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Analysis of a Retrial Queue With Two-Type Breakdowns and Delayed Repairs

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ABSTRACT This article studies an M/G/1 retrial queue with two types of breakdowns. When the server is idle, it is subject to breakdowns according to a Poisson process with rate δ and it cannot be repaired immediately. While when the server is busy, it may break down according to a Poisson process with rate θ and can be immediately repaired. Firstly, based on embedded Markov chain technique and probability generating function (PGF) method, we present the necessary and sufficient condition for the system to be stable and the PGF of the orbit size at the departure epochs. Secondly, we give the steady-state joint queue length distribution by supplementary variable method, and present some important performance measures and reliability indices. Thirdly, we provide the analysis of sojourn time of an arbitrary customer in the system when the system is in stable state. Finally, some numerical examples are presented to illustrate the effect of the some system parameters on important performance measures and reliability indices.

INDEX TERMS Retrial queue, performance measures, reliability indices, passive breakdowns, active breakdowns.

I. INTRODUCTION

Retrial queues with unreliable servers have been investigated extensively, due to their applications in various fields, such as telephone switching systems, call centers, computer communication and telecommunication networks, manufacturing systems etc. On one hand, retrial queues can reflect the characteristics of customer service requirements, i.e., arriving customers who find the server unavailable may join into a retrial group (orbit) and ask for their services again some time later. For the survey papers, the books, the bibliographical information and recent literatures on retrial queues, readers are referred to Falin [10], Falin and Templeton [11], Artalejo and Gómez-Corral [4], Gómez-Corral [17], Artalejo [2], [3], Gao and Zhang [15], Zhang *et al.* [31] and references therein. On the other hand, due to some unexpected factors in reality, such as limited lifetime of the server, external interference, malfunctions of the server, starting failures, etc., the servers may break down and need repair during idle period or busy period. Severs' failures and repairs were introduced by Aissani [1] and Kulkarni and Choi [21]. Since then, related studies regarding retrial queues with unreliable servers and repairs have been carried out successively from queuing and reliability viewpoints. In earlier relevant papers, the types of breakdowns of the servers may be divided into as follows:

(1) **active breakdowns**, i.e., the server is subject to breakdowns when it is busy. In this case, the server's life time is often assumed to be exponential distributed. Wang [23] studied both queueing characteristics and reliability issues for an M/G/1 retrial queue with server breakdowns and general retrial times. Falin [12] dealt with an unreliable M/G/1 retrial queue, in which the server's lifetime follows exponential distribution and the repair time is generally distributed. Different from classical retrial queues with only one orbit queue, the retrial queue in Falin [12] has two waiting queues, one is normal waiting queue which is formed by the arriving primary customers who find the server unavailable at their arrival epochs, the other is orbit queue which is formed by those customers whose services are interrupted by the failures of the server. Chang *et al.* [6] considered a multi-server retrial

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queue with customer feedback and impatient, in which the server's breakdown is incurred by exponentially distributed lifetime when it is working. Yang *et al.* [29] considered an unreliable retrial queue with J optional vacations, where the server is subject to random breakdowns and repairs when he is working. Gao *et al.* [13] treated an M/M/1 retrial queue with an unreliable server from the economic viewpoint.

(2) **passive breakdowns**, i.e., when the server is idle, the server may break down and needs immediately repair. Taleb and Aissani [22] considered the performance measures and reliability indices for a new unreliable M/G/1 retrial queue, in which persistent and impatient customers, active and passive failures and preventive maintenances are both taken into account. Performance analysis was considered by Krishna Kumar *et al.* [19] for a Markovian retrial queue with passive and active breakdowns.

(3) **catastrophic failures**, i.e., the sever breakdowns are caused by external attacks or shocks (called as negative customers). In such retrial queues, if a negative arrives at a system, it removes one or all present customers in the system at once (called as individual or complete removal) and makes the server breakdown and repair. Many studies on such retrial queues have been carried out from queuing and reliability and economic viewpoints. Interested readers are referred to Wang *et al.* [24], Wang and Zhang [25], Wu and Lian [26], Wu and Yin [27], Gao and Wang [14] and references therein.

(4) **starting failures**, i.e., when the server is idle, an arriving (new or returning) customer must start the server. If the server is successfully started with a certain probability, the customer receives service immediately. Otherwise, the server undergoes repair immediately. Yang and Li [28] presented an M/G/1 retrial queue with the sever subject to starting failures. Krishna Kumar *et al.* [18] addressed the performance analysis of an M/G/1 retrial queue with feed back and starting failures. Atencia *et al.* [5] developed a discrete-time Geo/G/1 retrial queue with general retrial times, Bernoulli feedback and starting failures. Recently, Yang *et al.* [30] generalized the model of Krishna Kumar *et al.* [18] to a multi-server retrial system with feed back and starting failures. For more retrial queues with breakdowns and repairs, the readers are referred to the recent survey paper by Krishnamoorthy *et al.* [20].

Most unreliable retrial queues assume that the server can be immediately repaired when it breaks down. For example, Zhang [32] studied an M/M/1 retrial queue with passive breakdowns and active breakdowns from economic point, in which whenever any type of breakdowns occurs, the sever immediately enters a repair stage and the repair times for these two types of breakdowns are identical and exponential distribution. Zirem *et al.* [33] dealt with a batch arrivals retrial queue with active breakdowns, where the sever can be immediately repaired when breakdown happens and reserved service schedule is considered for the interrupted customer. However, in many realistic situations, such as in the area of computer communication networks and flexible manufacturing systems, etc, it may not be possible to start the repair process immediately due to non-availability of the repairman or of the apparatus needed for the repairs or due to being undetected timely. Recently, Choudhury and Tadj [9] studied the steady-state behavior of an unreliable retrial queue with a second optional service phase and delayed repair. Choudhury and Ke [7], [8], respectively, studied a batch arrival and a single arrival unreliable retrial queue with general retrial times under Bernoulli vacation schedule, in which the server is subject to active breakdown and delaying repair, i.e., when the server's failure occurs, it can begin its repair after some delaying time. For such retrial queues, the authors obtained some important performance measures and reliability indices.

In this article, we analyze an M/G/1 retrial queue with passive and active breakdowns and delayed repairs for passive breakdowns. To the best of the authors' knowledge, studies for such retrial queue do not yet exist. The motivation of this work is that such retrial queue arises in various practical fields, such as in communication networks and manufacturing systems, it not only characterizes the retrial phenomenon of customers, but takes the delayed repairs for passive breakdowns into consideration. Moreover, another motivation for considering such retrial model is to obtain analytical solution in term of closed form expression by supplementary variables technique and evaluate the performance measures and the reliability of the considered system which may be suited to many communication networks. The basic findings of the paper and their significance are outlined as follows:

- We introduce a new repairable M/G/1 retrial queue with passive and active breakdowns, in which passive active breakdowns are subject to delayed repair. Such model has potential applications in packet-switching networks.
- We give the stable condition of the system, stationary analysis of joint distribution of the orbit size and the server's state. Based on these analysis, we can give the expressions of important performance measures of the system.
- Sojourn time of an arbitrary customer can reflect the quality of service of the system, so we present the expression of Laplace transform of the sojourn time of an arbitrary customer, and prove that Little's law still hold in our model.
- Reliability indices including the steady state availability of the server, the failure frequency of the server and the mean time to first failure of the server are provided.

The rest of this article is organized as follows. Section 2 gives the system description and a practical example. Section 3 presents the stable condition of the system and the steady-state analysis, and gives some system measures. Section 4 focuses on the reliability indexes of the system. Section 5 studies the distribution of the sojourn time in the system of any customer. Section 6 gives some numerical examples to illustrate the features of our model.

II. MODEL FORMULATION AND A PRACTICAL EXAMPLE

A. MODEL DESCRIPTION

In this section, we consider an unreliable retrial queue with two types of breakdowns and delayed repairs due to passive breakdowns. Assumptions of the queueing system are as follows.

- Arriving process and general service times. Customers from outside arrive at the system according to a Poisson processes with rate λ. The service time B of each customer follows an arbitrary distribution with cumulative distribution function (c.d.f.) $B(x)$, probability density function (p.d.f.) $b(x)$, finite first two moments β_1, β_2. If an arriving customer finds the server idle, the customer obtains service immediately. Otherwise the arriving customer who finds the server busy or inoperative because of failures will produce a source of unsatisfied customers, who may retry several times for service. Such unsatisfied customers are said to be in "orbit" and form a queue according to FCFS discipline.
- Two types of breakdowns and delayed repairs. The sever is subject to passive and active breakdowns, respectively, in idle period and busy period. When the server is idle, it breaks down at an exponential rate δ (called as a passive breakdown). When the server is busy serving a customer, it breaks down at an exponential rate θ (called as an active breakdown). When an active breakdown occurs, the server can be immediately repaired and the repair time R follows general distribution with c.d.f. $R(x)$, p.d.f. $r(x)$, finite first two moments ν_1, ν_2. However, due to lack of monitoring of the server in the idle period, when a passive failure happens, the server can not obtain immediate repair and stays there until a customer arrives at the service station from outside or the orbit if any. The repair time G for a passive failure follows general distribution with c.d.f. $G(x)$, p.d.f. $g(x)$, finite first two moments μ_1, μ_2. It is assumed that, when the service of a customer is interrupted by an active breakdown, the customer in service waits there to accept its remaining service as soon as the repair is completed. While the customer who starts the repair for a passive failure doesn't leave the service facility and can immediately obtain its service after the completion of the repair.
- Constant retrial policy. Under such retrial policy, only the first customer in the orbit is permitted to apply for service when the server becomes idle and the retrial time follows exponential distribution with rate α.
- All random variables defined above are assumed to be mutually independent.

