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Analysis of an EOQ Inventory Model for Deteriorating Items with Different Demand Rates

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Abstract

In this article, we proposed and analyzed the EOQ inventory model for constant deteriorating items with various demand rates such as constant, linear and quadratic function of time and time dependent holding cost. The main objective of this model is to minimize the total cost without shortages. Finally, through numerical examples and sensitivity analysis, we depict the significance of the total cost and the cycle period.

Keywords: Deterioration, Quadratic demand, Holding cost, Shortages

I. Introduction

The study of effect of deteriorating items in inventory models becoming a growing interest since last two decade. Due to this, we can make the proper planning to maintain the inventory economically to increase the revenue of industry. Deteriorating items can be classified in to two categories. The first category refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time like meat, vegetables, fruit, medicine, flowers, film and so on: the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives,

like computer chips, mobile phones, fashion and seasonal goods, and so on. Both the two categories have the characteristic of short life cycle.
Shah and Jaiswal (1977) proposed an order-level inventory model for items with a constant rate of deterioration. Aggarwal (1978), Dave and Patel (1981), Giri and Chaudiri (1998), Chang and Dye (1999), Khanra and Chaudhuri (2003), Teng, Chang and Goyal (2005) have used constant deterioration rate in inventory models. Anil kumar sharma *et al*. (2012) considered an inventory model with time dependent holding cost. Chaitanya Kumar Tripathy and Umakanta Mishra (2010) developed a inventory model for Weibull deteriorating items for quadratic demand with permissible delay in payments. R. Begum, S. K. Sahu and R. R. Sahoo (2010) established an order level inventory models for deteriorating items with quadratic demand. Gobinda Chandra *et al*. (2012) analysed constant deteriorating inventory management with quadratic demand rate. R. Begum, S. K. Sahu and R. R. Sahoo (2012) developed an inventory model for deteriorating with quadratic demand and partial backlogging. Trailokyanth Singh and Hadibandhu Pattanayak (2013) presented an EOQ model for a deteriorating item with time dependent quadratic demand and variable deterioration under delay in payments. Recently, Venkateswarlu. R and Mohan. R (2014) studied an inventory model with Weibull deterioration and quadratic demand under inflation. In this paper, we have taken deterioration rate as constant and demand rate is quadratic function of time. Shortages are not allowed and time dependent holding cost.

II. Assumptions

The following assumptions are made in developing the model:

1.The demand rate for the item is represented by a quadratic and continuous function of time.

2.Shortages are not allowed.

3.The lead time is zero.

4.A finite planning horizon is assumed.

5.Replenishment rate is infinite.

III. Notations

The following notations have been used in developing the model:
1.D(t):The demand rate $D(t) = a + bt + ct^2$, where a,b and c are the positive constants.

2.I(t): Inventory level at time *t*.

3.Q: Inventory level at time $t = 0$.

4.HC:Holding Cost is linear function of time H(t) = α +βt , α>0, β>0

5.C_1: Unit purchase cost of an item.

6.θ: θ = θ(t) is the constant rate of deterioration of an item.

7.TC: Total Cost per unit time

8. T : The time interval between two successive orders.

9. A: The order cost per unit order is known and constant.

10.DC: Deterioration Cost per cycle.

11.SC: Set up Cost or Ordering Cost per cycle.

IV. Mathematical Formulation and Solution of the Model

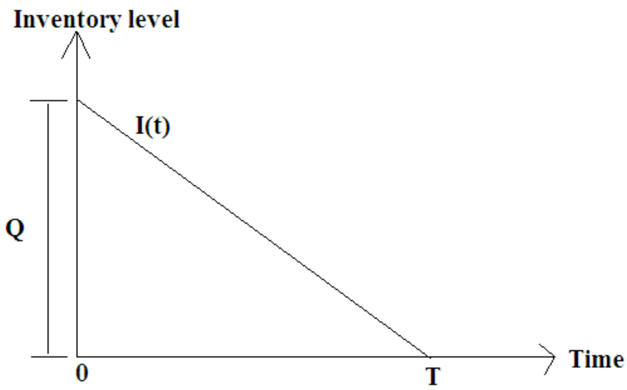


Figure 1

The instantaneous inventory level I(t) at any time t during the cycle time T is governed by the following differential equation

$$\frac{dI(t)}{dt} + \theta(t)I(t) = -D(t), \qquad 0 \leq t \leq T$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a + bt + ct^2), \qquad 0 \leq t \leq T$$

