

Analysis and Design of Three-Stage Concatenated Color-Shift Keying

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Abstract—Visible light communication (VLC) relies on abundant unlicensed bandwidth resources. As an attractive high-data-rate modulation scheme designed for VLC, color shift keying (CSK)-assisted modulation is analyzed. We commence our study from an uncoded M -CSK scheme relying on the joint maximum-likelihood (ML) hard detection (HD) of three colors when communicating over an AWGN channel, where both empirical and analytical results are provided. We invoke extrinsic information transfer (EXIT) charts for designing a Maximum *a posteriori* probability (MAP)-based soft detection (SD)-aided iterative receiver jointly detecting the three colors. Based on the EXIT characteristics of M -CSK, we design different signal labeling strategies for diverse color constellations and detection schemes, which are capable of achieving a substantially improved bit-error-rate (BER) performance. Thus, given fixed transmission power, a CSK system using our proposed signal labeling is capable of increasing the reliable data transmission distance by about 30%.

Index Terms—Color constellation, color shift keying (CSK), extrinsic information transfer (EXIT) chart, iterative receiver, near capacity, signal labeling.

I. INTRODUCTION

TO minimize the risk of the impending capacity crunch imposed by the exponentially increased mobile data explosion, heterogeneous networks (HetNets) composed of multiple radio access technologies (RATs), relying on diverse cellular architectures and adaptive transmission modes, have been recognized as one of the most promising solutions. Hence, the fifth-generation (5G) wireless system is expected to rely on the HetNet philosophy [1], [2]. Among others, millimeter-wave communications (MWC) [3] and visible light communications (VLC) [4]–[9] constitute a pair of important complementary access technologies in addition to the development of traditional RAT along the Long-Term Evolution “avenue” [10]. In particular, VLC has attracted substantial research attention across both the academic and industrial communities since it provides abundant license-free bandwidth, which is free from

microwave pollutions, while supporting the dual purpose of both communications and illumination.

As an appealing optical wireless communication technology, both high-brightness white and colored light-emitting diodes (LEDs) [5], [12]–[14] may be used for conveying information. As a benefit of their long life expectancy, high lighting efficiency, and small size, LED devices are routinely used by car manufacturers in both headlights and taillights. This provides a valuable opportunity for exploiting VLC as a part of the road infrastructure for conveying the teletraffic with the aid of traffic lights, streetlights, and for vehicle-to-vehicle communications using the headlights and taillights. The development of VLC schemes has led to the definition of an IEEE standard [5], [16], which is known as 802.15.7, where the so-called color shift keying (CSK) is used for high-data-rate transmission. To elaborate a little further, Drost and Sadler proposed a CSK constellation design using the so-called Billiards algorithm of [19] and expanded their research to an arbitrary number of LEDs in [20]; as a further development, Monterio and Hranilovic proposed another design by invoking the interior point method of [21] and [22], where optimized color constellations were proposed. Furthermore, Singh *et al.* [23] conceived a novel QuadCSK arrangement by invoking four source lights. Finally, a multiuser CSK scheme was presented by Luna-Rivera *et al.* in [24]. Nonetheless, at the time of writing, there is a paucity of research on iterative CSK transceiver design; hence, in this paper, we analyze the performance of the M -CSK system’s color constellation and design its bit-to-symbol mapping. Our contributions are thus twofold.

- We conceive the maximum *a posteriori* probability (MAP)-based soft detection (SD) of CSK and characterize its *extrinsic* mutual information improvement achieved by our iterative receiver with the aid of extrinsic information transfer (EXIT) charts [25]–[27].
- We characterize the performance limitations of the IEEE standard’s constellation labeling, and we circumvent them by designing improved signal labeling strategies with the aid of EXIT charts [28]. As a result, we demonstrate that the power gain attained by this signal labeling improves the achievable transmission distance and/or enhances the link’s reliability in vehicular VLC applications.

The remainder of this paper is organized as follows. In Section II, we briefly introduce the CSK modulation philosophy, including its color coding and intensity conversion techniques. Both ML-based hard detection (HD) and MAP-based SD are characterized. In Section III, we focus our attention

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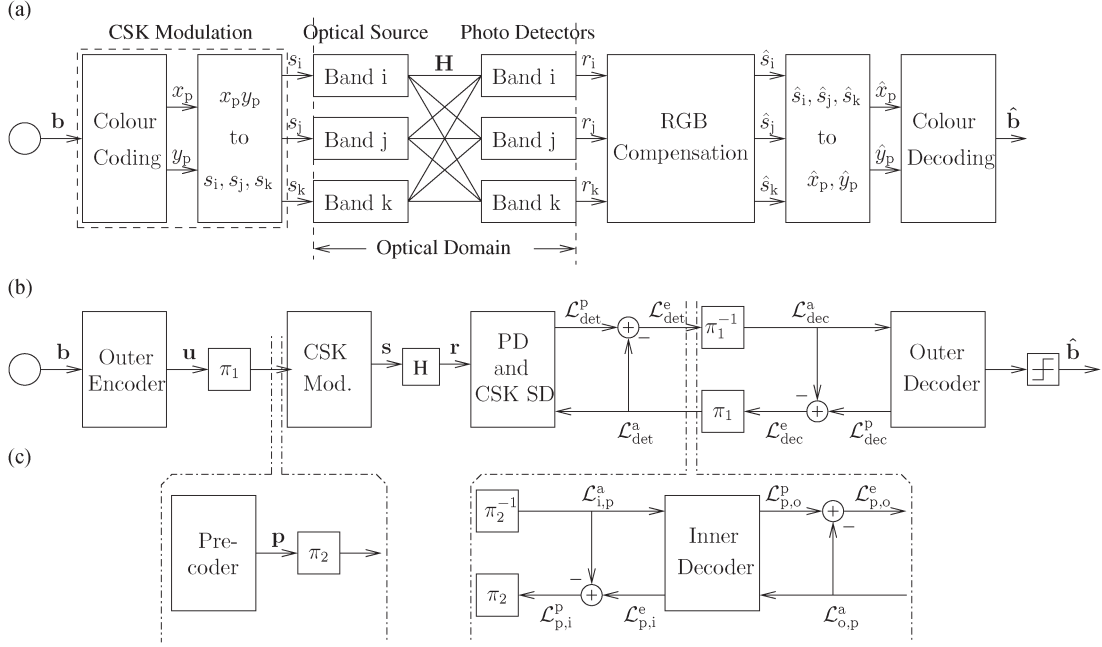


Fig. 1. Transceiver of M -CSK using the joint ML-based HD in (a), the two-stage iterative joint MAP-based SD in (b), and the three-stage iterative joint MAP-based SD with URC precoder in (c).

88 on EXIT chart-based constellation labeling. We then de-
 89 sign sophisticated channel coding schemes for achieving a
 90 near-capacity performance. Our performance versus coding
 91 complexity results and the practically achievable physical trans-
 92 mission distance are also discussed. Finally, we conclude in
 93 Section IV.

94 *Notations:* In this paper, we use lowercase bold fonts to
 95 represent a vector, and uppercase bold fonts to denote a matrix,
 96 where we let the superscript in bracket $(\cdot)^{(n)}$ denote the n th
 97 candidate in all possible legitimate intensity vectors. Addition-
 98 ally, the operator \mathbb{E} stands for expectation and \mathbb{T} for the SD
 99 transfer function; \mathcal{Q} is the Q -function, which is used for error
 100 probability calculation; $(\cdot)^T$ represents the matrix transpose;
 101 and $\|\cdot\|$ denotes the Euclidean norm. Finally, we employ \mapsto to
 102 represent the misinterpretation of a received symbol as another
 103 one in our pairwise error probability (PEP) calculation.

104

II. COLOR SHIFT KEYING SYSTEM

105 Fig. 1 shows the transceiver architecture of M -ary CSK.
 106 Explicitly, in Fig. 1(a), the joint ML-based HD-aided system
 107 is presented, whereas our two-stage joint MAP-based SD-
 108 aided system is shown in Fig. 1(b). As proposed in the IEEE
 109 802.15.7 standard [16], the M -ary color coding scheme groups
 110 the consecutive $N_b = \log_2 M$ incoming bits \mathbf{b} into M -ary CSK
 111 symbols mapped onto a 2-D color constellation point in the
 112 form of the xy color coordinate set $(x_p, y_p) \in \chi$, where χ
 113 contains all the xy color coordinate set. The color coordinates
 114 $\mathbf{x} = (x_p, y_p)$ are generated by the intensity of the three light
 115 sources; hence, these xy color coordinates are converted to the
 116 three-element optical intensity vectors $\mathbf{s}_n = [s_i, s_j, s_k]^T$ before
 117 transmission, which represent the power of the light sources.

