Delta_a)$
- SINR is monotonically increasing in terms of dynamic range ρ :

$$\gamma = \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a} \leq \Delta_a \Delta_d \frac{P_S}{P_I}$$

- ▷ Thus, system design should always aim at maximizing ρ irrespective of P_S , P_I , Δ_a and Δ_d (as they do not affect ρ)
- ▷ Signal type, ADC properties and p define ρ via α^2/g and P_N
 - OFDM transmission
 - Uniform quantization
 - ADC resolution
 - AGC bias

Dynamic Range for Uniform Quantization

- Input–output relation for uniform b -bit quantization ($Q = 2^b$):

$$\mathcal{Q}(y) = \begin{cases} 1, & \text{if } \frac{Q-2}{Q-1} < y \\ \frac{2q-2}{Q-1} - 1, & \text{if } \frac{2q-3}{Q-1} - 1 < y \leq \frac{2q-1}{Q-1} - 1 \\ -1, & \text{if } y \leq \frac{2-Q}{Q-1} \end{cases}$$

- After calculating α^2/g and P_N for OFDM signal:

$$\rho = \left[\frac{2\pi \left[\Phi\left(\frac{1}{\sqrt{p}} \frac{2-Q}{Q-1}\right) + \sum_{q=2}^{Q-1} \left(\frac{2q-2}{Q-1} - 1\right)^2 \left[\Phi\left(\frac{1}{\sqrt{p}} \left(\frac{2q-1}{Q-1} - 1\right)\right) - \Phi\left(\frac{1}{\sqrt{p}} \left(\frac{2q-3}{Q-1} - 1\right)\right) \right] \right]}{\left(2e^{-\frac{1}{2p} \left(\frac{2-Q}{Q-1}\right)^2} + \sum_{q=2}^{Q-1} \left(\frac{2q-2}{Q-1} - 1\right) \left[e^{-\frac{1}{2p} \left(\frac{2q-3}{Q-1} - 1\right)^2} - e^{-\frac{1}{2p} \left(\frac{2q-1}{Q-1} - 1\right)^2} \right] \right)^2} \right]^{-1}$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution

- $\rho = \rho(b, p)$: The ADC affects achieved dynamic range (and SINR) only through its resolution and VGA setting (or AGC bias)

Numerical Results

Dynamic Range vs. VGA Setting

- Dynamic range can be maximized by proper AGC:

$$\rho^*(b) = \max_{p'} \rho(b, p')$$

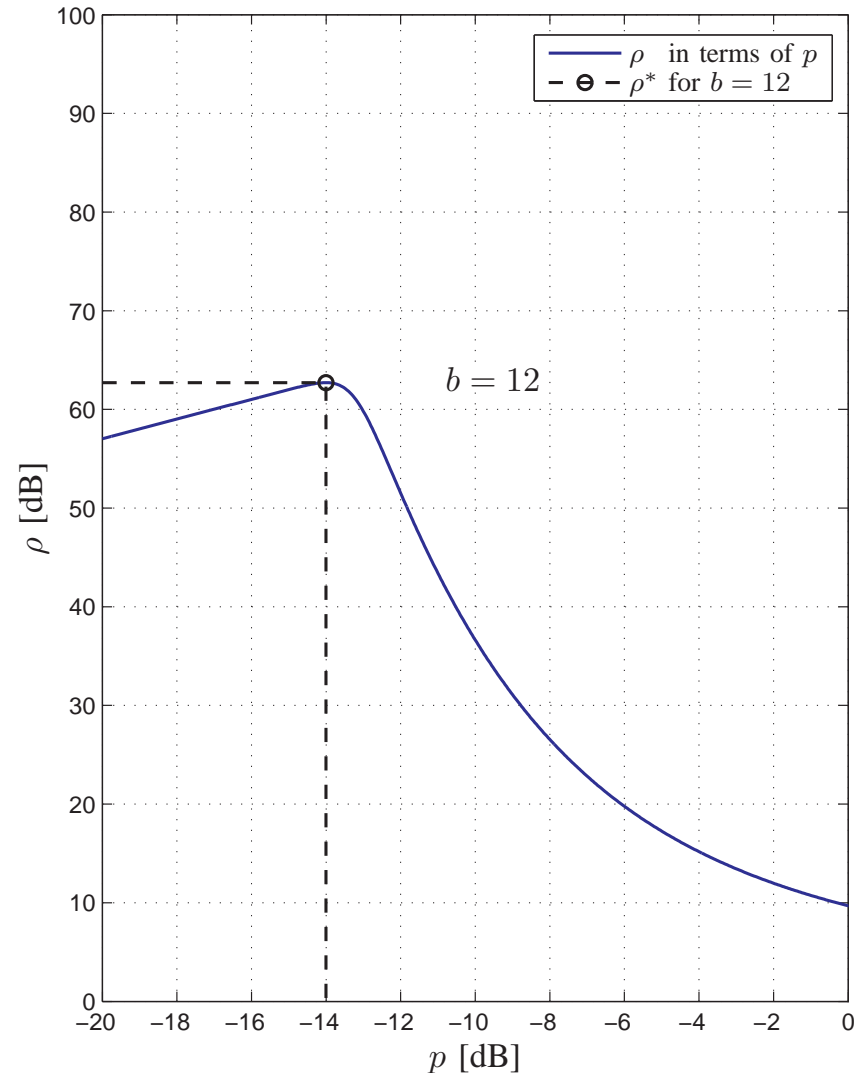
- ▷ results in maximal SINR with any P_S , P_I , Δ_a and Δ_d

