# ANALYSIS OF A CLOSED LOOP CONTROL FOR CURRENT UNBALANCE COMPENSATION BASED ON AC/AC CONVERTERS

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## *ABSTRACT*

*In this paper a stability analysis method for unbalance compensation system based on AC-Chopper is proposed. Employing Ac-Choppers and Steinmetz theory it's possible to obtain a dynamic compensation system. This system's performance and stability are not depending only on the cause of unbalance but also on grid parameters such as line impedances and other connected loads. If a quasi-static sinusoidal steady-state approximation is hypothesized, through the multivariable control theory it's possible to study the compensation system stability propriety in relation with grid parameters.*

## INTRODUCTION

In three-phase power systems the generated voltages are sinusoidal and equal in magnitude, with the individual phases 120 degree apart. However, the resulting power system at the distribution end and the point of utilization can be unbalanced for several reasons. The nature of the unbalance includes unequal voltage magnitudes at the fundamental system frequency (under-voltages and over-voltages), fundamental phase angle deviation, and unequal levels of harmonic distortion between the phases. A major cause of voltage unbalance is the uneven distribution of single-phase loads, that can be continuously changing across a three-phase power system. Example problem areas can be traction, where some technical reasons encourage the adoption of a single-phase feeding system. The single-phase railway system usually causes unbalance problems due to large unbalanced traction loads. These unbalance may cause extra losses, motor overheating and malfunction of electronic devices. The results are unacceptable if they affect the power system behaviour significantly. International standards give limits for the unbalance ratio < 2% for LV and MV systems and <1% for HV systems. A solution to mitigate the unbalances is the so-called "Steinmetz Circuit" [1] [2]that consists of a capacitor and an inductor properly rated in order to compensate the current inverse-sequence component. This solution is not able to compensate a dynamic unbalance. In [3][4] an unbalance compensation technique based on Ac/Ac regulators has been proposed. Studying the validity range of each Steinmetz Circuits it is possible to obtain a variable compensation net by employing AC-choppers [5] instead of capacitor and inductor. Considering a situation as in Figure 1, where an ohmic-inductive load cause unbalance in a three-phase grid, a control law is defined in order to rebalance the line currents system.

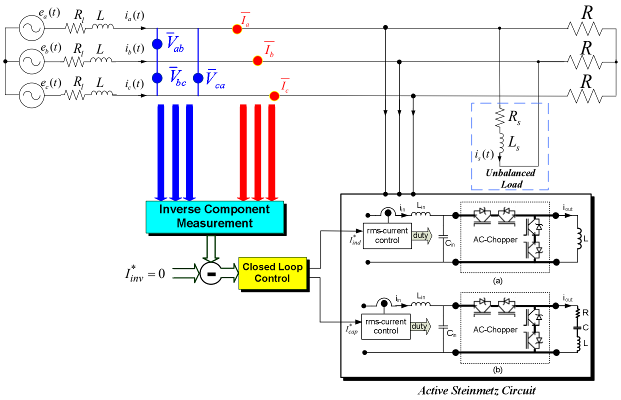


**Figure 1 Compensation Scheme**

In [4], m easuring the rms current value and the current phase of the unbalanced RL load a feed-forward action is used to compensate the symmetrical inverse component generated in the line currents. Due to uncertainties presence, the only feed-forward action is not able to completely re-balance the system current, so in addition, a closed-loop control is used. In this work an analysis of the this control is performed focusing on the system modelling and on the stability study.

## PROBLEM DESCRIPTION

As shown in Figure 1, the line impedance presence in addition to an unbalanced current system leads to unbalanced voltage supply. Complete modelling of all effects present on the grid is quite complex. Therefore, in order to a better comprehension of the results, before studying the detailed model of the system, intermediate simplified model will be considered and described in the next sections.