A. CSK Modulation

118

1) *Color Coding:* The color constellations¹ of 8-CSK and 120
 16-CSK are displayed in Fig. 2, where the aforementioned xy 121
 color coordinates represent the specific locations of the CSK 122
 symbols (marked by c_ξ , $\xi = 1, \dots, M$). The operating light 123
 intensity I is normalized to unity. Since CSK operates at a 124
 fixed intensity, a constraint is imposed on the maximum output 125
 light intensity I [19]. Furthermore, for the sake of fixing the 126
 perceived color, we impose an average color constraint on the 127
 CSK symbols for the color as follows [21]: 128

$$\sum_n \omega_n \mathbf{s}_n = \mathcal{C}_{\text{avg}} \quad (1)$$

where ω_n is the probability of \mathbf{s}_n , and \mathcal{C}_{avg} is a vector represent- 129
 ing the average perceived color of the transmitted light, which 130
 is known as *color balance*. To elaborate a little further, there 131
 is a pair of 2-D color constellations discussed in this paper. 132
 Specifically, in *Type I*, the color constellation obeys the IEEE 133
 standard of [16], whereas in *Type II*, we rely on a color-balanced 134
 constellation, where \mathcal{C}_{avg} represents a specific perceived color. 135
 Specifically, in this paper, we employed a *white-balanced* (WB) 136
 color constellation. 137

The 8-CSK and 16-CSK *Type I* color constellations are 138
 portrayed in Fig. 2(a) and (b). By contrast, in Fig. 2(c) and (d), 139
 the 8-CSK and 16-CSK *Type II* color constellations maintain- 140
 ing WB [19], [21] are presented. Owing to the constellation 141

¹The constellation is a collection of xy coordinates, which is defined by the international commission on illumination in CIE 1931 color space of [29], where each location corresponds to a color with a specific wavelength, as perceived by the human eye. The location of constellation points may be on the edge, on the vertices, or within a triangle where the vertices represent the optical light sources, as detailed in [17].

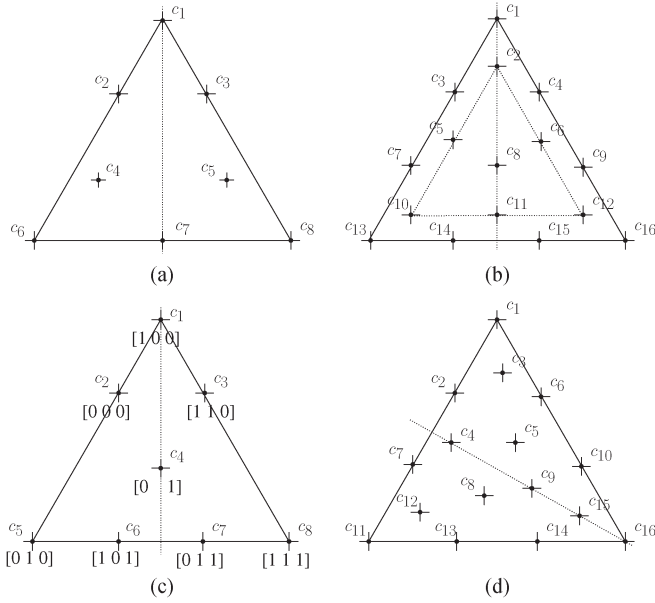


Fig. 2. Color constellation of 8-CSK and 16-CSK, where the operating intensity is normalized to 1. The CSK color constellations specified by the IEEE standard [16] *Type I* are detailed in (a) and (b), whereas those of (c) and (d) are the WB constellations *Type II* proposed in [21]. The details of (c) will be further explained in Table I as an example.

142 simplicity of 4-CSK, the standard color constellation auto-
143 matically retains the WB property. Having considered these
144 two types of constellations, the bits \mathbf{b} are mapped to the
145 corresponding xy color coordinates according to the signal
146 labeling strategy of $\mu^l(\mathbf{b}) \rightarrow (x_p, y_p)$, where l denotes the l th
147 possible signal labeling strategy of all possible candidates,
148 and our specific signal labeling design will be detailed in
149 Section III-A.

150 2) *Light Intensity Conversion*: The 2-D xy color coordi-
151 nates of the symbols are now converted to the 3-D optical signal
152 intensities of light bands. We let \mathcal{S} denote the alphabet of the
153 transmitted intensity vector \mathbf{s}_n . The subsequent conversion to
154 the optical signal intensities of the source light is achieved by
155 simultaneously solving a simple system of three equations, i.e.,
156 $\{\mathbf{x}_p = \mathbf{x}_c^T \mathbf{s}; y_p = \mathbf{y}_c^T \mathbf{s}; \sum \mathbf{s} = I\}$, where we define the coordi-
157 nate vector of the three light sources as $\mathbf{x}_c = [x_i, x_j, x_k]^T$
158 and $\mathbf{y}_c = [y_i, y_j, y_k]^T$ [16]. Thus, the entire CSK modulation
159 process may be interpreted as a one-to-one mapping function.

160 *Example*: For the 8-CSK *Type II* color constellation of
161 Fig. 2(c), given the color band combinations of $[110, 010, 000]$,²
162 the basic coordinates are $\mathbf{x}_c = [0.734, 0.402, 0.169]$ and $\mathbf{y}_c =$
163 $[0.265, 0.597, 0.007]$, whereas the corresponding color constel-
164 lations χ in the dimensions $\mathbf{x} = (x_p, y_p)$ are shown in Table I.
165 Then, upon solving the given system of equations, we arrive
166 at the alphabet of intensity vectors \mathbf{s} , where an example of the
167 8-CSK's intensity vector alphabet \mathcal{S} (columns of $s_i, s_j,$ and s_k)
168 is included in Table I. Furthermore, the notations $\mathcal{S}_{u_1}^0$ and $\mathcal{S}_{u_1}^1$
169 are only relevant for SD-aided CSK, which will be detailed in

²The IEEE standard [16] proposed nine valid sets of color band combinations, where each combination represents a set of three potential color schemes to be used in the CSK system. For example, the color band $[110, 010, 000]$ corresponds to the specific color source, which has the center wavelength of 753, 564, and 429 nm [17].

TABLE I
ALPHABET OF 8-CSK *Type II* INTENSITY VECTORS \mathcal{S} , USING THE COLOR BAND COMBINATION OF $[110, 010, 000]$

	\mathbf{b}	label	$\mathbf{x} = (x_p, y_p)$	s_i	s_j	s_k
$\mathcal{S}_{u_1}^0$	000	c_2	(0.435, 0.299)	0	0.666	0.334
	001	c_4	(0.513, 0.486)	0.112	0.278	0.610
	010	c_5	(0.324, 0.400)	0.610	0.279	0.111
	011	c_7	(0.402, 0.597)	0.501	0	0.499
$\mathcal{S}_{u_1}^1$	100	c_1	(0.357, 0.093)	0	1	0
	101	c_6	(0.546, 0.179)	0	0	1
	110	c_3	(0.169, 0.007)	0.334	0.666	0
	111	c_8	(0.734, 0.265)	1	0	0

Section II-D. As for other CSK modulation schemes, a similar 170
procedure will be implemented. 171

B. Optical Domain Propagation 172

In this paper, we consider a point-to-point transmission 173
system, where the optical signal intensities $\{s_i, s_j, s_k\}$ of the 174
three light bands $\{i, j, k\}$ emitted from the M -CSK modulator 175
block of Fig. 1(a) propagate through an optical channel. At the 176
receiver, a dedicated photodetector (PD) corresponding to each 177
of the three light bands is used for converting the received color 178
component into their electronic representations. Explicitly, the 179
output \mathbf{r} is a distorted and noise-contaminated intensity vector 180
of the color light bands formulated as 181

