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ANALYSIS OF CIRCULARLY POLARIZED DIELECTRIC RESONATOR ANTENNA EXCITED BY A SPIRAL SLOT

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Abstract—The hemispherical dielectric resonator antenna (DRA) excited by a single-arm spiral slot is studied theoretically in this paper. The Green's function technique is employed to formulate an integral equation for the spiral slot current. The moment method with piecewise sinusoidal (PWS) basis and testing functions is used to convert the integral equation into a matrix equation by using a delta-gap exciting source. The input impedance, return loss, axial ratio and radiation pattern are calculated. Numerical results demonstrate that the analysis is efficient.

1. INTRODUCTION

DRAs have been investigated extensively because they have several inherent advantages such as freedom from metallic loss, small size, lightweight, low cost, relatively wider bandwidth, high radiation efficiency. On the other hand, the circularly polarized (CP) antenna is insensitive to transmitter and receiver orientation, offers less sensitivity to propagation effects and is suitable for satellite communications. Thus, CP excitation for DRA becomes an interesting problem. However, theoretical analysis of such structure is very limited. So far works about this topic are mainly experimental. In this paper, a hemispherical DRA excited by a single-arm spiral slot is studied theoretically. The spiral slot is cut on an infinite ground plane and used to couple the energy om a coaxial cable to the DRA. The Green's function technique is employed to formulate an integral equation for the spiral slot current. The moment method with piecewise sinusoidal (PWS) basis and testing functions is used to convert the integral equation into a matrix equation by using a delta-gap exciting source. After the equivalent magnetic current for the spiral slot is solved, the input impedance, return loss and axial ratio are calculated. Numerical results demonstrate that the analysis is efficient. It is feasible to excite the circularly polarized hemispherical DRA using a spiral slot.

2. THEORY

The geometry of the structure is shown in Figure 1. A single-arm spiral slot is cut on an infinite ground plane and used to couple the energy from a coaxial cable to the DRA. The coaxial cable is mounted beneath the ground plane with its inner conductor soldered across the slot. The radius and dielectric constant of the DRA are denoted by a_r and ε_d, respectively. The radial distance ρ from the origin to the center-line of the slot arm is defined by an Archimedean spiral function of $\rho = a\phi + \Delta$, where a is the spiral constant, ϕ is the winding angle (starting at ϕ_S rad and ending at ϕ_E rad), and Δ is the winding constant. The slot arm width is designated as W and assumed to be very small compared to a free space wavelength. In the following formulation, the fields are assumed to vary harmonically as $e^{j\omega t}$, which is suppressed. Moreover, $r(\rho, \theta, \phi)$ and $r'(\rho', \theta', \phi')$ refer to the field and source points, respectively. The superscripts f and d are used to denote the regions of the free space and DRA, respectively.

In this analysis, the delta-gap source is used to model the slot excitation. Enforcing the boundary condition across the slot, an integral equation is formulated for the unknown magnetic current

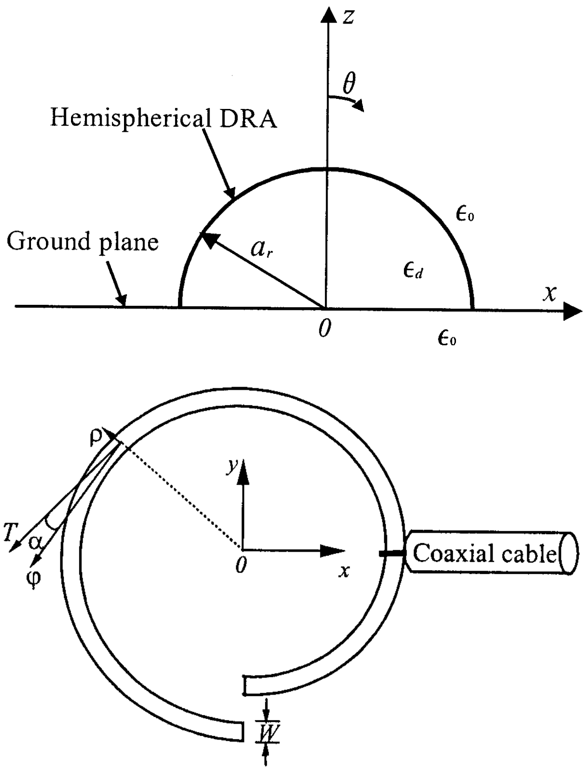

Figure 1. Geometry of DRA excited by a single-arm spiral slot.

flowing in a direction tangential to the spiral arm. Let J_s be the excitation current density, we have

$$H_T^d - H_T^f = J_s. \tag{1}$$

The following integral equation for the magnetic current $M_T(\phi')$ is then obtained:

$$\iint_{S'} 2G_T^d M_T(\phi')dS' - \iint_{S'} 2G_T^d[-M_T(\phi')]dS' = J_s, \tag{2}$$

where S is the slot surface, G_T^d and G_T^f are the magnetic type Green's functions of the spherical DRA and the free space, respectively. Considering the effect of the large ground plane we invoke image theory so that the two equivalent magnetic currents should be multiplied by factor "2". And $J_s = -\left[I_0/\sqrt{a^2 + (a\phi_0 + \Delta)^2}\right]\delta(\phi - \phi_0)$, where I_0

and ϕ_0 are the current amplitude and ϕ coordinate at the feed point, respectively. For convenience I_0 is set to unity.

