K_H	=	adsorption constant for hydrogen	[cm³/mol]
k	=	reaction rate constant	[mol/cm³-cat·sec]
L	=	thickness of catalyst layer	[cm]
p_H	=	hydrogen pressure	[atm]
r	=	reaction rate	[mol/cm³-cat·sec]
X	=	dimensionless distance	[—]
x	=	distance from catalyst surface	[cm]
η	=	catalytic effectiveness factor	[—]
θ	=	porosity of catalyst	[—]
ρ_p	=	apparent density of catalyst	[g/cm³]
ρ_t	=	true density of catalyst	[g/cm³]
τ	=	tortuosity factor	[—]
ϕ_K	=	modulus	[—]

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ANALYSIS OF BATCH ELECTROKINETIC FILTRATION

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Filtration under constant D. C. electric field with constant hydraulic pressure, what is termed electrokinetic filtration, was discussed in this paper. Under these conditions it is considered that electrophoresis occurs in the slurry and causes less cake formation, and that electroosmosis occurs at the same time in the filter cake. Therefore, the flow rate of electrokinetic filtration is increased in comparison to filtration at the same hydraulic pressure. An equation taking account of both effects is presented. Experimental investigation of the characteristics of electrokinetic filtration is performed according to the theoretical equation. As a result, the flow rate of filtration is significantly increased for calcium carbonate slurry and white clay slurry. For instance, at the electric field of 30 volt/cm under hydraulic pressure of 163.2 G/cm², Ruth's filtration coefficient for calcium carbonate slurry was about 9.4 times as large as that of the filtration at the same hydraulic pressure, and Ruth's coefficient for white clay slurry was about 15 times at 12volt/cm and 163.2 G/cm². It is confirmed that the equation of electrokinetic filtration proposed in this paper is practically available.

Introduction

Interfacial electrokinetic phenomena such as electroosmosis, electrophoresis and electrodialysis have industrially been applied to refining, separation, electrodeposition, ore dressing by electric separation and dewatering[1].

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Recently, electrokinetic phenomena were applied to filtration by Moulik *et al.*[3,4] and Jorden[2]. It was observed by Moulik *et al.* that the cake formation was decreased and the flow rate of filtration was increased by the electrophoresis. It was investigated by Jorden to improve the clearness of filtrate by enlarging the interfacial electrokinetic potential and increasing the amount of adsorption of particles on the surface of filter medium. It was reported by the authors[6] that particle bed containing liquid was more effectively dewatered by electroosmosis caused by applying a D. C. electric field to the particle bed, and the electrokinetic filtration was an effective separation method for solid-liquid dispersion.

In the case of electrokinetic filtration, the flow rate of filtration is larger than that of conventional filtration. Electrophoresis which occurs in slurry reduces the rate of cake formation and at the same time the rate of permeation is increased by electroosmosis occurring inside the filter cake. In this paper, an equation for electrokinetic filtration, in which both effects are taken into account to the conventional equation for batch filtration of constant pressure, is derived on basis of the authors' work[6] on electroosmosis in particle bed. Adaptability of the equation for electrokinetic filtration is also examined experimentally.

1 Equation for Batch Electrokinetic Filtration

The equation for batch filtration of constant hydraulic filtration pressure ΔP_H is expressed as follows:

$$\frac{1}{A}\cdot\frac{dV}{d\theta}=\frac{\Delta P_H\cdot g_c}{\mu(R_{c0}+R_{m0})}=\frac{\Delta P_H\cdot g_c}{\frac{\alpha_0\mu\nu_0}{A}V+\mu R_{m0}} \tag{1}$$

$$R_{c0}=\frac{\alpha_0\nu_0}{A}V=\alpha_0\frac{W}{A} \tag{2}$$

where V is the volume of filtrate, θ the filtration time, A the filtration area, g_c the conversion factor, μ the viscosity of filtrate, R_{c0} and R_{m0} the filtration resistances of cake and medium respectively, α_0 the average specific resistance, ν_0 the dry cake mass per unit volume of filtrate and $W=\nu_0 V$ the mass of dry cake formed with the volume of filtrate V.