Throughout the rest of the paper, for a c.d.f. $F(x)$, we denote $\overline{F}(x) = 1 - F(x)$ as the tail of $F(x)$, $\widetilde{F}(s) = \int_0^\infty e^{-sx}dF(x)$ as the Laplace-Stieltjes transform (LST) of $F(x)$ and $\overline{F}^*(s) = \int_0^\infty e^{-sx}\overline{F}(x)dx$ as the Laplace transform of function $\overline{F}(x)$. Obviously, we can obtain that $\overline{F}^*(s) = \frac{1-\widetilde{F}(s)}{s}$.

Define the functions $\beta(x)$, $\mu(x)$ and $\nu(x)$ as the conditional completion rates for service time, for repair time for an active breakdown and repair time for a passive failure, respectively, i.e.,

$$\beta(x) = \frac{b(x)}{\overline{B}(x)}, \quad \mu(x) = \frac{g(x)}{\overline{G}(x)}, \quad \nu(x) = \frac{r(x)}{\overline{R}(x)}.$$

B. A PRACTICAL APPLICATION EXAMPLE

Besides its theoretical interest, our retrial queue has potential applications in a packet-switching network, in which messages are divided into IP packets before they are sent. For instance, most modern Wide Area Network (WAN) protocols, including TCP/IP, X.25, and Frame Relay, are based on packet-switching technologies. The router is an interconnected device over which a packet is transmitted from a source host to to a destination host in a packet switching network. If the source host wishes to send a package to a destination host, it first sends the package to the router to which it is connected, and then the package is transmitted to the destination host. Assume packages arrive at the source host from outside according to a Poisson process. Upon receiving a package, the host immediately sends it to its router. If the router is available, the package is accepted and is transmitted immediately and the transmission time is assumed to be generally distributed. Otherwise the package is blocked by the router due to limitations in the TCP/IP network path MTU (Maximum Transmission Unit) or active breakdowns, in this case, the blocked package is stored in the buffer of the source host (called as orbit) and has to be retransmitted some time later according to FCFS. Besides, due to external attacks or other technical faults, the router may break down during idle period or during the packet transmission period. We assume that the network administrator who is responsible for failure management of the network always does some secondary auxiliary jobs when the router is idle until a packet arrives at the router and always is on duty when the router is busy. If the router fails when it is transmitting a packet, it can be immediately repaired by the network administrator and resumes the transmission of the interrupted packet as soon as its repair process is completed. While if the router breaks down when it is idle, the repair may be delayed till the arrival epoch of the next packet from outsider or the orbit at which the network administrator returns and immediately begins the repair process of the router. The time interval from the epoch at that the passive failure occurs to the epoch at which the next packet arrives is called as delayed period. Here the packet who arrives during the delayed period can be transmitted immediately after the completion of the repair for the passive failure. This scenario can be modelled as our retrial queueing system with two-type failures and delayed repairs.

III. STABILITY CONDITION AND STEADY-STATE ANALYSIS

This section focuses on investigating the stability condition of the system and deriving some steady state distributions of the system, respectively, by embedded Markov chain technique and supplementary variable method.

A. STABILITY CONDITION

Let S_B be the generalized service time interval of a customer from the beginning of his service to the end of his service, with c.d.f $S_B(x)$, LST $\widetilde{S}_B(s)$. Taking into account the possible occurrence of active breakdowns in the service process, we have that $\widetilde{S}_B(s) = \widetilde{B}(s + \theta(1 - \widetilde{R}(s)))$, which leads to $E[S_B] = \beta_1(1+\theta\nu_1) \triangleq \beta_1^*, E\left[S_B^2\right] = \beta_2(1+\theta\nu_1)^2 + \theta\beta_1\nu_2 \triangleq \beta_2^*$.

In the following, we give some useful notations:

$$a_k = \int_0^\infty \frac{(\lambda t)^k}{k!} e^{-\lambda t} dG(t), \quad k = 0, 1, 2, \cdots,$$

$$h_k = \int_0^\infty \frac{(\lambda t)^k}{k!} e^{-\lambda t} dS_B(t), \quad k = 0, 1, 2, \cdots,$$

where a_k is the probability that there are k customers who enter into the orbit during the repair time for a passive failure, h_k is the probability that k customers who join the orbit during the generalized service time.

Let $A(z) = \sum_{k=0}^\infty z^k a^k, H(z) = \sum_{k=0}^\infty z^k h^k$, and $\Lambda(z) = \lambda(1-z)$, then

$$A(z) = \widetilde{G}(\Lambda(z)),$$

$$A'(1) = \left.\frac{dA(z)}{dz}\right|_{z=1} = \lambda\mu_1 \triangleq \rho_1,$$

$$A''(1) = \left.\frac{d^2A(z)}{dz^2}\right|_{z=1} = \lambda^2\mu_2,$$

$$H(z) = \sum_{k=0}^\infty z^k h^k = \widetilde{B}\left(\lambda(1-z) + \theta(1 - \widetilde{R}(\lambda(1-z)))\right),$$

$$H'(1) = \left.\frac{dH(z)}{dz}\right|_{z=1} = \lambda\beta_1^* \triangleq \rho,$$

$$H''(1) = \left.\frac{d^2H(z)}{dz^2}\right|_{z=1} = \lambda^2\beta_2^*.$$

Define $c_k = \sum_{i=0}^k a_i h_{k-i}, k \geq 0$, then c_k is the probability that k customers enter into the orbit during the passive repair time and generalized service time. Then $C(z) = \sum_{k=0}^\infty z^k c_k = A(z)H(z)$.

To develop the necessary and sufficient condition for the system to be stable. we first establish the embedded Markov chain of the system at departure epochs.

Let T_k ($T_0 = 0$) be the time epoch at which the k-th customer leaves the system, $N_k = N(T_k)$ be the orbit size at the time of the kth departure, then the process $\{N_k, k \geq 0\}$ is a Markov chain with state space $\mathbb{N}$. Then we have the following theorem.

Theorem 3.1: (1) The Markov chain $\{N_k, k \geq 0\}$ is ergodic if and only if $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 < \frac{\alpha}{\lambda+\alpha}$.

(2) Under the condition $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 < \frac{\alpha}{\lambda+\alpha}$, let $\pi_n = \lim_{k\to\infty} P(N_k = n), n \geq 0$, be the stationary probabilities of the Markov chain $\{N_k, k \geq 0\}$, then the PGF $\Pi(z) = \sum_{n=0}^\infty z^n \pi_n$ is given as follows:

$$\Pi(z) = \frac{\alpha}{\lambda+\delta}\pi_0 \times \frac{\delta C(z)[(\lambda+\delta)(z-1)+(\lambda+\alpha)z]-(\lambda+\alpha)(\delta+\Lambda(z))H(z)}{(\lambda+\alpha)\left[(\delta+\lambda+\alpha)z-(\alpha+\lambda z)H(z)\right]-\delta(\alpha+\lambda z)C(z)} \tag{1}$$

where

$$\pi_0 = \frac{(\lambda+\delta)[(\lambda+\alpha)(\lambda+\alpha+\delta)]}{\alpha\big((\lambda+\delta)(\lambda+\alpha+\delta)+\delta(\lambda+\alpha)\rho_1\big)} \times \left(\frac{\alpha}{\lambda+\alpha} - \rho - \frac{\delta}{\lambda+\alpha+\delta}\rho_1\right).$$

Proof: (1) From the assumptions of our model, the one-step transition probabilities are given as follows:

$$q_{m,n} = P(N_{k+1} = n | N_k = m) = \begin{cases} \frac{\lambda}{\lambda+\delta}h_n + \frac{\delta}{\lambda+\delta}c_n, & m = 0, n \geq 0, \\ \frac{\alpha}{\lambda+\delta+\alpha}h_0 + \frac{\delta}{\lambda+\delta+\alpha}\cdot\frac{\alpha}{\lambda+\alpha}c_0, & m > 0, n = m-1, \\ \frac{\lambda}{\lambda+\delta+\alpha}h_{n-m} + \frac{\alpha}{\lambda+\delta+\alpha}h_{n-m+1} + \frac{\delta}{\lambda+\delta+\alpha}\left(\frac{\lambda}{\lambda+\alpha}c_{n-m} + \frac{\alpha}{\lambda+\alpha}c_{n-m+1}\right), & m > 0, n > m-1, \\ 0, & \text{otherwise}, \end{cases}$$

Obviously, the Markov chain $\{N_k, k \geq 0\}$ is irreducible and aperiodic. And the mean drift

$$x_m = E[N_{k+1} - N_k | N_k = m] = \begin{cases} \rho + \frac{\delta}{\lambda+\delta}\rho_1, & m = 0, \\ \rho + \frac{\delta}{\lambda+\delta+\alpha}\rho_1 - \frac{\alpha}{\lambda+\alpha}, & m > 0. \end{cases}$$

Then from Foster's criterion (see Gómez-Corral [16]), we know that the inequality $\rho + \frac{\delta}{\lambda+\delta+\alpha}\rho_1 < \frac{\alpha}{\lambda+\alpha}$ is a sufficient condition for the system to be stable.

The same inequality is also the necessary condition for ergodicity. Assume that $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 \geq \frac{\alpha}{\lambda+\alpha}$, which implies that $x_m \geq 0$ for all $m \geq 0$. Furthermore, according to the one-step transition probabilities, we know that the down drift

$$D_m = \sum_{n<m}(n-m)P(N_{k+1} = n | N_k = m) = \begin{cases} 0, & m = 0, \\ -\left(\frac{\alpha}{\lambda+\delta+\alpha}h_0 + \frac{\delta}{\lambda+\delta+\alpha}\cdot\frac{\alpha}{\lambda+\alpha}c_0\right), & m > 0, \end{cases}$$

which implies that the Markov chain $\{N_k, k \geq 0\}$ satisfies Kaplan's condition namely if the sequence $\{D_m, m \geq 0\}$ is bounded below. Thus the Markov chain $\{N_k, k \geq 0\}$ is not ergodic, and then the necessity of the ergodicity is proven.

