This paper isthe result of deliberations of the Society's discussion group on
SEISMIC DESIGN OF DUCTILE MOMENT RESISTING REINFORCED CONCRETE FRAMES

SECTION C

ANALYSIS FOR TORSION EMPLOYING PROVISIONS OF NZRS 4203:1974

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C1.0 SCOPE

The section considers the torsional provisions of NZS 4203:1976. It describes proposed amendments to the "static" provisions of the Code, it gives a step by step procedure for following these provisions, and it discusses the use of modal analysis.

C2.0 NZS 4203 TORSION PROVISIONS

In its present form Clause 3.4.7.1 allows the use of a "static method" for buildings not worse than "moderately unbalanced" which by inference includes regular and reasonably symmetric buildings. The last sentence of Clause 3.5.2.2.2 in effect requires that the "static method" must be used for buildings of this category.

It is proposed to issue the first substantial amendment to NZS 4203 in early 1978. A proposal to include a revision of Clause 3.4.7.1. in the 1978 Amendment will be circulated for comment by SANZ in the latter half of 1977. This paper assumes that the revision adopted by SANZ will be as follows:

"3.4.7.1 Horizontal Torsional Moments : Methods of Assessment Applicable to Various Building Types

3.4.7.1.1

For structures not more than four storeys high or for reasonably regular structures more than four storeys high which are symmetric or of moderate eccentricity, horizontal torsion effects shall be taken into account either by the static method of Clause 3.4.7.2, or by the two-dimensional modal analysis method of Clause 3.5.2.2.1 which also uses Clause 3.4.7.2, or by the three-dimensional modal analysis method of Clause 3.5.2.2.2.

3.4.7.1.2

For reasonably regular structures more than four storeys high with a degree of eccentricity, horizontal torsional effects shall be taken into account either by the static method of Clause 3.4.7.2, or by the two-dimensional modal analysis method of Clause 3.5.2.2.1 which also uses Clause 3.4.7.2, or by the three-dimensional modal analysis method of Clause 3.5.2.2.2. However, it is recommended that the three-dimensional modal analysis of Clause 3.5.2.2.2 be used for such structures.

3.4.7.1.3

For irregular structures more than four storeys high, horizontal torsional effects shall be taken into account by the three-dimensional modal analysis method of Clause 3.5.2.2.2."

The corresponding amendments to Commentary Clause C3.4.7.1. proposed are:

Paragraph 4 is amended to read:

"Reasonably regular buildings are square, circular or rectangular structures which have no major re-entrant angles and which are substantially uniform in plan; that is, the eccentricity and the plan position of the centre of rigidity should be essentially constant from floor to floor throughout the height of the building."

Replace the existing 5th paragraph of C3.4.7.1 beginning "If a substantial..." with the following paragraphs:

"Structures of moderate eccentricity are structures for which the average eccentricity e_s is not greater than 0.3b. Any structure with an average eccentricity e_s greater than 0.3b has a high degree of eccentricity or imbalance.

For exceptionally flexible buildings which are highly irregular and not more than four storeys high (Clause 3.4.7.1.1), it is recommended that a three-dimensional modal analysis should be used, as the dynamic behaviour in such cases is likely to be more complex than for stiff buildings. It should be noted that even a three-dimensional modal analysis may not always give good predictions of the dynamic behaviour of very irregular buildings, and may indeed seriously underestimate earthquake effects in some cases."

Clauses C.3 to C.7 outline in detail the application of the "Static Method".

Torsional analysis of highly eccentric buildings and of irregular buildings of more than four storeys using three-dimensional dynamic modal analysis is discussed in Clause C.8 below.

C3.0 FLOOR MASSES AND EQUIVALENT STATIC FORCES

The method of calculating these masses and equivalent static forces is outlined in the paper on Analysis (Section B).

C4.0 CENTRE OF MASS

When considering a particular storey in a multi-storey building the centre of mass is the point on the floor plan through which the inertia forces act. Thus for the storey under consideration the centre of mass is found by calculating the centroid through which the resultant of the inertia forces, i.e. equivalent static

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forces, of the floors above act.