When hydraulic pressure ΔP_H and a D. C. electric field are applied at the same time to the slurry, the cake and the filter medium, Eq. (1) must be modified as follows. If the ζ-potential of suspended particles and that of filter medium are the same polarity, the direction of electric field can be selected as the filtration rate is increased. In the case of only filtration pressure ΔP_H, the mass of cake corresponding to filtrate volume V is $\nu_0 V$. In the electrokinetic filtration, the mass of filter cake is decreased because suspended particles in the slurry are moved by electrophoresis in the reverse direction of filtrate flow. When a certain strength of electric field is applied to the electrodes, suspended particles become stationary because the electrophoretic velocity becomes equal to the velocity of filtrate flow. This strength of electric field is defined as the critical value E_{cr}. If the strength of electric field E is equal to E_{cr} or greater than E_{cr}, the filter cake is not formed on the filter medium and therefore R_{c0} is zero. When E is smaller than E_{cr}, the dry cake mass formed on the filter medium W can be expressed as follows:

$$W=\nu V \tag{3}$$

When u is defined as the superficial velocity of filtrate in slurry and u_E the superficial electrophoretic velocity of particles, the dry cake mass per unit time is represented as

$$d(\nu V)/d\theta=\nu_0(u-u_E)A$$

and in the case of no electric field,

$$d(\nu_0 V)/d\theta=\nu_0 uA$$

Substituting the relation of $u_E=kE$[6] in which k is the electrophoretic coefficient and $u=u_{Ecr}$ into these equations, the following equation is obtained when ν and ν_0 are considered as constant during filtration.

$$\nu=\nu_0\left(\frac{E_{cr}-E}{E_{cr}}\right) \tag{4}$$

The filtration resistance of cake R_c may be expressed by the following equation.

$$R_c=\alpha\cdot\frac{W}{A}=\alpha\cdot\frac{\nu V}{A}=\alpha\nu_0\left(\frac{E_{cr}-E}{E_{cr}}\right)\frac{V}{A} \tag{5}$$

where α is the average specific filtration resistance in the electrokinetic filtration. Then the effect of electroosmosis generated in the filter cake and filter medium on the filtration rate must be considered. When the pressure difference at filter medium is ΔP_m and the filtration resistance of filter medium is R_m, the flow rate of filtrate is expressed as

$$\left.\begin{aligned}&\frac{1}{A}\cdot\frac{dV}{d\theta}=\frac{\Delta P_m\cdot g_c}{\mu R_m}=\frac{(\Delta P_{mH}+\Delta P_{mE})g_c}{\mu R_m}\\&\Delta P_m=\Delta P_{mH}+\Delta P_{mE}\end{aligned}\right\} \tag{6}$$

and the flow rate in the filter cake is obtained by using Eq. (5) as follows:

$$\left.\begin{aligned}&\frac{1}{A}\cdot\frac{dV}{d\theta}=\frac{\Delta P_c\cdot g_c}{\mu R_c}=\frac{(\Delta P_{cH}+\Delta P_{cE})g_c}{\mu\alpha\nu_0\left(\frac{E_{cr}-E}{E_{cr}}\right)\frac{V}{A}}\\&\Delta P_c=\Delta P_{cH}+\Delta P_{cE}\end{aligned}\right\} \tag{7}$$

Where ΔP_c is the pressure difference at the filter cake, ΔP_{mH} and ΔP_{cH} are the hydraulic pressure differences at the filter medium and the filter cake respectively, and ΔP_{mE} and ΔP_{cE} are the electroosmotic filtration pressure differences respectively. The total filtration pressure ΔP taking account of electroosmosis may be expressed by the following equation.