(2) Denote τ_n and ω_n as follows

$$\begin{aligned}\tau_n &= \frac{\lambda}{\lambda+\delta}h_n + \frac{\delta}{\lambda+\delta}c_n, n \geq 0,\\ \omega_n &= \frac{\lambda}{\lambda+\delta+\alpha}h_{n-1} + \frac{\alpha}{\lambda+\delta+\alpha}h_n\\ &\quad + \frac{\delta}{\lambda+\delta+\alpha}\left(\frac{\lambda}{\lambda+\alpha}c_{n-1} + \frac{\alpha}{\lambda+\alpha}c_n\right), \quad n \geq 0,\end{aligned}$$

where $h_{-1} = c_{-1} = 0$. From the expression of the one-step probabilities, we can get the one-step probability Matrix $Q = (q_{i,j})$ of the Markov chain $\{N_k, k \geq 0\}$ as follows

$$\mathbf{Q} = \begin{bmatrix} \tau_0 & \tau_1 & \tau_2 & \tau_3 & \cdots \\ \omega_0 & \omega_1 & \omega_2 & \omega_3 & \cdots \\ & \omega_0 & \omega_1 & \omega_2 & \cdots \\ & & \ddots & \ddots & \ddots \end{bmatrix}$$

By the matrix equation $(\pi_0\pi_1\pi_2\cdots) = (\pi_0\pi_1\pi_2\cdots)\mathbf{Q}$, and normalization condition $\sum_{n=0}^{\infty}\pi_n = 1$, we can obtain the expression of $\Pi(z)$ as given by (1). □

Remark 1 (Special Case): Suppose that no passive breakdown occurs in the retrial system, i.e., $\delta = 0$, then our system is reduced to the M/G/1 retrial queue with active breakdowns and constant retrial times, which is a special case by taking $A(x) = 1 - e^{-\alpha x}, x > 0$ in Wang [23].

B. STEADY STATE ANALYSIS

In this subsection, we study the steady state distribution of the system by using supplementary variable method.

At time t, the state of the system can be described by the Markov process $\{(N(t), J(t), \xi_1(t), \xi_2(t), \xi_4(t)), t \geq 0\}$, where $N(t)$ is the number of customers in the orbit, $J(t)$ denotes the state of the server defined as:

$$J(t) = \begin{cases} 0, & \text{the server is idle}\\ 1, & \text{the server is busy}\\ 2, & \text{the server is under repair for an active breakdown}\\ 3, & \text{the server is during the delayed period}\\ 4, & \text{the server is under repair for a passive breakdown}\end{cases}$$

when $J(t) = 1$, $\xi_1(t)$ is the elapsed service time; when $J(t) = 2$, $\xi_2(t)$ is the elapsed repair time for an active breakdown; when $J(t) = 4$, $\xi_4(t)$ denotes the elapsed repair time for a passive failure.

Since the arrival stream is a Poisson process, from Burke's theorem, we can see that the steady state probabilities of the Markov process $X(t) = \{(N(t), J(t), \xi_0(t), \xi_1(t), \xi_2(t), \xi_4(t))\}$ exist and are positive if and only if $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 < \frac{\alpha}{\lambda+\alpha}$.

Therein after, we assume that the stationary condition $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 < \frac{\alpha}{\lambda+\alpha}$ always holds. Let $\{(N, J, \xi_0, \xi_1, \xi_2)\}$ be the limit of the Markov process $\{(N(t), J(t), \xi_0(t), \xi_1(t), \xi_2(t)), t \geq 0\}$.

Define the following joint steady-state probabilities and steady-state probability densities:

$$\begin{aligned} P_{n,j} &= P(N = n, J = j)\\ &= \lim_{t\to\infty} P_{n,j}(t), n \geq 0, \quad j = 0, 3,\\ P_{n,j}(x)dx &= P(N = n, J = j, x < \xi_j \leq x + dx)\\ &= \lim_{t\to\infty} P_{n,j}(t, x)dx, \quad n \geq 0,\ j = 1, 4, x \geq 0,\\ &\quad P_{n,2}(x, y)dxdy\\ &= P(N = n, J = 2, x < \xi_1 \leq x + dx,\\ &\quad y < \xi_4 \leq y + dy)\\ &= \lim_{t\to\infty} P_{n,j}(t, x, y)dxdy, \quad n \geq 0,\ x, y \geq 0.\end{aligned}$$

Based on the method of supplementary variable technique, we formulate the basic equations describing steady state as follows:

$$(\lambda+\delta)P_{0,0} = \int_0^{\infty} P_{0,1}(x)\beta(x)dx, \tag{2}$$

$$(\lambda+\delta+\alpha)P_{n,0} = \int_0^{\infty} P_{n,1}(x)\beta(x)dx, \quad n \geq 0, \tag{3}$$

$$\begin{aligned}\frac{d}{dx}P_{n,1}(x) &= -\left(\lambda+\theta+\beta(x)\right)P_{n,1}(x) + \lambda P_{n-1,1}(x)\\ &\quad + \int_0^{\infty} P_{n,2}(x, y)\nu(y)dy, \quad n\geq 0,\ x>0,\end{aligned} \tag{4}$$

$$\begin{aligned}&\frac{\partial}{\partial y}P_{n,2}(x, y)\\ &= -(\lambda+\nu(y))P_{n,2}(x, y) + \lambda P_{n-1,2}(x, y),\\ &\quad n \geq 0,\ x, y > 0,\end{aligned} \tag{5}$$

$$\lambda P_{0,3} = \delta P_{0,0}, \tag{6}$$

$$(\lambda+\alpha)P_{n,3} = \delta P_{n,0}, n \geq 1, \tag{7}$$

$$\begin{aligned}&\frac{d}{dx}P_{n,4}(x)\\ &= -(\lambda+\mu(x))P_{n,4}(x) + \lambda P_{n-1,4}(x),\\ &\quad n \geq 0,\ x > 0,\end{aligned} \tag{8}$$

where $P_{-1,1}(x) = P_{-1,2}(x, y) = P_{-1,4}(x) = 0$.

Equations (2)-(8) are to be solved under the boundary conditions:

$$P_{n,1}(0) = \lambda P_{n,0} + \alpha P_{n+1,0} + \int_0^{\infty} P_{n,4}(x)\mu(x)dx, n \geq 0, \tag{9}$$

$$P_{n,2}(x, 0) = \theta P_{n,1}(x), n \geq 0, \quad x \geq 0, \tag{10}$$

$$P_{n,4}(0) = \lambda P_{n,3} + \alpha P_{n+1,3}, \quad n \geq 0, \tag{11}$$

and the normalization condition is

$$\begin{aligned}&\sum_{n=0}^{\infty}(P_{n,0} + P_{n,3}) + \sum_{n=0}^{\infty}\int_0^{\infty}\Big(P_{n,1}(x) + P_{n,4}(x)\Big)dx\\ &\quad + \sum_{n=0}^{\infty}\int_0^{\infty}\int_0^{\infty} P_{n,2}(x, y)dxdy = 1.\end{aligned} \tag{12}$$

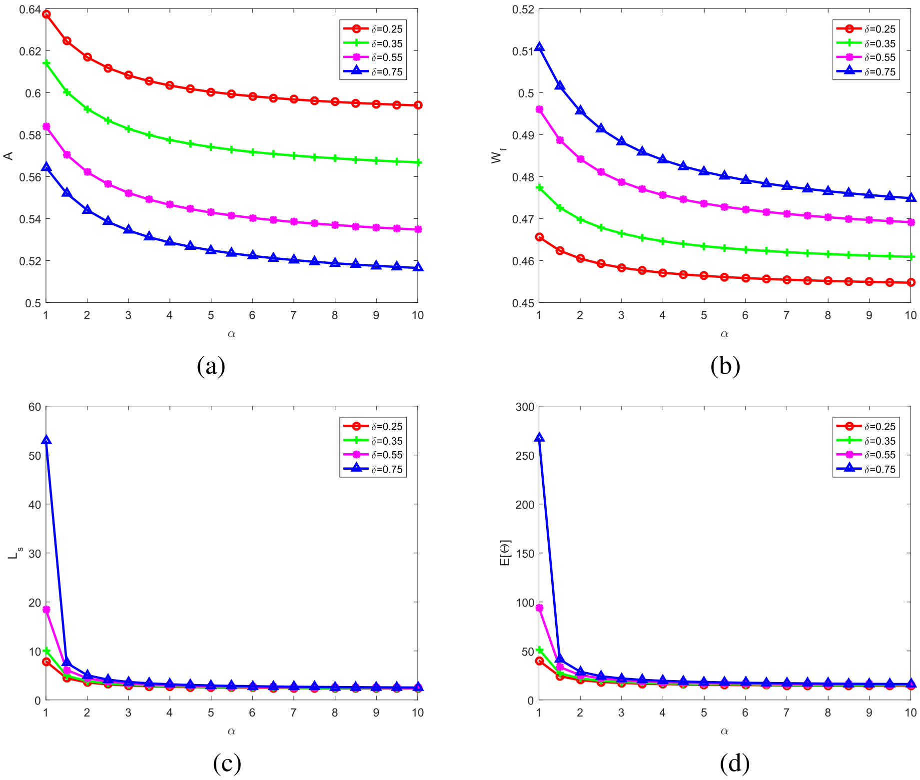

FIGURE 1. **The effect of α on $A, W_f, L_s, E[\Theta]$ for different values of $\delta = 0.25, 0.35, 0.55, 0.75$.**

Define the following generating functions

$$P_j(z) = \sum_{n=0}^{\infty} z^n P_{k,j}, \quad j = 0, 3,$$

$$P_j(x; z) = \sum_{n=0}^{\infty} z^n P_{n,j}(x), \quad j = 1, 4,$$

$$P_2(x, y; z) = \sum_{n=0}^{\infty} z^n P_{n,2}(x, y).$$

From Eqs.(2)-(12) and by some algebraic manipulations, we can have the following Theorem.