The solution of above equation with boundary conditions I(0) = Q and I(T) = 0, is

$$I(t) = \left[a(T-t) + \frac{(b+a\theta)(T^2-t^2)}{2} + \frac{(c+b\theta)(T^3-t^3)}{3} + \frac{(c\theta)(T^4-t^4)}{4}\right], 0 \leq t \leq T \qquad (1)$$

The optimum order quantity is given by

$$I(0) = Q = aT + \frac{(b+a\theta)T^2}{2} + \frac{(c+b\theta)T^3}{3} + \frac{(c\theta)T^4}{4} \tag{2}$$

The Total Cost (TC) per unit time consists of the following costs:

(i) The total Carrying cost/ holding cost (HC) per cycle [0,T] is given by

$$HC = \frac{1}{T}\int_0^T (\alpha + \beta t) I(t)dt$$

$$HC = \frac{\alpha aT}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta T^4}{5} + \frac{\beta aT^2}{6} + \frac{\beta(b+a\theta)T^3}{8} + \frac{\beta(c+b\theta)T^4}{10} + \frac{\beta c\theta T^5}{12}$$

(ii) The number of deteriorated units (NDU) during the cycle period [0, T] is given by $NDU = Q - \int_0^T D(t)dt$ where $D(t) = a + bt + ct^2$

(iii) The Deterioration Cost (DC) for the cycle [0,T] is given by

$$DC = \frac{C_1}{T} \times (NDU)$$

$$DC = \frac{C_1}{T}\left[Q - \int_0^T D(t)dt\right] = \frac{C_1 a\theta T}{2} + \frac{C_1 b\theta T^2}{3} + \frac{C_1 c\theta T^3}{4}$$

(iv) The Ordering Cost/Setup Cost (SC) per cycle [0, T] is given by

$$SC = \frac{A}{T}$$

The Total Cost per unit time is given by

$$TC = HC + DC + SC$$

$$TC = \frac{\alpha aT}{2} + \frac{\alpha(b+a\theta)T^2}{3} + \frac{\alpha(c+b\theta)T^3}{4} + \frac{\alpha c\theta T^4}{5} + \frac{\beta aT^2}{6} + \frac{\beta(b+a\theta)T^3}{8} + \frac{\beta(c+b\theta)T^4}{10} + \frac{\beta c\theta T^5}{12} + \frac{C_1 a\theta T}{2} + \frac{C_1 b\theta T^2}{3} + \frac{C_1 c\theta T^3}{4} + \frac{A}{T} \tag{3}$$

The necessary condition for total cost (TC) to be minimize is

$$\frac{\partial(TC)}{\partial T} = 0 \; and \; \frac{\partial^2(TC)}{\partial T^2} > 0 \quad for\ all\ T > 0$$

$$\frac{\partial(TC)}{\partial T}=\frac{\alpha a}{2}+\frac{2\alpha(b+a\theta)T}{3}+\frac{3\alpha(c+b\theta)T^2}{4}+\frac{4\alpha c\theta T^3}{5}+\frac{2\beta aT}{6}+\frac{3\beta(b+a\theta)T^2}{8}+\frac{4\beta(c+b\theta)T^3}{10}+$$
$$\frac{5\beta c\theta T^4}{12}+\frac{C_1 a\theta}{2}+\frac{2C_1 b\theta T}{3}+\frac{3C_1 c\theta T^2}{4}-\frac{A}{T^2}=0 \qquad (4)$$

$$\frac{\partial^2(TC)}{\partial T^2}=\frac{2\alpha(b+a\theta)}{3}+\frac{6\alpha(c+b\theta)T}{4}+\frac{12\alpha c\theta T^2}{5}+\frac{2\beta a}{6}+\frac{6\beta(b+a\theta)T}{8}+\frac{12\beta(c+b\theta)T^2}{10}+\frac{20\beta c\theta T^3}{12}+$$
$$\frac{2C_1 b\theta}{3}+\frac{6C_1 c\theta T}{4}-\frac{2A}{T^3}>0$$

By solving equation (4) the value of T can be obtained and then from equation (3) and (2), the optimal value of TC and Q can be find out respectively.

CASE I:

If c = 0, then the equation (1) represents the instantaneous inventory level at any time *t* for the linear demand rate $(i.e., D(t) = a + bt)$.

CASE II:

If b=0 and c = 0, then the equation (1) represents the instantaneous inventory level at any time *t* for the constant demand rate $(i.e., D(t) = a)$.