Unless the building is completely symmetrical in its mass distribution for its complete height the centre of mass will vary from storey to storey. Engineering judgement is necessary but for reasonably regular frame structures it is reasonable to calculate an average centre of mass near the mid-height of the structure and use this for all storeys.

C5.0 CENTRE OF RIGIDITY

The centre of rigidity at a particular storey has generally been obtained in the past by considering the elements within that storey only. For both principal axes the stiffnesses of the various elements are determined. The centre of rigidity has been taken to be the centroid of these stiffnesses.

A more general method of establishing the centre of rigidity at each level is illustrated by using the model shown in figure 8. This involves the use of a plane frame computer program and employs the model shown in which the various frames of the structure in one direction are placed end to end and connected by rigid bars at each floor. The structure is subjected to lateral loads distributed vertically as specified by the Code (Clause 3.4.6) and is analysed statically. This is done in both principal directions. The centroid of the shear forces at each storey derived from this analysis is the centre of rigidity at that storey.

For reasonably regular frame structures it would not normally be necessary to use this more elaborate procedure: an average centre of stiffness for the building would suffice, computed on the basis of individual storey stiffnesses.

C6.0 ECCENTRICITIES e_s, e_{d1}, e_{d2}

Calculate the static and design eccentricities for each principal direction as defined in NZS 4203, Clause 3.4.7.2, using the formulae

$$e_{d1} = 1.7e_s - e_s^2/b + 0.1b$$

$$e_{d2} = e_s - 0.1b$$

C7.0 CALCULATION OF COLUMN ELASTIC SHEAR FORCES

Consider the two principal directions, X and Y.

Determine the elastic shear force for each lateral load resisting element in direction X by considering the translational shear force and torsional shear force for that element separately as follows:

(1) For loading in direction X, calculate the translational shear in the element by distributing the storey shear to all elements having stiffness in that direction, in proportion to their stiffnesses.

(2) For loading in direction X, calculate the two storey torques corresponding with the eccentricities e_{d1} and e_{d2}. For each storey torque calculate the shear in direction X in the element using the method outlined in many standard texts by authors such as Benjamin$^{(6)}$ and Blume, Newmark, Corning$^{(4)}$.

(3) Combine the shear obtained in step 1 with those in step 2 to give the two elastic shears in the element due to an earthquake in direction X. (Sometimes the shears will be of opposite sign, but they should be summed algebraically).

(4) For loading in direction Y, calculate the two torsional shear forces in the element in direction X, as in step 2. These are the two elastic shears in the element in direction X due to an earthquake in direction Y. Note that in some buildings with a high degree of eccentricity the maximum shear in an element in direction X may be due to the torsional shears induced by an earthquake in direction Y.

(5) The elastic code level shear for the element in direction X is the most severe of the four shears derived in steps 3 and 4.

The elastic shear for lateral load resisting elements in the Y direction can then be found following a similar procedure.

C8.0 TORSIONAL EFFECTS AND THREE-DIMENSIONAL ANALYSIS

The proposed revision of Clause 3.4.7.1 requires that a three-dimensional modal analysis as described in Clause 3.5.2.2.2 be used to determine torsional effects for irregular structures over four storeys, and recommends its use for highly eccentric or "unbalanced" regular structures.

C9.0 BUILDING EXAMPLES : CODE TERMINOLOGY AND METHODS OF ANALYSIS

Commentary clause C.C.9 discusses a number of example buildings which illustrate the code terminology and recommends methods of analysis.

COMMENTARY

CC2.0 NZS 4203 TORSION PROVISIONS

The torsion provisions of NZS 4203 are designed to produce a distribution of internal actions which will then be used as the basis for the capacity design of structures. The distribution of the internal force system on which the capacity design procedure is based is not critical; consequently a reasonable degree of approximation in the initial elastic analysis of the structure is justifiable. This in turn justifies the various approximations upon which the torsional provisions of the Code are based.

The basic justification for the provisions of NZS 4203 in regard to torsion is to enable design for strength of members to be better related to the probable load distribution induced by torsional effects, in particular those caused by the stiffness distribution of members. If this approach is not followed it is likely that some members will yield at load levels which could be substantially less than those prescribed by the Code, and substantially less than those at which other members will yield. Such a progressive failure effect, apart from the possibility of introducing other torsional effects by virtue of reduced stiffness associated with yielding, results in greater energy absorption requirements in some members and the possibilities of further stiffness degradation and ultimately failure of the "weaker" members.