$$\begin{aligned}\Delta P\cdot g_c&=(\Delta P_m+\Delta P_c)g_c\\&=\{(\Delta P_{mH}+\Delta P_{mE})+(\Delta P_{cH}+\Delta P_{cE})\}g_c\\&=\{(\Delta P_{mH}+\Delta P_{cH})+(\Delta P_{mE}+\Delta P_{cE})\}g_c\\&=(\Delta P_H+\Delta P_E)g_c\end{aligned} \tag{8}$$

where ΔP_E is the total electroosmotic filtration pressure. Substituting Eqs. (6) and (7) into Eq. (8), the following equation is obtained:

$$\begin{aligned}\frac{d\theta}{dV}&=\frac{\mu\alpha\nu_0}{(\Delta P_H+\Delta P_E)g_cA^2}\left(\frac{E_{cr}-E}{E_{cr}}\right)V\\&+\frac{\mu R_m}{(\Delta P_H+\Delta P_E)g_cA}=\frac{2}{K_R}(V+V_0)\end{aligned} \tag{9}$$

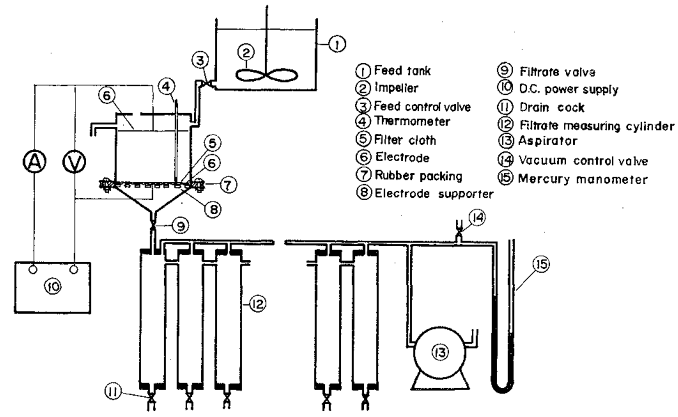

Fig. 1 Outline of experimental apparatus

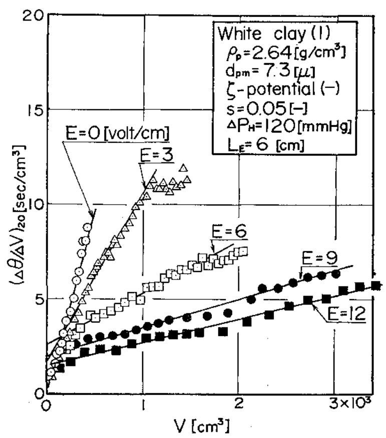

Fig. 3 Ruth's plots of electrokinetic filtration for white clay slurry (1)

where

$$\left.\begin{aligned} K_R&=\frac{2(\Delta P_H+\Delta P_E)g_cA^2}{\mu\alpha\nu_0\left(\dfrac{E_{cr}-E}{E_{cr}}\right)}=\frac{2(\Delta P_H+\Delta P_E)g_cA^2}{\mu\alpha\nu} \\ V_0&=\frac{R_mA}{\alpha\nu_0\left(\dfrac{E_{cr}-E}{E_{cr}}\right)}=\frac{R_mA}{\alpha\nu} \end{aligned}\right\} \tag{10}$$

Eq. (9) is the fundamental equation for batch electrokinetic filtration. It is considered that E_{cr} is constant under the condition of constant concentration of slurry, and that ΔP_E is constant under the constant voltage applied between both electrodes. Furthermore, in the case of incompressible cake, α is constant. Therefore K_R and V_0 become to be independent of filtration time θ under constant temperature of slurry, and a similar equation to Ruth's equation for batch filtration is obtained by integrating Eq. (9). E_{cr} is estimated by substituting the observed values of ν and ν_0 into Eq. (4). E_{cr} is generally a function of slurry concentration[5].

2 Experimental Apparatus and Procedure

Fig. 1 shows the outline of the experimental apparatus. The filter consists of an acrylic resin cylinder