- Optimal VGA setting yields

$$p = p^*(b) = \arg \max_{p'} \rho(b, p')$$

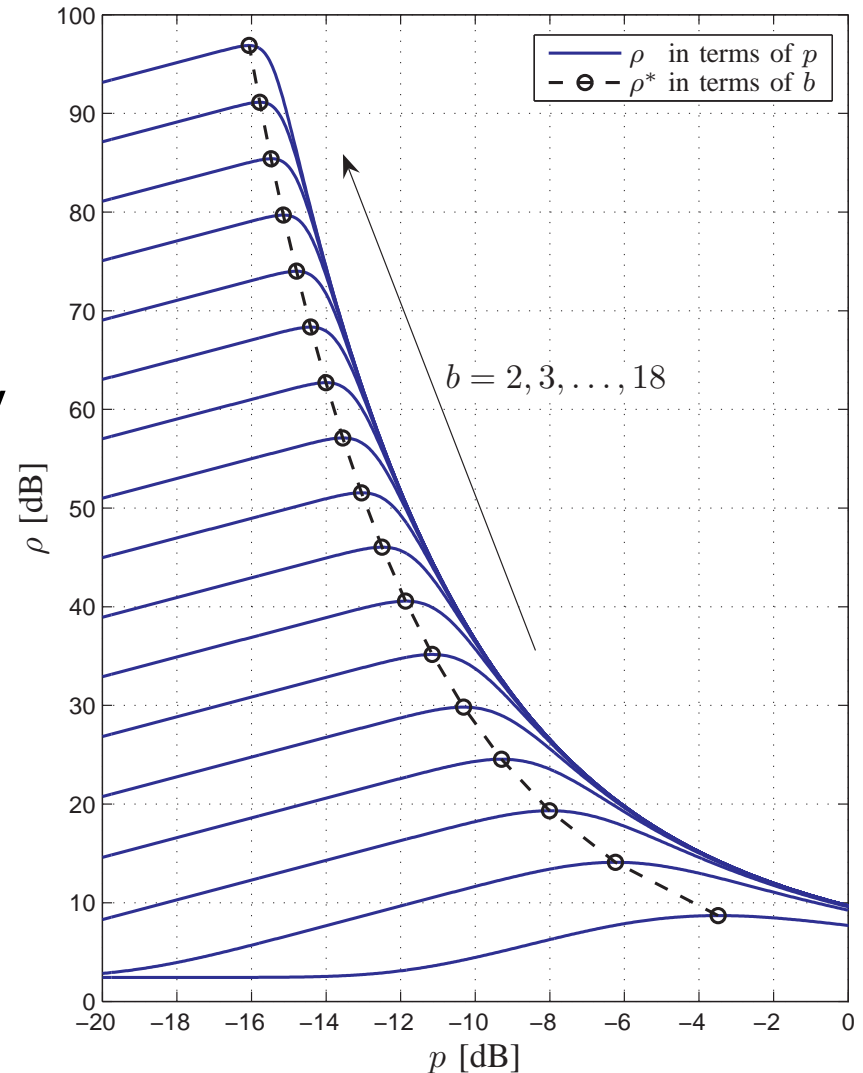
- AGC bias:

- ▷ when $p < p^*(b)$, quantization dominates
- ▷ when $p > p^*(b)$, clipping dominates

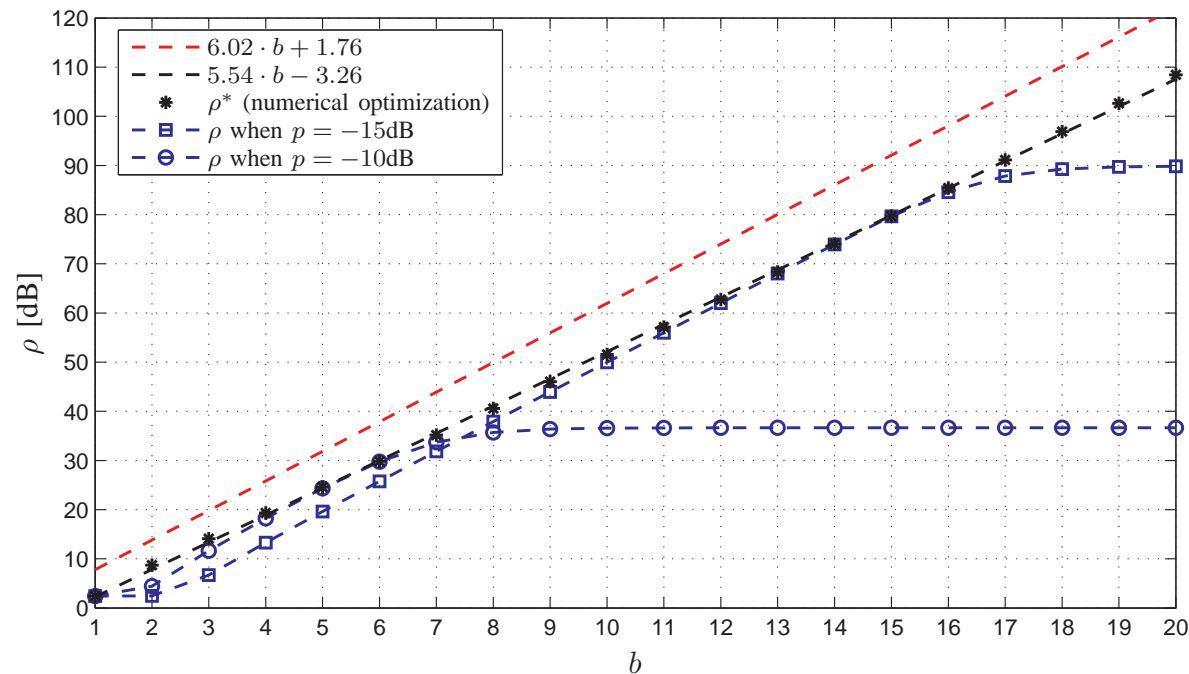


Optimal VGA Setting vs. ADC Resolution

- $\rho^*(b)$ increases in terms of b
 - ▷ Higher ADC resolution allows to trade off quantization noise level for lower clipping probability
- $p^*(b)$ decreases in terms of b
 - ▷ AGC should be designed by choosing VGA setting based on ADC resolution
 - ▷ Constant VGA setting would inevitably result in significant AGC bias and loss of dynamic range