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{v}. \quad (2)$$

The optical channel is subjected to potential color-band interfer- 182
ence; hence, \mathbf{H} is a (3×3) -element matrix, where the diagonal 183
entries of \mathbf{H} represent the channel gain of the corresponding 184
band, whereas the other entries host the interference imposed 185
by the other bands. As pointed out in [24], other propagation 186
factors, such as diffuse multipath, dispersion, and the non- 187
LOS components, should also be considered in the future, but 188
again, 90% of the received power is concentrated in the LOS 189
component, whereas the contribution of the power received 190
from reflections is small enough to be neglected. Therefore, the 191
channel matrix may be assumed an idealized identity matrix. 192
Based on these discussions, in this paper, we assume that the 193
system experiences an ideal AWGN channel. 194

Furthermore, \mathbf{v} represents the noise imposed in the electronic 195
domain at the receiver, with each entry having a zero mean and 196
a variance of $\sigma_0^2 = \sigma^2/3$, where we define the electronic SNR 197
as $1/\sigma^2$ and σ^2 is the total electronic domain noise power at the 198
receiver. Thus, we have $E_{b_{elec.}}/N_0 = 1/(N_b\sigma_0^2)$. Furthermore, 199
we may also conveniently define the optical SNR as $\gamma_o = \kappa\gamma_e$, 200
where $\kappa = \sigma\mathbb{E}\{\|\mathbf{s}\|\}/\mathbb{E}\{\|\mathbf{s}\|^2\}$. 201

C. Joint ML-Based CSK HD of the Three Colors 202

In practice, all the aforementioned detrimental channel 203
effects can be mitigated by the RGB compensation block of 204
Fig. 1(a) for retrieving the transmitted optical intensities.³ 205

³Note that we implicitly assumed here lossless conversion from the elec-
tronic domain to the optical domain at the transmitter and from the optical
domain to the electronic domain at the receiver. Hence, we may use the optical
signal intensity and electronic signal amplitude interchangeable.

206 When we assume perfect channel state information (CSI) at
 207 the receiver, the estimated intensity vector $\hat{\mathbf{s}}$ is obtained by
 208 the joint ML-based HD of the three colors. When an order
 209 value of $M = 4, 8, 16$ is considered, the full-search-based joint
 210 ML detection complexity remains moderate, and the estimated
 211 intensity vector may then be expressed as

$$\hat{\mathbf{s}} = \arg \min_{\tilde{\mathbf{s}} \in \mathcal{S}} \|\mathbf{r} - \mathbf{H}\tilde{\mathbf{s}}\|^2 \quad (3)$$

212 which implies that the estimated intensity vector $\hat{\mathbf{s}}$ is given by
 213 that specific legitimate intensity vector $\tilde{\mathbf{s}} \in \mathcal{S}$, which exhibits
 214 the lowest distance between \mathbf{r} and $\mathbf{H}\tilde{\mathbf{s}}$. The estimated intensity
 215 vector $\hat{\mathbf{s}}$ is then finally converted back to the resultant bits.

216 Let us now characterize the analytical performance of our
 217 joint ML-based HD by the union-bound-based approach for
 218 deriving an upper bound, which may be deemed tight at high
 219 SNRs, where the average bit error probability may be written
 220 as [30]

$$P_{e,\text{union}} \leq \frac{1}{MN_b} \sum_{\xi=1}^M \sum_{\zeta=1, \zeta \neq \xi}^M d(\tilde{\mathbf{s}}^{(\xi)}, \tilde{\mathbf{s}}^{(\zeta)}) P(\tilde{\mathbf{s}}^{(\xi)} \mapsto \tilde{\mathbf{s}}^{(\zeta)}) \quad (4)$$

221 with $d(\tilde{\mathbf{s}}^{(\xi)}, \tilde{\mathbf{s}}^{(\zeta)})$ representing the Hamming distance of the le-
 222 gitimate intensity vectors $\tilde{\mathbf{s}}^{(\xi)}$ and $\tilde{\mathbf{s}}^{(\zeta)}$. Furthermore, $P(\tilde{\mathbf{s}}^{(\xi)} \mapsto$
 223 $\tilde{\mathbf{s}}^{(\zeta)})$ denotes the PEP of these legitimate intensity vectors,
 224 which is given by [30]

$$\begin{aligned} & P(\tilde{\mathbf{s}}^{(\xi)} \mapsto \tilde{\mathbf{s}}^{(\zeta)}) \\ &= P \left[\left\| \mathbf{r} - \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\|^2 > \left\| \mathbf{r} - \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} \right\|^2 \right] \\ &= P \left[\left\| \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\|^2 / 2 - \mathbf{r}^T \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} > \left\| \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} \right\|^2 / 2 - \mathbf{r}^T \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} \right] \\ &= P \left[\left(\mathbf{H}\tilde{\mathbf{s}}^{(\xi)} + \mathbf{v} \right)^T \mathbf{H} \left(\tilde{\mathbf{s}}^{(\zeta)} - \tilde{\mathbf{s}}^{(\xi)} \right) \right. \\ &\quad \left. > \frac{\left\| \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} \right\|^2 - \left\| \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\|^2}{2} \right] \\ &= P \left[\mathbf{v}^T \mathbf{H} \left(\tilde{\mathbf{s}}^{(\zeta)} - \tilde{\mathbf{s}}^{(\xi)} \right) > \Psi \right] \end{aligned} \quad (5)$$

225 where we have

$$\Psi = \left(\left\| \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} \right\|^2 - \left\| \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\|^2 \right) / 2 - \left(\mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right)^T \mathbf{H} \left(\tilde{\mathbf{s}}^{(\zeta)} - \tilde{\mathbf{s}}^{(\xi)} \right). \quad (6)$$

226 Since $\mathbf{H}(\tilde{\mathbf{s}}^{(\zeta)} - \tilde{\mathbf{s}}^{(\xi)})$ is a matrix that contains constant num-
 227 bers, the term $\mathbf{v}^T \mathbf{H}(\tilde{\mathbf{s}}^{(\zeta)} - \tilde{\mathbf{s}}^{(\xi)})$ obeys the Gaussian distribu-
 228 tion with $\mathcal{N}(0, \left\| \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} - \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\|^2 \sigma_0^2)$. As a result, the PEP
 229 between $\tilde{\mathbf{s}}^{(\xi)}$ and $\tilde{\mathbf{s}}^{(\zeta)}$ can be written as

$$P(\tilde{\mathbf{s}}^{(\xi)} \mapsto \tilde{\mathbf{s}}^{(\zeta)}) = \mathcal{Q} \left(\Psi / \left\| \mathbf{H}\tilde{\mathbf{s}}^{(\zeta)} - \mathbf{H}\tilde{\mathbf{s}}^{(\xi)} \right\| \sigma_0 \right). \quad (7)$$

D. Joint MAP-Based CSK SD of the Three Colors

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Let us now discuss the more powerful joint MAP-based
 SD of M -CSK and its iterative receiver shown in Fig. 1(b).
 When a sophisticated channel-coded system is considered, the
 information bit sequence \mathbf{b} is first channel encoded, yielding
 the coded bit sequence \mathbf{u} and then bit-interleaved by π_1 ,
 before entering into the M -CSK block. The subsequent one-
 to-one mapping of Fig. 1(b) from the incoming bits to the
 resultant optical light intensities follows the procedure detailed
 in Section II-A. Following the PDs employed at the receiver's
 front end in Fig. 1(b), a joint MAP-based SD block is used
 for directly converting the received optical signal intensities
 back to channel-coded soft values, rather than going through
 the consecutive stages of RGB compensation, demapping, and
 color decoding, which was required when joint ML-based HD
 was employed.

The joint MAP-based SD block of Fig. 1(b) exchanges
extrinsic information with the channel decoder, and the final
 HD is carried out by the channel decoder once a predefined
 stopping criterion, such as the maximum affordable number of
 iterations, has been met. For ease of explanation, the following
 terms are defined:

- $\mathcal{L}_{\text{det}}^p$, $\mathcal{L}_{\text{det}}^a$, and $\mathcal{L}_{\text{det}}^e$: the *a posteriori*, *a priori*, and *ex-*
trinsic log-likelihood ratio (LLR) of the detection block,
 which are detailed in [25] and [27].
- $\mathcal{L}_{\text{dec}}^p$, $\mathcal{L}_{\text{dec}}^a$, and $\mathcal{L}_{\text{dec}}^e$: the *a posteriori*, *a priori*, and
extrinsic LLR of the channel decoder block.