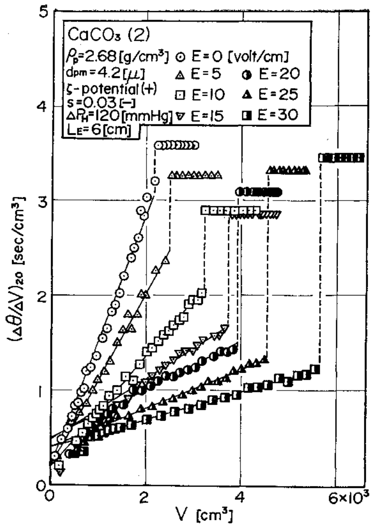

Fig. 2 Ruth's plots of electrokinetic filtration for calcium carbonate slurry (2) and results of permeation accompanied just after filtration

of 135 mm inside diameter, of which the lower part is made up of a filter cloth (polyethylene fiber, Pylen SP #7, plain fabrics) ⑤, a copper wire gauze electrode (the opening of 1.0 mm) ⑥ and a perforated supporting plate ⑧. The upper electrode ⑥ is a perforated carbon plate or perforated copper plate. The distance between the electrodes is 6 or 10 cm. The slurry concentration of 3 or 5wt% is suspended in the city water and sufficiently agitated in a feed tank ①. The outlet side of filtrate is decompressed by an aspirator ⑬ with closing valve ⑨, and filtration pressure is kept at 110 mmHg (149.6G/cm²) or 120 mmHg (163.2G/cm²) by controlling the valve ⑭. Slurry is fed into the filter from the feed tank and then a constant voltage is applied to the electrodes, and the experiment is immediately started by opening the valve ⑨. The filtrate volume, the slurry temperature and the electric current are recorded at a given time interval. Slurry materials used are calcium carbonate (1) (ρ_p=2.75 g/cm³, d_{pm}=5.8 μ, positive ζ-potential), calcium carbonate (2) (ρ_p=2.68 g/cm³, d_{pm}=4.2 μ), and white clay (1) (ρ_p=2.64 g/cm³, d_{pm}=7.3 μ, negative ζ-potential).

3 Experimental Results and Discussion

Figs. 2 and **3** show Ruth's plots with the strength of electric field as parameter for calcium carbonate slurry (2) and white clay slurry (1), respectively. The abscissas represent the filtrate volume V and the ordinates show $(\Delta\theta/\Delta V)_{20}$ corrected for 20°C, that is, reciprocal of the flow rate of filtration. The strength of electric field E is the value of the voltage drop between the electrodes divided by the distance between them. The correlations between $(\Delta\theta/\Delta V)_{20}$ and V are linear for calcium carbonate slurry (2) except for

Table 1 Characteristics of electrokinetic filtration for calcium carbonate slurry (1) (s=0.05, ΔP_H=149.6 G/cm²)

E [volt/cm]	K_R [cm⁶/sec]	V_0 [cm³]	ν [g-dry cake/cm³-filtrate]	$(\Delta P_H+\Delta P_E)/\alpha$ [(G/cm²)/(cm/g)]	α [cm/g]
0	2.55×10^3	1.49×10^3	0.0741	4.72×10^{-8}	3.17×10^8
0	2.37×10^3	1.63×10^3	0.0719	4.26×10^{-8}	3.51×10^8
5	3.08×10^3	1.60×10^3	0.0703	5.41×10^{-8}	
10	3.52×10^3	1.74×10^3	0.0685	6.03×10^{-8}	
15	4.88×10^3	2.28×10^3	0.0633	7.72×10^{-8}	
20	8.26×10^3	3.36×10^3	0.0614	$12.7\ \times10^{-8}$	

Table 2 Characteristics of electrokinetic filtration for calcium carbonate slurry (2) (s=0.03, ΔP_H=163.2 G/cm²)

E [volt/cm]	K_R [cm⁶/sec]	V_0 [cm³]	ν [g-dry cake/cm³-filtrate]	$(\Delta P_H+\Delta P_E)/\alpha$ [(G/cm²)/(cm/g)]	rate of permeation [cm³/sec]	ΔP_E [G/cm²]	α [cm/g]
0	1.47×10^3	1.18×10^2	0.1018	3.74×10^{-8}	0.300	0	4.36×10^9
5	2.17×10^3	2.39×10^2	0.0810	4.39×10^{-8}	0.344	19.3	4.16×10^9
10	3.39×10^3	3.05×10^2	0.0589	4.99×10^{-8}	0.403	31.2	3.90×10^9
15	5.88×10^3	$14.1\ \times10^2$	0.0572	8.41×10^{-8}	0.459	39.8	2.41×10^9
20	7.41×10^3	$17.8\ \times10^2$	0.0565	10.47×10^{-8}	0.489	52.1	2.06×10^9
25	$10.0\ \times10^3$	$20.0\ \times10^2$	0.0538	13.45×10^{-8}	0.500	84.3	1.84×10^9
30	$13.8\ \times10^3$	$26.2\ \times10^2$	0.0459	15.84×10^{-8}	0.520	97.9	1.65×10^9