The torsional behaviour of structures in earthquakes is complex and still not well understood. The static torsional provisions of NZS 4203 attempt to take some of the more important effects into account in a reasonably simple way. An account of the basis of this is given in a paper by Elms(2).

NZS 4203 has been written on the assumption that the use of dynamic analysis and computers is not yet widespread in New Zealand and has aimed to allow the use of manual static analysis in as many cases as possible. Thus the static method of accounting for torsional effects given in Clause 3.4.7.2 is intended to apply to symmetric buildings of regular plan and unsymmetric buildings of regular plan including those which can be described as highly eccentric. In line with the final paragraph of the 1973 Amendments to the SEAOC Code(1) it is now proposed in the revised Clause 3.4.7.1 to allow these provisions to be used for small but highly irregular buildings, of up to 4 storeys. Buildings which are excluded, referred to as "irregular" in Clause 3.4.7.1, include those which vary in plan with height, vary markedly in stiffness with height, vary markedly in strength with height, and those structures with unusual features.

The Dynamic Analysis provisions of NZS 4203 derive from the SEAOC commentary referred to above(1). In particular refer to the Commentary on Section 2313 (d) 2 on pages 141 to 144 of the SEAOC Code. In essence this commentary defines irregular structures by guidelines and recommends that these structures be analysed by dynamic analysis. This recommendation recognizes that the "triangular" distribution of equivalent static forces is only valid for regular structures which vibrate principally in a first mode.

There has been some criticism of the torsion provisions by some engineers in that the provisions are not well suited to computer analysis. The possibility of simpler versions of formulae 31A and 31B is under review.

CC4.0 CENTRE OF MASS

The definition of centre of mass used in Clause C3.0 is more general than that usually employed which considers the mass of the floor above the storey under consideration only. Referring to Building 1 in figure 1 it can be seen that if one considers all the masses of the various floors above the storey under consideration the influence of the eccentric penthouse reduces as one progresses down the building. Since one is in fact interested in inertia forces and their actions on the storey being considered it is more correct to consider the inertia forces associated with all the masses above. The inertia forces are the "equivalent static" inertia forces calculated using the Code specified vertical distribution. The use of inertia forces rather than masses produces the following two effects: Firstly for a regular building such as Building 1 the variation in the centre of mass with height is more rapid at the top of the building and so the error in using an "average" value calculated at mid-height will involve less error for the bulk of the building than when floor masses are used to calculate the centre of mass. Secondly the weighting of the inertia forces means that irregularities of mass distribution towards the top of the building will cause more variation in the centre of mass than irregularities of mass distribution towards the bottom of the building.

For regular symmetric buildings the traditional method of calculating the centre of mass using the mass of the floor above the storey under consideration is quite satisfactory. For a regular building with an eccentric penthouse such as Building 1 or a building with a symmetric podium having set-backs complying with Clause 3.4.11 of NZS 4203 the use of an average centre of mass calculated near mid-height of the building is reasonable because firstly it will be conservative for the lower and more heavily loaded storeys and secondly further accuracy hardly seems justified when employing Clause 3.4.7.2.

CC5.0 CENTRE OF RIGIDITY

The traditional method of establishing the centre of rigidity for a regular multi-storey building has been to consider an isolated storey. The simplest method involves consideration of column stiffness assuming fixed ends. The Muto method(3) introduces a more realistic assessment of column stiffness by considering the actual end fixity associated with the beams. A similar refinement of "stiffness" is used by Blume, Newmark and Corning(4).

The more general method advocated is associated with a more general definition of centre of rigidity. When considering a given storey we are strictly interested in the point at which the inertia forces above the storey must be applied so that there is no rotation within the storey.

