

ANALYSIS OF BENDING RIGIDITY  
OF NONWOVEN FABRICS

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ANALYSIS OF BENDING RIGIDITY OF NON-WOVEN FABRICS

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## SUMMARY

Theoretical analysis of the bending rigidity of the nonwovens are carried out in this thesis. The effects of fiber diameter, fiber tensile modulus, fabric density, fabric thickness, and fiber orientation distribution function on the bending rigidity of the nonwovens are studied. A rectangular, planar unit cell model assuming complete freedom and no freedom of relative fiber motion are used in the analysis. For the case of complete freedom of relative fiber motion, moment equilibrium method is used and for the case of no freedom of relative fiber motion, energy approach is used. The bending rigidity is predicted for the spunbonded, Reemay (2.15 oz/sq. yd.), nonwoven fabric. The predicted and experimentally observed bending rigidities of the nonwoven fabric are compared. It is found that bending rigidity of the nonwoven fabric is accurately predicted by the rectangular unit cell model assuming no freedom of relative fiber motion.



## CHAPTER 1

### INTRODUCTION

Nonwoven fabrics may be defined as structures produced by bonding or the interlocking of fibers, or both, accomplished by mechanical, chemical, thermal or solvent means. The term nonwovens does not include fabrics which are woven, knitted, tufted or made by the wool felting processes.

The basic difference between these nonwovens and woven fabrics is the greater bending rigidity of nonwovens. This is due to the different mechanical principles of deformations in the two structures. The present analysis of bending rigidity of the nonwovens will be based on the contribution of the fiber component rather than on the influence of the bond.

Theoretical analysis of the effects of fiber diameter, fiber tensile modulus, fabric density, fabric thickness, and fiber orientation distribution function on the bending rigidity of nonwovens are carried out in this thesis. The bending rigidity in the two extreme cases of complete freedom and no freedom of individual fiber motion during fabric bending are analyzed. The predicted and experimentally observed bending rigidity of spunbonded Reemay (2.15 oz/sq. yd.), are compared. Also, the bending rigidity of the nonwoven is predicted for several fiber orientation distribution functions.

Much has been claimed for the future of the nonwoven industry, but little has been known about the design criteria for the nonwovens.

These analyses should provide a guide to effective utilization of fibers in nonwovens. It is also shown here how bending rigidity in the two extreme cases of complete freedom and no freedom of individual fiber motion during fabric bending can be predicted by a different approach.

<sup>1</sup>  
Cox carried out an analysis on the effect of orientation of fibers on the stiffness and strength of paper and other fibrous materials. It was shown that these effects might be represented completely by the first few coefficients of the distribution function for the fibers in respect of orientation, the first three Fourier coefficients for a planar matrix and the first fifteen spherical harmonics for a solid medium. He hypothesized that the behavior of a planar material tested at various angles could be represented by a composition of sets of parallel fibers in appropriate ratios. The analysis also showed that the effect of short fibers might be represented merely by use of a reduced value for their modulus of elasticity.

<sup>2</sup>  
Backer and Petterson compared the mechanical behavior of woven and nonwoven fabrics. In their study, they outlined the principal mechanisms by which woven fabrics deform and recover in manufacture and in end usage. They formulated parallel sequence of events in the distortion process for nonwovens and showed how these actions influence the ability of nonwovens to fit smoothly on three-dimensionally curved surfaces, to hang and drape freely, and to recover from bends and tensile extension. It was demonstrated how knowledge of fiber properties and web structure can be used effectively to predict web behavior.

<sup>3</sup>  
Abbot and Coplan analyzed the fabric flexure, relating fabric

behavior to fiber properties through the geometry of the structure, particularly as it is affected by fiber interactions. They developed equations for the flexural rigidity of the fabric, based on the degree of fiber association within the yarn and the effect of the yarn intersections within the fabric in producing a cyclic fluctuation of rigidity along the length of the yarn.

Petterson and Backer<sup>4</sup> found out that the orthotropic theory developed for rigid materials predicted well the behavior of selected flexible textile-fiber nonwoven structures. The predicted properties agreed closely to the measured properties of the test samples. The orthotropic theory was useful in the characterization of nonwoven structures as engineering materials. Also the prediction of angular properties of nonwoven based on knowledge of principal-direction properties was made possible with this theory.

Freeston and Platt<sup>5</sup> presented an analysis on the mechanisms determining the bending rigidity of nonwovens. Equations were developed to predict the bending rigidity in the two extreme cases of complete freedom and no freedom of fiber motion during fabric bending. Effects of fiber orientation, fiber bending, and torsion were considered in the case of complete freedom of fiber motion. Effects of fiber orientation, tension, and compression were considered in the case of no freedom of fiber motion. Different methods of production were shown for making textile nonwovens more flexible.

In this thesis, a rectangular, planar unit cell model assuming complete freedom and no freedom of relative fiber motion is used in the prediction of bending rigidity of the nonwoven fabric. The total number

of fibers in the unit cell is determined by utilizing the denier of the fiber and denier of the rectangular unit cell itself. Instead of actually counting the number of fibers, the fiber orientation distribution function of the nonwoven fabric is determined by another method. Also, energy approach is used in the prediction of bending rigidity of the nonwoven fabric in the case of no freedom of relative fiber motion.

## CHAPTER 2

## THEORETICAL ANALYSIS

Point of Interest

One of the basic differences between paper like nonwovens and woven fabrics is the greater bending rigidity of the nonwovens. For most textile fabrics used for apparel purpose, a low level of bending rigidity is required. So if we want to design a nonwoven fabric with low bending rigidity to replace a woven cloth for apparel uses, the mechanisms of deformation during bending of woven and nonwoven fabrics must be known. If we compare the mechanisms of deformation during bending, the following differences are found.

## Bending deformation mechanisms for woven fabrics:

1. Local crimp interchange
2. Yarn buckling at the bend
3. Yarn flattening
4. Yarn compaction
5. Fiber slippage
6. Fiber motion
7. Fiber extension
8. Fiber bending

## Bending deformation mechanisms for nonwovens:

1. Fiber motion
2. Fiber straightening in the fabric plane
3. Bond rotation

4. Fiber buckling at the bend
5. Fiber bending out of the fabric plane
6. Bond bending

The low bending rigidity of woven fabric is due to the high degree of freedom of individual fiber motion during bending, and the high rigidity of the nonwoven is due to less freedom of individual fiber motion. The fibers in the nonwovens are actually bonded together at fiber cross-overs during manufacturing.

#### The Analysis

In this thesis, the bending rigidities in the two extreme cases of complete freedom and no freedom of individual fiber motion during fabric bending are predicted. The former case considers the effects of fiber orientation distribution and fiber bending. The latter considers fiber orientation distribution and tensile and compressive fiber strains.

In order to determine the strains in the fibers during bending, the nonwoven fabric is assumed to be composed of a large number of individual, planar, rectangular unit cells (Figure 1) with boundaries parallel and perpendicular to the axis of curvature  $\vec{k}$ . The boundaries AB and CD of the unit cell remain straight and parallel during fabric bending. By choosing a unit cell of sufficient size, the variation in the number and orientation of fibers from point to point in the unit cell can be neglected.

To facilitate the analysis, the fibers are assumed to be straight and continuous. In addition, the following assumptions are made:

1. The fibers in the nonwovens are of circular cross-section.

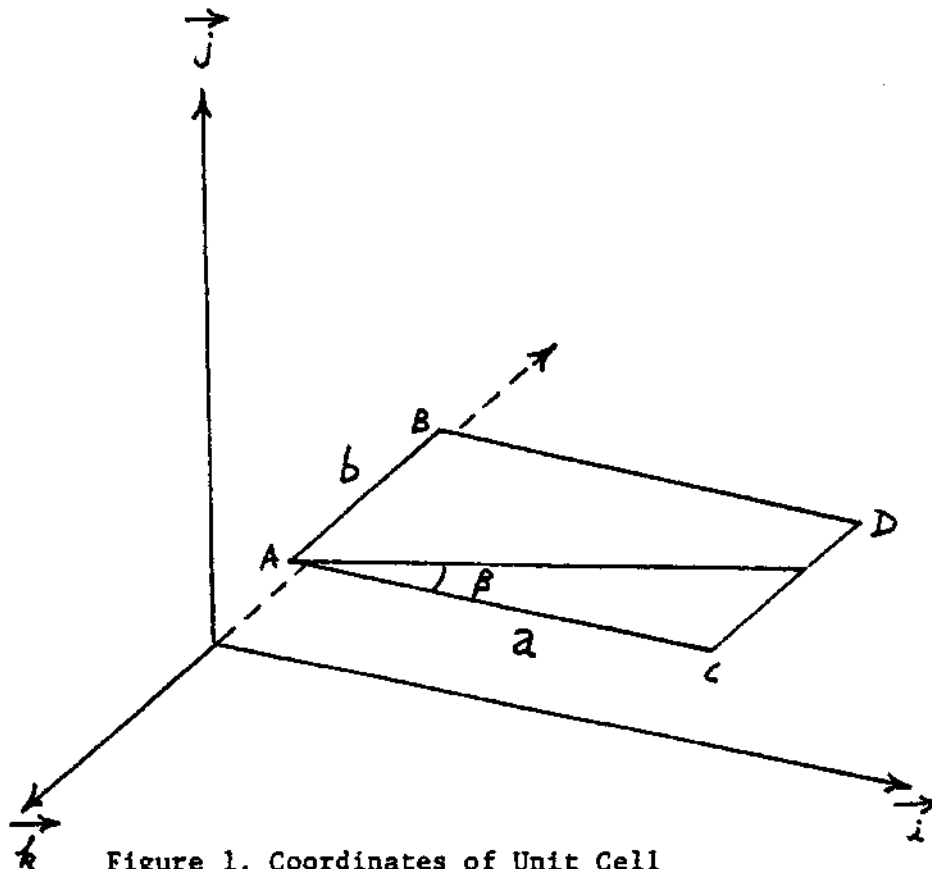


Figure 1. Coordinates of Unit Cell

2. The diameter of the fibers is constant and uniform along its length.
3. The fiber diameter is small compared to fiber length.
4. The fiber diameter and fabric thickness are small compared to the radius of curvature.
5. The fibers exhibit a complete elastic behavior.
6. The fiber does not buckle during bending.
7. The nonwoven fabric is of uniform thickness.

#### Determination of Total Number of Fibers in a Unit Cell

The orientation of any fiber before bending is defined by the angle it makes with the  $\vec{i}$  axis (see Figure 1). The nonwoven fabric is assumed to be composed of straight and continuous fibers with different

angles of orientation  $\beta$ , and fiber-to-fiber normal distance  $c$ . It is further assumed that the fiber-to-fiber normal distance  $c$  is constant between the fibers of the same orientation angle  $\beta$  (see Figure 2).

Consider a rectangular unit cell of length  $a$  (measured parallel to the  $\vec{i}$  axis) and width  $b$  (see Figure 3). The fiber-to-fiber distance  $\ell$  (measured parallel to the  $\vec{i}$  axis) is:

$$\ell = \frac{c}{\sin \beta} \quad \dots \dots \dots (1)$$

Also, the distance  $w$  is given by

$$w = \frac{b}{\sin \beta} \quad \dots \dots \dots (2)$$

Therefore, the total fiber length  $L(\beta)$  in a cell of width  $b$  and unit length measured along the  $\vec{i}$  axis with an orientation angle  $\beta$  is:

$$L(\beta) = \frac{w}{\ell} \quad \dots \dots \dots (3)$$

Substituting Equations (1) and (2) into Equation (3),

$$\begin{aligned} L(\beta) &= \frac{b}{\sin \beta} / \frac{c}{\sin \beta} \quad \dots \dots \dots (4) \\ &= \frac{b}{c} \end{aligned}$$

If, however, the unit cell is of length  $a$ , Equation (4) becomes



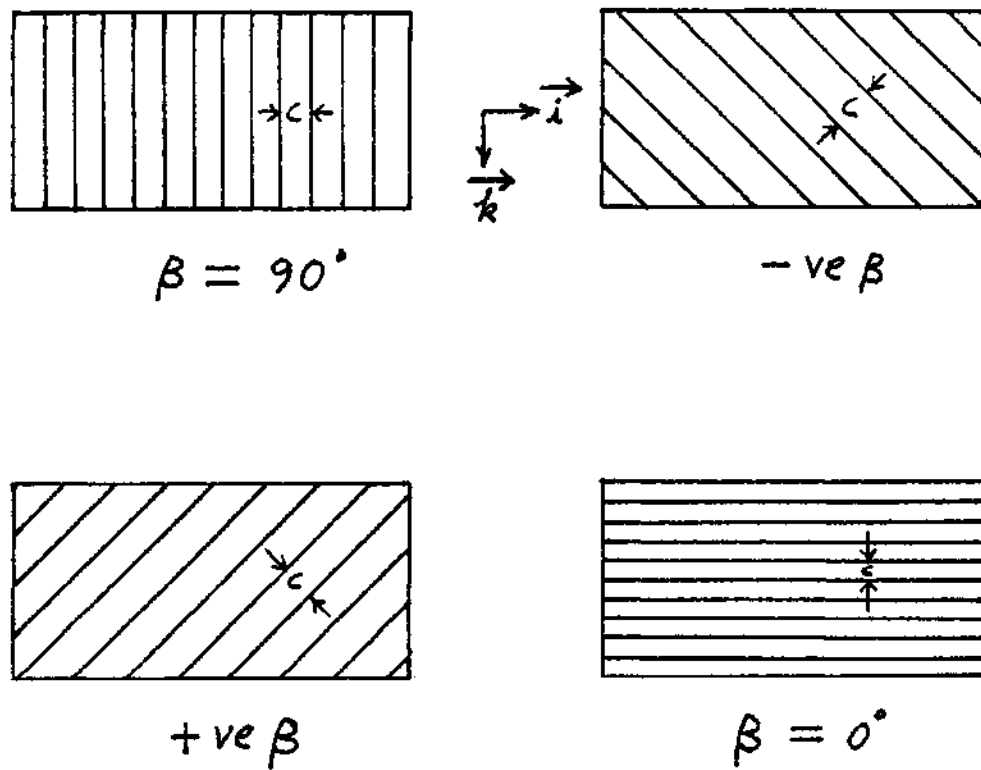


Figure 2. Different Orientation Angles and Fiber-to-Fiber Normal Distances of Fibers in the Unit Cell