Dynamic Range vs. ADC Resolution

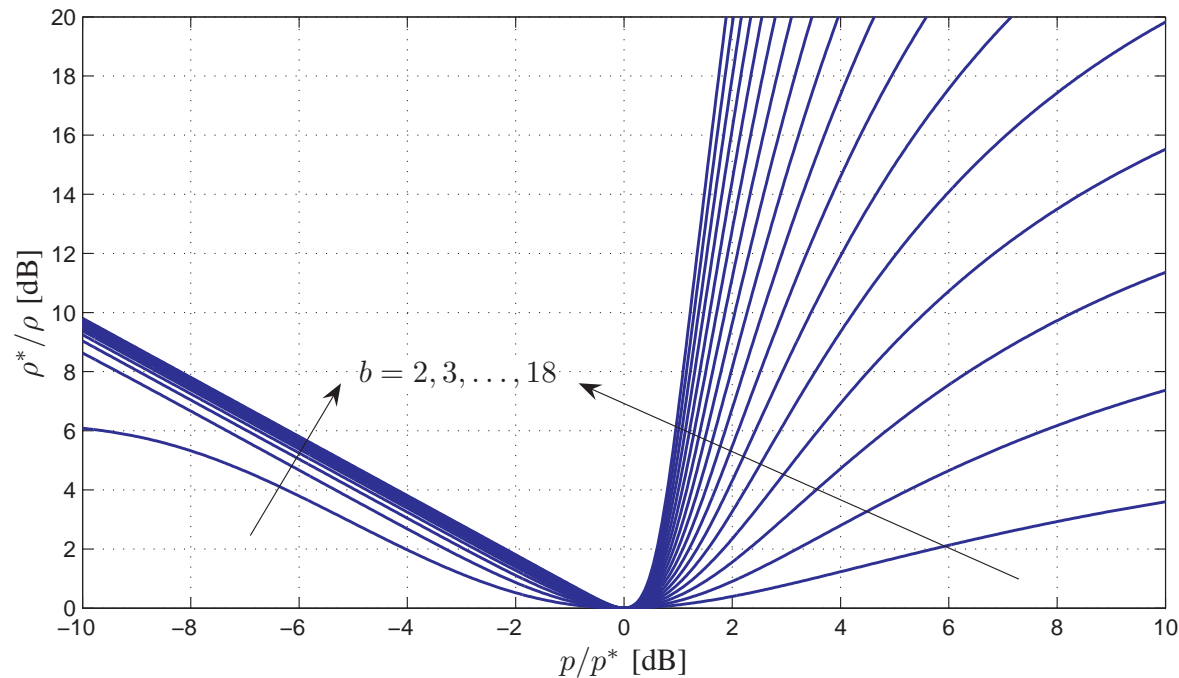


- Least-squares fit at $b = 1, 2, \dots, 20$ shows almost linear relation:

$$\rho^*(b) \simeq 5.54 \cdot b - 3.26 \text{ [dB]}$$

- The classic rule-of-thumb, $6.02 \cdot b + 1.76$ [dB], is too optimistic
 - ▷ Not intended for OFDM signals, e.g., clipping neglected

Loss of Dynamic Range from AGC Bias



- The loss of dynamic range due to AGC bias is increased when the ADC resolution is increased
 - ▷ AGC bias may eat away the benefit of using better ADC
- Low VGA setting is a safe choice: linear loss in terms of AGC bias
- Too high VGA setting causes ADC saturation due to clipping

SINR vs. Dynamic Range (1)

- Signal to interference and noise ratio (SINR) versus dynamic range ρ :