Referring to Building 8 in Figure 8, the relative stiffness of the frames in each direction are obtained by placing the frames end to end connected by rigid bars at each floor, and by applying lateral loads distributed vertically as specified by the Code. The rigid bars dictate that at each floor level all the frames have the same displacement, i.e. the structure is displaced laterally with no rotation in a horizontal plane. The shear forces in each frame indicate the relative stiffnesses of the frames at each level, and the centroid of the shear forces provides us with the location of the centre of rigidity as defined above.

CC6.0 ECCENTRICITIES e_s, e_{d1}, e_{d2}

The code formulae 31A and 31B are the result of work by Elms(2) and are similar to formulae proposed by Newmark and Rosenbleuth(5). A diagram illustrating e_{d1} and e_{d2} is given in Figure 1 of Ref (2).

Although formula 31A was derived from studies on single storey buildings it has been employed on several very eccentric buildings of up to 8 storeys similar to Building 2 in Figure 2. It was found that the results using Eq. (31A) compared very closely with those from a three-dimensional dynamic modal analysis employing the RSS combination of modes.

The provisions of the formulae are twofold as outlined in the Code commentary. Firstly the 0.1b term covers "accidental torsion" due to inaccuracies in calculating the centres of mass and rigidity, construction variation, torsional ground excitation, unanticipated foundation variation, and partial asymmetric yielding during earthquake. Secondly the term $1.7e_s - e_s^2/b$ provides primarily for the possibility of the coupling of translational and torsional modes and secondarily for what can be loosely called "dynamic magnification" associated with horizontal torsion and the effective shifting of the centre of mass. It has been suggested quite reasonably that this formula should be period dependent because coupling occurs when translational and torsional modes have similar periods. However at this stage further research is necessary.

If three-dimensional analysis is employed the mode shapes will generally include both translational and torsional displacements. Only in very regular and symmetrical structures will modes occur that are wholly translational or rotational. Skinner has recommended in private correspondence that when employing this type of analysis, modes that have periods within 0.1 seconds should be added algebraically rather than combined by the RSS method. This allowance is similar to that for coupling associated with planar analysis in that modes with very close periods will both be stimulated by a given earthquake and thus their effects should be added algebraically.

CC7.0 CALCULATION OF COLUMN ELASTIC SHEAR FORCES

When calculating the storey torques, the equivalent static forces are all applied at the e_{d1} position for all floors in one principal direction and then for all floors at the e_{d2} position and so on. It is not necessary to consider cases where equivalent static forces are applied at e_{d1} positions for some floors and at e_{d2} positions for others.

To determine the elastic column shear forces associated with the storey torques, as described in Steps 2 and 4, it is necessary to know the relative stiffnesses of all the elements in all directions. For a reasonably regular frame structure it will generally suffice to take the relative stiffness of the elements within one storey, but if the more rigorous general method illustrated in Fig. 8 has been used to calculate the centre of rigidity at each level, the same procedure can be used to determine the relative stiffnesses of all the elements. The procedure will be the same as that described in Section C5.0, but with all the frames in both directions tied together with rigid bars at each floor level for analysis with a plane frame computer program.

CC8.0 TORSIONAL EFFECTS AND THREE-DIMENSIONAL MODAL ANALYSIS

In regard to clause 3.5.2.2.2 some brief comments on three-dimensional modal analysis may be helpful. The various periods and mode shapes of the normal modes of vibration of a building are functions of the mass and stiffness of the building. When considering a three-dimensional model, matrices are employed and the mode shapes generally have deflection components in three directions viz. X, Y and rotation about the Z axis. When a building has an axis of symmetry it is usual for several of the modes to have deflection components predominantly along this axis of symmetry. These modes appear in a plane frame analysis as the 1st, 2nd, 3rd etc. modes in the direction of the axis of symmetry.

Using a three-dimensional model and considering earthquake stimulation in say the X direction, mode shapes which are predominantly in the X direction together with torsional modes about the Z axis will be stimulated whereas modes which are predominantly Y will have little effect.

Clause 3.5.2.2.2 is in need of review but unfortunately there was not sufficient time to include this in the proposed 1978 Amendment. The intent of the first sentence of the Clause is to ensure that all significant modes are considered in a three-dimensional modal analysis. It is recommended that 8 or 9 modes be considered. It is then likely that 2 or 3 will have predominant displacements in each of the 3 directions X, Y and Z. Thus when the building is stimulated in, say, the X direction the major response will be produced by the 2 or 3 predominantly X modes with any significant torsional modes.