Recall that the M -CSK scheme groups a set of $N_b = \log_2 M$
 consecutive incoming bits, when we consider $\mathbf{u}_{(1:N_b)}$ to be
 a binary sequence constituted by N_b consecutive bits. Then,
 for the v th bit of $\mathbf{u}_{(1:N_b)}$, its bit-wise *a posteriori* informa-
 tion $\mathcal{L}_{\text{det}}^p(u_v)$ can be derived by the max-log approximation,
 i.e., [31]

$$\begin{aligned} \mathcal{L}_{\text{det}}^p(u_v) &= \mathcal{L}_{\text{det}}^a(u_v) + \max_{\tilde{\mathbf{s}} \in \mathcal{S}_{u_v}^1} [-\|\mathbf{r} - \mathbf{H}\tilde{\mathbf{s}}\|^2 / 2\sigma^2 + A] \\ &\quad - \max_{\tilde{\mathbf{s}} \in \mathcal{S}_{u_v}^0} [-\|\mathbf{r} - \mathbf{H}\tilde{\mathbf{s}}\|^2 / 2\sigma^2 + A] \end{aligned} \quad (8)$$

where we introduce the shorthand of $A = \sum_{\tau=1, \tau \neq v}^{N_b} u_\tau \mathcal{L}_{\text{det}}^a(u_\tau)$
 for compactness. Additionally, we define $\mathcal{S}_{u_v}^0$ and $\mathcal{S}_{u_v}^1$ as two
 subsets of \mathcal{S} , namely as $\mathcal{S}_{u_v}^0 = \{\tilde{\mathbf{s}} \in \mathcal{S} | u_v = 0\}$ and $\mathcal{S}_{u_v}^1 =$
 $\{\tilde{\mathbf{s}} \in \mathcal{S} | u_v = 1\}$, where a simple subset example is given in
 Table I.

As a result, the *extrinsic* LLRs $\mathcal{L}_{\text{det}}^e$ gleaned from the detec-
 tion block of Fig. 1(b) are deinterleaved, and then they are fed
 as the *a priori* LLRs $\mathcal{L}_{\text{dec}}^a$ into the outer decoder of Fig. 1(b).
 Similarly, as shown in Fig. 1(b), the updated *extrinsic* LLRs
 $\mathcal{L}_{\text{dec}}^e$ are fed back and reinterleaved, before being processed as
 the *a priori* LLRs $\mathcal{L}_{\text{det}}^a$ by the detection block [32].

III. EXTRINSIC INFORMATION TRANSFER-CHART-AIDED ANALYSIS

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Having obtained the soft LLRs of the joint MAP-based SD,
 let us now invoke EXIT charts [25] for conveniently analyzing
 the convergence behavior of our iterative-detection-aided coded

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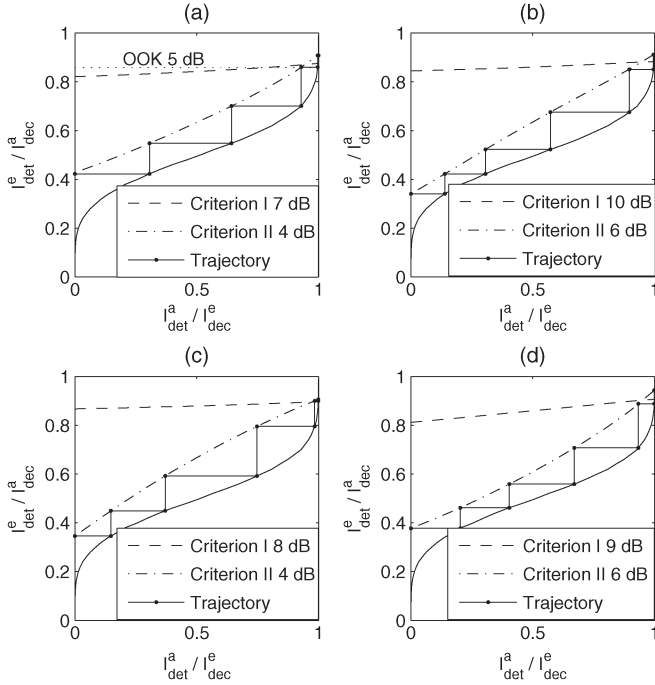


Fig. 3. EXIT characteristics of CSK joint MAP-based SD using different color constellations in an AWGN channel and a half-rate RSC outer decoder (solid line). In all figures, the dashed line and the dash-dotted line represent the approximately required minimum electronic $E_{b,elec.}/N_0$ (detailed in legend) to provide an open tunnel for Criteria I and II, and ON-OFF keying (OOK) is added as benchmarker. (a) 8-CSK *Type I*. (b) 16-CSK *Type I*. (c) 8-CSK *Type II*. (d) 16-CSK *Type II*.

280 M -CSK system by examining the exchange of the input/output
 281 mutual information I_{det}^a and I_{det}^e between the inner and outer
 282 decoders [25]. To be more explicit, by investigating the EXIT
 283 characteristics and the area of open EXIT of the tunnel between
 284 the EXIT curves of the inner and outer decoders, we were able
 285 to find the most beneficial signal labeling for diverse design
 286 objectives. In the remainder of this paper, we assume familiarity
 287 with EXIT charts; see [27].

288 A. Design Criterion I

289 Fig. 3 portrays the EXIT characteristics of 8-CSK and
 290 16-CSK employing both types of color constellations of Fig. 2
 291 for transmission over a LOS AWGN channel, where the
 292 S-shaped solid lines represent the EXIT characteristics of a
 293 half-rate recursive systematic convolutional (RSC) outer de-
 294 coder, whereas the dashed or dash-dotted lines represent the
 295 specific SNR values, where an open EXIT chart tunnel emerges.
 296 For the time being, let us focus our attention on the dashed
 297 lines in Fig. 3. The most important observation related to
 298 M -CSK is that its EXIT characteristic exhibits a modest but
 299 perceivable slope, which implies that the employment of an
 300 iterative receiver is capable of providing some iteration gain
 301 in M -CSK. By contrast, as shown in Fig. 3(a), OOK does not
 302 exhibit any iteration gain. This is because the OOK bits of
 303 the three emitters are independent of each other; hence, they
 304 provide no extrinsic information for each other. On the other
 305 hand, the bits of the CSK symbols are not independent of each
 306 other; hence, a modest iteration gain is achieved. However, the
 307 gradient of the joint MAP-based SD's curve is low. This is

TABLE II
 LABELING MAPS PRESENTED FOR DIFFERENT CONSTELLATION
 AND DETECTION SCHEMES

name	labels ($c_1 \cdots c_{N_b}$) in Fig. 2
8-CSK <i>Type I</i> , Crit. I	(4,0,6,1,2,5,3,7)
8-CSK <i>Type I</i> , Crit. II	(2,5,1,0,4,3,7,6)
8-CSK <i>Type II</i> , Crit. I	(3,2,1,0,6,4,5,7)
8-CSK <i>Type II</i> , Crit. II	(2,1,7,4,5,6,3,0)
16-CSK <i>Type I</i> , Crit. I	(0,1,3,5,2,7,10,6,4,11,14,13,9,15,12,8)
16-CSK <i>Type I</i> , Crit. II	(7,1,5,8,9,15,0,3,14,11,10,4,12,2,6,13)
16-CSK <i>Type II</i> , Crit. I	(1,0,3,5,4,2,7,13,12,6,15,9,10,14,11,8)
16-CSK <i>Type II</i> , Crit. II	(8,14,13,4,1,3,10,7,11,6,9,12,0,15,5,2)

because the signal labeling design of [16] minimizes the sum
 of Hamming distances between the pair of the closest adjacent
 color constellation points of 8-CSK and 16-CSK. To elaborate
 a little further, the bit-to-symbol mapping rule, which is also
 referred to as the constellation labeling μ of the 8-CSK and
 16-CSK schemes obeying Criterion I, is formulated as

$$\text{Criterion I: } \tilde{\mu}_I^l = \arg \min_{\mu^l \in \mathbb{M}} \sum_1^M d_{adj}(\mu^l) \quad (9)$$

where \mathbb{M} denotes an $(M! \times M)$ -element matrix, each row of
 the matrix contains a set of legitimate signal labelings (signal
 labeling examples can be found in Table II), d_{adj} represents
 the Hamming distance between the adjacent color constellation
 points, and μ^l is the specific signal labeling. We launch a
 full search within the set \mathbb{M} and choose the specific signal
 labeling that meets the requirement of (9). We invoke this
 criterion for designing a Gray-coded bit-to-symbol mapping
 for *Type II* color constellations, noting that, due to the specific
 arrangement of the color constellation points shown in Fig. 2,
 it is impossible to realize a perfect and consistent Gray bit-
 to-symbol mapping. Explicitly, our proposed signal labeling
 designed for the *Type II* color constellation using Criterion I
 is listed in Table II, where the decimal numbers represent the
 corresponding bit combinations, which are assigned to their
 corresponding locations (c_1, c_2, \dots, c_8) in Fig. 2. Additionally,
 when using this criterion, we obtained the same signal labeling
 scheme as the IEEE standard constellation labeling of [16].