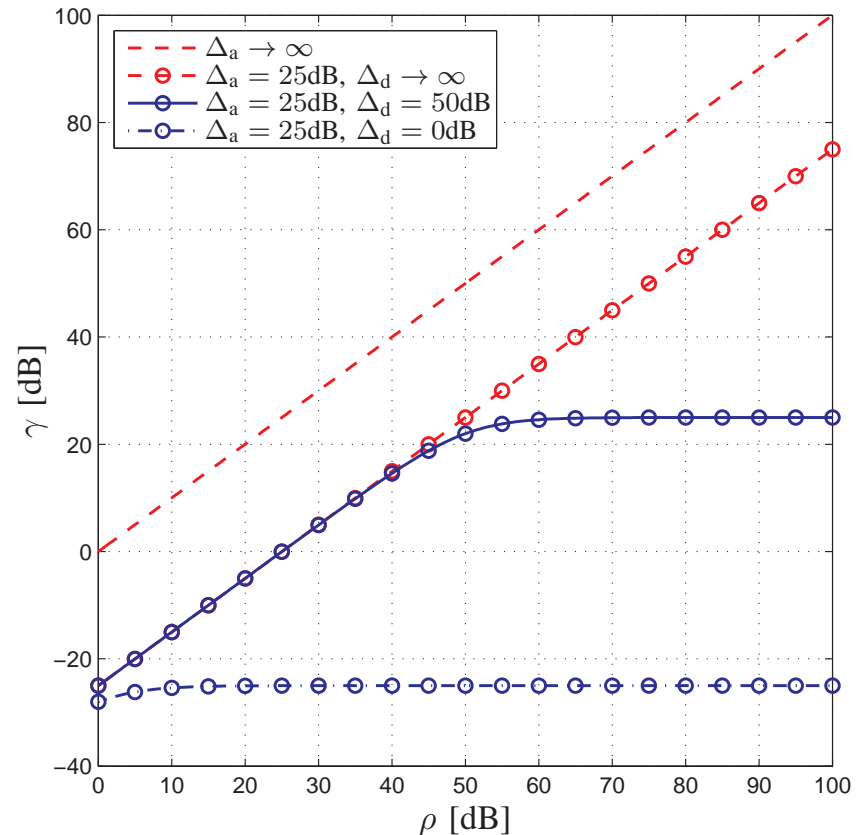
$$\gamma = \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a}$$

- On the right: Example when
 - ▷ SIR before AC $\frac{P_S}{P_I} = -50\text{dB}$
- Tight bounds if $\Delta_a \frac{P_S}{P_I} < 1$:

$$\gamma \leq \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + 1} \cdot \frac{P_S}{P_I/\Delta_a} \leq \rho \cdot \frac{P_S}{P_I/\Delta_a}$$

$$\gamma \leq \frac{\rho}{\rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a} \leq \Delta_d \cdot \frac{P_S}{P_I/\Delta_a}$$

- Thus, $\gamma \approx \min\{\rho, \Delta_d\} \cdot \Delta_a \frac{P_S}{P_I}$ is a good approximation in practical situations with limited dynamic range (imperfect AC)



SINR vs. Dynamic Range (2)

- Signal to interference and noise ratio (SINR) versus dynamic range ρ :

$$\gamma = \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a}$$

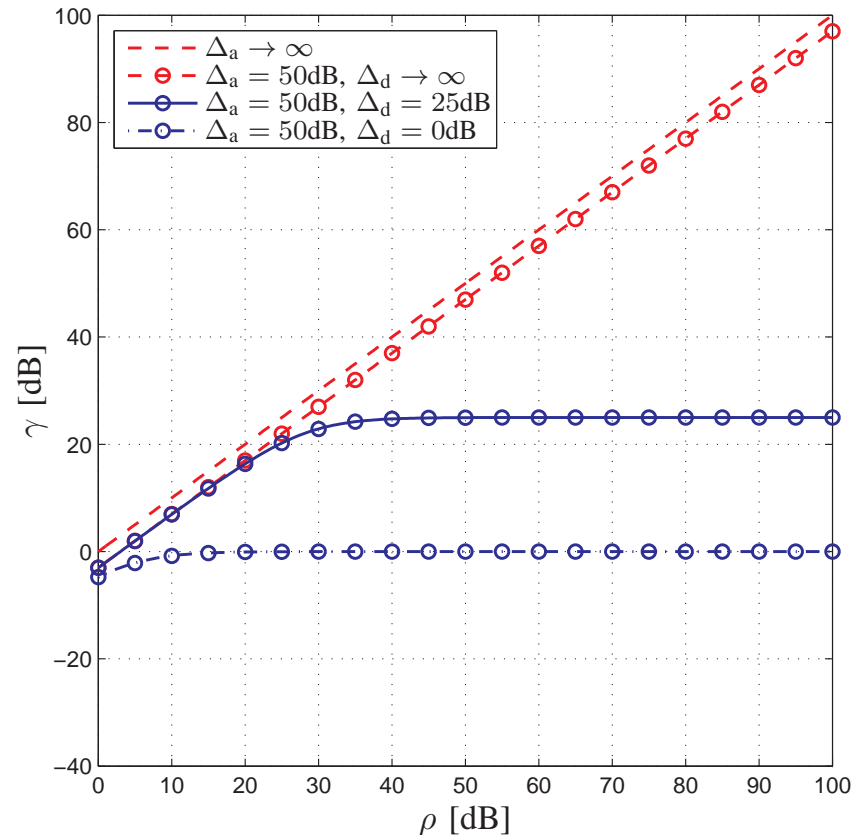
- On the right: Example when
 - ▷ SIR before AC $\frac{P_S}{P_I} = -50\text{dB}$