The second requirement to include the effects of ± 0.1b accidental torsion in the analysis is not simple to achieve. One can offset the masses by ±0.1b and run separate analyses in each direction but each new mass position produced new mode shapes and periods and a large amount of design data is accumulated which can only be sifted efficiently by a computer. This should not be a difficult service for a Computer Bureau to provide.

The last sentence of 3.5.2.2.2 was included in the same spirit as Clauses 3.5.2.4.1 and 3.5.2.5.1 to ensure that dynamic analysis is not used as a means of reducing seismic shears. Most buildings when analysed by a modal analysis have shears approximately 80% to 90% of the static analysis shears. The Committee felt that dynamic analysis gives an improved distribution of shears but this does not in itself justify large reductions in storey shears. The difficulty with the last sentence of 3.5.2.2.2 is that it in effect requires that if a dynamic analysis is done on a moderately unbalanced building then a static analysis must also be done. This is a fairly time consuming requirement and seems unfortunate when the same requirement is not stipulated for a highly eccentric building. Hopefully an amendment of the complete dynamic section of the code will reduce the amount of work whilst achieving the committee's aim.

CC9.0 BUILDING EXAMPLES : CODE TERMINOLOGY AND METHODS OF ANALYSIS

Building 3 in figure 3 illustrates an irregular structure for which the static provisions of Clause 3.4.7.2 are inappropriate and a three-dimensional dynamic analysis is required. This example also illustrates some of the difficulties of defining centres of mass and rigidity in a meaningful manner.

The masses of the individual floors are shown. In the tower the traditional method of determining the centre of mass by considering the floor at the top of the storey under consideration is reasonable in that it represents quite accurately the centroid of the inertia forces which are transferred across the storey. In the lower 3 storeys, however, the traditional method gives a quite erroneous answer.

If it is assumed that the members of frames A and B are considerably lighter than those of frames C, D and E, as is likely, then the centre of stiffness as defined by the general method will probably be between frames C and D, close to D. If the traditional method of calculating the centre of rigidity is used in the tower little eccentricity will result which seems reasonable whereas in the podium the same result would seem erroneous and the general method more appropriate. Whether or not the general method proposed produces a reasonable result is not known and until such time as sufficient research of case studies is completed it is prudent to employ the three-dimensional dynamic modal analysis approach.

Building 4 in figure 4 is a more extreme case of the problems illustrated by Example Building 3. It is probable that the basic mode shapes will be of two patterns: firstly modes where the predominant deflections are along the diagonal line of symmetry and secondly modes which have a centre of rotation somewhere along this line of symmetry and the deflections are predominantly rotational about this centre. For buildings of this nature use of a three-dimensional dynamic modal analysis programe is required. For a building of this size use of a program such as TABS would be reasonably cheap, of the order of $700, and could include runs with equivalent static loads applied with and without eccentricities. TABS is a program developed at Berkeley and basically follows the typical manual process of considering the building as a series of plane frames in 2 directions. There are limitations to a program of this nature but for most New Zealand buildings TABS is quite adequate.

Building 5 in figure 5 does not justify use of sophisticated computer techniques but is strictly an irregular building and has moderate to high eccentricity or "unbalance". In this case it is recommended that the building be analysed floor by floor using the general definition of centre of mass i.e. the centroid of the inertia forces above the floor under consideration but the traditional definition of centre of rigidity considering only the floor under consideration. If these assumptions are made and the Code provisions applied it is felt that a sufficiently conservative structure will result.