Table 3 Characteristics of electrokinetic filtration for white clay slurry (1) (s=0.05, ΔP_H=163.2G/cm²)

E [volt/cm]	K_R [cm⁶/sec]	V_0 [cm³]	ν [g-dry cake/cm³-filtrate]	$(\Delta P_H+\Delta P_E)/\alpha$ [(G/cm²)/(cm/g)]
0	1.06×10^2	1.32×10	—	—
3	2.21×10^2	1.92×10^2	0.0514	2.84×10^{-9}
6	7.28×10^2	9.53×10^2	0.0627	11.41×10^{-9}
9	$13.2\ \times10^2$	1.32×10^3	0.0564	18.61×10^{-9}
12	$16.0\ \times10^2$	1.18×10^3	0.0566	22.64×10^{-9}

the initial period of filtration as shown in Fig. 2, and the similar result as this has been obtained for calcium carbonate slurry (1). The correlations for white clay slurry (1) are approximately linear as shown in Fig. 3. Fig. 2 shows the results of permeation processes under the same pressure as ΔP_H. The permeations are continued in succession to the electrokinetic filtration for 60 minutes in order to estimate the electroosmotic filtration pressure ΔP_E and the specific resistance of filtration α. These results show that Ruth's coefficient K_R and the fictitious volume of filtrate V_0 for these slurry are constant during filtration except for the initial period. Therefore, Eq. (9) is practically available for slurries used. The values of K_R and V_0 obtained at 20°C are listed in **Tables 1, 2** and **3** together with the observed values of ν and other factors.

Figs. 4 and **5** represent the correlations between $(\Delta P_H+\Delta P_E)/\alpha$ and E, and between ν and E, listed in Table 1 and 3. $(\Delta P_H+\Delta P_E)/\alpha$ for each value of E is calculated by substituting the values of K_R and ν into Eq. (10). It is shown in Figs. 4 and 5 that the relations between ν and E are linear for these slurries and it is confirmed that the relation between ν and E in Eq. (4) is reasonable. The following experimental equations are obtained from the results:

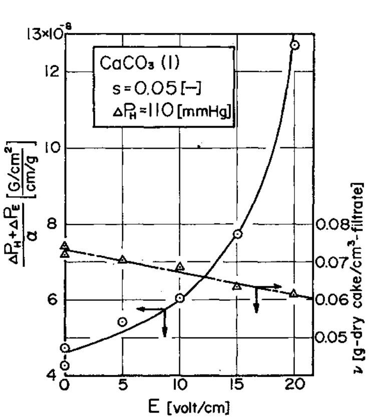

Fig. 4 Correlation between $(\Delta P_H+\Delta P_E)/\alpha$ and E, and relation between ν and E for calcium carbonate slurry (1)

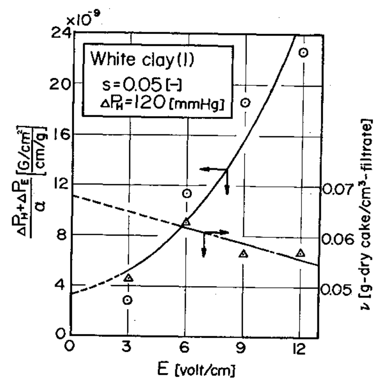

Fig. 5 Correlation between $(\Delta P_H+\Delta P_E)/\alpha$ and E, and relation between ν and E for white clay slurry (1)

$\nu=0.0732(1-E/123)$ for calcium carbonate slurry (1)

$\nu=0.0891(1-E/55.5)$ for calcium carbonate slurry (2)

$\nu=0.0677(1-E/66.6)$ for white clay slurry (1)

viz. E_{cr}=123, 55.5 and 66.6 volt/cm respectively.