V. Numerical Analysis

We assume suitable values for A,a,b,c,d, θ, α, β and C_1 with appropriated units,

From the equations (2),(3) and (4) We get the optimal values of T, Q and TC.

Examples:

1. Let A=1000, a=25, b=40, c=20, θ =0.02, α =0.5, β =0.01, C_1 =1.5. From equations (2), (3), and (4), we get T = 2.8598,Q = 405.9664, TC = 497.1807
2. Let A=1000, a=25, b=40, c=0, θ =0.02, α =0.5, β =0.01, C_1 =1.5. From equations (2), (3), and (4), we get T = 3.8235, Q = 406.5309, TC = 400.5971
3. Let A=1000, a=25, b=0, c=0, θ=0.02, α=0.5, β=0.01, C_1=1.5. From equations (2), (3), and (4), we get T = 10.3133, Q = 284.4235, TC = 179.2686.

VI. Sensitivity Analysis

Sensitivity Analysis is performed by increasing the value of parameter θ by 0.02 to 0.1 and keeping the remaining parameters at their original values.

Table-1
Sensitivity of the deterioration parameter θ

Model Type	θ	T	Q	HC	DC	SC	TC
Quadratic Demand	0.02	2.8598	405.9664	139.5439	7.8520	349.7848	497.1807
	0.04	2.7803	394.6662	135.8320	14.7167	359.6734	510.2221
	0.06	2.7112	385.3137	132.7680	20.8388	368.8403	522.4471
	0.08	2.6502	377.4238	130.1900	26.3812	377.3300	533.9012
	0.10	2.5957	370.6772	127.9896	31.4589	385.2525	544.7010

Table-2
Sensitivity of the deterioration parameter θ

Model Type	θ	T	Q	HC	DC	SC	TC
Linear Demand	0.02	3.8235	406.5309	131.7753	7.2814	261.5404	400.5971
	0.04	3.6772	395.6453	128.6988	13.5753	271.9460	414.2201
	0.06	3.5546	386.9754	126.3227	19.1611	281.3256	426.8094
	0.08	3.3911	367.8640	120.2390	23.4858	294.8895	438.6143
	0.10	3.2962	361.0354	118.2924	27.9101	303.3796	449.5821

Table-3
Sensitivity of the deterioration parameter θ

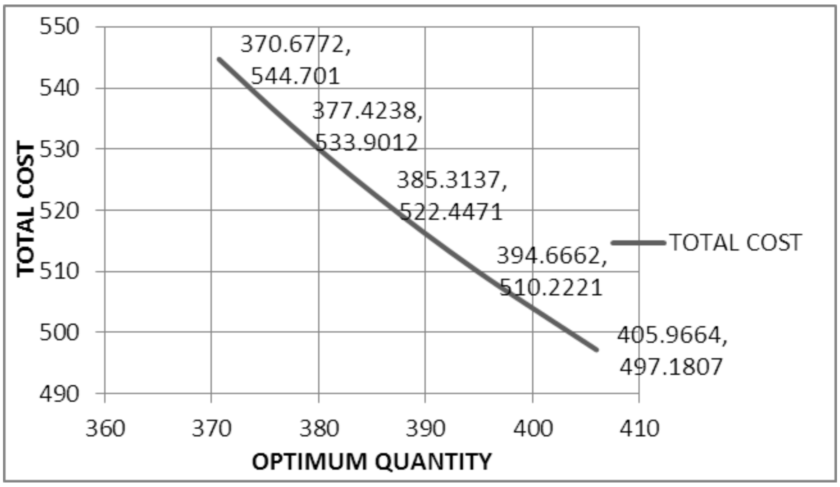
Model Type	θ	T	Q	HC	DC	SC	TC
Constant Demand	0.02	10.3133	284.4235	78.4391	3.8674	96.9621	179.2686
	0.04	9.4242	280.0127	78.4505	7.0681	106.1098	191.6284
	0.06	8.7916	277.7591	78.7651	9.8905	113.7449	202.4005
	0.08	8.3057	276.6271	79.2121	12.4585	120.3992	212.0698
	0.10	7.9141	276.1437	79.7188	14.8389	126.3567	220.9144