Building 6 is L shaped in plan but not highly eccentric. If the mass is assumed to be uniformly distributed and the columns are assumed to all be of equal stiffness in each direction then the centres of mass and rigidity are both at the re-entrant angle. The assumption of uniform column stiffness is perhaps a little artificial but a building nearly satisfying these assumptions is quite feasible and the eccentricity will be very small. It is not entirely clear whether the comments on Clause 3.4.7.2 would include this building whereas they could clearly include Building 7. The concern regarding buildings of this nature is also expressed in the SEAOC code(1), page 142, in recommending that these buildings should be subjected to a dynamic analysis. The concern is that the wings may tend to vibrate independently, presumably because of the marked difference in stiffness in the wings, and that "these buildings usually generate high torsional or twisting effects". Example Buildings 6 and 7 demonstrate that the latter assumption is not necessarily the case but the problem will increase with "unbalance" between the wings. The valid concern, confirmed in correspondence by Bertero of Berkeley, is that for buildings of these plan shapes, which are eccentric, stresses at the junction can be high. Presumably if the wings are very long the earthquake imput could vary but this seems an unlikely situation in New Zealand with our normal scale of building. Using a three-dimensional computer program it is not difficult to analyse these stresses, nor is it prohibitively expensive, so it seems reasonable that this approach could be adopted as a viable alternative to the Code recommendation of separating the wings. Apart from the difficulty and expense of detailing seismic separations the many uncertainties in our analysis and design techniques by no means guarantee separation as a better solution in the event of an earthquake.

REFERENCES

1. S.E.A.O.C. Recommended Lateral Force Requirements and Commentary, 1973.
2. D. G. Elms. Seismic Torsional Effects on Buildings. Bulletin of the N.Z. National Society for Earthquake Engineering. Vol. 9, No. 1, March, 1976.
3. K. Muto. Seismic Analysis of Reinforced Concrete Buildings.
4. J. A. Blume, N. M. Newmark and L. H. Corning. Design of Multistorey Reinforced Concrete Buildings for Earthquake Motions. P.C.A. 1961.
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6. J. R. Benjamin. Statically Indeterminate Structures. McGraw-Hill, 1959.

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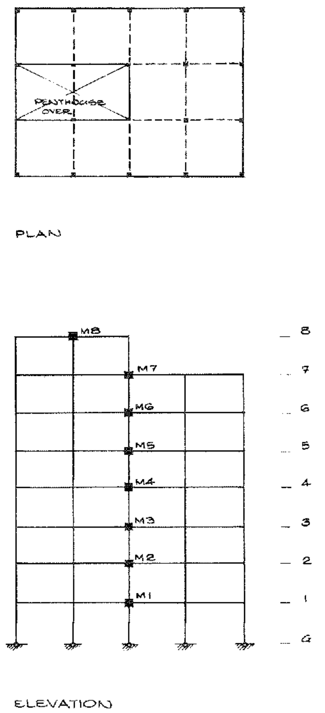

FIGURE 1: BUILDING 1.
REGULAR MINOR ECCENTRICITY
STATIC ANALYSIS

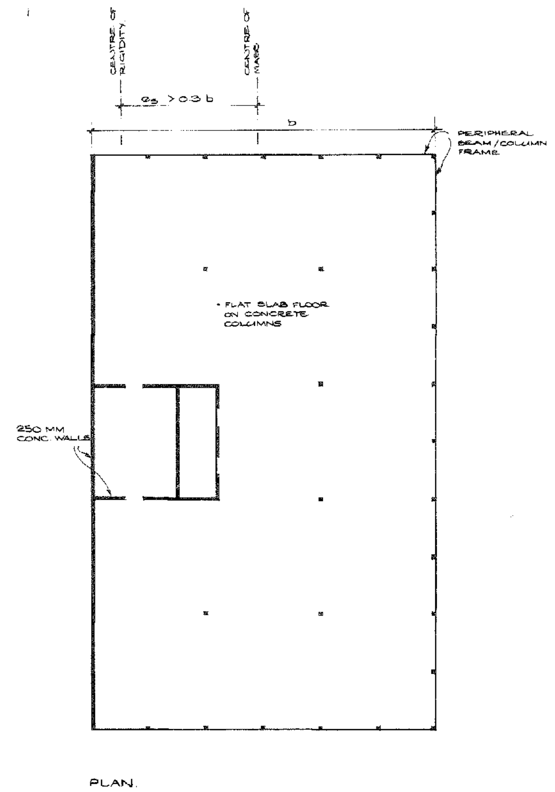

FIGURE 2: BUILDING 2
(REGULAR, IF FLOOR PLAN CONSTANT,
HIGHLY ECCENTRIC es > 0.3 b.
STATIC ANALYSIS ACCEPTABLE
BUT 3D MODAL ANALYSIS
RECOMMENDED)