Table 4 Values of a and n in $V=aW_p^n$ (V expressed in cm^3 and W_p in watt·hr)

$CaCO_3$(1)			$CaCO_3$(2)			White clay (1)		
E [volt/cm]	a [cm^3]	n [−]	E [volt/cm]	a [cm^3]	n [−]	E [volt/cm]	a [cm^3]	n [−]
5	938	0.615	5	1200	0.663	3	603	0.565
10	450	0.615	10	676	0.662	6	339	0.647
15	241	0.686	15	536	0.635	9	146	0.810
20	167	0.703	20	292	0.729	12	115	0.822
			25	202	0.774			
			30	177	0.768			

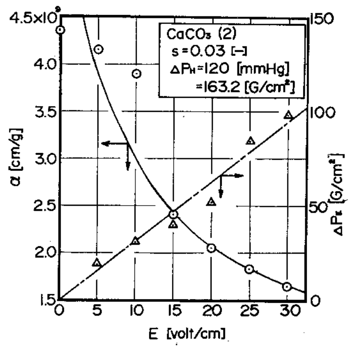

Fig. 6 Correlation between α and E, and relation between ΔP_E and E for calcium carbonate slurry (2)

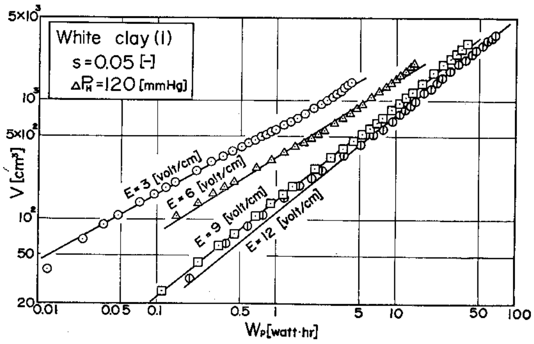

Fig. 7 Correlations between V and W_p with strength of electric field as parameter for white clay slurry (1)

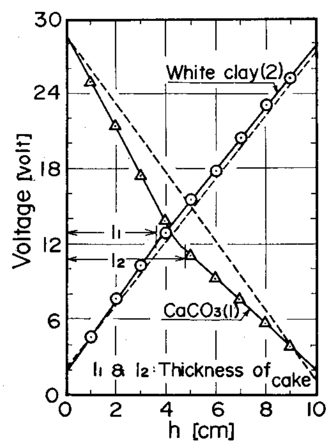

Fig. 8 Voltage distribution in liquid and cake for calcium carbonate (1) and white clay (2)

The degree of increase of $(\Delta P_H+\Delta P_E)/\alpha$ with increase of E is larger than the degree of decrease of ν with increase of E, and therefore the rate of filtration increases with E as shown in Eq. (9).

Fig. 6 shows the relation between α and E and that between ΔP_E and E for calcium carbonate slurry (2). ΔP_E is calculated from the equation of $\Delta P_E=\Delta P_H(q_E/q_H)$[6]. The value of q_H is the arithmetical mean between the flow rate of permeation under the permeation pressure of ΔP_H and that of electrokinetic filtration under both ΔP_H and ΔP_E just before permeation, because the structure of the filter cake is changed during permeation and becomes to be different from the structure of cake of electrokinetic filtration. The value of ΔP_E increases in proportion to E[6]. The degree of decrease in α becomes gradually to be less as E increases and α approaches a certain value. It is considered that α decreases with increasing E owing to electrophoresis acting on particles being formed as cake. Therefore, the effect of electrophoresis on ν and α in K_R is larger than that of electroosmosis on ΔP_E.