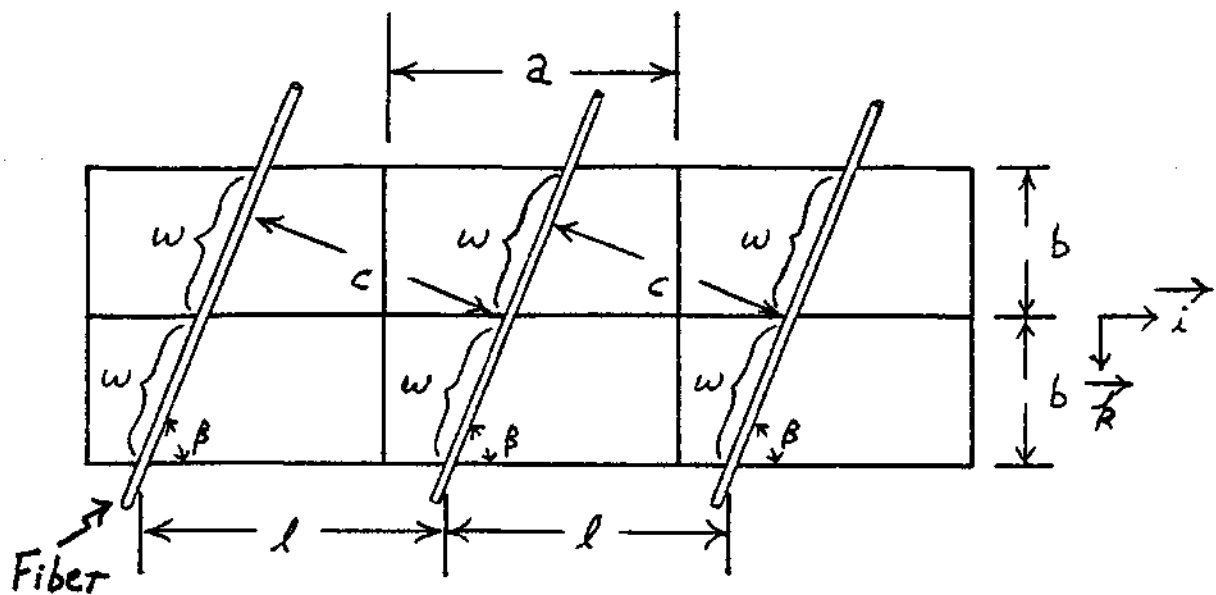


Figure 3. Relationship between Fibers of Same Orientation Angle in the Unit Cell

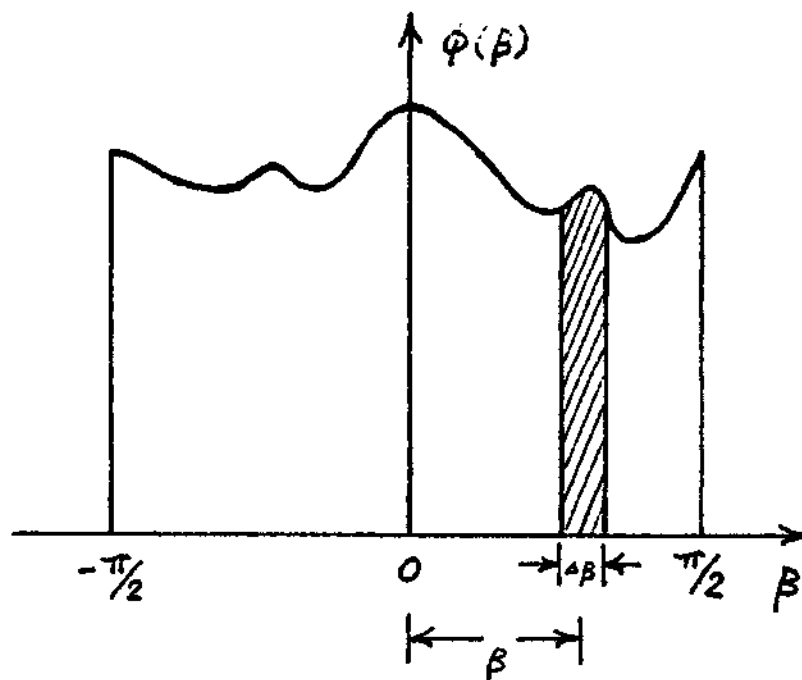


Figure 4. Generalized Fiber Orientation Distribution Function  $\phi(\beta)$

$$L(\beta) = \left( \frac{b}{c} \right) a \dots \dots \dots (5)$$

The generalized fiber orientation distribution function  $\phi(\beta)$ , shown in Figure 4, is defined mathematically as follows:

(1) Probability of finding a fiber with orientation angle of  $\beta \pm \Delta\beta/2$  is  $\phi(\beta)\Delta\beta$ , and,

(2) the mathematical condition of  $\int_{-\pi/2}^{\pi/2} \phi(\beta) d\beta = 1$  is satisfied.

Therefore, for a given fiber orientation distribution function  $\phi(\beta)$ , the total number of fibers  $N(\beta)$  with orientation angle  $\beta \pm \Delta\beta/2$  in the unit cell is given by:

$$N(\beta) = N_f \cdot \phi(\beta)\Delta\beta \dots \dots \dots (6)$$

Where  $N_f$  is the total number of fibers in the unit cell.

Thus the total length  $L$  of all fibers in the unit cell is given by

$$\begin{aligned} L &= \sum L(\beta) \\ &= \sum N_f \cdot \phi(\beta)\Delta\beta \cdot w \dots \dots \dots (7) \end{aligned}$$

If  $D$  is the denier of the unit cell of width  $b$  and length  $a$ , and  $d$  is the fiber denier, the total length of fibers in the unit cell is thus

$$L = \frac{D}{d} \cdot a \quad \dots \dots \quad (8)$$

Combining Equations (7) and (8), we have

$$\begin{aligned} L(\beta) &= \sum N_f \cdot \phi(\beta) \Delta\beta \cdot w \quad \dots \dots \quad (9) \\ &= \frac{D}{d} \cdot a \end{aligned}$$

So, for a given orientation distribution function and cell width  $b$ , the only unknown parameter in the above Equation (9) is  $N_f$ --total number of fibers in the unit cell.

#### Bending Rigidity of Nonwoven Fabric with Complete Freedom of Fiber Motion

The nonwoven fabric is said to have complete freedom of relative fiber motion if no bonding is assumed to occur at fiber cross-overs. Therefore, the nonwoven fabric can be considered as made up of continuous filaments with complete freedom of motion. Since during bending, the unit cell forms a segment of the surface of a cylinder, the fibers in the bent unit cell are assumed to follow a cylindrically helical path. Also as the fiber diameter and fabric thickness are assumed small compared to the radius of curvature, the neutral axis of bending of the nonwoven fabric is taken to coincide with the geometric center line of the fabric.

When considering the extreme case of complete freedom of individual fiber motion during bending, the effect of fiber orientation distribution and fiber bending moment are considered. Considering the bent unit cell to behave as an isotropically elastic thin plate, its moment equilibrium equation is:

$$M_b = (EI)_c \cdot K \quad \dots \dots \dots (10)$$

Where  $(EI)_c$  is the cross sectional bending rigidity of the unit cell and  $K$  is its curvature.

The component parallel to the  $\vec{k}$  axis of the bending moment acting on a fiber in the unit cell with the orientation angle  $\beta$  is

$$\begin{aligned} (M_f)_\beta &= (EI)_f \frac{\cos^2 \beta}{\rho} \cdot \cos \beta \quad \dots \dots \dots (11) \\ &= (EI)_f \cdot \cos^3 \beta / \rho \end{aligned}$$

Where  $(EI)_f$  is the bending rigidity of the fiber,  $\rho$  is the radius of curvature of the bent unit cell.

If  $N(\beta)$  is the total number of fibers in the unit cell with orientation angles within the range  $\beta \pm \Delta\beta/2$ , the sum of the  $\vec{k}$  components of the bending moments of all such fibers is given by:

$$M_t(\beta) = N(\beta) \cdot (EI)_f \cdot \cos^3 \beta / \rho \quad \dots \dots \dots (12)$$

The total bending moment  $M_c$  of all the fibers in the unit cell is therefore

$$\begin{aligned} M_c &= \sum M_t(\beta) = \sum N(\beta) \cdot (EI)_f \cdot \cos^3 \beta / \rho \quad \dots \dots \dots (13) \\ &= \frac{(EI)_f}{\rho} \cdot \sum N(\beta) \cdot \cos^3(\beta) \end{aligned}$$

The bending rigidity  $(EI)_c$  of the unit cell is thus:

$$(EI)_c = \frac{M_c}{K} = M_c \cdot \rho \quad \dots \dots \dots (14)$$

$$= (EI)_f \cdot \sum N(\beta) \cos^3(\beta)$$

The above equation gives the bending rigidity of the unit cell assuming complete freedom of relative fiber motion and neglecting lateral contraction.

#### Bending Rigidity of Nonwoven Fabric with No Freedom of Fiber Motion

In the above analysis, the fibers are assumed to have complete freedom of relative motion. There is no restriction of any kind to the fiber motion. But in the actual case, the fibers are bonded together at fiber cross-overs in the manufacture of the nonwoven to achieve sufficient strength. As a result, the fiber motion is restricted and the bending rigidity of the nonwovens is greatly increased. The other extreme case of no freedom of relative fiber motion is investigated in order to find out which of the two cases is a closer approximation of the actual bending behavior of the nonwoven fabric.

In pure bending of elastic, completely bonded nonwovens, as shown in Figure 5, the rectangular unit cell is considered to behave like an isotropically elastic beam. Plane sections through the unit cell taken normal to its  $\vec{i}$  axis remain plane after the unit cell is subjected to bending around the  $\vec{k}$  axis. The strain in the fibers in the bent unit cell is going to vary linearly, or directly, as their respective distances from the neutral surface. The strain  $\epsilon_i$  in the  $\vec{i}$  direction of any layer in the bent unit cell is given by

$$\epsilon_i = y/\rho \quad \dots \dots \dots (15)$$

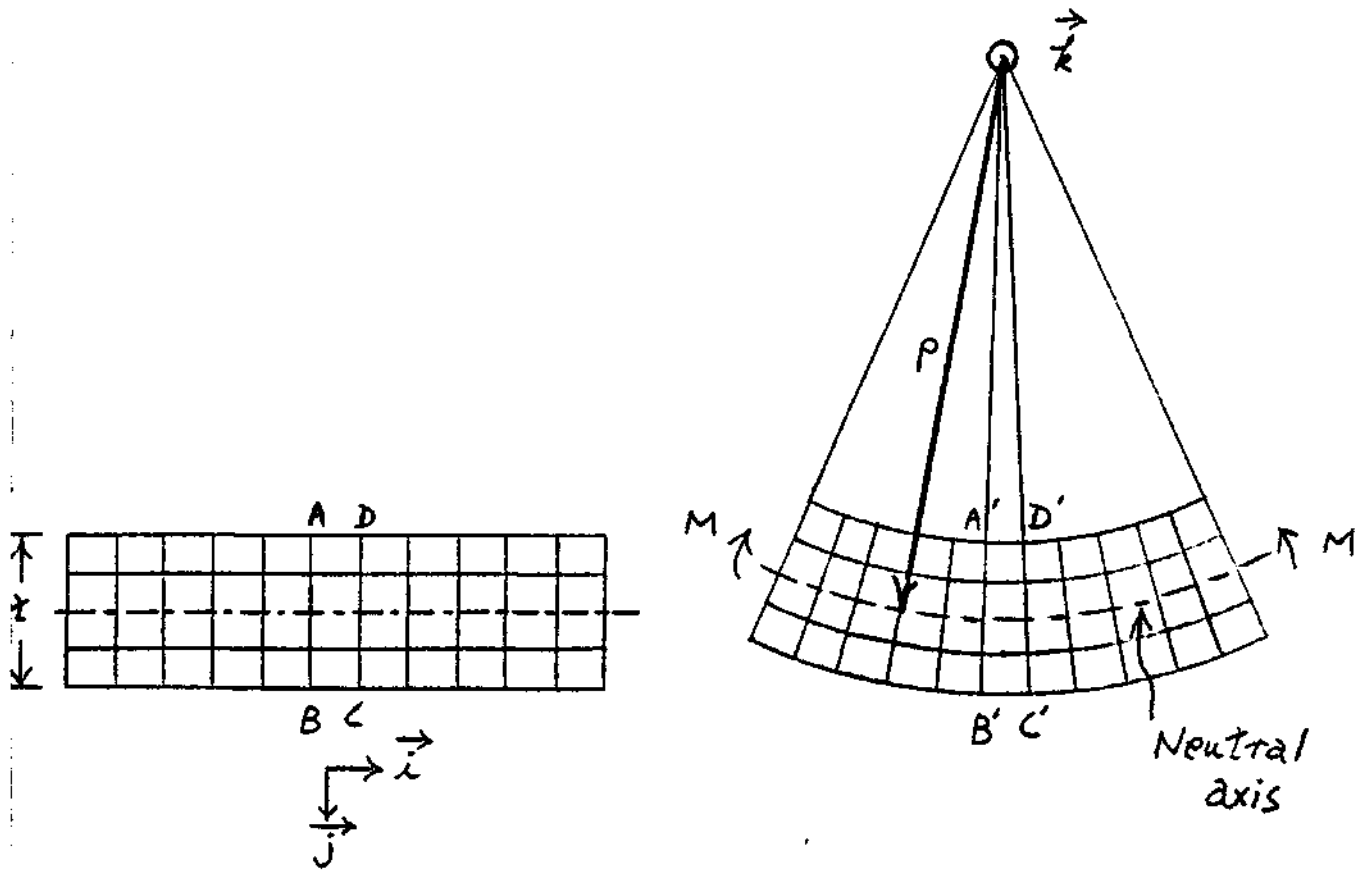


Figure 5. Deformation of a Nonwoven Fabric Segment in Bending

Where  $y$  is the distance of the layer from the neutral surface and  $\rho$  is the radius of curvature of the bent unit cell. The strain  $\epsilon_f$  of a fiber with an orientation angle  $\beta$ , located in such a layer of the unit cell can be shown (see Figure 6) to be

$$\epsilon_f = \epsilon_1 \cdot \cos^2 \beta \quad \dots \dots \dots (16)$$

The strain energy for a fiber located at a distance  $y$  from the

neutral surface and with orientation angle  $\beta$  is thus

$$(S.E.)_{f\beta} = 1/2 E_f \cdot \Sigma_f^2 \cdot \pi r_f^2 \cdot w_{f\beta} \quad \dots \quad (17)$$