Fig. 4 portrays the BER performance of the joint ML-based
 HD-aided CSK system of Fig. 1(a) using different color con-
 stellations and signal labeling strategies in an AWGN channel.
 Observe in Fig. 4 that all our simulation results recorded for
 the *Type I* color constellation of $M = 4, 8$, and 16-CSK using
 Criterion I are well matched to their analytical union bound
 represented in dashed line.

B. Design Criterion II

1) *Design Criterion*: When we design the signal labeling
 for our iterative joint MAP-based SD-aided receiver, the bit-
 to-symbol mapping is different from the Gray-like mapping of
 Criterion I. In general, when an iterative receiver is invoked,
 the free squared Euclidean distance condition (FEDC) on having
 ideal feedback usually dominates the performance of iterative
 detection system [33], [34]; hence, a signal labeling having a
 large FEDC performs well.

Additionally, in [33], an alternative objective function (OF) is
 proposed for the constellation labeling search procedure relying

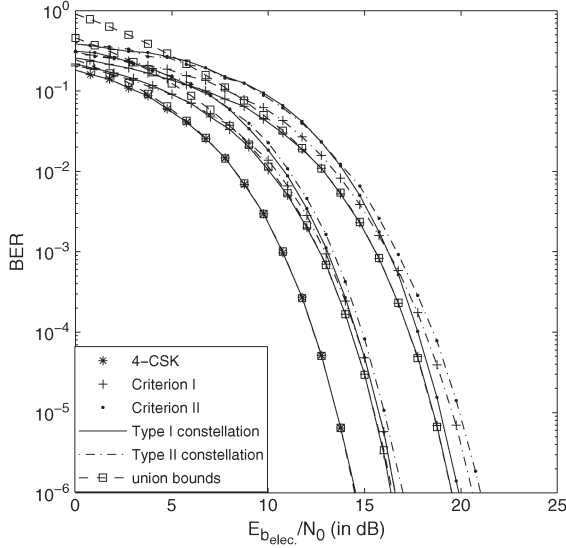


Fig. 4. Analytical and empirical BER performance of the joint ML-based HD CSK schemes using both types of color constellations and different signal labeling strategies in an AWGN channel. The corresponding three- and four-bits/symbol constellation layouts are described in Fig. 2, and the various signal labeling schemes are detailed in Table II. The analytical union bounds are represented by the dashed lines and squares.

351 on the *harmonic mean* of the minimum squared Euclidean
352 distance after the feedback, which is formulated as [34]

$$d_h^2(\mu) = \left(\frac{1}{N_b 2^{N_b}} \sum_{n=1}^{N_b} \sum_{b=0}^1 \sum_{\mathbf{x} \in \chi_b^n} \frac{1}{\|\mathbf{x} - \mathbf{z}\|^2} \right)^{-1} \quad (10)$$

353 where χ_b^n is the specific subset of χ , whose label has the
354 binary value b at the n th bit position, whereas \mathbf{z} is the same
355 as \mathbf{x} , except that its n th bit is inverted. As pointed out in [33]
356 and [34], a specific signal labeling associated with a large d_h^2
357 usually also has a large FEDC; hence, we rely on the d_h^2 OF
358 value for characterizing a signal labeling, rather than on the
359 computationally more complex evaluation of the FEDC.

360 However, purely relying on evaluating the FEDC value alone
361 is insufficient because it does not guarantee finding near-
362 capacity signal labelings, as also pointed out in [35]. Namely,
363 our goal is to find that specific signal labeling, which leads
364 to the smallest open tunnel area in the EXIT chart [34], [35].
365 Hence, in this paper, we first evaluate all legitimate the signal
366 labeling schemes based on (10) and then verify the result with
367 the aid of EXIT chart analysis.

368 *Example:* Given the 8-CSK *Type II* color constellation of
369 Fig. 2(c), based on studying the Hamming distance between
370 the closest-neighbor adjacent constellation points, we carried
371 out a full search in the space of all possible $M!$ signal labeling
372 possibilities and used the one associated with the largest d_h^2 OF
373 value, while relying on the design criterion of

$$\text{Criterion II: } \tilde{\mu}_{\text{II}}^l = \arg \max_{\mu^l \in \mathbb{M}} d_h^2(\mu^l). \quad (11)$$

374 The resultant signal labeling designed for the for 8-CSK
375 *Type II* color constellation relying on our iterative joint MAP-
376 based SD-aided receiver is also shown in Table II. For other

TABLE III
EXAMPLE OF OPTIMIZING SIGNAL LABELING CANDIDATES FOLLOW
CRITERION II FOR 8-CSK TYPE II COLOR CONSTELLATION

name	labels ($c_1 \cdots c_{N_b}$)	d_h^2	open tunnel	$\mathcal{A}_{\text{diff}}$
Label 1	(7,3,6,0,2,1,5,4)	0.0653	6.5 dB	0.3166
Label 2	(4,1,2,6,7,0,3,5)	0.1217	4.0 dB	0.1625
Label 3	(4,1,5,0,3,2,6,7)	0.0519	7.5 dB	0.3694
Label 4	(7,4,2,0,3,5,6,1)	0.0934	4.5 dB	0.1951
Label 5	(4,5,7,0,3,6,2,1)	0.0763	5.5 dB	0.2582
proposed	(2,1,7,4,5,6,3,0)	0.1280	3.5 dB	0.1333

CSK color constellations using joint MAP-based SD, the signal 377
labeling design follows the same procedure, which has also 378
been detailed in Table II. Inspired by the method of [28], we 379
use EXIT charts for comparing the performance of the signal 380
labeling schemes employed. 381

2) *EXIT-Chart-Based Verification of the d_h^2 -Aided Labeling* 382
Optimization: It was shown in [28] that the area under the 383
EXIT curve of a joint MAP-based SD's EXIT chart approxi- 384
mately equals to the system's achievable throughput, which is 385
directly related to its electronic SNR, but remains unaffected by 386
its signal labeling, i.e., [25] 387

$$\mathcal{A}(\gamma_e) = \int_0^1 I_{\text{det}}^e dI_{\text{det}}^a \quad (12)$$

$$= \int_0^1 \mathbb{T}_{\text{det}}(I_{\text{det}}^a, \mu^l, \gamma_e) dI_{\text{det}}^a. \quad (13)$$

Furthermore, the area under the outer decoder's EXIT curve 388
equals to its coding rate R . Again, if we expect the system to 389
achieve a vanishingly low BER, it has to exhibit an open tunnel 390
all the way, leading to the (1.0, 1.0) point. Then, the open tunnel 391
area $\mathcal{A}_{\text{diff}}$ between these two curves characterizes the system's 392
binary data-rate loss in comparison to its true capacity, and the 393
size of this area depends on the specific the signal labeling [28] 394

$$\mathcal{A}_{\text{diff}}(\mu^l, \gamma_e) = \mathcal{A}(\gamma_e) - R. \quad (14)$$