- Tight bounds if $\Delta_a \frac{P_S}{P_I} > 1$:

$$\gamma \leq \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d} \cdot \frac{P_S}{P_I/\Delta_a} \leq \rho$$

$$\gamma \leq \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d} \cdot \frac{P_S}{P_I/\Delta_a} \leq \Delta_d \cdot \frac{P_S}{P_I/\Delta_a}$$

- Thus, $\gamma \approx \min\{\rho, \Delta_a \Delta_d \frac{P_S}{P_I}\}$ is a good approximation when analog cancellation works almost perfectly

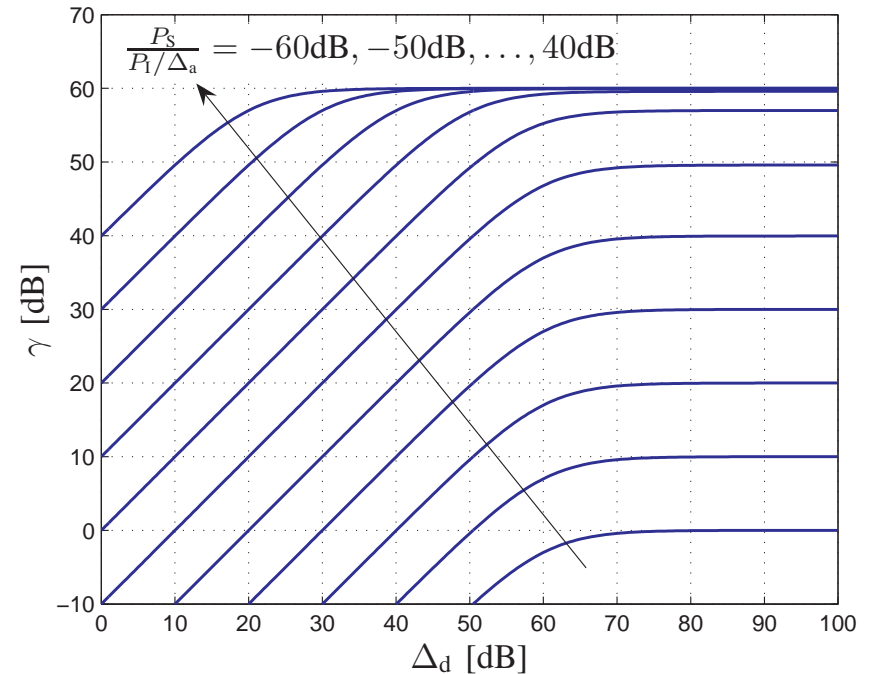


SINR vs. Digital Cancellation

- Signal to interference and noise ratio (SINR) versus digital suppression Δ_d :

$$\gamma = \frac{\rho}{\frac{P_S}{P_I/\Delta_a} + \rho/\Delta_d + 1} \cdot \frac{P_S}{P_I/\Delta_a}$$

- On the right: Example when
 - ▷ dynamic range $\rho = 60\text{dB}$, e.g., 12-bit ADC resolution with small AGC bias (3dB loss of dynamic range)
- SINR increases linearly in terms of digital suppression until performance is limited by the ADC dynamic range or imperfect analog cancellation