Where  $E_f$  is the tensile modulus of the fiber,  $w_{f\beta}$  is the length of the fiber,  $r_f$  is the radius of the fiber. Substituting Equation (15) and Equation (16) into Equation (17) results in

$$(S.E.)_{f\beta} = 1/2 E_f \cdot \frac{y^2 \cos^4 \beta}{\rho^2} \cdot \pi r_f^2 \cdot w_{f\beta} \quad \dots \quad (18)$$

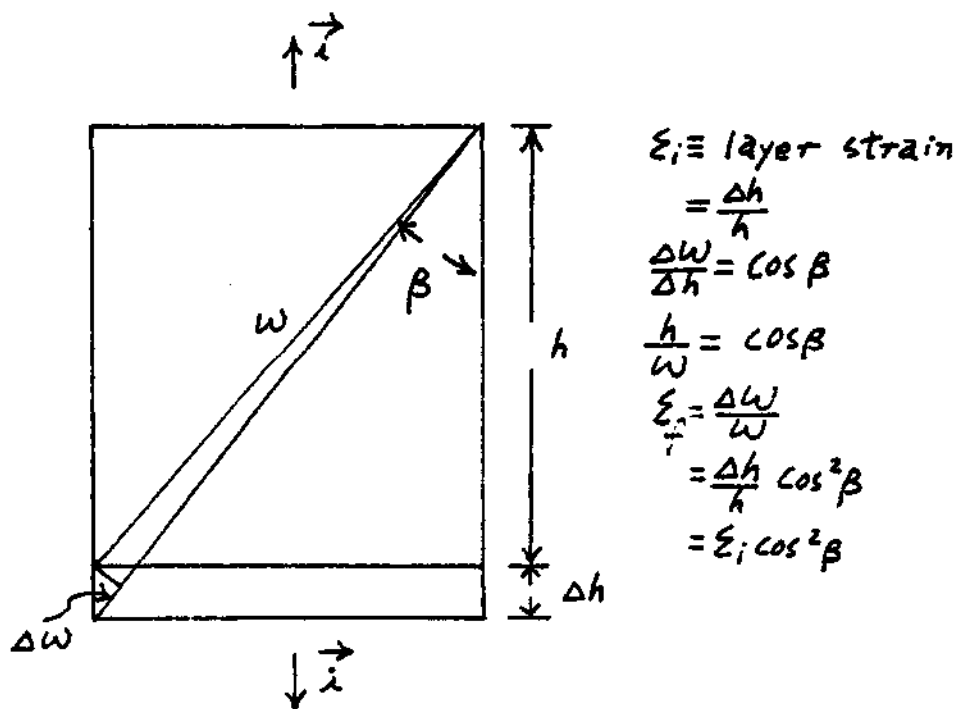


Figure 6. Strain  $\epsilon_f$  of a Fiber with an Orientation Angle  $\beta$



If  $N(\beta)_y$  is the total number of fibers in the unit cell with orientation angle within the range  $\beta \pm \Delta\beta/2$  and at distance  $y$  from the neutral axis, the total strain energy of these fibers is given by

$$(S.E.)_{\beta y} = 1/2 E_f \cdot \frac{y^2 \cos^4 \beta}{\rho^2} \cdot \pi r_f^2 \cdot w_{f\beta} \cdot N(\beta)_y \quad (19)$$

Thus, the total strain energy of all the fibers at a distance  $y$  from the neutral axis is given by

$$(S.E.)_y = \sum 1/2 E_f \cdot \frac{y^2 \cos^4 \beta}{2} \cdot \pi r_f^2 \cdot w_{f\beta} \cdot N(\beta)_y \quad \dots (20)$$

The nonwoven fabric is assumed to be composed of many layers of fibers of equal thickness, which are located at distances  $y_1, y_2, y_3, y_4, \dots, y_{n-1},$  and  $y_n$  from the neutral axis. The total strain energy of the unit cell is thus:

$$(S.E.)_c = \sum_{i=1}^n (S.E.)_{y_i} \quad \dots (21)$$

Also, the strain energy of the bent unit cell is:

$$(S.E.)_c = 1/2 (EI)_c \cdot K^2 \cdot a \quad \dots (22)$$

Where  $(EI)_c$  is the bending rigidity of the unit cell,  $K$  is its curvature and  $a$  is the length of the unit cell. Combining Equations (21)

and (22) gives the bending rigidity of the bent unit cell as follows

$$(EI)_c = 2 \sum_{i=1}^n (S.E.)_{y_i} / K^2 a \quad \dots \dots \dots (23)$$

Application of Theoretical Analysis to Nonwoven Fabrics with Different Fiber Orientation Distribution Functions

In order to find out which of the two freedom of fiber motion extremes is a closer approximation of the actual bending behavior of the nonwoven fabric, the predicted and experimentally measured bending rigidities of spunbonded nonwoven are compared.

Spunbonded products were developed in 1965 by Du Pont. Basically, these products are nonwoven fabrics produced by combining the spinning of fibers and formation of the sheet together. These fabrics are sheet structures of continuous filaments of fibers bonded at filament cross-overs. There are no "free fiber ends" in the structure.

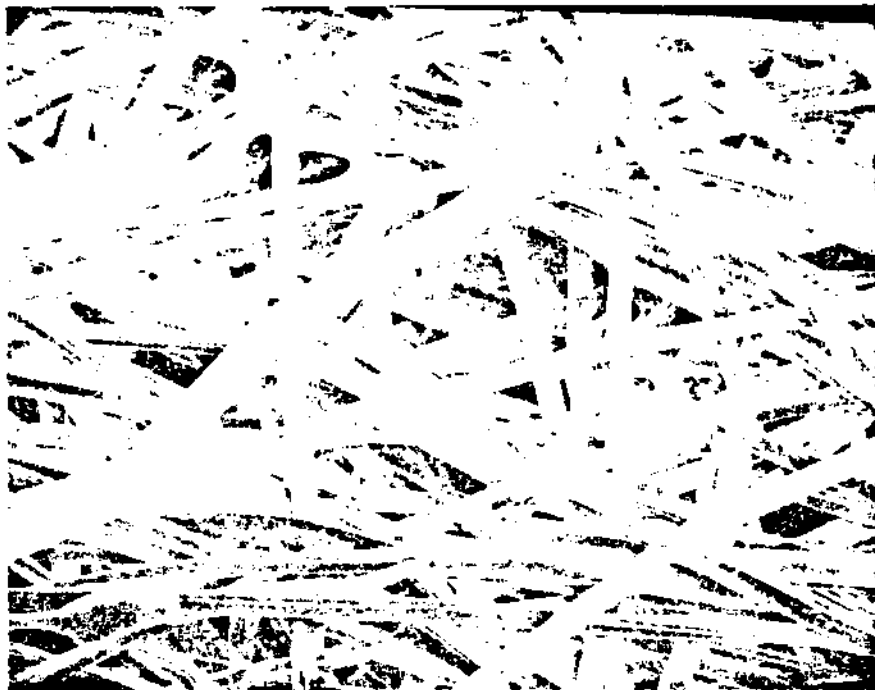
Comparison between the predicted and experimentally observed bending rigidities is made on Reemay (see Figure 7), a spunbonded nonwoven fabric consisting of a web of continuous polyester filaments of uniform cross-sections, similar to conventional polyester fibers in properties and appearance.

In order to predict the bending rigidity of the nonwoven fabric for both cases of complete freedom and no freedom of relative fiber motion, the fiber diameter, fiber tensile modulus, fabric thickness, and fiber orientation distribution function must be determined first.

The diameter of the fibers in the nonwoven was measured by means of the MINI-SEM Scanning Electron Microscope. The average diameter of



(400X)



(100X)

Figure 7. Structure of Reemay (2.15 oz/sq.yd)

the fibers was found to be 0.0010856 in.

The tensile modulus  $E_f$  of the fibers was determined by the tensile test of the fibers removed from the nonwoven fabric. Fibers of gauge length 1/2 in. were tensile tested at cross head speed of 0.2 in./min. by the Instron tensile tester. Modulus at 1% elongation was found to be 262528.7 psi.

The fabric thickness  $t$  was measured by the Frazier Compressometer. A pressure of 0.50 lb/in<sup>2</sup> was applied on the nonwoven fabric during the measurement of the thickness. Five layers of nonwoven fabric were measured together and the average thickness of a single layer was found to be 0.0108 in.

Assuming the cross-section of the individual fibers to be circular, the moment of inertia  $I_f$  of the fiber is given by

$$I_f = \frac{\pi d_f^4}{64} \quad \dots \dots (24)$$

Where  $d_f$  is the diameter of the fiber in the fabric. Since  $d_f$  was found to be 0.0010856 in.,  $I_f$  is therefore equal to  $6.8179 \times 10^{-14}$  in<sup>4</sup>. The bending rigidity of the fiber  $(EI)_f$  is therefore equal to  $1.7899 \times 10^{-8}$  lb-in<sup>2</sup>.

The denier of the fibers in the nonwoven fabric can be determined by the following equation, if the density of the fiber is known.

$$\text{Denier} = \frac{900,000 \cdot \text{Density} \cdot \pi d^2}{4} \quad \dots \dots (25)$$

Where  $d$ , the diameter of the fiber, is expressed in centimeter. The

density for polyester fiber is  $1.38 \text{ gm/cm}^3$ , so the denier of the fiber was determined to be 7.417 den.

The denier of the rectangular unit cell of  $1/2$  in. wide and 1 in. long must be determined in order to find the total number of fibers in the unit cell (see Figure 8). If the weight of the unit cell of the above dimensions is found to be  $G$  gm, the denier of this unit cell is given by the following equation

$$\text{Den} = \frac{900,000 \cdot G}{2.54} \dots \dots \dots (26)$$

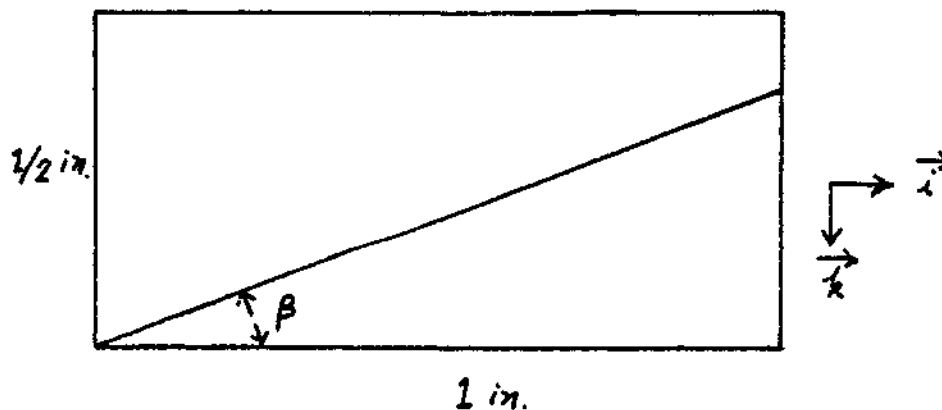


Figure 8. Dimension of Unit Cell

The weight per square yard of the nonwoven is 2.15 oz. Therefore the denier of the unit cell of  $1/2$  in. wide and 1 in. long was found to be 8339.6558 den. So from Equation (8), the total length of

fibers in the unit cell (1in. X 1/2 in.) was 1124.40 in.

At this point, sufficient information was acquired to predict the bending rigidity of the nonwoven with any fibre distribution function. Instead of actually finding the fiber orientation distribution function of the nonwoven itself, four different fiber orientation distribution functions were assumed. The fibers were assumed to be distributed evenly in every direction (random), parallel laid, cross laid, and following an elliptical orientation distribution function. These different fiber orientation distribution functions are shown on the polar diagram of Figure 9.

The bending rigidity of the unit cell (1 in. X 1/2 in.) in the case of complete freedom of relative fiber motion at various orientation angles are predicted for the above fiber orientation distribution functions according to Equation (14). The results were plotted on the polar diagrams of Figure 10.

Since the fabric thickness and fiber diameter were determined as 0.0108 and 0.0010856 inches respectively, the fabric was most appropriately assumed to be made up of ten layers of fibers, five layers above and below the neutral surface. It was also assumed that each layer consists of equal number of fibers and the fiber orientation distribution function of each layer is the same. The bending rigidity of the unit cell (1 in. X 1/2 in.) in case of no freedom of relative fiber motion are predicted for the above fiber orientation distribution functions according to Equation (23). The results for the assumption that the fabric was made up of ten layers of fibers were plotted on the polar diagrams of Figure 11.