## SYSTEM MODELING

The aim of this section is to obtain an equivalent model scheme for system in Figure 1. The multivariable system obtained is reported in Figure 2. The system's input are the real and imaginary part of the desired inverse component phasor for $i_a(t)$, $i_b(t)$, $i_c(t)$ (normally the two reference values are zero but it's also possible to consider a different value that is acceptable for the standard, limiting the Ac-Chopper input current). Outputs are real and imaginary part of the inverse component present on the three-phase system. Each part of the scheme is discussed in the next paragraphs.

## Controller

In Figure 2, the controller is represented with matrix $\overline{\overline{C}}$ and $\overline{\overline{B_{eq}}}$. The first matrix contains the $K(s)$ controller transfer functions on the principle diagonal. In this paper a classic PI controller has been considered as reported in (1)

$$K(s) = K_p + \frac{K_i}{s} \quad \textbf{(1)}$$

Matrix $\overline{\overline{B_{eq}}}$ explicitly reported in Figure 2 is introduced in [4] and it's a linear combination of the controller output that gives references $I^*_{cap}$ and $I^*_{ind}$.

## Variable Impedance

As reported in [4], the variable impedance control is a rms-input-current control. $\overline{\overline{G_c}}$ contains on the principal diagonal the closed-loop transfer function of the variable impedance system. It's possible to consider $G(s)$ a first order transfer function as in (2)

$$G(s) = \frac{1}{1+s\tau} \quad \textbf{(2)}$$

on condition that time constant $\tau$ is relatively big in order to first order system approximation. Next paragraph will show that the $G(s)$ passband has to be small enough to guarantee the validity of the grid used model.

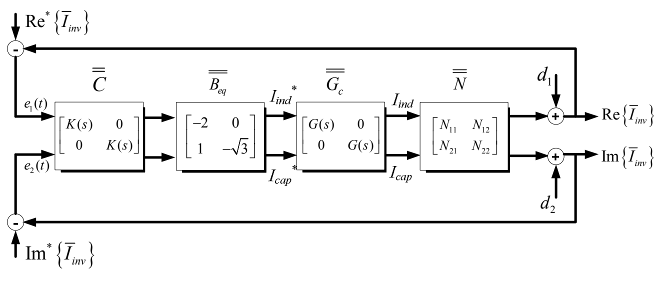


**Figure 2 – forward path**

## Grid modelling

With aim of obtaining a grid model, the equivalent circuit reported in Figure 3 is considered. The variable capacitive and inductive impedances are schematized as current generators. In this way, the grid can be seen as a system with input $I_{cap}$, $I_{ind}$ (Ac-Chopper rms input current value) and output the real and imaginary part of the inverse component $\overline{I}_{inv}$. In order to define the symmetrical component decomposition, a quasi static sinusoidal steady-state have to be hypothesized. In this way the grid is described as a static 2x2 matrix $\overline{\overline{N}}$ with 2 disturbs $d_1$ and $d_2$ acting at the output.

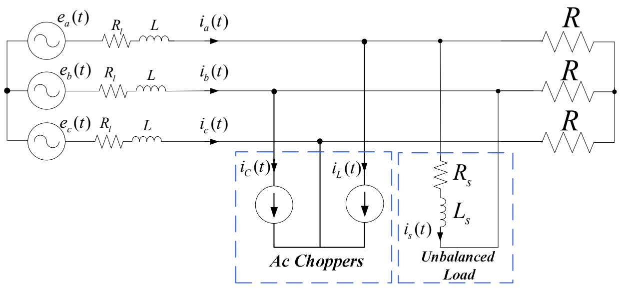


**Figure 3 – Grid model circuit**

This approximation is acceptable if $I_{cap}$, $I_{ind}$ variations are slow in such a way that the grid behaviour can be considered as a succession of sinusoidal steady-states. This is true if the Ac-Choppers rms-input current control is slow respect to the grid dynamics. Regarding the grid model, three cases will be studied. First, in case (a) line impedances will be neglected and will not be considered. In this way voltage supply is considered always balanced. In case (b) the presence of line impedance and the effect of unbalanced voltage on three-phase load R is been considered. In case (c), a further non-ideality in introduced considering the Ac-Choppers input currents not perfectly inductive and capacitive, with -90° and +90° phase shift respect to the voltage, and two phase error $\alpha$ and $\beta$ are introduced.