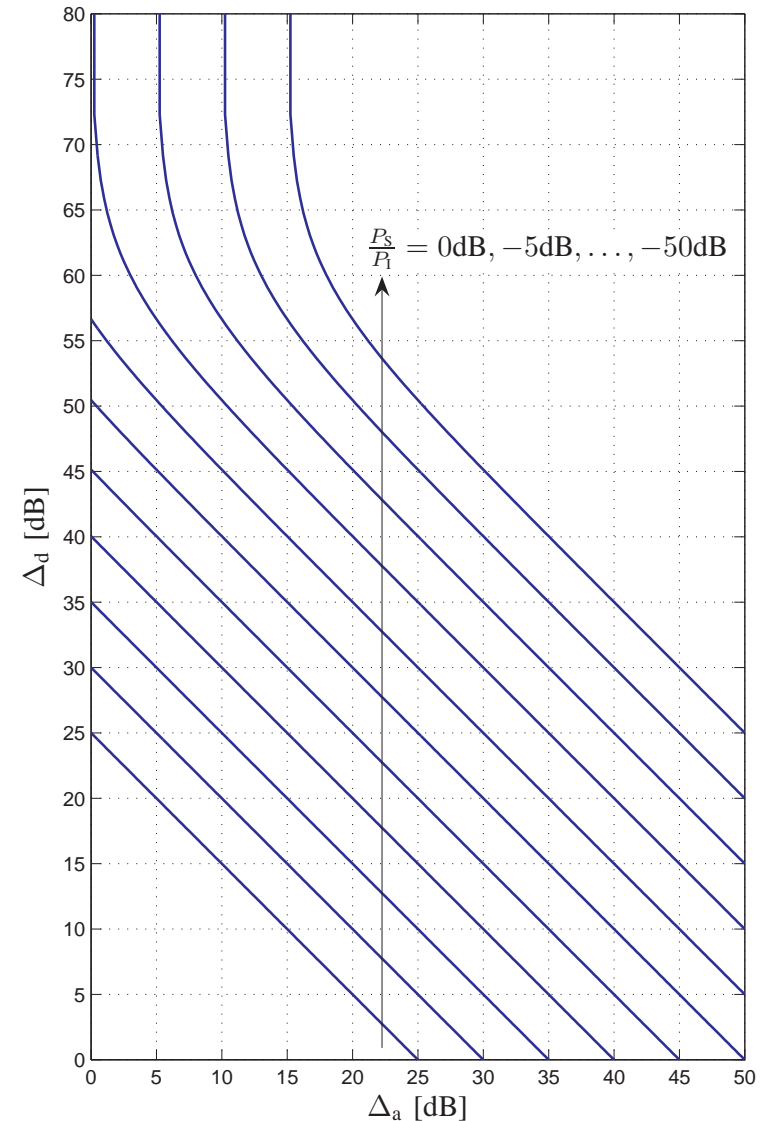


Suppression Requirements

- Minimal digital suppression needed to achieve $\gamma \geq \gamma_t$:

$$\Delta_d \geq \frac{\rho}{\frac{P_S}{P_I/\Delta_a} \left(\frac{\rho}{\gamma_t} - 1 \right) - 1}$$

- On the right: Example when
 - dynamic range $\rho = 60\text{dB}$
 - target SINR $\gamma_t = 25\text{dB}$
- Digital cancellation is efficient if target SINR $\gamma_t \ll \rho$ (obviously) and SIR after AC $\Delta_a \frac{P_S}{P_I} \geq \frac{\gamma_t}{\rho}$
- Then requirement for combined analog and digital suppression becomes simply $\Delta_a \Delta_d \geq \frac{\gamma_t}{P_S/P_I}$



Conclusion

Conclusion

- Wireless full-duplex: A progressive frequency-reuse concept!
 - ▷ Generic MIMO-OFDM transceivers considered herein
- Challenging implementation: strong self-interference combined with limited dynamic range, i.e., practical A/D conversion
 - ▷ Residual self-interference due to non-ideal cancellation
 - ▷ Quantization noise due to limited b -bit ADC resolution
 - ▷ Clipping noise due to high peak-to-average power ratio
- Analytical expressions for desired signal power to residual interference and clipping-plus-quantization noise power ratio
 - ▷ Optimal adaptive gain control for maximal dynamic range
 - ▷ Bias in variable gain amplifier setting
 - ▷ Analog vs. digital cancellation



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