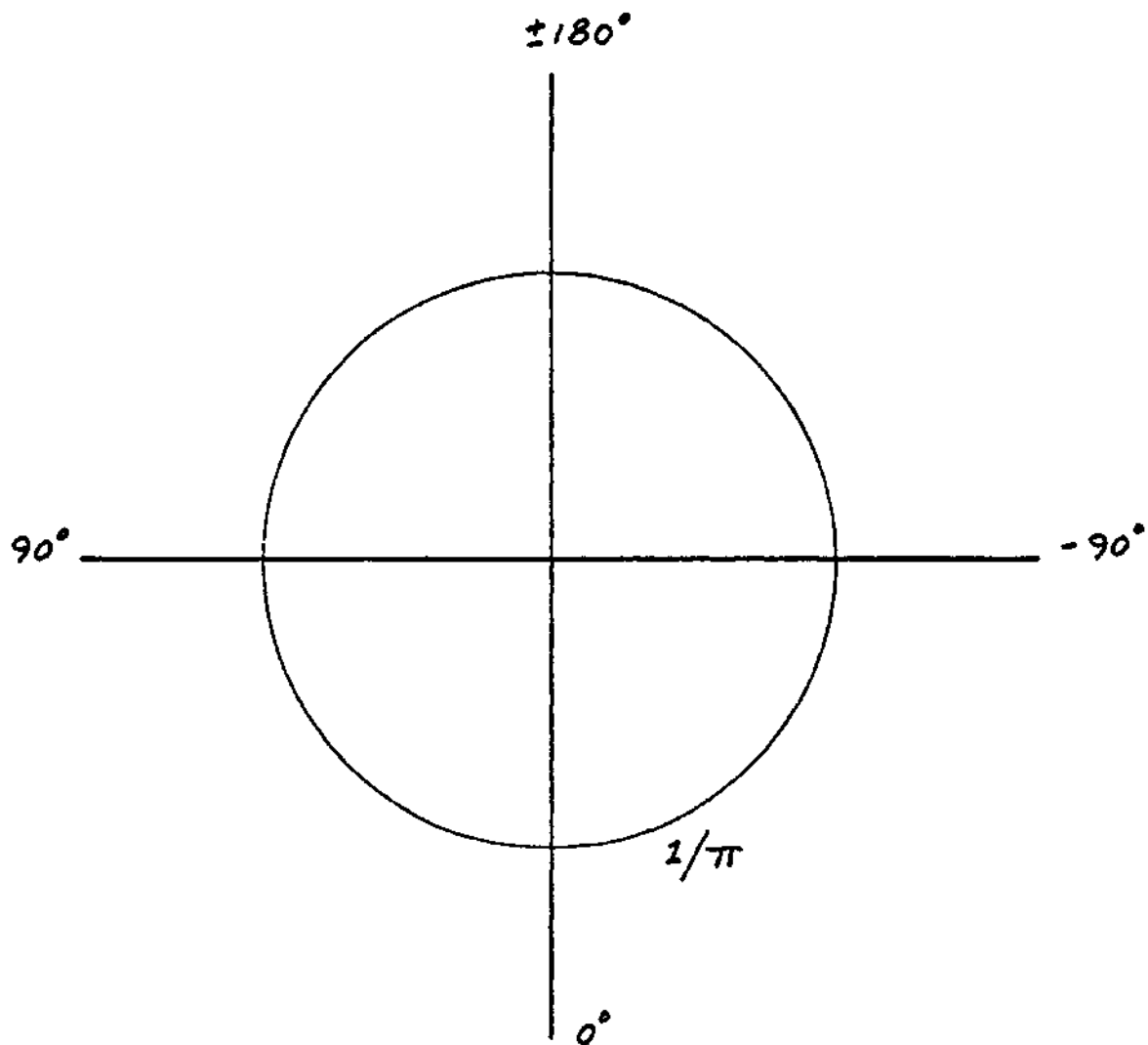


Figure 9(a). Fiber Orientation Distribution Function--Random

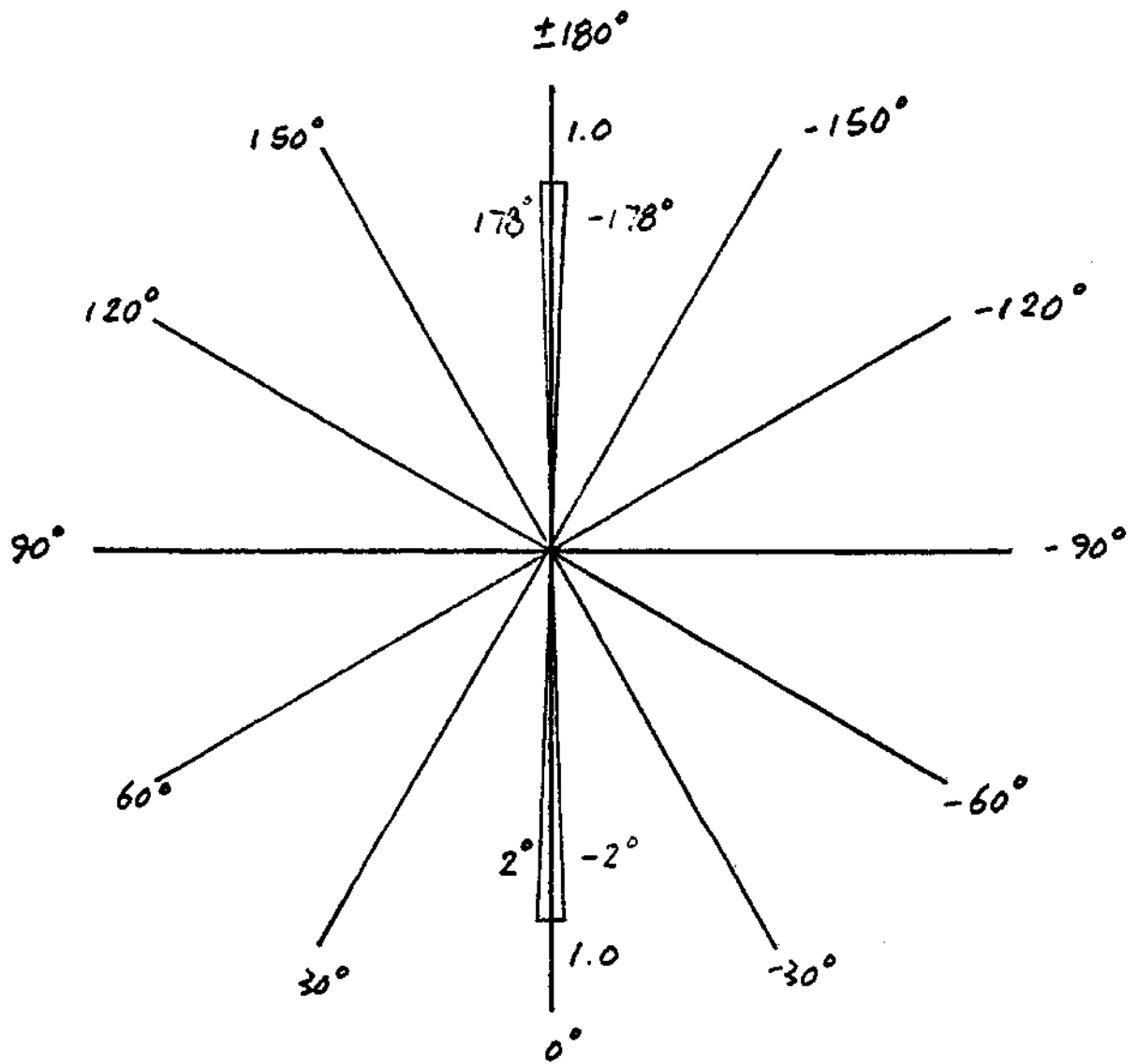


Figure 9(b). Fiber Orientation Distribution Function--Parallel Laid



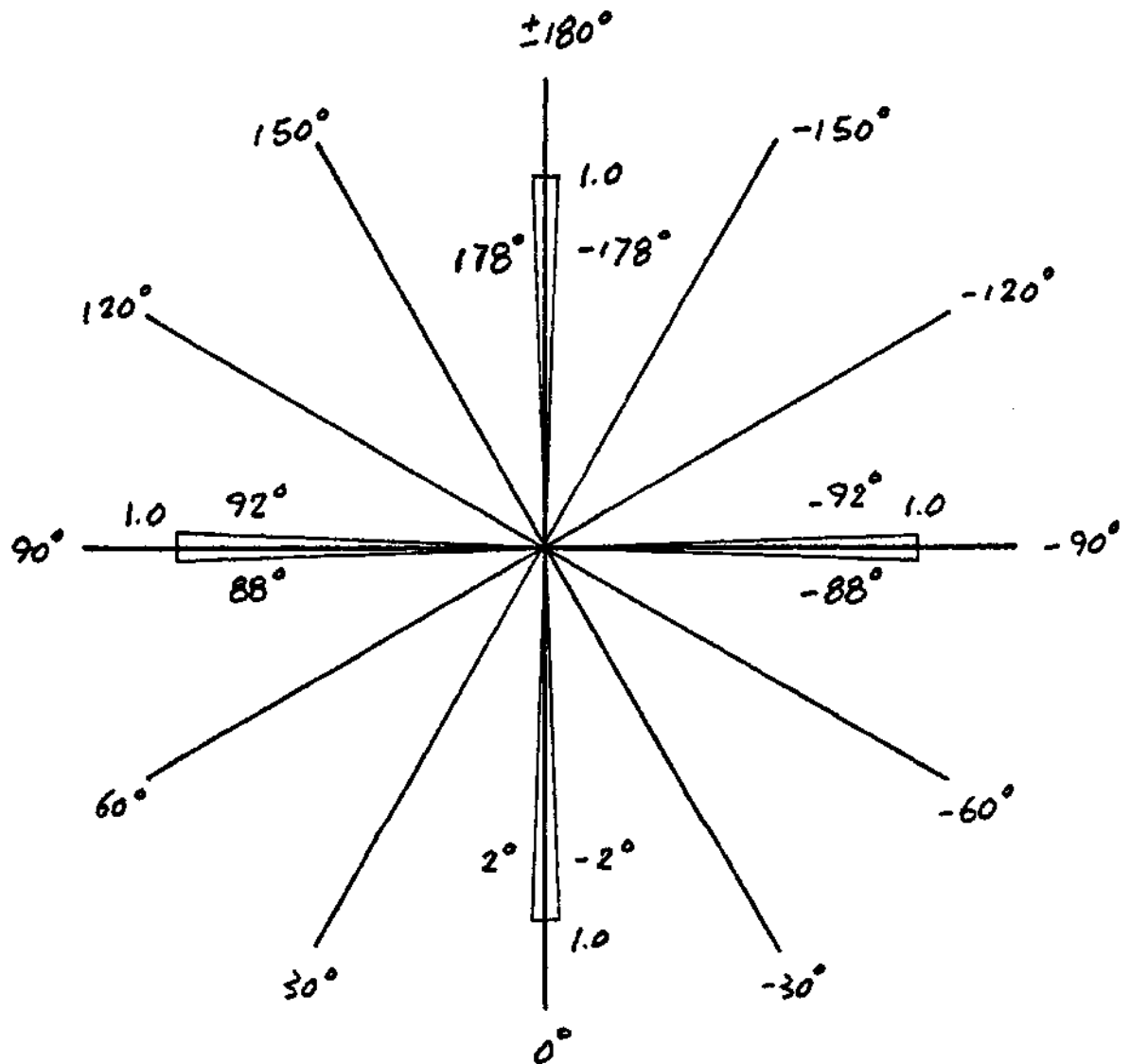


Figure 9(c). Fiber Orientation Distribution Function--Cross Laid

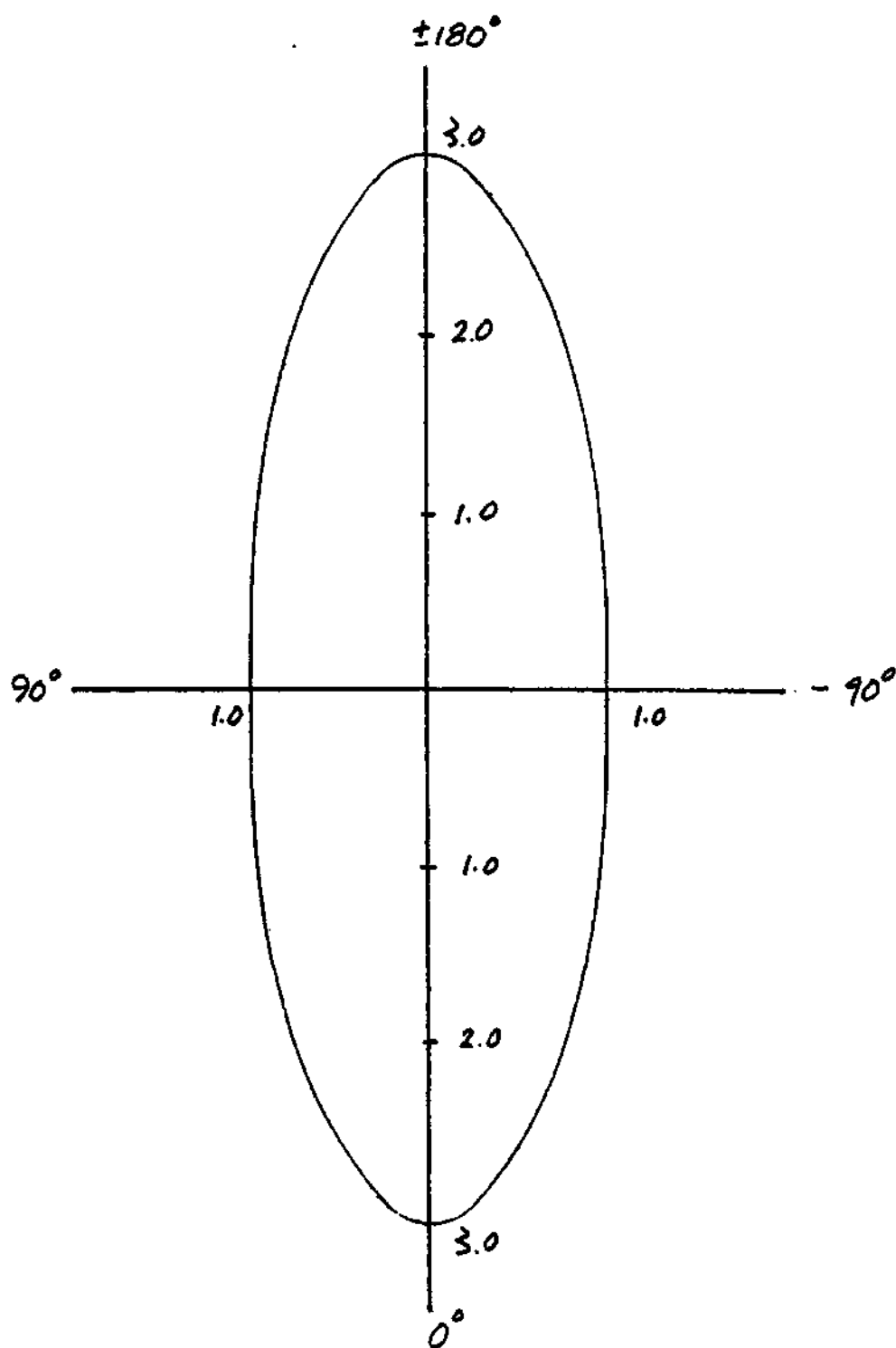


Figure 9(d). Fiber Orientation Distribution Function--Ellipse

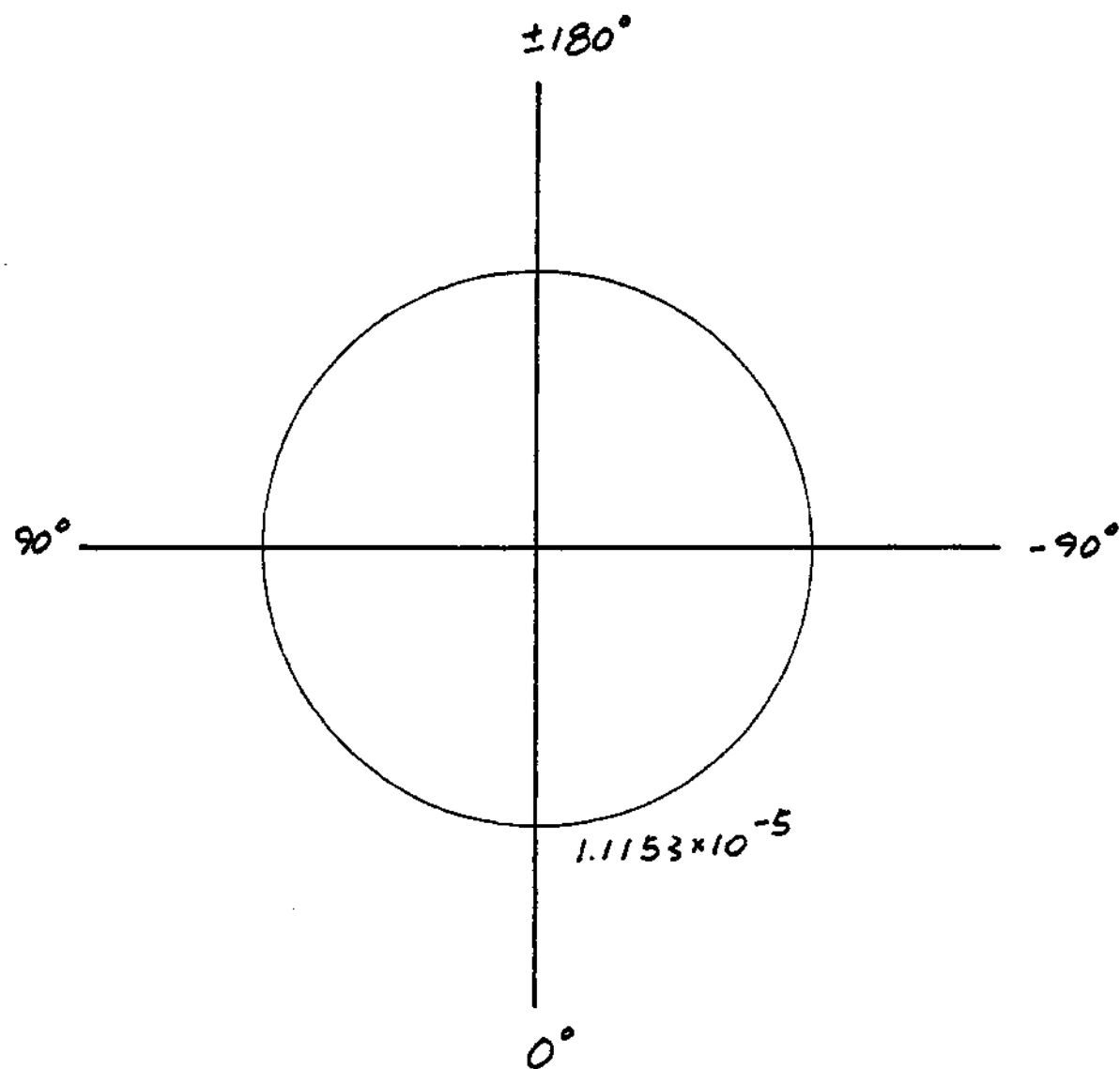