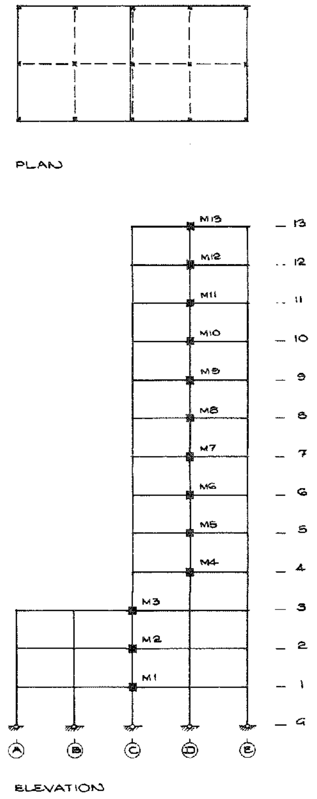

FIGURE 3: BUILDING 3
(IRREGULAR, LOW ECCENTRICITY
IN TOWER, MODERATE ECCENTRICITY
IN PODIUM. 3D MODAL ANALYSIS)

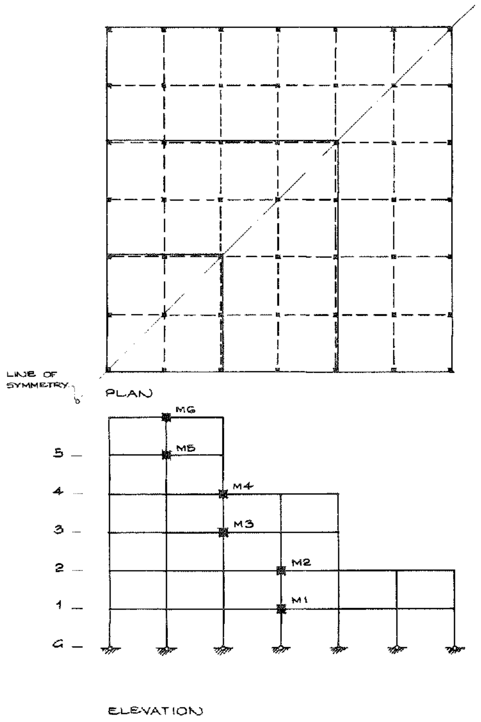

FIGURE 4: BUILDING 4
(IRREGULAR, MODERATE TO
HIGH ECCENTRICITY.
3D MODAL ANALYSIS)

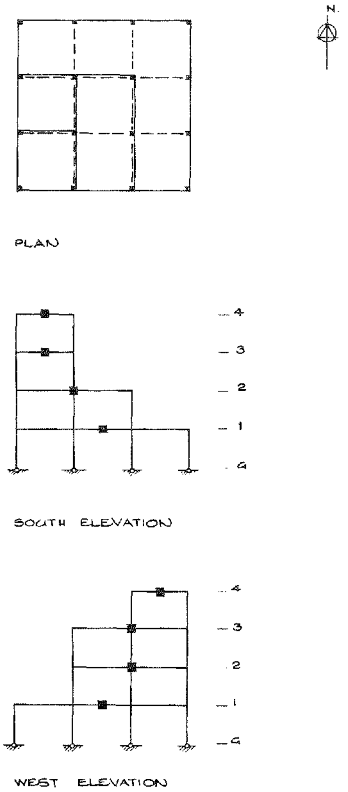

FIGURE 5: BUILDING 5 (IRREGULAR, MODERATE TO HIGH ECCENTRICITY. STATIC ANALYSIS AS LESS THAN 4 STOREYS BUT 3D MODAL ANALYSIS RECOMMENDED).