If we aim to attain a near-capacity performance, the area $\mathcal{A}_{\text{diff}}$ 395
should be small. Based on (14), we proceed by finding the 396
specific signal labeling strategy that results in having an open 397
EXIT tunnel at the lowest possible electronic SNR, which 398
leads to a vanishingly low BER at the lowest SNR, provided 399
that the interleaver is sufficiently long to ensure that the soft 400
information is near Gaussian. For practical finite-length inter- 401
leavers, this is not the case; hence, usually, a slightly higher 402
SNR is required for the stair-case-shaped decoding trajectory 403
to pass through the EXIT chart's constriction. To demonstrate 404
the associated $\mathcal{A}_{\text{diff}}$ differences, we consider the 8-CSK *Type II* 405
color constellation as an example and randomly opt for the 406
signal labeling schemes of Table III. When we invoke a half- 407
rate RSC code, the corresponding results are shown in Fig. 5, 408
whereas some specific numerical results are listed in Table III. 409
Observe in Fig. 5 that the EXIT characteristics of the randomly 410
selected Labels 1–5 exhibit different gradients and that our 411
proposed signal labeling has the smallest open tunnel, as well 412

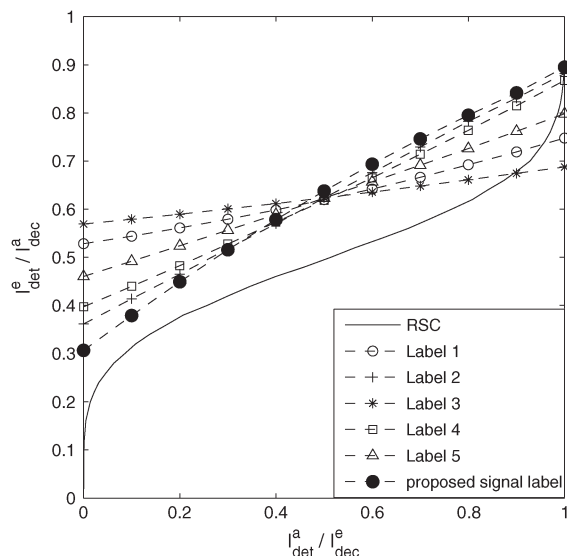


Fig. 5. EXIT characteristics of our proposed signal labeling candidates for 8-CSK *Type II* at $E_{b_{elec.}}/N_0 = 3.5$ dB (dashed lines) for joint MAP-based SD for our iterative receiver of Fig. 1(b) in an AWGN channel, where the solid S-shaped line represents the half-rate RSC decoder, whereas the index of the different signal labeling strategies has been detailed in Table III.

413 as a crossover point closest to the (1.0, 1.0) point, which implies
 414 having the lowest residual BER, as detailed in [25]–[27]. The
 415 tunnel opens at $E_{b_{elec.}}/N_0 = 3.5$ dB, whereas for the other
 416 signal labeling schemes, the EXIT characteristics intersect with
 417 that of the outer decoder. This suggests that our proposed signal
 418 labeling is capable of achieving a significant iteration gain.
 419 Following this approach, our other proposed signal labeling
 420 schemes are listed in Table II.

421 3) *Performance Comparison of Criteria I and II*: Let us
 422 now return to Fig. 3 and observe the dash-dotted lines in
 423 each subfigure, which represent the EXIT characteristics of the
 424 labeling schemes obeying Criterion II. For all color constella-
 425 tions, the dash-dotted style curves are associated with a steeper
 426 gradient and a lower open-tunnel area than for Criterion I.
 427 This implies that the reliability of the *extrinsic* LLRs improves
 428 more substantially upon iterating since more reliable *a priori*
 429 LLRs are gleaned from the outer decoder. Furthermore, the
 430 dash-dotted lines cross the S-shaped RSC decoder curves closer
 431 to the (1.0, 1.0) point; hence, a lower residual BER floor is
 432 expected. Having said this, we also note that these benefits are
 433 achieved at the cost of an increased number of iterations, i.e., at
 434 higher complexity.

435 Furthermore, as for the performance of Criterion II using HD
 436 shown in Fig. 4, we can observe that, although the differences
 437 between the designs relying on Criteria I and II are relatively
 438 small, the performance of the signal labeling obeying Criterion I
 439 is consistently better than that of Criterion II. This is because
 440 having the lowest Hamming distance between the points of
 441 closest-neighbor adjacent constellation points ensures having
 442 the least number of bit errors between the adjacent constellation
 443 points, when decision errors occur in an AWGN channel.
 444 Moreover, regardless of the specific signal labeling criterion
 445 used, the *Type II* color constellation is outperformed by the
 446 color constellation *Type I*. Hence, in line with [21] and [22], the

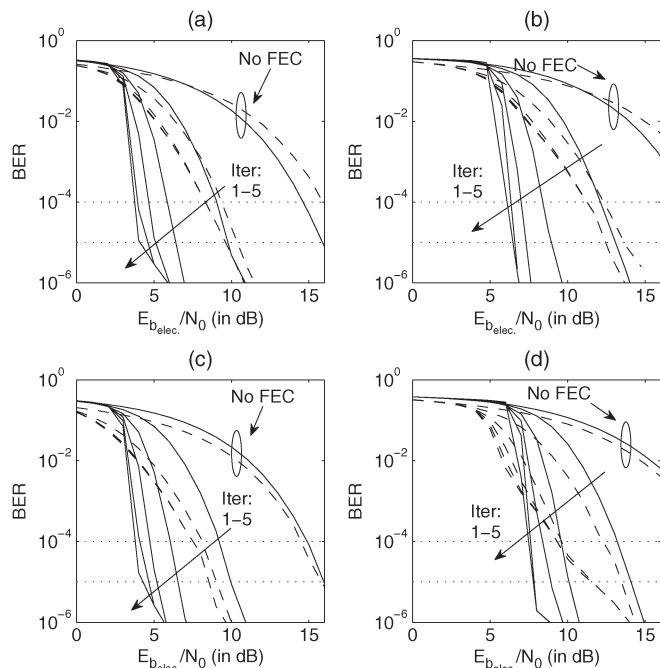


Fig. 6. BER performance of the two-stage half-rate RSC coded CSK system of Fig. 3 using our joint MAP-based SD-aided iterative receiver, where different color constellations are considered. Criterion I is represented by the dashed line and Criterion II by the solid line. From right to left, the multiple solid and dashed lines represent the BER curves associated with an increasing number of iterations from 1 to 5 (link to Fig. 3). (a) 8-CSK *Type I*. (b) 16-CSK *Type I*. (c) 8-CSK *Type II*. (d) 16-CSK *Type II*

color constellation *Type I* is confirmed to be a better solution 447
 448 for an uncoded CSK system.

449 Additionally, the corresponding BER characteristics of Fig. 3
 450 are shown in Fig. 6,⁴ where both the *Types I* and *II* color
 451 constellations of Fig. 2 are characterized in an AWGN channel,
 452 and both Criteria I and II are considered. Similarly to our EXIT-
 453 chart-based predictions shown in Fig. 3, the different *M*-CSK
 454 schemes exhibit rather different iteration gains in Fig. 6. The
 455 maximum attainable iteration gain of the systems relying on
 456 Criterion I is essentially achieved during the first two rounds,
 457 which is due to the limited slope of the related EXIT curves
 458 shown by the dashed lines in Fig. 3. By contrast, the systems
 459 relying on Criterion II benefit from a more substantial iteration
 460 gain. Consequently, the shape of the BER curves exhibits a
 461 “turbo-cliff” effect. After five iterations, the electronic SNR
 462 difference between the two different signal labeling schemes
 463 is around 5–7 dB in all subfigures.

Another important observation in Fig. 6 is that there is a 464
 465 gradient change in the BER curves around 10^{-5} , as mentioned
 466 in Section III-C. This point is associated with the intercept
 467 point observed in the EXIT chart of Fig. 3. Hence, the EXIT
 468 characteristic of the joint MAP-based SD scheme fails to reach
 469 the (1.0, 1.0) point. We will demonstrate in Section III-C that
 470 this performance limitation is almost completely eliminated by
 471 our three-stage system.

⁴Although introducing a redundant forward error correction (FEC) code reduce the effective data rate of the system, it results in considerable performance gains. Furthermore, in [16], half-rate or even lower rate Reed–Solomon codes were considered, which is outperformed by the FEC scheme employed in this paper.