Figure 10(a). Bending Rigidity ( $\text{lb-in}^2$ ) with Special Fiber  
Orientation Distribution Function--Random (C.F.)

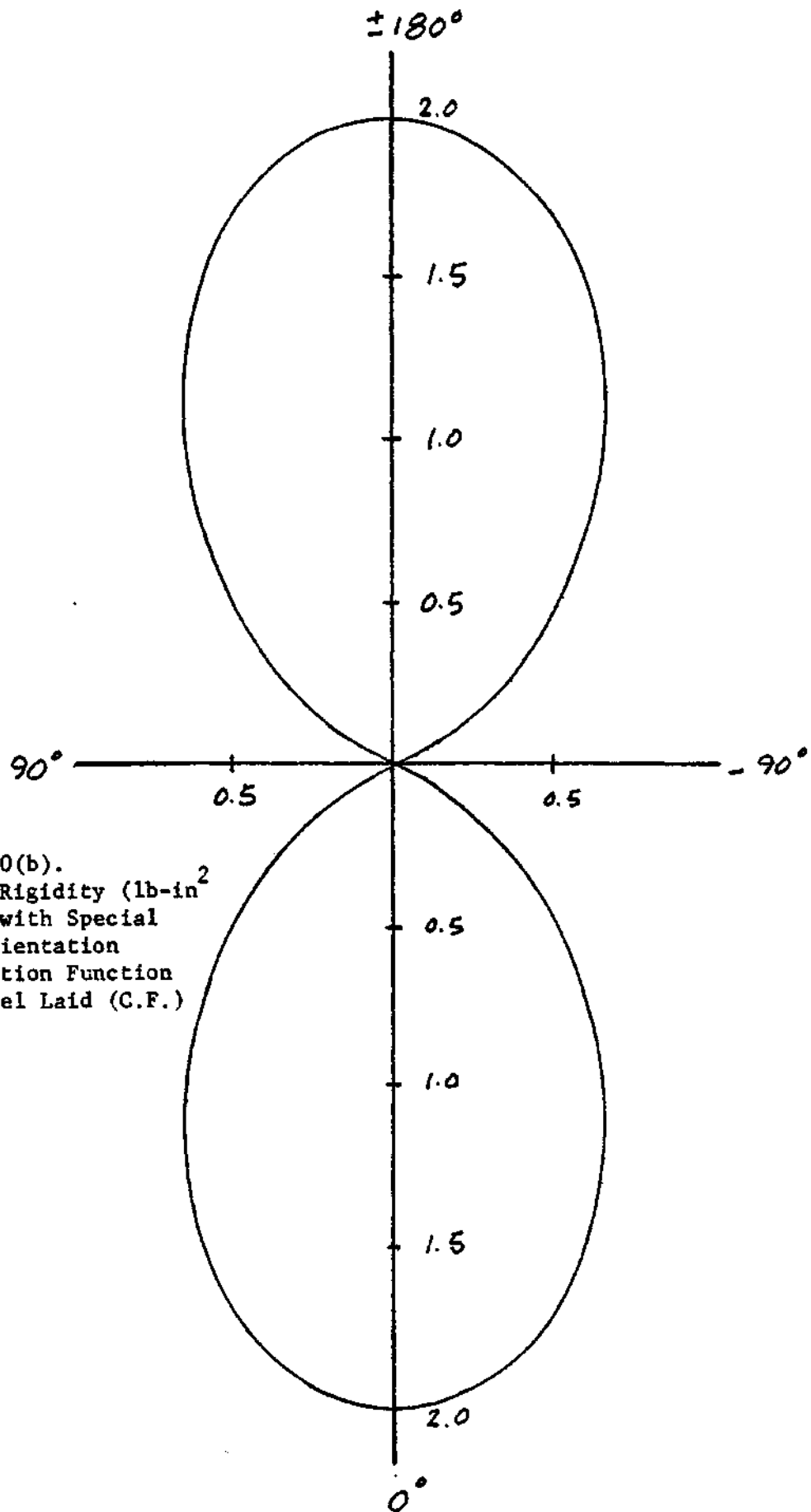


Figure 10(b).  
 Bending Rigidity ( $\text{lb-in}^2$   
 $\times 10^{-5}$ ) with Special  
 Fiber Orientation  
 Distribution Function  
 --Parallel Laid (C.F.)

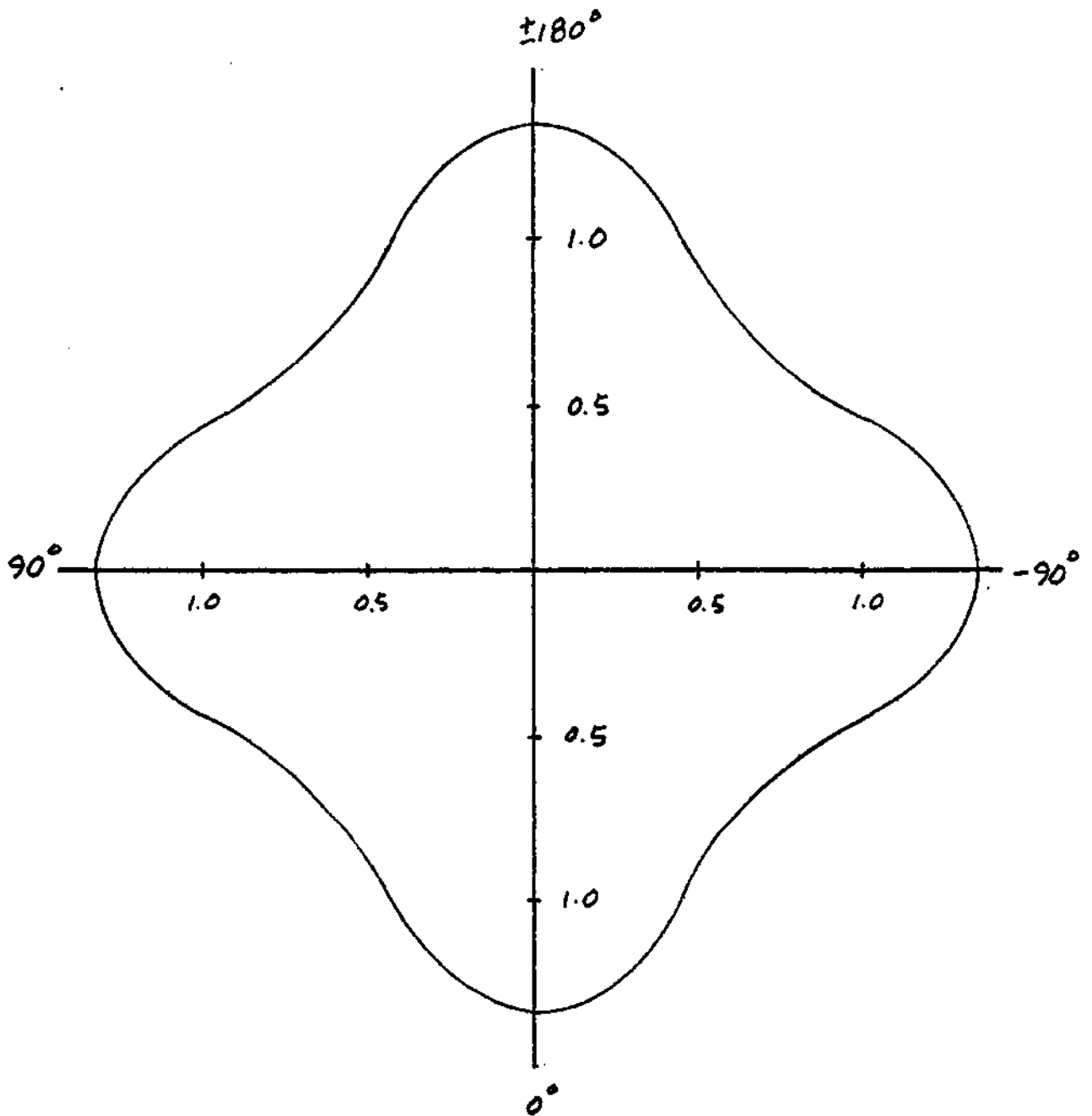


Figure 10(c). Bending Rigidity (lb-in<sup>2</sup> X 10<sup>-5</sup>) with Special Fiber Orientation Distribution Function--Cross Laid (C.F.)