Theorem 3.2: The generating functions of the stationary joint distribution of the orbit size and the server state are given by:

$$\begin{aligned}
P_0(z) &= \frac{\alpha(\lambda+\alpha)\,(z-H(z)) - \delta C(z)(1-z)}{\mathcal{D}(z)} P_{0,0},\\
P_1(x,z) &= \frac{\mathcal{N}_1(z)}{\mathcal{D}(z)}(1-B(x))\alpha P_{0,0}\\
&\quad \times \exp\left\{-\left(\Lambda(z)+\theta(1-\widetilde{R}(\Lambda(z)))\right)x\right\},\\
P_2(x,y,z) &= \theta P_1(x,z)(1-R(y))\exp\{-\Lambda(z)y\},\\
P_3(z) &= \frac{\delta P_{0,0}}{\lambda+\alpha}\\
&\quad \times \left[\frac{\alpha}{\lambda} + \frac{\alpha(\lambda+\alpha)\,(z-H(z)) - \delta\alpha C(z)(1-z)}{\mathcal{D}(z)}\right],\\
P_4(x;z) &= \frac{\mathcal{N}_2(z)}{\mathcal{D}(z)}(1-G(x))\exp\{-\Lambda(z)x\}\delta\alpha P_{0,0},
\end{aligned}$$

where

$$\begin{aligned}
P_{0,0} &= \frac{\lambda}{\alpha}\frac{(\lambda+\alpha)(\lambda+\alpha+\delta)}{(\lambda+\delta)(\lambda+\alpha+\delta)+\delta(\lambda+\alpha)\rho_1}\\
&\quad \times \left(\frac{\alpha}{\lambda+\alpha} - \rho - \frac{\delta}{\lambda+\alpha+\delta}\rho_1\right),\\
\mathcal{D}(z) &= (\lambda+\alpha)\big((\lambda+\alpha+\delta)z - (\alpha+\lambda z)H(z)\big)\\
&\quad -\delta(\alpha+\lambda z)C(z),\\
\mathcal{N}_1(z) &= \delta A(z)\big(\alpha+\lambda z+(\lambda+\alpha+\delta)(z-1)\big)\\
&\quad -(\alpha+\lambda)(\delta+\Lambda(z)),\\
\mathcal{N}_2(z) &= (\alpha+\lambda z)(1-H(z)) + (\lambda+\alpha+\delta)(z-1).
\end{aligned}$$

Next our interest is to give marginal orbit size distribution due to the server state being in states $J = 1, 2, 4$, respectively. Let $P_1(z) = \int_0^\infty P_1(x;z)dx, P_2(z) = \int_0^\infty \int_0^\infty P_2(x,y;z)dxdy, P_4(z) = \int_0^\infty P_4(x;z)dx$, then we have the following results.

Theorem 3.3: (1) The marginal PGF of the orbit size when the server is busy is give by

$$P_1(z) = \frac{\mathcal{N}_1(z)}{\mathcal{D}(z)}\frac{1-H(z)}{\Lambda(z)+\theta(1-\widetilde{R}(\Lambda(z)))}\alpha P_{0,0}.$$

(2) The marginal PGF of the orbit size when the server is under repair for an active breakdown is given by

$$P_2(z) = \frac{\mathcal{N}_1(z)}{\mathcal{D}(z)}\frac{1-H(z)}{\Lambda(z)+\theta(1-\widetilde{R}(\Lambda(z)))}\frac{1-\widetilde{R}(\Lambda(z))}{\Lambda(z)}\theta\alpha P_{0,0}.$$

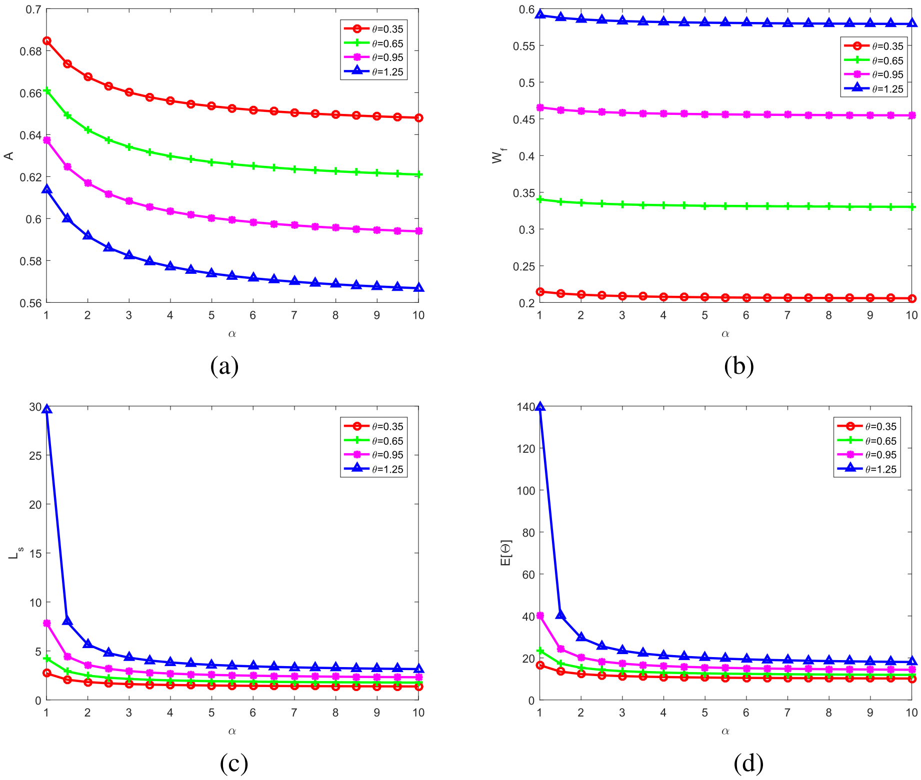

FIGURE 2. The effect of α on $A, W_f, L_S, E[\Theta]$ for different values of $\theta = 0.35, 0.65, 0.95, 1.25$.

(3) The marginal PGF of the orbit size when the server is under repair for a passive breakdown is given by

$$P_4(z) = \frac{\mathcal{N}_2(z)}{\mathcal{D}(z)}\frac{1 - A(z)}{\Lambda(z)}\delta\alpha P_{0,0}.$$

Let $\Phi(z) = E[z^N]$, which denotes the PGF of the number of customers in the orbit, let N_S be the number of customers in the system at arbitrary time under stability condition, with PGF $\Psi(z) = E[z^{N_S}]$. Then by $\Phi(z) = \sum_{j=0}^{4} P_j(z)$ and $\Psi(z) = P_0(z) + zP_1(z) + zP_2(z) + P_3(z) + zP_4(z)$, we can obtain the following Corollary.

Corollary 3.1: The PGFs $\Phi(z)$ and $\Psi(z)$, respectively, of N and N_S, are given by as follows

$$\Phi(z) = \frac{\mathcal{N}(z)}{\mathcal{D}(z)}\frac{\alpha}{\lambda}P_{0,0},$$

$$\Psi(z) = \frac{\begin{pmatrix}\delta C(z)\big((\lambda+\delta)(z-1)+(\lambda+\alpha)z\big) \\ -(\lambda+\alpha)(\delta+\Lambda(z))H(z)\end{pmatrix}}{D(z)}\frac{\alpha}{\lambda}P_{0,0}, \tag{13}$$

where $\mathcal{N} = (z-1)(\lambda+\delta+\alpha)(\lambda+\delta)+\delta(\alpha+\lambda z)\,(C(z) - H(z))\,.$

Remark 2: Comparing Eq.(1) and Eq.(13), we see that the PGF $\Psi(z)$ of the number of customers in the system at an arbitrary time coincides with the PGF $\Pi(z)$ of the embedded Markov chain at departure epochs, as shown in Gómez-Corral [16].

C. PERFORMANCE MEASURES OF THE SYSTEM

Based on the results given in section 3.2, the main purpose of this subsection is to provide main performance measures of the queueing system. By direct calculation through L'Hospital's rule and routine differentiation, we can have the following Theorem 3.4.