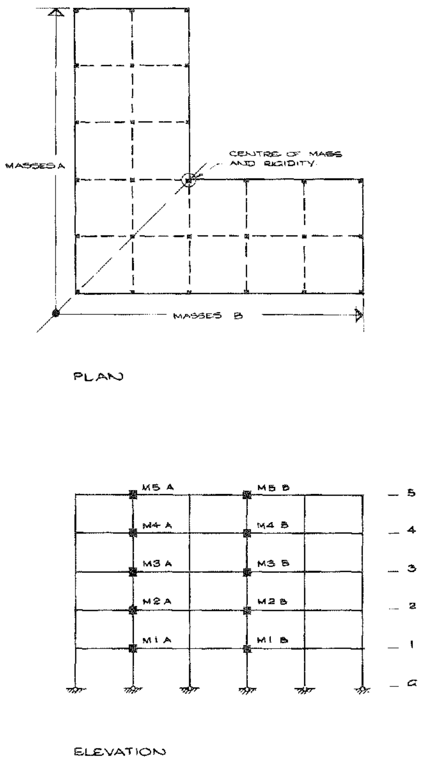

FIGURE 6: BUILDING 6 (REGULAR, LOW ECCENTRICITY)

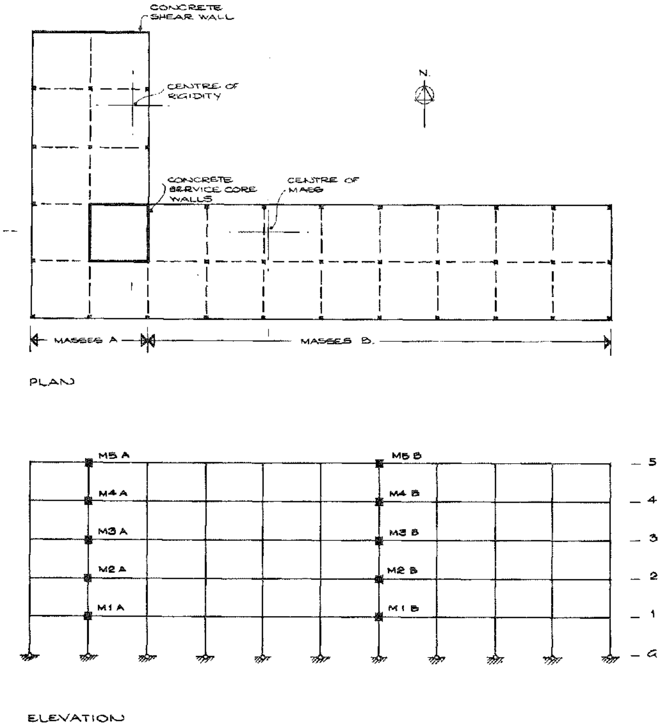

FIGURE 7: BUILDING 7 (REGULAR, MODERATE ECCENTRICITY EAST-WEST, HIGH ECCENTRICITY IN NORTH-SOUTH. 3D MODAL ANALYSIS IS RECOMMENDED)

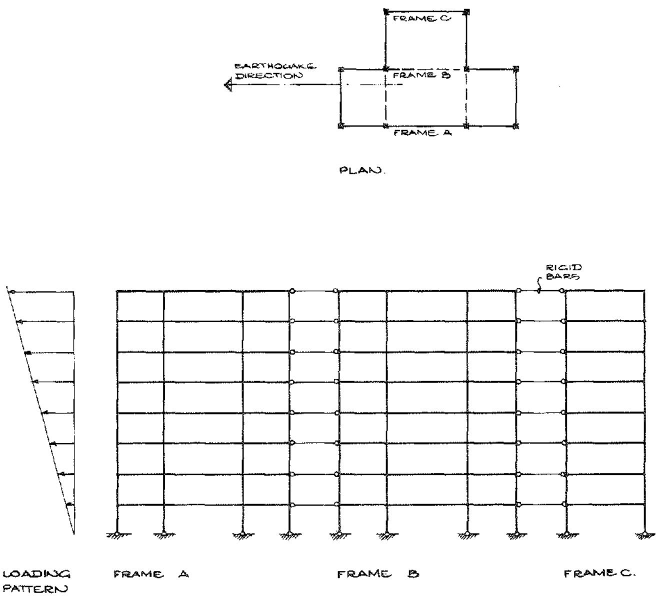

FIGURE 8: BUILDING 8