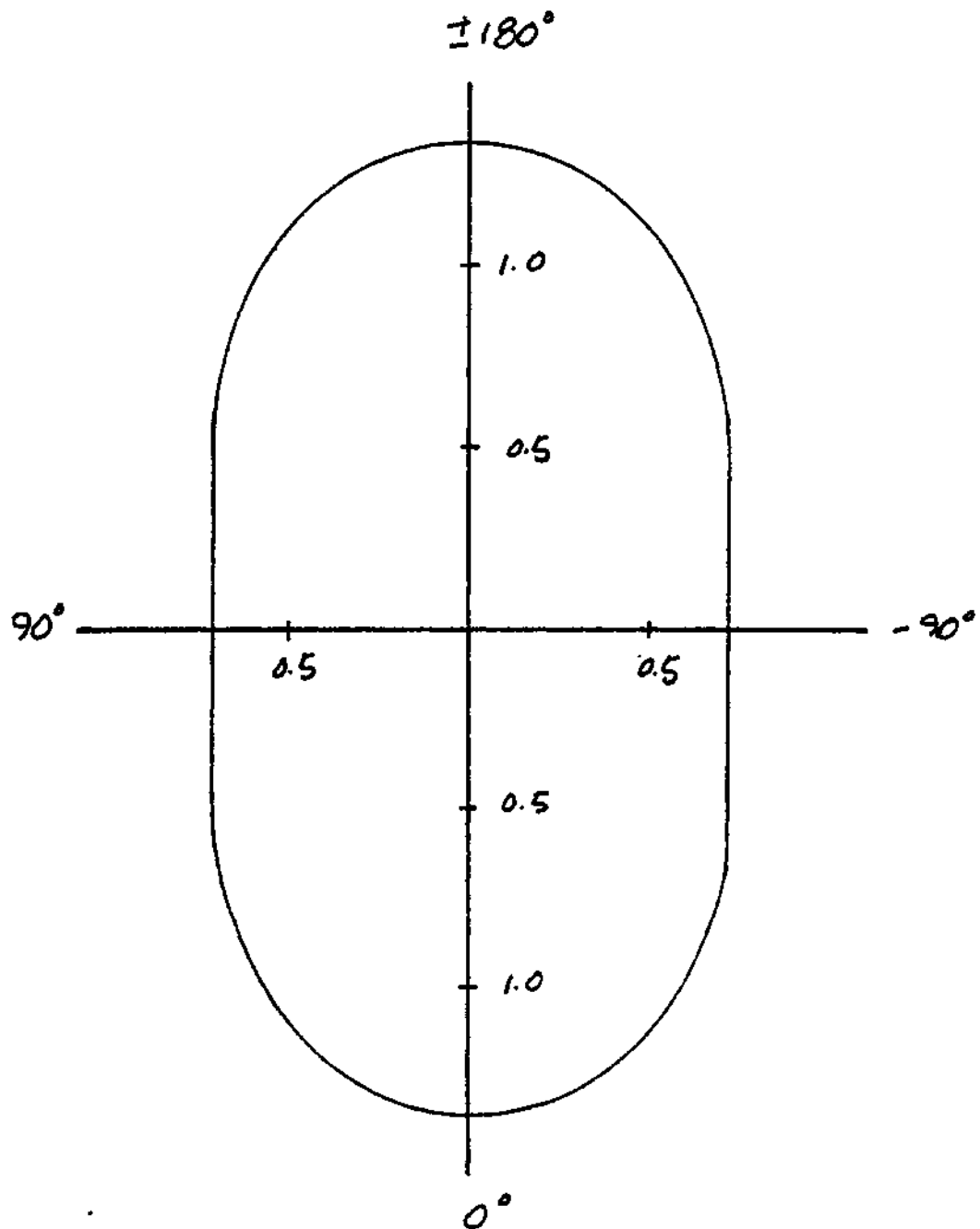


Figure 10(d). Bending Rigidity ( $\text{lb-in}^2 \times 10^{-5}$ ) with Special Fiber Orientation Distribution Function--Ellipse (C.F.)

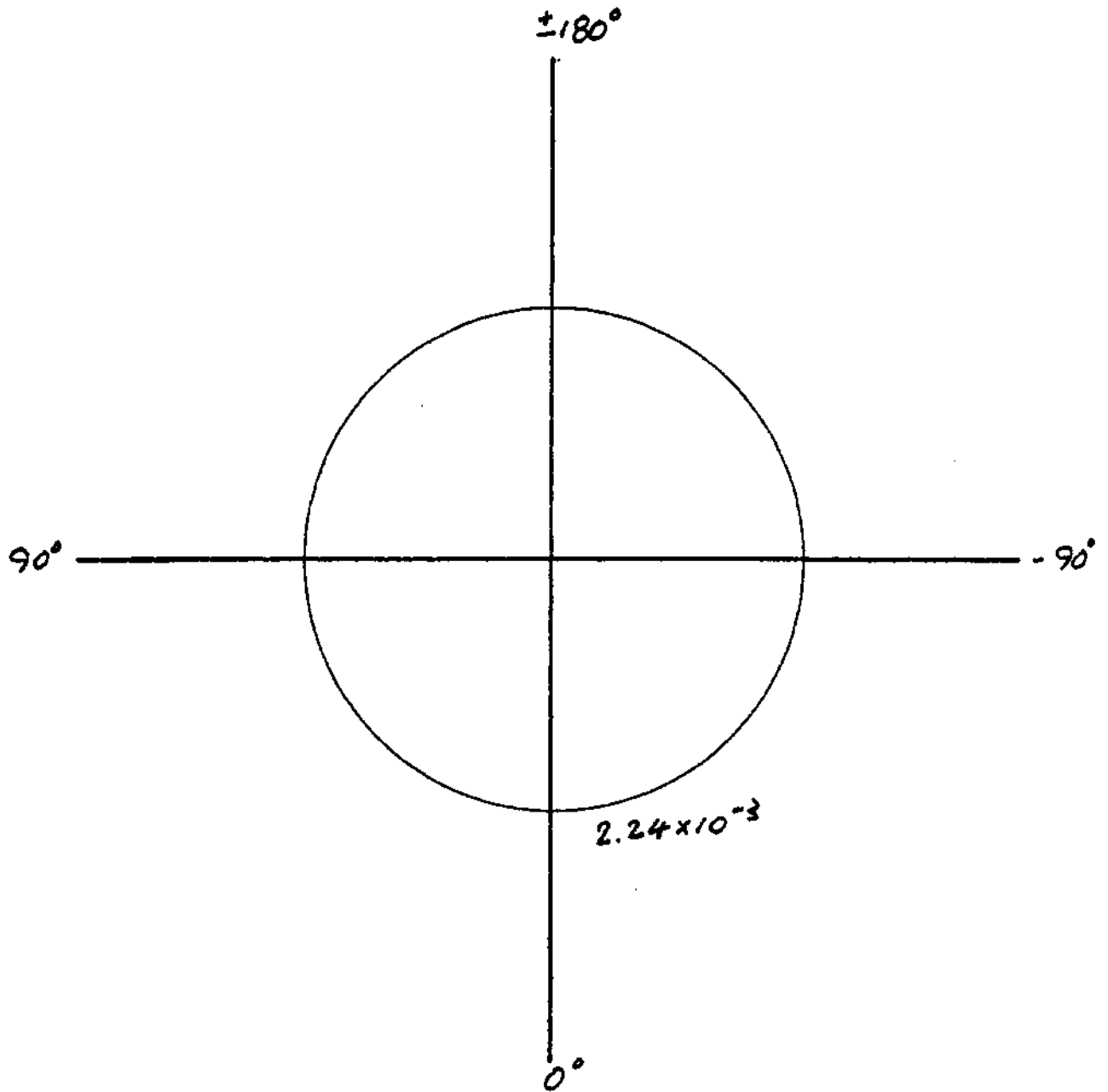


Figure 11(a). Bending Rigidity ( $\text{lb-in}^2$ ) with  
Special Fiber Orientation  
Distribution Function--Random  
(N.F.)

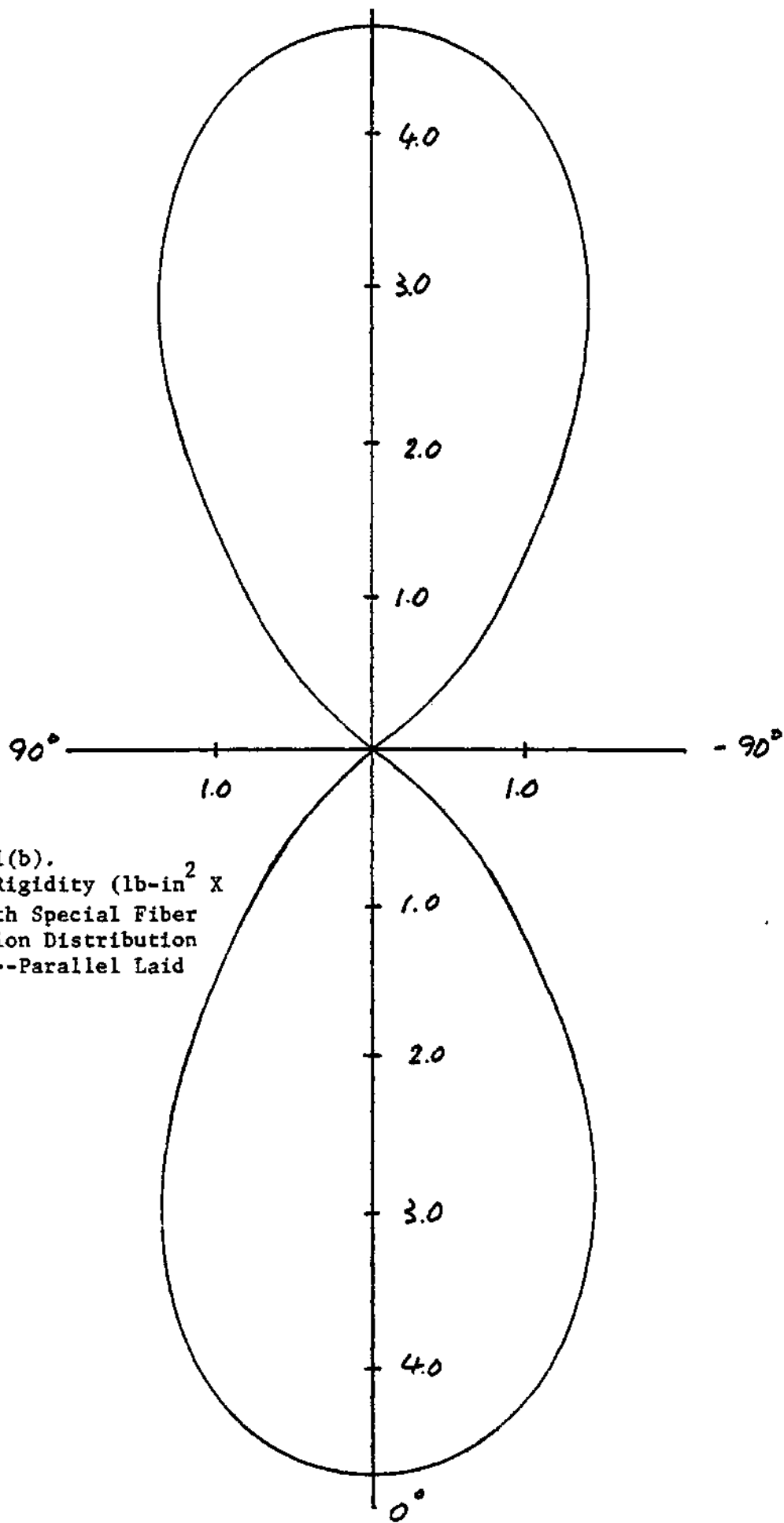


Figure 11(b).  
Bending Rigidity (lb-in<sup>2</sup> X  
10<sup>-3</sup>) with Special Fiber  
Orientation Distribution  
Function--Parallel Laid  
(N.F.)



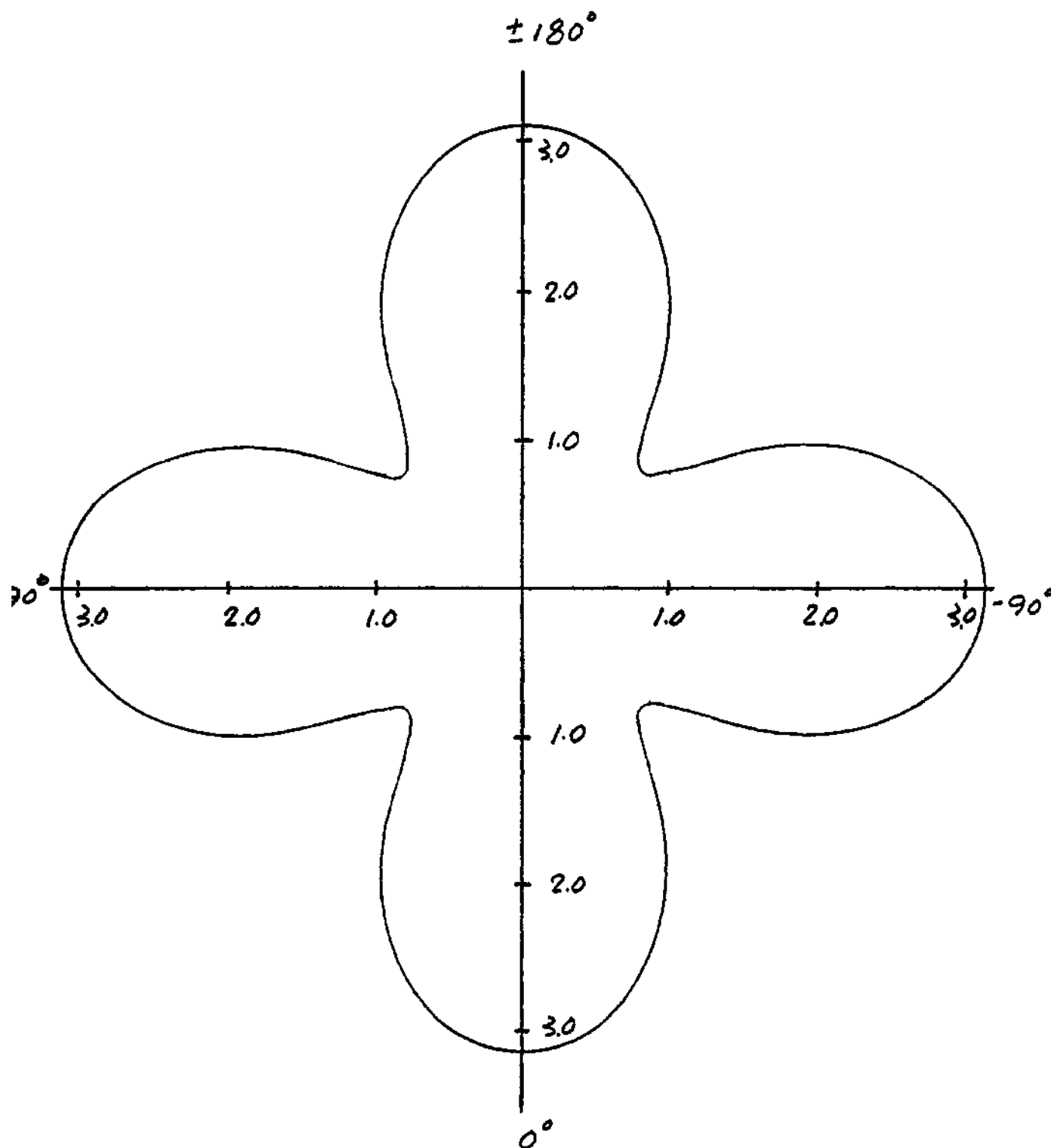


Figure 11(c). Bending Rigidity ( $\text{lb-in}^2 \times 10^{-3}$ ) with Special Fiber Orientation Distribution Function--Cross Laid (N.F.)