472 C. Three-Stage Concatenated Error Correction Coding

473 Again, despite the aforementioned benefits, all the EXIT
 474 characteristics of the iterative joint MAP-based SD schemes
 475 fail to reach the (1.0, 1.0) point at the top-right corner of the
 476 EXIT chart, which implies the potential emergence of an error
 477 floor in the low BER range. Suffice it to say here that it was
 478 demonstrated in [25]–[27] that an open EXIT tunnel leading to
 479 the (1.0, 1.0) point of the EXIT chart is a sufficient condition for
 480 achieving iterative decoding convergence to a vanishingly low
 481 BER. Invoking an additional unity-rate code (URC) is capable
 482 of eliminating this problem, as detailed in [25]–[27], because it
 483 has an infinite impulse response and, hence, efficiently spreads
 484 the *extrinsic* information without increasing the system's inter-
 485 leaver delay. Due to its unity rate and simple two-stage trellis,
 486 this is achieved without reducing the throughput while only
 487 modestly increasing the complexity.

488 Hence, this becomes a three-stage concatenated system,
 489 where a URC encoder and decoder along with their interleaver
 490 and deinterleaver are inserted at the transmitter and receiver,
 491 respectively, as shown in Fig. 1(c). At receiver end, we first
 492 perform a sufficiently high number of iterations between the
 493 joint MAP-based SD and the URC decoder, which we refer to
 494 as inner iterations. The output $\mathcal{L}_{p,o}^p$ of this combined block is
 495 then forwarded to the outer RSC decoder for exchanging the
 496 LLRs. The outer decoder then provides *extrinsic* information in
 497 form of the *a priori* information $\mathcal{L}_{o,p}^a$ fed to the inner decoder.
 498 These iterations continue until the maximum affordable number
 499 of outer iterations has been reached.

500 Again, invoking the URC imposes slight extra complexity,
 501 but fortunately, the number of inner iterations required remains
 502 low. The associated performance of the 8-CSK *Type II* color
 503 constellation of Fig. 2(c) is shown in Fig. 7 when we employ
 504 two inner iterations between the SD and URC decoder.

505 In Fig. 7, we use the solid line marked by the square and
 506 circle markers for representing the different signal labeling
 507 strategies and operating with the aid of the URC precoder.
 508 We also portray the corresponding results achieved by our
 509 two-stage concatenated system operating without the URC,
 510 which is represented by the dashed and dash-dotted lines for
 511 comparison. Upon introducing the URC precoder of Fig. 1(c),
 512 all the EXIT curves become capable of reaching the (1.0, 1.0)
 513 point in Fig. 7. Although, we note that, consequently, we have
 514 a starting point in the EXIT curve, which is at a lower value
 515 than that of the two-stage system. However, the introduction of
 516 this URC does not reduce the overall code rate. Hence, it does
 517 not change the area under the EXIT curve of its corresponding
 518 three-stage system either. Its ultimate benefit is that it allows the
 519 curve to reach the (1.0, 1.0) point. To elaborate a little further,
 520 this URC will beneficially affect the shape of the original
 521 EXIT curve, making the curve steeper, which may result in a
 522 smaller open tunnel area in the EXIT chart, hence requiring
 523 a reduced SNR. We note however that it might also increase
 524 the risk of intersecting with the outer decoder's curve before
 525 it reaches the (1.0, 1.0) point of decoding convergence to an
 526 infinitesimally low BER. Hence, it necessitates an increased
 527 number of iterations for approaching the (1.0, 1.0) point asso-
 528 ciated with an infinitesimally low BER. For example, observe
 529 furthermore in Fig. 7 that having an open tunnel for Criterion II

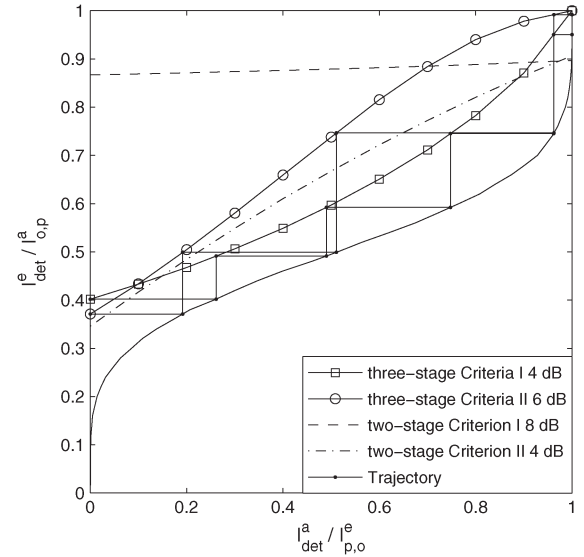


Fig. 7. EXIT characteristics of our joint MAP-based SD assisted 8-CSK *Type II* color constellation with the aid of the URC precoder after two inner iterations using different color constellations in the AWGN channel and the half-rate RSC outer decoder (solid line), where the solid lines with square and circle markers represent the approximately required minimum electronic $E_{b,elec}/N_0$ (detailed in legend) to achieve an open tunnel for Criteria I and II, respectively. The dashed and dash-dotted lines denote the EXIT characteristics of the signal labeling adopting Criteria I and II without URC from Fig. 3(c) as our benchmarker.

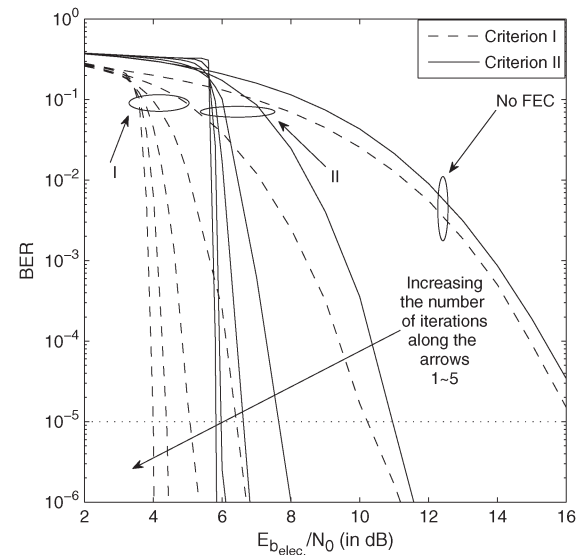


Fig. 8. BER performance of the three-stage half-rate RSC-URC coded CSK system of Fig. 7 using the joint MAP-based SD-aided iterative receiver (two inner iterations), where different color constellations are considered. Criterion I is represented by the dashed line and Criterion II by the solid line. The performance of the system operating without RSC coding is highlighted by an ellipse circle (link to Fig. 7).

requires an electronic SNR of 6 dB, whereas Criterion I has an
 530 open tunnel at 4 dB for our three-stage system. The same trends
 531 are valid for the other three color constellations, which will not
 532 be specified here. Correspondingly, the BER performance of
 533 Fig. 7 recorded in an AWGN channel is displayed in Fig. 8,
 534 where the performance of adopting Criterion I is represented by
 535 the dashed line, whereas the solid lines represent Criterion II.
 536 The most important result is that the URC precoder eliminates
 537

TABLE IV
CODING GAIN (dB) OF M -CSK AT BER LEVEL OF 10^{-6}

Constellation labeling	2-stage iterations: 1-5 in [dB]					3-stage iterations: 1-5 in [dB]				
4-CSK	6.4	7.6	7.6	7.6	7.6	5.2	9.6	11.1	11.9	12.2
8-CSK <i>Type-I</i> Criterion I	6.5	7.2	7.2	7.2	7.2	6.3	10.1	11.7	12.4	13.0
8-CSK <i>Type-I</i> Criterion II	5.7	10.6	10.7	10.7	10.7	4.6	9.0	10.0	10.7	10.9
8-CSK <i>Type-II</i> Criterion I	6.8	7.2	7.2	7.2	7.2	5.6	10.0	11.0	11.7	11.9
8-CSK <i>Type-II</i> Criterion II	5.9	9.7	11.0	11.2	11.2	4.8	8.4	9.6	10.3	10.6
16-CSK <i>Type-I</i> Criterion I	6.5	8.0	8.0	8.0	8.0	5.0	9.0	10.5	11.1	11.5
16-CSK <i>Type-I</i> Criterion II	5.7	10.1	13.0	13.0	13.0	4.6	8.9	10.8	11.5	11.8
16-CSK <i>Type-II</i> Criterion I	9.6	10.0	10.0	10.0	10.0	6.2	10.8	12.7	13.4	13.7
16-CSK <i>Type-II</i> Criterion II	8.3	12.5	13.5	14.4	14.4	7.3	11.6	12.8	13.5	13.6

538 the performance limitations of the two-stage system. Further-
539 more, the BER results closely match the EXIT-chart-based
540 performance predictions shown in Fig. 7. However, compared
541 to the results shown in Fig. 6(c), the BER performance was not
542 improved but actually degraded for all the systems relying on
543 Criterion II. On the other hand, with the aid of the URC pre-
544 coder, Criterion I attained a substantial iteration gain in the low
545 SNR range, which is an explicit benefit of its capability of effi-
546 ciently spreading the *extrinsic* information. Again, for the other
547 color constellation schemes of Fig. 2, the BER performance
548 would exhibit the same trend, which will not be detailed
549 here.