Fig. 7 shows the correlation between the filtrate volume V and the electric power consumption W_p with E as parameter for white clay slurry (1). The empirical equation

$$V=a\cdot W_p^n \tag{11}$$

is obtained for each value of E, and a and n in this equation are experimental constants. The value of a and n for these slurries are listed in **Table 4**. The value of n approaches unity as E increases. When L_E is the distance between electrodes and I is the electric current, the following equation is formed.

$$dW_p/d\theta=EL_EI \tag{12}$$

Substituting Eq. (12) into Eq. (9), the following equation is obtained.

$$\frac{dV}{dW_p}=\frac{K_R}{2(V+V_0)EL_EI} \tag{13}$$

If E is equal to E_{cr}, $2(V+V_0)/K_R$ becomes constant because V/K_R and V_0/K_R becomes zero and constant respectively, and then n is equal to unity under condition of constant value of I. A larger intercept value of a means that the appointed volume of filtrate is obtained by less electric power consumption. As K_R increases with increasing E for these slurries, there is the strength of electric field which is not economical in viewpoint of electric power consumption.

It is important to know the voltage distribution in the cake. The distribution is measured by using other equipment. The cake used in this experiment was made by sedimentation. The precipitates are calcium carbonate (1) and white clay (2) (ρ_p=2.69g/cm^3, d_{pm}=3.5 μ, negative ζ-potential). **Fig. 8** shows the distribution in which the broken line corresponds to city water. The drop in voltage are 3.71 volt/cm in the cake and 1.75 volt/cm in the clarified liquid for calcium carbonate (1), and the slope of 3 volt/cm involving contact potential differences for city water. The slope in the cake for white clay (2) is slightly larger than that of city water.

Conclusion

1) The fundamental equation (9) for batch electrokinetic filtration is proposed and formally coincides with Ruth's equation for batch filtration under constant pressure.

2) Electrokinetic filtration has two remarkable effects that i) the filtration rate is increased by the electroosmotic filtration pressure ΔP_E generated in the cake layer and the filter medium, and ii) the mass of filter cake formed on the filter medium is decreased by the electrophoresis in the slurry.

3) The electroosmotic filtration pressure ΔP_E is proportional to the strength of electric field. In the case of cake filtration the contribution of electrophoresis to the modified filtration coefficient K_R is larger than that of electroosmosis.

4) The relation between the filtrate volume V and the electric power consumption W_p is represented as $V=a\cdot W_p^n$. With increasing the strength of electric field, a is decreased and n is increased to unity.

Acknowledgment

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Nomenclature

A	=	filtration area	[cm^2]
a	=	experimental constant in Eq. (11)	[cm^3]
d_{pm}	=	median diameter of particles in slurry	[micron]
E	=	strength of electric field	[volt/cm]
E_{cr}	=	critical strength of electric field	[volt/cm]
g_c	=	conversion factor	[dyne/g-force]
K_R	=	modified Ruth's filtration coefficient	[cm^6/sec]
k	=	electrophoretic coefficient	[cm^2/volt·sec]
L_E	=	distance between electrodes	[cm]
n	=	experimental constant in Eq. (11)	[—]
ΔP_c	=	pressure drop in filter cake	[g-force/cm^2]
ΔP_E	=	electroosmotic filtration pressure	[g-force/cm^2]
ΔP_H	=	hydraulic filtration pressure	[g-force/cm^2]
ΔP_m	=	pressure drop in filter medium	[g-force/cm^2]
q_E	=	electroosmotic flow rate	[cm^3/sec]
q_H	=	hydraulic flow rate	[cm^3/sec]
R_c	=	cake resistance	[1/cm]
R_m	=	filter medium resistance	[1/cm]
s	=	slurry concentration	[g-solid/g-suspension]
u	=	superficial velocity of filtrate	[cm/sec]
u_E	=	superficial electrophoretic velocity of particles	[cm/sec]
V	=	filtrate volume	[cm^3]
V_0	=	fictitious filtrate volume	[cm^3]
W	=	dry cake mass	[g]
W_p	=	electric power consumption	[watt·hr]
α	=	average specific filtration resistance	[cm/g]
ζ	=	ζ-potential of particles	[volt]
θ	=	filtration time	[sec] or [hr]
μ	=	viscosity of filtrate	[g/cm·sec]
ν	=	dry cake mass per unit volume of filtrate	[g-solid/cm^3-filtrate]
ρ_p	=	density of solid particles	[g/cm^3]

〈Subscripts〉

c	=	value of filter cake
E	=	value of electroosmosis
H	=	value of hydraulic pressure
m	=	value of filter medium

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