From **Table-1**, we observed the following points
(i) Increase in the value of parameter θ then the value of T is less than the values of T for Linear and Constant demand.
(ii) The values of SC, TC are more than the values of SC, TC for Constant and Linear demands.
From **Table-2**, we observed the following points
(i) Increase in the value of parameter θ then the value of T is more than the value of T for Quadratic demand and less than the value of T for Constant demand.
(ii) The values of SC, TC are more than the values of SC, TC for Constant demand and less than the values of SC, TC for Quadratic demand.
From **Table-3**, we observed the following points

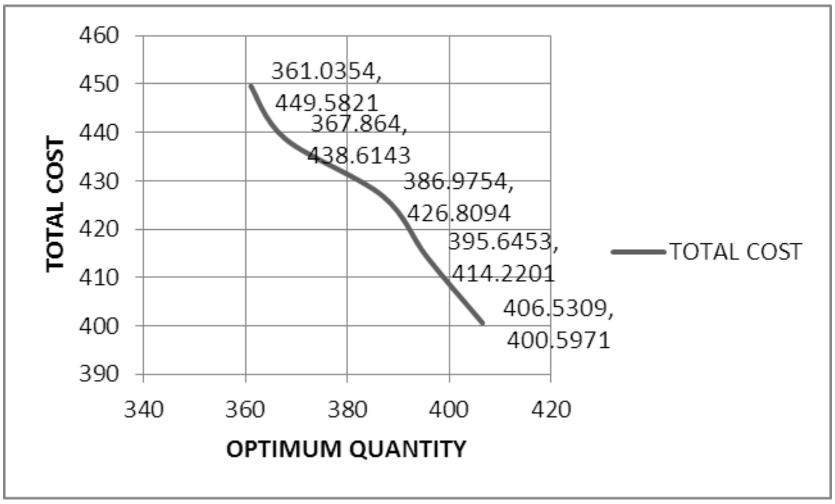
(i) Increase in the value of parameter θ then the value of T is very high than the values of T for other demands.
(ii) The value of Q is less than the value of Q for other demands.
(iii) The values of HC, DC, SC and TC are very low than the values of HC, DC, SC and TC for other demands.

From the above all tables, we observed the following points
(i) Increase in the value of parameter θ then the values of T, Q are decreased for all demands.
(ii) The value of HC is decreased for quadratic and linear demands and increased for constant demand.
(iii) The values of DC, SC and TC are increased for all demands.

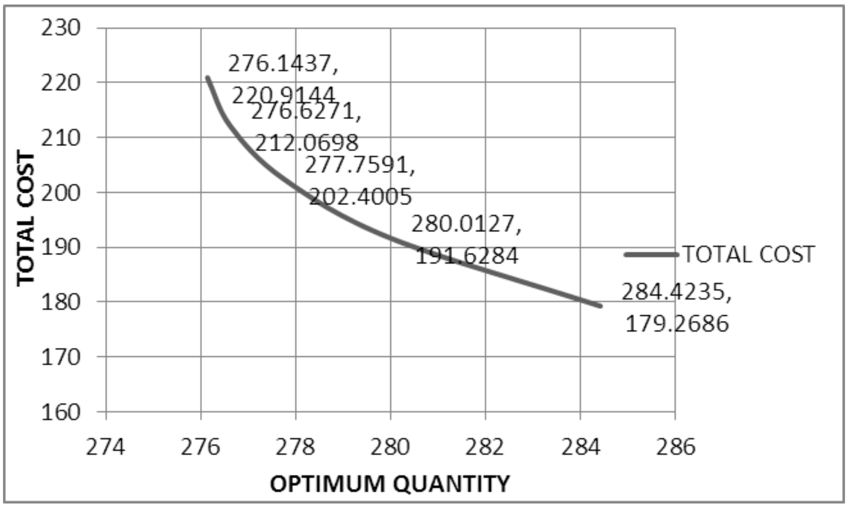
The following graph is drawn on the basis of the results of the above table-1:



The following graph is drawn on the basis of the results of the above table-2:



The following graph is drawn on the basis of the results of the above table-3:



VII. Conclusion and Future Research

In this paper, we have developed and analyzed the inventory model for deteriorating item with time dependent holding cost and different demand rates and established analytical solution of the model that minimize the total inventory cost. Here we considered that shortages are not allowed. From the sensitivity analysis we concluded that if the demand rate is taken as constant then the total cost is very low and cycle period is long. If the demand rate is taken as linear or quadratic then the total cost is high but cycle period is short. Finally, using higher powers of time in demand rate we may develop various inventory models. In future we shall develop several models with various considerations like fuzzy, stochastic variable in the demand including various deterioration rates, with and without shortages in the model.

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