### Case (a)

Matrix $\overline{\overline{N^a}}$ and disturbs $d_1$ and $d_2$ are reported in (3)

$$\overline{\overline{N^a}} = \begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{\sqrt{3}}{6} & -\frac{\sqrt{3}}{3} \end{bmatrix} \quad \begin{bmatrix} d_1^a \\ d_2^a \end{bmatrix} = \begin{bmatrix} \frac{|\overline{I}_s|\left(\sqrt{3}\sin\phi + \cos\phi\right)}{2\sqrt{3}} \\ \frac{|\overline{I}_s|\left(\sqrt{3}\cos\phi - \sin\phi\right)}{2\sqrt{3}} \end{bmatrix} \quad \textbf{(3)}$$

Where $\overline{I}_s = |\overline{I}_s|\angle\phi$ is the unbalanced load current phasor.

If (4) is supposed

$$L_0(s) = K(s)G(s) \quad \textbf{(4)}$$

Forward path matrix of the system is calculated in (5)

$$\overline{\overline{L^a}} = \overline{\overline{N^a}} \cdot \overline{\overline{G_c}} \cdot \overline{\overline{B_{eq}}} \cdot \overline{\overline{C}} = \begin{bmatrix} L_0(s) & 0 \\ 0 & L_0(s) \end{bmatrix} \quad \textbf{(5)}$$

It's important to note that matrix $\overline{\overline{N^a}}$ outputs are real and imaginary parts of the inverse component injected by the Ac-Choppers compensation net. Disturbs $d_1$ and $d_2$ are real and imaginary parts of the inverse component related to the unbalancing. The aim of the control is to reject these disturbs.

All the theory about the unbalance compensation control is reported in [4]. Next cases will present a more complex formulation due to the presence of the line impedance and the $\overline{I_{cap}}$ and $\overline{I_{ind}}$ phase displacements. Anyway the physical sense of the terms remain the same.

**Case (b)**

Considering the line impedance, Matrix $\overline{\overline{N}}$ and disturbs $d_1$ and $d_2$ are function of all grid parameters.

$$\overline{\overline{N^b}}(R_l,L,R) \quad \begin{bmatrix} d_1^b(R_l,L,R_s,L_s,R) \\ d_2^b(R_l,L,R_s,L_s,R) \end{bmatrix} \tag{6}$$

Assuming (7), the forward path is reported in (8)

$$k=\frac{R}{(\omega L)^2+(R+R_l)^2} \qquad \overline{\overline{M_b}}=\begin{bmatrix} R_l+R & \omega L \\ -\omega L & R_l+R \end{bmatrix} \tag{7}$$

$$\overline{\overline{L^b}}=\overline{\overline{N^b}}\cdot\overline{\overline{G_c}}\cdot\overline{\overline{B_{eq}}}\cdot\overline{\overline{C}}=k\,L_0(s)\overline{\overline{M_b}} \tag{8}$$

It's possible to note that matrix $\overline{\overline{M_b}}$ structure is such that it's always non-singular (except in case of all zero element).

**Case (c)**

In this case line impedance and unbalanced voltage effect are considered. Moreover, input current Ac-Choppers phase shift have been introduced. Using particular $\alpha$ and $\beta$ values it's possible to model power losses in the converters and phase deviation due to any input filter.

$$\overline{\overline{N^c}}(R_l,L,R_s,L_s,R,\alpha,\beta) \quad \begin{bmatrix} d_1^c(R_l,L,R_s,L_s,R) \\ d_2^c(R_l,L,R_s,L_s,R) \end{bmatrix} \tag{9}$$

As in the previous case, it's possible to determine a matrix $\overline{\overline{M_c}}$ such that the forward path is:

$$\overline{\overline{L^c}}=\overline{\overline{N^c}}\cdot\overline{\overline{G_c}}\cdot\overline{\overline{B_{eq}}}\cdot\overline{\overline{C}}=k\,L_0(s)\overline{\overline{M_c}} \tag{10}$$

## STABILITY ANALYSIS

In this paragraph a stability analysis methodology for systems (5), (8) e (10) will be proposed. The stability is studied considering is the generalized Nyquist criteria [6]. For each case, the eigenvalues of the system matrix are calculated and the Nyquist diagram is evaluated.