Theorem 3.4: (1) Under the steady state condition, we have the following results:

- *The Probability P_0 that the server is idle is given by*

$$P_0 = P_0(1) = \frac{\lambda[\delta + (\alpha+\lambda)(1-\rho)]}{(\lambda+\delta)(\lambda+\alpha+\delta)+\delta(\lambda+\alpha)\rho_1}.$$

- *The Probability P_1 that the server is busy is given by*

$$\begin{aligned} P_1 &= P_1(1) \\ &= \frac{\lambda(\alpha+\lambda)+\delta\big((\alpha+\lambda)\rho_1+\delta+\alpha+2\lambda\big)}{(\lambda+\delta)(\lambda+\alpha+\delta)+\delta(\lambda+\alpha)\rho_1}\lambda\beta_1. \end{aligned}$$

- *The Probability P_2 that the server is under repair for an active breakdown is given by*

$$P_2 = P_2(1) = \theta\nu_1 P_1.$$

- *The Probability P_3 that the server is during delayed period is given by*

$$P_3 = P_3(1) = \delta\frac{(\lambda+\alpha+\delta)(1-\rho)-\lambda\rho-\delta\rho_1}{(\lambda+\delta)(\lambda+\alpha+\delta)+\delta(\lambda+\alpha)\rho_1}.$$

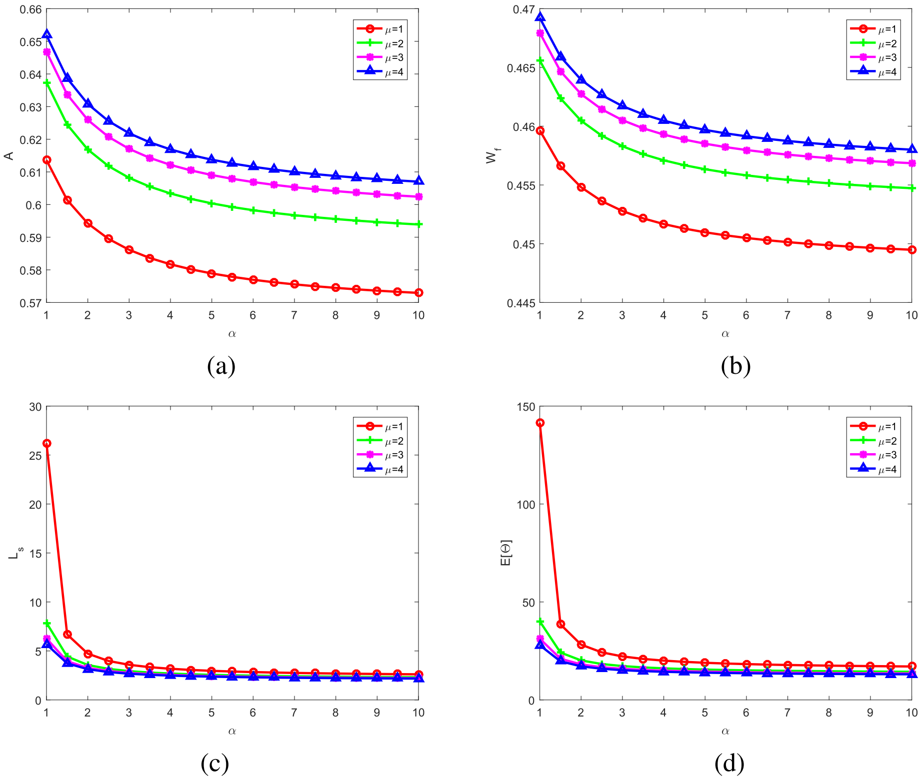

FIGURE 3. The effect of α on $A, W_f, L_S, E[\Theta]$ for different values of $\mu = 1, 2, 3, 4$.

- *The Probability P_4 that the server is under repair for a passive breakdown is given by*

$$P_4 = P_4(1) = \delta\rho_1 \frac{\delta + (\alpha+\lambda)(1-\rho)}{(\lambda+\delta)(\lambda+\alpha+\delta) + \delta(\lambda+\alpha)\rho_1}.$$

(2) Let L_O and L_S be, respectively, the mean orbit size and system size, i.e., $L_O = E[N] = \frac{d}{dz}\Phi(z)|_{z=1}$ and $L_S = E[N_S] = \frac{d}{dz}\Phi(z)|_{z=1}$, then we have

$$L_O = \frac{N''(1)}{2N'(1)} - \frac{D''(1)}{2D'(1)},$$
$$L_S = L_O + P_1(1) + P_2(1) + P_4(1),$$

where

$$\mathcal{D}'(1) = (\lambda+\alpha)(\lambda+\alpha+\delta)\left[\frac{\alpha}{\lambda+\alpha} - \rho - \frac{\delta}{\lambda+\alpha+\delta}\rho_1\right],$$
$$D''(1) = -(\lambda+\alpha)\left[2\lambda\rho + (\alpha+\lambda)\lambda^2\beta_2^*\right] - \delta\left[2\lambda(\rho+\rho_1) + (\alpha+\lambda)(\lambda^2\mu_2 + 2\rho\rho_1 + \lambda^2\beta_2^*)\right],$$
$$N'(1) = (\delta+\lambda)(\delta+\lambda+\alpha) + \delta(\lambda+\alpha)\rho_1,$$
$$N''(1) = 2\delta\lambda\rho_1 + \delta(\lambda+\alpha)(\lambda^2\mu_2 + 2\rho\rho_1).$$

Next, we make the analysis of a cycle of the system. A cycle of the system Θ is defined to be the length of the period that starts at the epoch when the server completes a service and the orbit is empty, and ends at the epoch at which the server becomes idle and the orbit is empty once again. Obviously, $\Theta = \Theta_{0,0} + \Theta_{0,1}\sum_{j=1}^{4}\Theta_j$, where $\Theta_{0,0}$ is the length of the server's idle period with empty orbit, $\Theta_{0,1}$ is the length of the server's idle period with nonempty orbit, Θ_1 is length of the server's busy period, Θ_2 is length of possible repair period for an active breakdown, Θ_3 is length of possible delayed period, Θ_4 is length of possible repair period for a passive breakdown. Taking into account the possible occurrence of a passive failure in server idle period, we have that $E[\Theta_{0,0}] = \frac{1}{\lambda+\delta}$. By applying the argument of an alternating renewal process, we know that

$$P_{0,0} = \frac{E[\Theta_{0,0}]}{E(\Theta)}, \quad P_0 - P_{0,0} = \frac{E[\Theta_{0,1}]}{E(\Theta)},$$
$$P_j = \frac{E[\Theta_j]}{E(\Theta)}, \quad j = 1, 2, 3, 4.$$

Then the expressions for $\Theta_{0,0}, \Theta_{0,1}, \Theta_j, j = 1, 2, 3, 4, \Theta$ are given as follows:

$$E[\Theta_{0,0}] = \frac{1}{\lambda+\delta}, \quad E[\Theta_{0,1}] = \frac{P_0 - P_{0,0}}{P_{0,0}(\lambda+\delta)},$$
$$E[\Theta_j] = \frac{P_j}{P_{0,0}(\lambda+\delta)}, \quad j = 1, 2, 3, 4,$$
$$E[\Theta] = \frac{1}{P_{0,0}(\lambda+\delta)}.$$

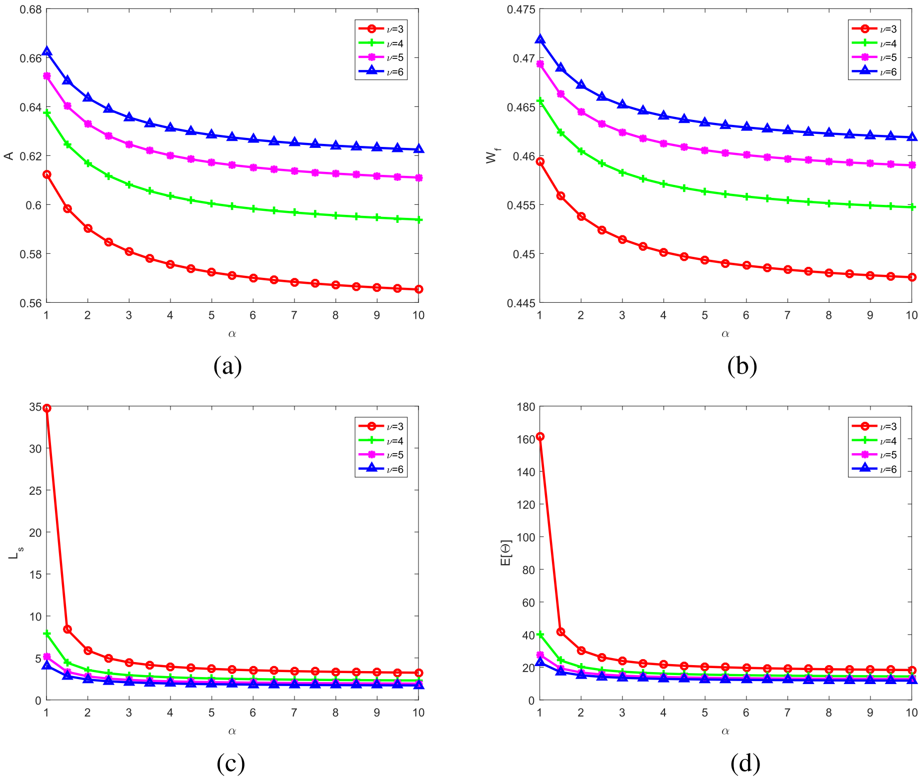

FIGURE 4. The effect of α on $A, W_f, L_S, E[\Theta]$ for different values of $\nu = 3, 4, 5, 6$.

IV. RELIABILITY ANALYSIS

In this section, we aim to provide some important reliability indexes of the queueing model based on the results obtained in Section III.

Suppose that the system is stable, let A be the steady state availability of the server, W_f be the failure frequency of the server, then we have that

$$A = P_0 + P_1, \quad W_f = \delta P_0 + \theta P_1. \tag{14}$$

Next, we focus on studying the mean time to first failure $MTTF$ of the server.

At initial time $t = 0$, the system is assumed to be empty and the server is idle, i.e., $P_{0,0}(0) = 1$. Let Y be the time to the first failure of the server, then the reliability function of the server is $U(t) = P(Y > t)$. The Laplace transform of $U(t)$ is denoted as $U^*(s) = \int_0^\infty e^{-st}U(t)dt$, $Res(s) > 0$, then $MTTF = E[Y] = \int_0^\infty U(t)dt = U^*(0)$. The expressions of $U^*(s)$ and $MTTF$ are given in the following Theorem.

Theorem 4.1: (1) The Laplace transform $U^*(s)$ is given by

$$U^*(s) = \frac{(s+\lambda+\alpha+\delta)\Big(1+\Lambda(\zeta(s))\overline{B}^*(s+\theta)\Big) - \Big(\alpha+\lambda\zeta(s)\Big)\widetilde{B}(s+\theta)}{\Big(s+\delta+\Lambda(\zeta(s))\Big)\Big(s+\lambda+(\lambda+\alpha)(1-\widetilde{B}(s+\theta))\Big)},$$

where $\zeta(s)$ is the minimum absolute value root of the equation

$$z(s+\lambda+\alpha+\delta) - (\alpha+\lambda z)\widetilde{B}(s+\lambda+\alpha+\delta) = 0$$

in the unit circle and $Res(s) > 0$.