550 Upon comparing Figs. 6(c) and 8, we observe that the BER
551 performance of the two-stage system relying on Criterion II is
552 similar to that of the three-stage system using Criterion I in the
553 BER range of 10^{-4} to 10^{-5} . The overall coding gain summary
554 of our coded M -CSK system has been listed in Table IV, where
555 we define the overall coding gain as the SNR improvement at
556 the BER level of 10^{-6} after each iteration. Hence, we can opt
557 for the most appropriate system according to the specific BER
558 versus complexity requirements.

559 D. Transmission Distance Improvement

560 Based on the BER results of Fig. 6(c), let us now investigate
561 how the different signal labeling designs would affect the attain-
562 able transmission distance. To demonstrate this, let us assume a
563 simple point-to-point transmission. Hence, the channel matrix
564 \mathbf{H} still remains a diagonal matrix, with each entry given by the
565 channel gain $H(0)$ formulated as [4]

$$H(0) = \frac{(m+1)A_r}{2\pi D^2} \cos^m(\phi) \cos(\psi) T_s(\psi) g(\psi) \quad (15)$$

566 where A_r represents the physical area of the PD, which is
567 exemplified by a value of 1 cm^2 according to [4]. Furthermore,
568 $m = -\log_2[\cos(\phi_{1/2})]$ is the order of Lambertian emission,
569 which is given by the semi-angle at half-illuminance of the
570 LED used, such as for example $\phi_{1/2} = 60^\circ$, and D denotes
571 the physical distance between the transmitter and receiver, and
572 ϕ and ψ represent the angle of irradiance and incidence, re-
573 spectively. For simplicity, we let $\phi = \psi = 0^\circ$. Still referring to
574 (15), $T_s(\psi) = 1$ is the gain of the optical filter, and we assume
575 having a field of view of $\psi_c = 60^\circ$ and a refractive index for
576 the lens at a PD, which is $n_{\text{PD}} = 1.5$, as well as an optical
577 concentrator gain of $g(\psi) = n_{\text{PD}}^2 / \sin^2(\psi_c) = 3$. Then, upon
578 considering two-stage 8-CSK *Type II* color constellation as
579 an example, according to Section II-B, the received electronic

SNR becomes $\gamma_e = (\eta P_x)^2 / \sigma^2 = [\eta P_{\text{tx}} H(0)]^2 / \sigma^2$, where P_{tx}
580 is the transmission power of the LED, whereas η represents the
581 responsivity of the PD assuming idealized lossless reception
582 associated with $\eta = 1$. Furthermore, given that the data rate of
583 8-CSK is 36 Mb/s [16], according to [4], we have $\sigma^2 = 5.0 \times 584$
 10^{-16} . Finally, if we let $P_{\text{tx}} = 10 \text{ mW}$ and aim for BER =
585 10^{-6} , while assuming that the total average power of the CSK
586 modulation is constant, the achievable transmit distance D can
587 eventually be expressed as 588

$$D = \sqrt{\frac{A_r P_{\text{tx}} g(\psi)}{\pi \sqrt{\gamma_e \sigma^2}}}. \quad (16)$$

Hence, based on (16), we investigate the beneficial effect
589 of different signal labeling design criteria on the attainable
590 transmit distance D , which is characterized in Fig. 9. Observe
591 that our proposed signal labeling using Criterion II has the
592 best performance, which is capable of increasing the trans-
593 mit distance while maintaining a BER of 10^{-6} . In practical
594 vehicle-to-vehicle communication and vehicle-to-infrastructure
595 communication, the transmission power is larger than the exper-
596 imental power that we used here. This implies that, by carefully
597 designing signal labeling according to proper FEC coding and
598 detection scheme, we could realize higher transmission quality
599 in limited range or longer but reliable data transmission. How-
600 ever, as predicted in Fig. 3, due to the limited iteration gain
601 of all other signal labeling strategies, except for the proposed
602 one using Criterion II, the distance improvement attained after
603 two iterations tends to be small, particularly for Criterion I.
604 By contrast, observe in Fig. 9 that for our proposed signal
605 labeling using Criterion II, the improvement between each
606 iteration is a more considerable. This distance improvement is
607 indeed expected for the two-stage system Criterion II because
608 we demonstrated in Fig. 8 that it needs 4-dB lower electronic
609 SNR than its Criterion I counterpart based on the BER versus
610 SNR curves of Fig. 6(c). 611

612 IV. CONCLUSION

Efficient signal labeling techniques were designed for
613 both types of CSK color constellations proposed in [16]
614 and [19]–[22]. We provided the EXIT-chart-based perfor-
615 mance analysis of both our two-stage and three-stage
616 iterative-detection-aided CSK systems. Commencing from the
617 conventional ML HD, we characterized both the empirical
618 and analytical BER performance of different CSK schemes,
619 adopting the IEEE standard constellation labeling of [16]. We
620

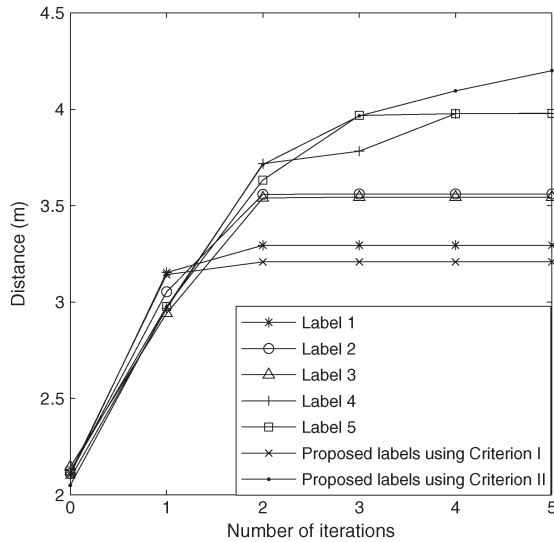


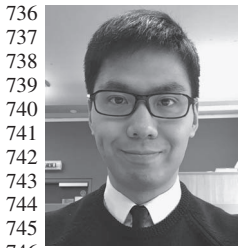
Fig. 9. Achievable transmit distance of a colored LED relying on the two-stage half-rate RSC coded 8-CSK *Type II* system of Fig. 1(b) using the different labeling strategies detailed in Fig. 5, where the LED's transmit optical power is 10 mW, and error tolerance level is $\text{BER} = 10^{-6}$. Additionally, the signal labeling using Criterion I is used as a benchmarker.

621 then demonstrated in Fig. 3 that all the standard M -CSK
 622 labeling schemes of Fig. 3(a) and (b) exhibit a limited iteration
 623 gain in the context of the joint MAP-based SD-aided iterative
 624 receiver of Fig. 1(b). Hence, more beneficial signal labeling
 625 schemes were proposed based on the design criterion of (10).
 626 The associated numerical BER performances were presented
 627 in Fig. 6. We discussed the potential distance and quality
 628 improvements of coded CSK in vehicle-related communication
 629 and concluded that Criterion II is more beneficial in a two-
 630 stage system, whereas Criterion I has the edge in a three-stage
 631 system.

632

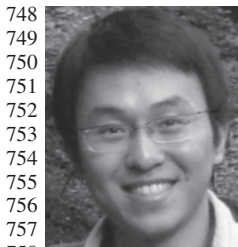
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