**Case (a)**

Due to the diagonal structure of $\overline{\overline{L^a}}$, the eigenvalues are easily calculated and reported in (11).

$$\lambda_1^a(s)=\lambda_2^a(s)=\lambda^a(s)=L_0(s) \tag{11}$$

**Case (b)**

$$\lambda_{1,2}^b(s)=k\cdot L_0(s)\cdot\lambda_{1,2}\left(\overline{\overline{M_b}}\right)= \\ =L_0(s)\cdot k\cdot(R+R_l\pm j\omega L) \tag{12}$$

**Case c)**

$$\lambda_{1,2}^c(s)=k\cdot L_0(s)\cdot\lambda_{1,2}\left(\overline{\overline{M_c}}\right) \tag{13}$$

In case (a), the eigenvalues $\lambda_1$ and $\lambda_2$ are coincident and equal to $L_0(s)$. Therefore, the system characteristic loci's and $L_0(s)$ have the same Nyquist diagram.

In case (b) and (c) eigenvalues can be written as in (14).

$$\lambda_{1,2}^b=L_0(s)\cdot A^b e^{\pm j\phi^b} \qquad \lambda_{1,2}^c=L_0(s)\cdot A^c e^{\pm j\phi^c} \tag{14}$$

Where:

$$A^b=\frac{R}{\sqrt{(\omega L)^2+(R+R_l)^2}} \qquad \phi^b=\text{atan}\left(\frac{\omega L}{R+R_l}\right) \\ A^c=A^c(R_l,L,R_s,L_s,R,\alpha,\beta) \qquad \phi^c=\phi^c(R_l,L,R_s,L_s,R,\alpha,\beta) \tag{15}$$

Due to the complexity, $A^c$ and $\phi^c$ are not reported here.

From (14) and (15), it's possible to affirm that both in case (b) and case (c), system's characteristic loci are an amplified and rotated version of $L_0(s)$ Nyquist diagram as shown in Figure 4.

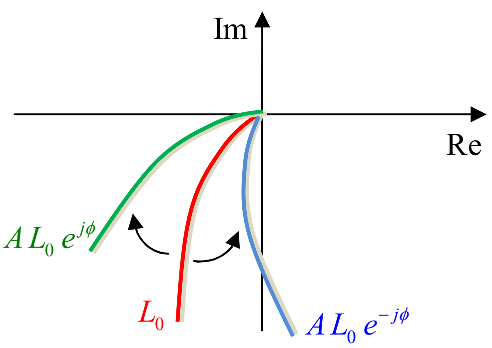


**Figure 4 - Nyquist diagram of Characteristic Loci**

In order to verify the system stability, the characteristic loci's phase margins have to be positive. It's clear that the stability is dependent on $L_0(s)$ structure and on the entity of $A$ and $\phi$. In case (b), in (15) it's shown that $A$ and $\phi$ are strongly dependent on R. Therefore it's interesting to study the stability when that parameter is varying. In case (c) the analysis is more complicated, and fixed the controller structure, line impedances and the unbalance entity, the varying parameters are R, $\alpha$ and $\beta$. Therefore an iterative algorithm is needed and it's shown in the next session.