(2) The expression of $MTTF$ is given by

$$MTTF = \frac{(\lambda+\alpha+\delta)\Big(1+\Lambda(\zeta(0))\overline{B}^*(\theta)\Big) - \Big(\alpha+\lambda\zeta(0)\Big)\widetilde{B}(\theta)}{\Big(s+\delta+\Lambda(\zeta(0))\Big)\Big(\lambda+(\lambda+\alpha)(1-\widetilde{B}(\theta))\Big)}.$$

Proof: To find $U(t)$, define the failure states $J = 2, 3, 4$ of the server are absorbing states. For the new system with absorbing states, using the same notations as in Section 3, we know that

$$U(t) = \sum_{n=0}^{\infty}\left(P_{n,0}(t) + \int_0^\infty P_{n,1}(t,x)dx\right),$$

and we have the following set of differential equations at time t:

$$\Big(\frac{d}{dt} + \lambda + (1-\delta_{0,n})\alpha + \delta\Big)P_{n,0}(t) = \int_0^\infty P_{n,1}(t,x)\beta(x)dx, n \geq 0, \tag{15}$$

$$\Big(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda + \theta + \beta(x)\Big)P_{n,1}(t,x) = (1-\delta_{0,n})\lambda P_{n-1,1}(t,x), \quad n \geq 0, \ x > 0, \tag{16}$$

$$P_{n,1}(t,0) = \lambda P_{n,0}(t) + \alpha P_{n+1,0}(t), \quad n \geq 0, \tag{17}$$

where $\delta_{0,n}$ is the Kronecker's symbol.

Define the Laplace transforms of $P_{n,0}(t), P_{n,1}(t,x)$ and corresponding generating functions as follows

$$P^*_{n,0}(s) = \int_0^\infty e^{-st} P_{n,0}(t)dt,$$
$$P^*_0(s;z) = \sum_{n=0}^\infty z^n p^*_{n,0}(s),$$
$$P^*_{n,1}(s;x) = \int_0^\infty e^{-st} P_{n,0}(t,x)dt,$$
$$P^*_1(s;x;z) = \sum_{n=0}^\infty z^n p^*_{n,1}(s;x),$$

Then

$$U^*(s) = P^*_0(s;1) + \int_0^\infty P^*_1(s;x;1)dx. \quad (18)$$

Based on the initial condition $P_{0,0}(0) = 1$ and taking the Laplace transform on both sides of Eq.s (15)-(17), we have

$$(s+\lambda+(1-\delta_{0,n})\alpha+\delta)P^*_{n,0}(s) = \delta_{0,n} + \int_0^\infty P^*_{n,1}(s;x)\beta(x)dx, \quad (19)$$
$$(s+\lambda+\theta+\beta(x))P^*_{n,1}(s;x) + \frac{\partial}{\partial x}P^*_{n,1}(s;x) = (1-\delta_{n,0})\lambda P^*_{n-1,1}(s;x), \quad n \geq 0, \quad (20)$$
$$P^*_{n,1}(s;0) = \lambda P^*_{n,0}(s) + \alpha P^*_{n+1,0}(s), \quad n \geq 0. \quad (21)$$

Multiplying on both sides of (19)-(21) by z^n and summing over $n \geq 0$, we have on simplification

$$(s+\lambda+\alpha+\delta)P^*_0(s;z) = 1 + \alpha P^*_{0,0}(s) + \int_0^\infty P^*_1(s;x;z)\beta(x)dx, \quad (22)$$
$$P^*_1(s;x,z) = P^*_1(s;0;z)e^{-(s+\theta+\Lambda(z))x}\overline{B}(x), \quad (23)$$
$$zP^*_1(s;0;z) = (\alpha+\lambda z)P^*_0(s;z) - \alpha P^*_{0,0}(s). \quad (24)$$

Combining (22)-(24) leads to

$$P^*_1(s;0;z) = \frac{\alpha+\lambda z - \alpha(s+\delta+\Lambda(z))P^*_{0,0}(s)}{z(s+\lambda+\alpha+\delta) - (\alpha+\lambda z)\widetilde{B}(s+\theta+\Lambda(z))}. \quad (25)$$

By Rouché's theorem, the denominator of (25) has exactly one zero point $z = \zeta(s)$ inside the unit circle and it is also the zero point for the numerator of (25), which leads to

$$P^*_{0,0}(s) = \frac{\alpha+\lambda\zeta(s)}{\alpha(s+\delta+\Lambda(\zeta(s)))}. \quad (26)$$

Then follows from (23)-(26), we have on simplification

$$P^*_0(s;z) = \frac{1}{s+\delta+\Lambda(\zeta(s))} \times \frac{z(s+\lambda+\alpha+\delta) - (\alpha+\lambda\zeta(s))\widetilde{B}(s+\theta+\Lambda(z))}{z(s+\lambda+\alpha+\delta) - (\alpha+\lambda z)\widetilde{B}(s+\theta+\Lambda(z))}, \quad (27)$$

$$P^*_1(s;x;z) = \frac{\lambda(s+\lambda+\alpha+\delta)}{s+\delta+\Lambda(\zeta(s))} \times \frac{z-\zeta(s)}{z(s+\lambda+\alpha+\delta) - (\alpha+\lambda z)\widetilde{B}(s+\theta+\Lambda(z))} \times \overline{B}(x)e^{-(s+\theta+\Lambda(z))x}. \quad (28)$$

Using (18),(27) and (28), we can get the expected results given in Theorem 4.1. □

V. ANALYSIS OF THE SOJOURN TIME IN THE SYSTEM

Sojourn time of an arbitrary customer can reflect the quality of service of the system. Based on this point, this section is devoted to discuss the distribution of the sojourn time T of any arbitrary tagged arriving customer, which is the length of the time interval from the epoch at which the tagged customer arrive at the system to the epoch at which the tagged customer leaves the system with his service completion. Let $\widehat{T}(s) = E[e^{-sT}]$, by conditioning on the system's state at the tagged customer's arrival epoch, we have that

$$\widehat{T}(s) = P_0\widetilde{S}_B(s) + P_3\widetilde{G}(s)\widetilde{S}_B(s) + \sum_{k=0}^\infty \int_0^\infty P_{k,1}(x)\widetilde{T}_{k,1}(x;s)dx + \sum_{k=0}^\infty \int_0^\infty \int_0^\infty P_{k,2}(x;y)\widetilde{T}_{k,2}(x,y;s)dydx + \sum_{k=0}^\infty \int_0^\infty P_{k,4}(x)\widetilde{T}_{k,4}(x;s)dx, \quad (29)$$

where

$$\widetilde{T}_{k,j}(x;s) \triangleq E\left[e^{-sT}|N=k, J=j, \xi_j = x\right], \quad j=1,4,$$
$$\widetilde{T}_{k,2}(x,y;s) \triangleq E\left[e^{-sT}|N=k, J=2, \xi_1 = x, \xi_2 = y\right].$$

To derive the explicit expression of $\widehat{T}(s)$, it is necessary to introduce two auxiliary random variables, one is the random variable T_1, which denotes the length of time interval calculated from the epoch when the server becomes idle and the tagged customer is at the head of the system to the epoch when the tagged customer leaves the system; the other is the random T_d, which denotes the length of time interval calculated from the epoch when a passive breakdown of the server occurs and the tagged customer is at the head of the system to the epoch when the tagged customer leaves the system. Denote $\widetilde{T}_1(s) = E[e^{-sT_1}]$, $\widetilde{T}_d(s) = E[e^{-sT_d}]$. With the help of the auxiliary variable T_d, we can derive the expression of $\widetilde{T}_1(s)$.

Lemma 5.1: The Laplace transform $\widetilde{T}_1(s)$ of T_1 and its mean value are given by as follows:

$$\widetilde{T}_1(s) = \frac{\frac{\alpha}{s+\lambda+\delta+\alpha}\left(1+\frac{\delta}{s+\lambda+\alpha}\widetilde{G}(s)\right)\widetilde{S}_B(s)}{1-\frac{\lambda}{s+\delta+\lambda+\alpha}\left(1+\frac{\delta}{s+\lambda+\alpha}\widetilde{G}(s)\right)\widetilde{S}_B(s)}, \quad (30)$$

$$E[T_1] = \left(\beta_1^* + \frac{1}{\lambda+\alpha} + \frac{\delta}{\lambda+\delta+\alpha}\mu_1\right)\left(1+\frac{\lambda}{\alpha}\right). \quad (31)$$

Proof: For $\widetilde{T}_1(s)$, by considering the order of the new arrival from outsider, the passive failure and the retrial time of the tagged customer who is at the head in the orbit, we have that

$$\begin{aligned}\widetilde{T}_1(s) &= \int_0^\infty \lambda e^{-\lambda x} e^{-(\delta+\alpha)x} e^{-sx}\widetilde{S}_B(s)\widetilde{T}_1(s)dx \\ &\quad + \int_0^\infty \alpha e^{-\alpha x} e^{-(\lambda+\delta)x} e^{-sx}\widetilde{S}_B(s)dx \\ &\quad + \int_0^\infty \delta e^{-\delta x} e^{-(\lambda+\alpha)x} e^{-sx}\widetilde{T}_d(s)dx \\ &= \frac{\lambda}{s+\lambda+\alpha+\delta}\widetilde{S}_B(s)\widetilde{T}_1(s) + \frac{\alpha}{s+\lambda+\alpha+\delta}\widetilde{S}_B(s) \\ &\quad + \frac{\delta}{s+\lambda+\alpha+\delta}\widetilde{T}_d(s). \end{aligned} \quad (32)$$

Similarly, for $\widetilde{T}_d(s)$, by conditioning on the beginning epoch of the repair for the passive breakdown whether at the epoch of the arrival from the outside or from the orbit, we have that

$$\widetilde{T}_d(s) = \frac{\lambda}{s+\lambda+\alpha}\widetilde{G}(s)\widetilde{S}_B(s)\widetilde{T}_1(s) + \frac{\alpha}{s+\lambda+\alpha}\widetilde{G}(s)\widetilde{S}_B(s). \quad (33)$$

Following from (32) and (33), we can obtain the result (30). By differentiating $\widetilde{T}_1(s)$ with respect to s and then taking limit $s \to 0$, i.e., $E[T_1] = -\frac{d}{ds}\widetilde{T}_1(s)|_{s=0}$, we can get (31). □

Now we can derive the expressions of $\widetilde{T}(s)$ and the mean value $E[T]$, which are given by Theorem 5.1.