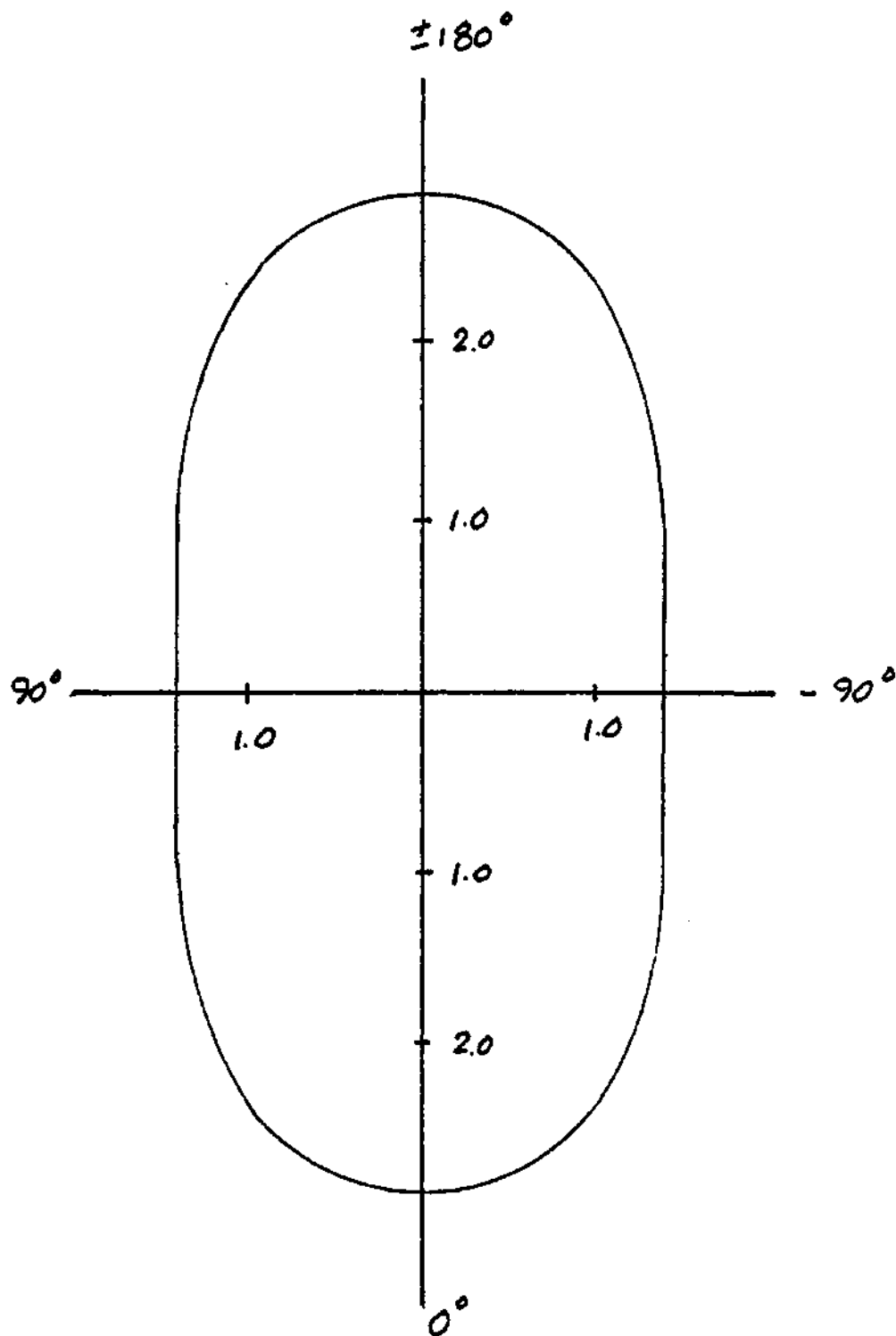


Figure 11(d). Bending Rigidity ( $\text{lb-in}^2 \times 10^{-3}$ ) with Special Fiber Orientation Distribution Function--Ellipse (N.F.)

## CHAPTER 3

## EXPERIMENTAL VERIFICATION

In the last chapter bending rigidity of the nonwoven fabric was predicted for the two extreme cases of fiber motion with four different assumed fiber orientation distribution functions. In this chapter bending rigidity of the nonwoven fabric was measured experimentally and also the bending rigidity was predicted again at the two extreme cases of fiber motion with the actual fiber orientation distribution function determined for the nonwoven fabric.

Experimental Measurement of the Bending  
Rigidity of the Nonwoven Fabric

The bending rigidity of the nonwoven fabric was measured experimentally by the FRL Cantilever Bending Tester. The nonwoven fabric test specimen of 1 in. wide and 8 in. long was used in the measuring of bending rigidity. The deflection was measured when the free length of the bent nonwoven fabric test specimen of the above dimensions was 4.5 in. long. The bending rigidity  $(EI)_{\text{exp}}$  measured experimentally is given by

$$(EI)_{\text{exp}} = \frac{W_0 \cdot L^4}{8 \delta_{\text{max}}} \quad \dots \quad (27)$$

Where  $W_0$  is load per unit length (lb/in.),  $L$  is the free length (in.) and  $\delta_{\text{max}}$  is the deflection (in.). Since the fabric weight was 2.15

oz/yd<sup>2</sup>, so  $W_0$  was 0.000103 lb/in. The bending rigidities of average of 10 measurements of the nonwoven fabric cut parallel to various orientation angles was

Table 1. Bending Rigidity of the Nonwoven Fabric Measured Experimentally

ORIENTATION ( $\beta$ )	DEFLECTION ( $\delta$ ) (inch)	(EI) <sub>exp</sub> (lb-in <sup>2</sup> )
90°	2.348	2.252 X 10 <sup>-3</sup>
60°	2.10	2.513 X 10 <sup>-3</sup>
30°	2.428	2.174 X 10 <sup>-3</sup>
0°	1.641	3.216 X 10 <sup>-3</sup>
-30°	2.256	2.339 X 10 <sup>-3</sup>
-60°	1.944	2.715 X 10 <sup>-3</sup>
-90°	2.334	2.252 X 10 <sup>-3</sup>

These results of the bending rigidities are shown in the polar diagram of Figure 12.

#### Predicted Bending Rigidity of the Nonwoven Fabric

As mentioned before, in order to predict the bending rigidity of the nonwoven fabric for both cases of complete freedom and no freedom of relative fiber motion, the fiber diameter, fiber tensile modulus, fabric thickness, and fiber orientation distribution function must be determined first. In the last chapter, all of the above parameters were determined except the fiber orientation distribution function.

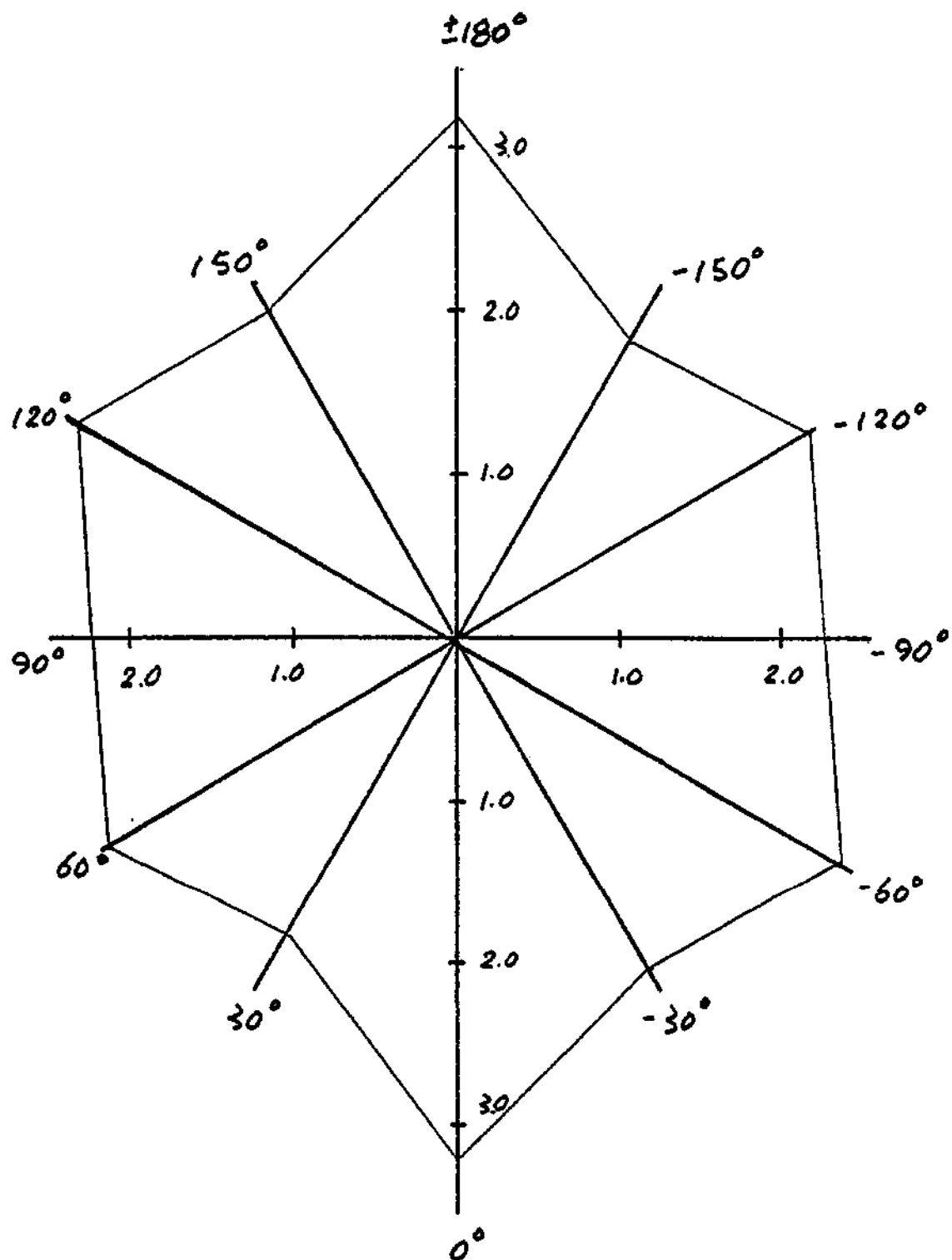


Figure 12. Bending Rigidity (lb-in<sup>2</sup> × 10<sup>-3</sup>) Measured by Cantilever Bending Tester

The actual fiber orientation distribution function was determined by tensile testing of the nonwoven fabric at various orientation angles. It was assumed that the higher the number of fibers orientated at an angle, the higher would be the tensile strength at that particular direction. In other words, the relative tensile strength of the fabric at any particular angle will be proportional to the relative number of fibers at that direction. The above is true if the test length of the sample is small. In the actual tensile test, gauge length was 1/8 in. long, sample width was 2 in. wide and cross head speed was 2 in./min. The following results were obtained during the test.

Table 2. Tensile Strength of the Nonwoven Fabric at Various Orientation Angle

ORIENTATION ( $\beta$ )	TENSILE STRENGTH (lb)
90°	41.53
60°	50.14
30°	43.98
0°	58.60
-30°	51.64
-60°	58.95
-90°	41.53

These results are plotted on a polar diagram of Figure 13. The strength diagram might then be treated as the fiber orientation distribution dia-