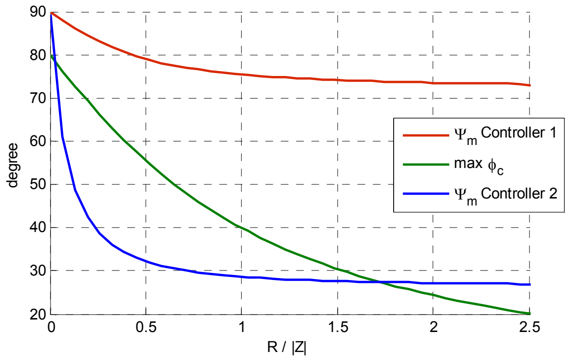


**Figure 5 - Numerical results**

**Numerical stability analysis**

In order to evaluate numerically the stability varying R, $\alpha$ and $\beta$, first a variation ranges have to be chosen. These ranges and all parameters considered in the numerical analysis are reported in table I. Since $L_0(s)$ present an infinite gain margin, it's possible to study the stability evaluating only the phase margin. The analysis steps are the following:

1. Parameters $R_l, L, R_S, L_S, K_p, K_i$ are fixed
2. A value of $R$ is chosen
3. $\phi_{\max} = \max\limits_{\alpha,\beta} \phi^c$ is evaluated
4. Phase margin $\psi_m\left[A^C L_0(s)\right]$ in calculated
5. If $\phi_{\max} < \psi_m$ then the closed-loop in stable
6. Repeating from 2, varying $R$, the numerical stability can be performed.

Numerical results are reported in Figure 5. Two controllers are considered. The first stable for all $R$ values, the other one stable only for some values.

It is possible to observe that increasing $R$, and so decreasing the power drawn by the loads connected to the three-phase grid, stability properties are improved. Vice versa, low values of $R$ are critical. In Figure 5, on the x-axis the values of $R$ is normalized on the line impedance module.

| Parameter | Value | Unit |
|---|---|---|
| Phase Voltage (RMS) | 63 | $kV$ |
| Line Frequency | 50 | $Hz$ |
| Line Resistance $R_l$ | 1.4 | $\Omega$ |
| Line Inductance $L$ | 25 | $mH$ |
| Unbalanced Load $R_s$ | 40 | $\Omega$ |
| Unbalanced Load $L_s$ | 0.26 | $H$ |
| AC/chopper control time costant $\tau$ | 1 | $s$ |
| Range for grid load R | [0 20] | $\Omega$ |
| Range for AC/Chipper phase displacement $\alpha$ and $\beta$ | [0 20] | *degree* |
| Controller 1 | Kp=0.5 Ki=1 | |
| Controller 2 | Kp=1 Ki=10 | |

**Table I**

## CONCLUSIONS

In this paper, a method for the stability analysis of an unbalance compensation control is been proposed. If a phasorial model of the grid can be considered, using the generalized Nyquist criteria and an iterative numerical analysis, it's possible to study the compensation control stability for a chosen variation range of parameters. This iterative analysis strategy is a trial-and-error criteria for the controller design.

## REFERENCES

[1] M. ChindriS, A. Cziker, S. Stefanescu, 2001 "Symmetrizing Steinmetz Circuitry", *CIRED 2001 Amsterdam (Netherlands) , June 2001.*

[2] L. Sainz, J. Pedra, M. Caro,"Steinmetz circuit influence on the electric system harmonic response", *Power Delivery*, IEEE Transactions on Volume 20, Issue 2, April 2005 Page(s): 1143 – 1150.

[3] B. Cougo, T. Meynard, "A Solution for Three-Phase System Unbalance Based on PWM AC Choppers", *IECON 2006* ,Paris (France), November 2006.

[4] P. Ladoux, P. Marino, G. Torre, 2008, "Power quality improvement by means of AC/AC choppers", *SPEEDAM 2008* , Ischia, On page(s): 77-8.

[5] L. Lowinsky, P. Ladoux, Y. Chéron, S. Alvarez: *3 MVAR Single Phase STATCOM based on AC Chopper Topology.* PCIM 2008 – Nurnberg (Germany), May 2008.

[6] J.M. Maciejowski, 1989, *Multivariable Feedback Design,* Wokingham, England.