Theorem 5.1: The Laplace transform $\widetilde{T}(s)$ of the sojourn time T and its mean value $E[T]$ are as follows

$$\begin{aligned}\widetilde{T}(s) &= \widetilde{S}_B(s)\big(P_0 + P_3\widetilde{G}(s)\big) + \Bigg[\frac{\mathcal{N}_1(\widetilde{T}_1(s))}{\mathcal{D}(\widetilde{T}_1(s))} \\ &\quad \times \frac{\widetilde{S}_B(s) - \widetilde{S}_B(\Lambda(\widetilde{T}_1(s)))}{\Lambda(\widetilde{T}_1(s)) - s} \\ &\quad \times \left(1+\theta\frac{\widetilde{R}(s)-\widetilde{R}(\Lambda(\widetilde{T}_1(s)))}{\Lambda(\widetilde{T}_1(s))-s}\right) \\ &\quad + \delta\frac{\mathcal{N}_3(\widetilde{T}_1(s))}{\mathcal{D}(\widetilde{T}_1(s))}\frac{\widetilde{G}(s)-\widetilde{G}(\Lambda(\widetilde{T}_1(s)))}{\Lambda(\widetilde{T}_1(s))}\widetilde{S}_B(s)\Bigg]\widetilde{T}_1(s)\alpha P_{0,0}, \end{aligned} \quad (34)$$

$$\begin{aligned}E[T] &= P_0\beta_1^* + P_3(\beta_1^* + \mu_1) \\ &\quad + (P_1+P_2)\left(E[T_1] + \frac{\beta_2^*(1+\lambda E[T_1])}{2\beta_1^*}\right) \\ &\quad + P_4\left(\beta_1^* + E[T_1] + \frac{\mu_2(1+\lambda E[T_1])}{2\mu_1}\right) \\ &\quad + \alpha P_{0,0}E[T_1]\left(\beta_1^*\frac{\mathcal{N}_1''(1)\mathcal{D}'(1) - \mathcal{N}_1'(1)\mathcal{D}''(1)}{2(\mathcal{D}'(1))^2}\right. \\ &\quad \left. + \delta\mu_1\frac{\mathcal{N}_3''(1)\mathcal{D}'(1) - \mathcal{N}_3'(1)\mathcal{D}''(1)}{2(\mathcal{D}'(1))^2}\right) \quad (35) \\ &= \frac{L_s}{\lambda}, \quad (36)\end{aligned}$$

where

$$\begin{aligned}\mathcal{N}_1'(1) &= \delta\rho_1(\lambda+\alpha) + (\lambda+\delta)(\delta+\alpha+\lambda), \\ \mathcal{N}_1''(1) &= \delta((\lambda+\alpha)(\lambda^2\mu_2 + 2\rho_1) + 2\rho_1(\delta+\lambda)), \\ \mathcal{N}_3'(1) &= \delta + (1-\rho)(\lambda+\alpha), \\ \mathcal{N}_3''(1) &= -2\lambda\rho - (\alpha+\lambda)\lambda^2\beta_2^*.\end{aligned}$$

Proof: Recall that in reliability theory, if a nonnegative random X denotes a life time of a unit, with p.d.f $f(x)$, c.d.f $F(x)$, then the random variable $X_x = X - x|X > x$ is called residual lifetime, and the p.d.f $f_x(y)$ of X_x is given by $f_x(y) = \frac{f(x+y)}{\overline{F}(x)}$.

In the following we first consider $\widetilde{T}_{k,1}(x;s) = E\left[e^{-sT}|N=k, J=1, \xi_1 = x\right]$.

Given that the tagged customer finds that the system is in the state $(N, J, \xi_1) = (k, 1, x)$ at its arrival epoch, then the tagged customer joins the $(k+1)$-th position of the orbit and its sojourn time is the sum of the three random variables: $B_x, R^{(M)}, T_1^{(K+1)}$, where B_x is the residual service, $T_1^{(K+1)}$ is the sum of $k+1$ independently and identically distributed (i.i.d.) random variables with generic random variable T_1, M is the number of active breakdowns occurring during B_x, and $R^{(M)}$ is the total repair times for M active breakdowns. Therefore we have that

$$\begin{aligned}&\widetilde{T}_{k,1}(x;s) \\ &= \int_0^\infty \frac{b(x+y)}{\overline{B}(x)}e^{-sy}\sum_{m=0}^\infty \frac{(\theta y)^m}{m!}e^{-\theta y}\big(\widetilde{R}(s)\big)^m\big(\widetilde{T}_1(s)\big)^{k+1}dy \\ &= \frac{\big(\widetilde{T}_1(s)\big)^{k+1}}{\overline{B}(x)}\int_x^\infty b(u)e^{-(s+\theta(1-\widetilde{R}(s)))(u-x)}du. \end{aligned} \quad (37)$$

Adopting similar analysis line to the above, we can obtain the expressions for $\widetilde{T}_{k,2}(x,y;s)$ and $\widetilde{T}_{k,4}(x;s)$ as follows:

$$\widetilde{T}_{k,2}(x,y;s) = \frac{\widetilde{T}_{k,1}(x;s)}{\overline{R}(y)}\int_y^\infty r(t)e^{-s(t-y)}dt, \quad (38)$$

$$\widetilde{T}_{k,4}(x;s) = \frac{\widetilde{S}_B(s)\big(\widetilde{T}_1(s)\big)^{k+1}}{\overline{G}(x)}\int_x^\infty g(t)e^{-s(t-x)}dt. \quad (39)$$

Substituting (37)-(39) into (29) and using Theorem 3.2, after some tedious calculations, we can obtain the result (34). By $E[T] = -\frac{d}{ds}\widetilde{T}(s)|_{s=0}$, we can have (35). Inserting the expressions of $P_{0,0}, P_i, i = 0,1,2,3,4$ and $E[T_1]$, and comparing the expression of L_s, we can have the Eq. (36). □

Remark 3: Eq.(36) shows that the Little's law still holds in our retrial queue system, which will also be shown by the following numerical examples.

TABLE 1. The effect of α on $E[T]$, $E[T]|_{Little}$, $MTTF$ for $\delta = 0.25, 0.35, 0.55, 0.75$.

	$\delta = 0.25$			$\delta = 0.35$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	31.4866	31.4866	2.8453	40.3130	40.3130	2.5132
2	14.2536	14.2536	2.8402	15.3672	15.3672	2.4256
3	11.7618	11.7618	2.8381	12.3660	12.3660	2.3861
4	10.7626	10.7626	2.8369	11.1970	11.1970	2.3636
5	10.2239	10.2239	2.8362	10.5745	10.5745	2.3491
6	9.8869	9.8869	2.8357	10.1879	10.1879	2.3389
7	9.6562	9.6562	2.8353	9.9243	9.9243	2.3314
8	9.4884	9.4884	2.8350	9.7332	9.7332	2.3257
9	9.3608	9.3608	2.8348	9.5882	9.5882	2.3211
	$\delta = 0.55$			$\delta = 0.75$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	73.8138	73.8138	2.1279	211.7915	211.7915	1.9112
2	17.6654	17.6654	1.9448	20.1584	20.1584	1.6744
3	13.5155	13.5155	1.8620	14.6556	14.6556	1.5672
4	11.9869	11.9869	1.8148	12.7376	12.7376	1.5061
5	11.1910	11.1910	1.7843	11.7599	11.7599	1.4667
6	10.7027	10.7027	1.7630	11.1665	11.1665	1.4391
7	10.3725	10.3725	1.7472	10.7679	10.7679	1.4187
8	10.1342	10.1342	1.7351	10.4817	10.4817	1.4030
9	9.9541	9.9541	1.7255	10.2661	10.2661	1.3906

TABLE 2. The effect of α on $E[T]$, $E[T]|_{Little}$, $MTTF$ for $\theta = 0.35, 0.65, 0.95, 1.25$..