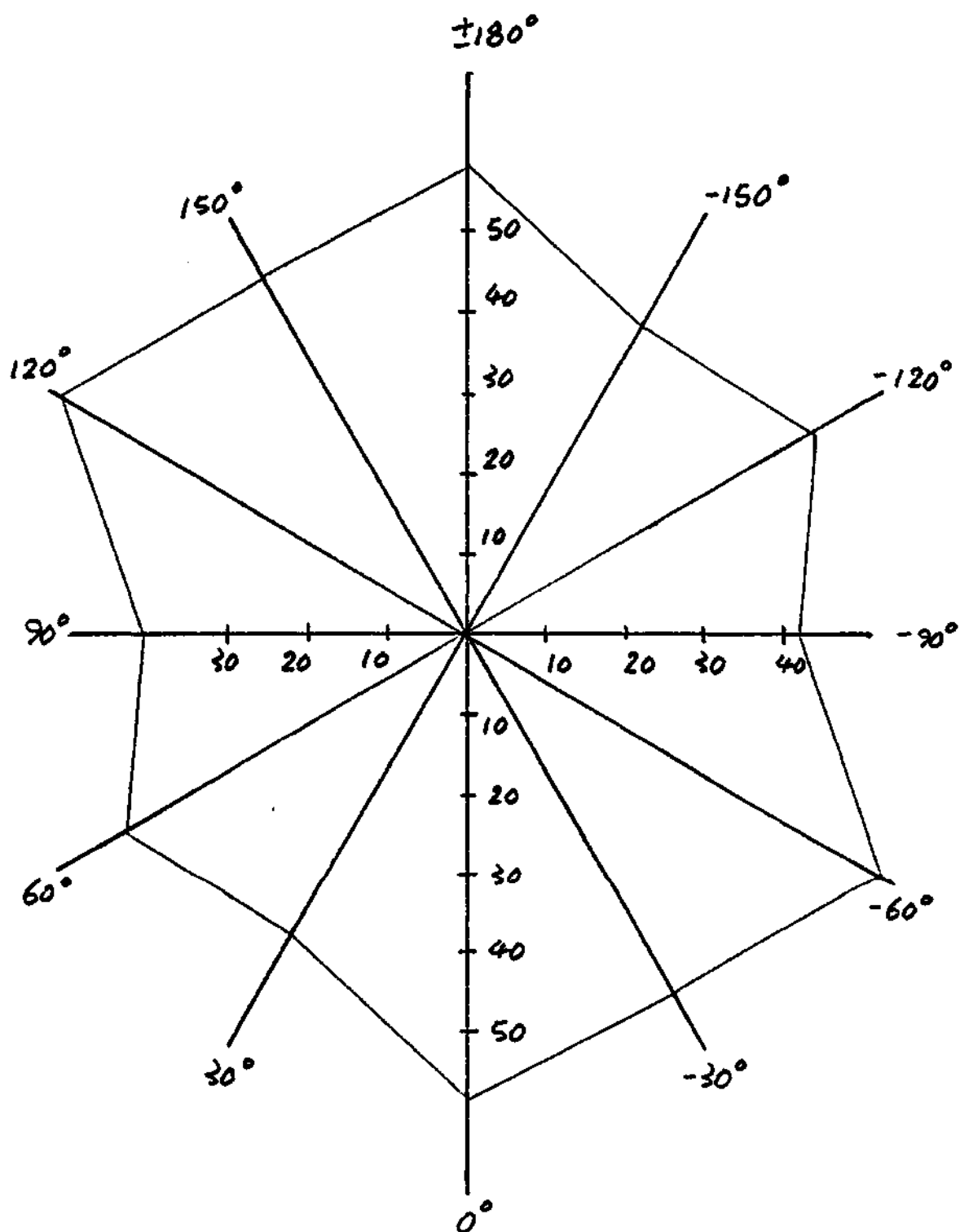


Figure 13. Tensile Strength (lb) at Various Orientation Angle

gram according to the above assumption. Any portion of the area under the curve between two angles over the total area is the fraction of the total number of fibers which lie at that particular direction.

Since the fiber orientation distribution had been determined for the nonwoven fabric, the bending rigidity of the unit cell (1 in. X 1/2 in.) in the case of complete freedom of relative fiber motion at various orientation angles were predicted according to Equation (14). The results are shown in the following table and plotted on the polar diagram of Figure 14.

Table 3. Bending Rigidity of the Nonwoven Fabric with Complete Freedom of Relative Fiber Motion

ORIENTATION ( $\beta$ )	BENDING RIGIDITY ( $lb\text{-}in^2$ )
$90^\circ$	$1.0798 \times 10^{-5}$
$60^\circ$	$1.0530 \times 10^{-5}$
$30^\circ$	$1.0931 \times 10^{-5}$
$0^\circ$	$1.1590 \times 10^{-5}$
$-30^\circ$	$1.1872 \times 10^{-5}$
$-60^\circ$	$1.1482 \times 10^{-5}$
$-90^\circ$	$1.0798 \times 10^{-5}$

For the same reason as mentioned in the last chapter, the fabric was also assumed to be made up of ten layers of fibers, five layers above and below the neutral surface. It was also assumed that each lay-



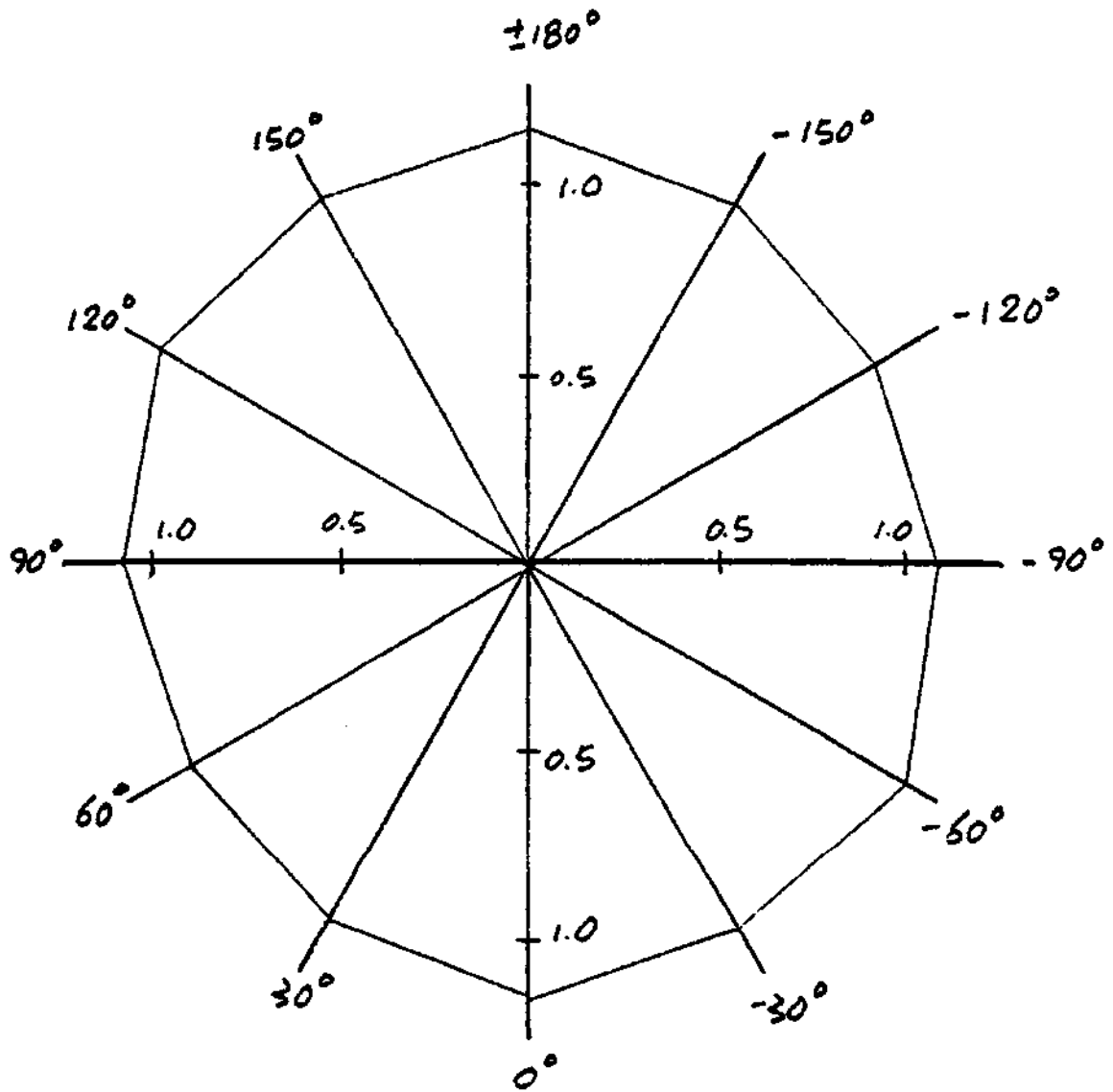


Figure 14. Predicted Bending Rigidity (lb-in<sup>2</sup> X 10<sup>-5</sup>) from Actual Fiber Orientation Distribution Function (C.F.)

er consists of equal number of fibers and the fiber orientation distribution function of each layer is same as that determined by the tensile test. The bending rigidity of the unit cell (lin. X 1/2 in.) in the case of no freedom of relative fiber motion is predicted according to Equation (23). The results for the assumption that the fabric was made up of ten layers of fibers are shown in the following table and plotted on a polar diagram of Figure 15.

Table 4. Bending Rigidity of the Nonwoven Fabric with No Freedom of Relative Fiber Motion

ORIENTATION ( $\beta$ )	BENDING RIGIDITY (lb-in <sup>2</sup> )
90°	2.1235 X 10 <sup>-3</sup>
60°	2.0627 X 10 <sup>-3</sup>
30°	2.1600 X 10 <sup>-3</sup>
0°	2.3116 X 10 <sup>-3</sup>
-30°	2.3816 X 10 <sup>-3</sup>
-60°	2.2907 X 10 <sup>-3</sup>
-90°	2.1235 X 10 <sup>-3</sup>

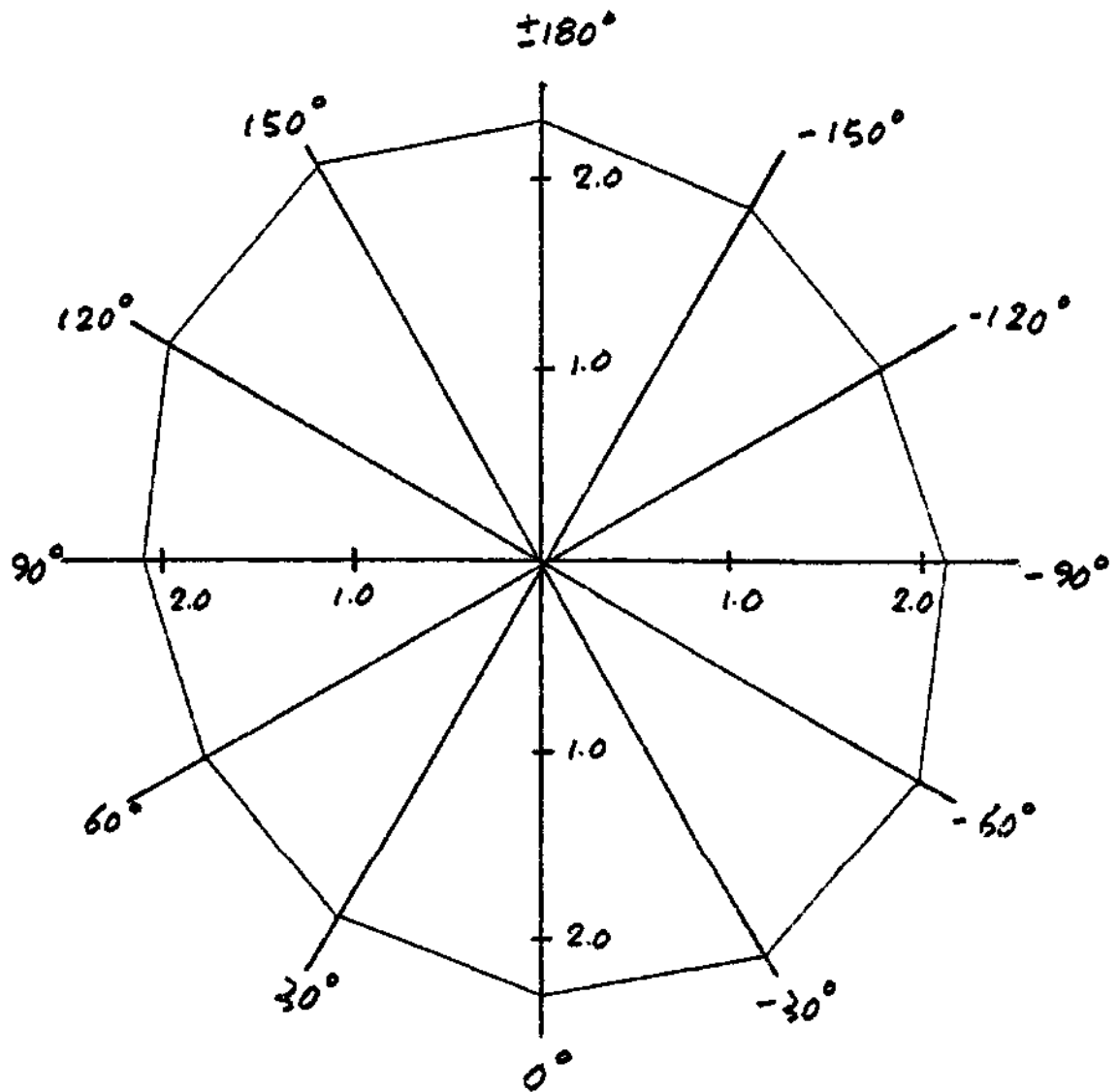


Figure 15. Predicted Bending Rigidity (lb-in<sup>2</sup> X 10<sup>-3</sup>) from Actual Fiber Orientation Distribution Function (N.F.)

## CHAPTER 4

## CONCLUSION AND RECOMMENDATION

In years past, we have seen great development and commercialization of nonwoven fabrics. It is always desirable to replace the thin apparel fabrics with the two dimensional, fabric-like nonwoven material. This is because the manufacturing cost for the nonwoven fabrics is low. However, one of the disadvantages of nonwoven fabrics over woven fabrics is their greater bending rigidity which limits their use in apparel applications. So if we want to design a nonwoven fabric with low bending rigidity, the mechanisms of deformation during bending of the nonwoven fabric must be known.

In this thesis it was found that the extreme case of no freedom of relative fiber motion is a closer approximation of the actual bending behavior of the nonwoven fabric studied. It was illustrated that the bending rigidity of the nonwoven fabric was accurately predicted by the rectangular unit cell model assuming no freedom of relative fiber motion, if the fiber diameter, fiber tensile modulus, fabric thickness and fiber orientation distribution function were determined. It was also shown that the high bending rigidity of the nonwoven fabrics was due to no freedom of individual fiber motion during fabric bending. In the actual manufacturing of the nonwoven fabrics, the fibers are bonded together at the fiber cross-overs to achieve sufficient strength. So in order to lower the bending rigidity of the nonwoven fabrics, less degree and frequency of bonding at fiber cross-overs, using of strong fi-

bers with low bending rigidity and more fiber length between bond points is going to be very helpful.

Finally, it is suggested that a three dimensional unit cell should be developed to give a better understanding of the bending behavior of the nonwoven fabrics. More theoretical analyses should be carried out, so that the nonwoven fabric could be engineered more accurately, scientifically, and economically to achieve our goal of replacing the expensive woven fabrics with the less expensive nonwoven fabrics.

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