	$\theta = 0.35$			$\theta = 0.65$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	10.8652	10.8652	3.6614	16.8851	16.8851	3.1154
2	7.2834	7.2834	3.6565	9.9249	9.9249	3.1090
3	6.4601	6.4601	3.6544	8.5599	8.5599	3.1063
4	6.0944	6.0944	3.6533	7.9773	7.9773	3.1048
5	5.8877	5.8877	3.6526	7.6543	7.6543	3.1039
6	5.7549	5.7549	3.6521	7.4489	7.4489	3.1033
7	5.6623	5.6623	3.6517	7.3069	7.3069	3.1028
8	5.5941	5.5941	3.6514	7.2027	7.2027	3.1024
9	5.5417	5.5417	3.6512	7.1231	7.1231	3.1021
	$\theta = 0.95$			$\theta = 1.25$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	31.4866	31.4866	2.8453	118.2750	118.2750	2.6824
2	14.2536	14.2536	2.8402	22.6347	22.6347	2.6785
3	11.7618	11.7618	2.8381	17.2366	17.2366	2.6768
4	10.7626	10.7626	2.8369	15.2984	15.2984	2.6759
5	10.2239	10.2239	2.8362	14.3009	14.3009	2.6753
6	9.8869	9.8869	2.8357	13.6929	13.6929	2.6749
7	9.6562	9.6562	2.8353	13.2836	13.2836	2.6746
8	9.4884	9.4884	2.8350	12.9892	12.9892	2.6744
9	9.3608	9.3608	2.8348	12.7673	12.7673	2.6742

VI. NUMERICAL EXAMPLES

As an numerical example, the arrival rate of external customers is taken as $\lambda = 0.25$, the service time follows the phase type distribution with representation $(\mathbf{d}_0, \Gamma)$ with $\mathbf{d}_0 = (0.65\ 0.35)$, $\Gamma = \begin{bmatrix} -0.8 & 0.1 \\ 0.2 & -0.6 \end{bmatrix}$, with d.f. $B(x) = 1 - \mathbf{d}_0 e^{\Gamma x}\mathbf{1}$, LST $\widetilde{B}(s) = \mathbf{d}_0(s\mathbf{I} - \Gamma)^{-1}\mathbf{t}$,, where $\mathbf{t} = -\Gamma\mathbf{1}$, $\mathbf{I}$ is a an identity matric with size 2×2, $\mathbf{1}$ is a column vector

TABLE 3. The effect of α on $E[T]$, $E[T]|_{Little}$, $MTTF$ for $\mu = 1, 2, 3, 4$..

	$\mu = 1$			$\mu = 2$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	104.8143	104.8143	2.8453	31.4866	31.4866	2.8453
2	18.8895	18.8895	2.8402	14.2536	14.2536	2.8402
3	14.3387	14.3387	2.8381	11.7618	11.7618	2.8381
4	12.7117	12.7117	2.8369	10.7626	10.7626	2.8369
5	11.8756	11.8756	2.8362	10.2239	10.2239	2.8362
6	11.3665	11.3665	2.8357	9.8869	9.8869	2.8357
7	11.0239	11.0239	2.8353	9.6562	9.6562	2.8353
8	10.7776	10.7776	2.8350	9.4884	9.4884	2.8350
9	10.5920	10.5920	2.8348	9.3608	9.3608	2.8348
	$\mu = 3$			$\mu = 4$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	25.0917	25.0917	2.8453	22.7090	22.7090	2.8453
2	13.0779	13.0779	2.8402	12.5427	12.5427	2.8402
3	11.0352	11.0352	2.8381	10.6936	10.6936	2.8381
4	10.1904	10.19046	2.8369	9.9178	9.9178	2.8369
5	9.7286	9.7286	2.8362	9.4911	9.4911	2.8362
6	9.4375	9.4375	2.8357	9.2211	9.2211	2.8357
7	9.2373	9.2373	2.8353	9.0348	9.0348	2.8353
8	9.0910	9.0910	2.8350	8.8986	8.8986	2.8350
9	8.9795	8.9795	2.8348	8.7947	8.7947	2.8348

TABLE 4. The effect of α on $E[T]$, $E[T]|_{Little}$, $MTTF$ for $\nu = 3, 4, 5, 6$..

	$\nu = 3$			$\nu = 4$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	139.1835	139.1835	2.8453	31.4866	31.4866	2.8453
2	23.5895	23.5895	2.8402	14.2536	14.2536	2.8402
3	17.8474	17.8474	2.8381	11.7618	11.7618	2.8381
4	10.4856	10.4856	2.8369	10.7626	10.7626	2.8369
5	14.7555	14.7555	2.8362	10.2239	10.2239	2.8362
6	14.1178	14.1178	2.8357	9.8869	9.8869	2.8357
7	13.6890	13.6890	2.8353	9.6562	9.6562	2.8353
8	13.3809	13.3809	2.8350	9.4884	9.4884	2.8350
9	13.1488	13.1488	2.8348	9.3608	9.3608	2.8348
	$\nu = 5$			$\nu = 6$		
α	$E[T]$	$E[T]\|_{Little}$	$MTTF$	$E[T]$	$E[T]\|_{Little}$	$MTTF$
1	20.4812	20.4812	2.8453	16.3149	16.3149	2.8453
2	11.1949	11.1949	2.8402	9.6821	9.6821	2.8402
3	9.5241	9.5241	2.8381	8.3665	8.3665	2.8381
4	8.8248	8.8248	2.8369	7.8035	7.8035	2.8369
5	8.4404	8.4404	2.8362	7.4910	7.4910	2.8362
6	8.1974	8.1974	2.8357	7.2921	7.2921	2.8357
7	8.0298	8.0298	2.8353	7.1545	7.1545	2.8353
8	7.9072	7.9072	2.8350	7.0536	7.0536	2.8350
9	7.8137	7.8137	2.8348	6.9764	6.9764	2.8348

with two 1's. The repair time for a passive failure follows the Erlang(3, μ) distribution, i.e., Erlang of order 3 with mean $\mu_1 = 3/\mu$, LST $\widetilde{G}(s) = (\frac{\mu}{s+\mu})^3$. The repair time for an active failure follows the Erlang(2, ν) distribution, i.e., Erlang of order 2 with mean $\nu_1 = 2/\nu$, LST $\widetilde{R}(s) = (\frac{\nu}{s+\nu})^2$.

Under the stationary condition $\rho + \frac{\delta}{\lambda+\alpha+\delta}\rho_1 < \frac{\alpha}{\lambda+\alpha}$, the base case for setting these system parameters is set below: $\delta = 0.25, \theta = 0.95, \mu = 2$, and $\nu = 4$. We assume that the values of the retrial rate α varies from 1 to 10 in the following Figures 1-4 and Tables 1-4, and each of the system parameters δ, θ, μ, and ν takes turn to change in a certain rang but keeps other system parameters fixed given in the base case. The purpose of this section is to illustrate the effect of these parameters on some important reliability indices, including steady-state availability A, the failure frequency W_f and mean time to first failure of the server $MTTF$, and queueing measures, including the mean system length L_s, the expected length of a cycle $E[\Theta]$ and the mean sojourn time of an arbitrary $E[T]$. In the following, we use the Matlab program to illustrate the numerical results of A, W_f, L_s, $E[\Theta]$ in Figures 1-4 and $E[T], E[T]\big|_{Little}, MTTF$ in Tables 1-4. In Tables 1-4, we list the mean values of sojourn time $E[T] = -\frac{d}{ds}\widetilde{W}_T(s)\big|_{s=0}$, and the corresponding values of $E[T]\big|_{Little} = \frac{L_s}{\lambda}$ obtained by Little's law, to show that Little's law still hold in our retrial queue. Fig.s 1-4 and Tables 1-4 graphically and numerically show that:

- Little's Law still holds in our retrial queueing system with two types of failures and delayed repairs, which is numerical shown in Tables 1-4.
- Given that values of other system parameters are fixed, all reliability indices (steady-state availability A, the failure frequency W_f and mean time to first failure of the server $MTTF$) and queueing measures (the mean system length L_s, the expected length of a cycle $E[\Theta]$ and the mean sojourn time of an arbitrary customer $E[T]$) decrease as the retrial rate α increases, which agrees with our expectations.
- The increase in the passive failure rate δ and active failure rate θ makes the server breakdown more frequently, and then decrease the steady-state availability A and the mean time to first failure of the server $MTTF$, but increase the failure frequency W_f, the system length L_s, the expected length of a cycle $E[\Theta]$ and the mean sojourn time of an arbitrary customer $E[T]$, which is shown in Fig.s 1, 2 and Tables 1,2.
- The increase in μ and ν can shorten the repair time of the server and makes the server more available, which increases the steady-state availability A, but decreases the failure frequency W_f, the system length L_s, the expected length of a cycle $E[\Theta]$ and the mean sojourn time of an arbitrary customer $E[T]$, which is shown in Fig.s 3, 4 and Tables 3,4. However, the changes in the values of μ and ν in the repair times of passive and active failures have no effect on the mean time to first failure of the server $MTTF$, because it is not calculated after the server fails for the first time, which can be seen from Tables 3,4.

VII. CONCLUSION

In this article, we have conducted an exhaustive study on an unreliable M/G/1 retrial queue with two-type breakdowns: one is passive failures with delayed repairs, the other is active breakdowns with immediately repair. Of course, such delayed repair process is different from that incurred by starting failures. The feature of starting failures is that the server may be broken down at the arrival epoch of a customer who arrives from outside or orbit and finds the server idle, in this case, the customer must start the server to receive its service. If the server is unsuccessfully started with some probability, it immediately accepts repair, otherwise, if it is successfully started with complimentary probability, it immediately renders service to the customer. However our delayed repair process is that when the server breaks down in idle period, i.e., a passive breakdown occurs, the server can begin its repair at the arrival epoch of a customer from outside or orbit. That is to say, the repair process of a passive failure is started by the next arriving customer (new or returning). For this model, we analyzed the sufficient and necessary condition for the system to be stable, the stationary queueing indexes, sojourn time in the system from the queueing viewpoint, and obtain reliability measures such as availability, server failure frequency, and mean time to first failure from reliability viewpoint. Some numerical examples were given to study the effect of some parameters on the important performance measures and reliability indices of the model. As one direction of further future research, it is very interesting to develop the discrete-time counterpart of our continuous-time retrial queue, the reason is that the discrete-time queueing system is more feasible to model computer and telecommunication systems. Another direction of future research, one can consider the equilibrium balking policy for the Markovian counterpart of our retrial queue from economic viewpoint.

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