Using the moment method, the magnetic current density is expanded in terms of unknown voltage coefficients V_n's as follows:

$$M_T(\phi') = \frac{1}{W}\sum_{n=1}^{N} V_n f_n(\phi'), \tag{3}$$

where $f_n(\phi')$'s are piecewise sinusoidal basis functions given by

$$f_n(\phi') = \begin{cases} \dfrac{\sin[k_e(\xi - \xi_{n-1})]}{\sin(k_e\Delta\xi)}, & \xi_{n-1} \le \xi \le \xi_n \\ \dfrac{\sin[k_e(\xi_{n+1} - \xi)]}{\sin(k_e\Delta\xi)}, & \xi_n \le \xi \le \xi_{n+1} \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

in which $k_e = k_0\sqrt{(1+\varepsilon_d)/2}$, $\xi(\phi') = \int_{\phi_S}^{\phi'} \sqrt{a^2 + (a\phi + \Delta)^2} d\phi$, $\xi_n = \int_{\phi_S}^{\phi_n} \sqrt{a^2 + (a\phi + \Delta)^2} d\phi$, $\phi_n = \phi_s + n \cdot (\phi_E - \phi_S)/(N+1)$, $\Delta\xi = \left[\int_{\phi_S}^{\phi_E} \sqrt{a^2 + (a\phi + \Delta^2} d\phi\right]/(N+1)$. Because the slot width is very small, it is reasonable to use the wire source approximation (independent of parameter ρ) instead of the planar source.

By applying the Galerkin's procedure and inserting (3) into (2), we can obtain

$$\frac{1}{W}\sum_{n=1}^{N} V_n \iint_S \iint_{S'} f_m(\phi)\left(G_T^d + G_T^f\right) f_n(\phi') dS' dS = \frac{-1}{2\sqrt{a^2 + (a\phi_0 + \Delta)^2}} \iint_S f_m(\phi)\delta(\phi - \phi_0) dS. \tag{5}$$

Equation (5) then becomes

$$\sum_{n=1}^{N} V_n \iint_l \iint_{l'} f_m(\phi)\left(G_T^d + G_T^f\right) f_n(\phi') dl' dl = \frac{-1}{2\sqrt{a^2 + (a\phi_0 + \Delta)^2}} \int_l f_m(\phi)\delta(\phi - \phi_0) dl. \tag{6}$$

That is,

$$\sum_{n=1}^{N} V_n \int_{\phi_{m-1}}^{\phi_{m+1}} \int_{\phi_{n-1}}^{\phi_{n+1}} f_m(\phi)\left(G_T^d + G_T^f\right) f_n(\phi') \cdot\sqrt{a^2 + (a\phi' + \Delta)^2}\sqrt{a^2 + (a\phi + \Delta)^2} d\phi' d\phi = -\frac{1}{2} f_m(\phi_0). \tag{7}$$

The resulting matrix equation can be written:

$$\left[Y_{mn}^d + Y_{mn}^f\right][V_n] = -\frac{1}{2} f_m(\phi_0), \quad m, n = 1, 2, \ldots, N \tag{8}$$

where

$$Y_{mn}^d = \int_{\phi_{m-1}}^{\phi_{m+1}} \int_{\phi_{n-1}}^{\phi_{n+1}} f_m(\phi) G_T^d f_n(\phi') \sqrt{a^2 + (a\phi' + \Delta)^2} \sqrt{a^2 + (a\phi + \Delta)^2} d\phi' d\phi,$$

$$Y_{mn}^f = \int_{\phi_{m-1}}^{\phi_{m+1}} \int_{\phi_{n-1}}^{\phi_{n+1}} f_m(\phi) G_T^f f_n(\phi') \sqrt{a^2 + (a\phi' + \Delta)^2} \sqrt{a^2 + (a\phi + \Delta)^2} d\phi' d\phi.$$

After V_n's are found, the input impedance can be easily calculated from $Z_{\text{in}} = \sum_{n=1}^{N} V_n f_n(\phi_0)$. Furthermore, the radiation fields E_θ, E_ϕ and axial ratio can be easily obtained.

Next we will discuss a technique to calculate Y_{mn}^d and Y_{mn}^f efficiently. For ease of computation, Green's function G_T^d is divided into its particular and homogeneous parts, that is, $G_T^d = G_T^{dP} + G_T^{dH}$. The former represents radiation of a source in an unbounded medium, while the latter accounts for the boundary discontinuity. Since the magnetic current under consideration lies on the x-y plane ($\theta' = \pi/2$) (see Figure 1), for simplification to formulate G_T^d, the T-direction current can be decomposed into the ρ-direction and ϕ-direction components:

$$\hat{T} = \hat{\rho} \sin\alpha' + \hat{\phi} \cos\alpha', \tag{9}$$

where $\sin\alpha' = a/\sqrt{a^2 + \rho'^2}$, $\cos\alpha' = \rho'/\sqrt{a^2 + \rho'^2}$, $\rho' = a\phi' + \Delta$. For convenience, we again mark $\rho = a\phi + \Delta$. After considerable algebraic manipulation we get

$$\begin{aligned}
G_T^{dP} = &\frac{1}{j\omega\mu_0} \left(\frac{1}{\sqrt{a^2 + \rho'^2}\sqrt{a^2 + \rho^2}}\right) \\
&\cdot \Big\{ (a^2 \cos\phi \cos\phi' - a\rho \sin\phi \cos\phi' - a\rho' \cos\phi \sin\phi' + \rho\rho' \sin\phi \sin\phi') \\
&\cdot [(\rho\cos\phi - \rho'\cos\phi')^2 \cdot (3 + 3jkR - k^2R^2) - (1 + jkR) \cdot R^2 + k^2R^4] \\
&+ [a(\rho + \rho')\cos(\phi + \phi') + (a^2 - \rho\rho')\sin(\phi + \phi')] \\
&\cdot [(\rho\cos\phi - \rho'\cos\phi')(\rho\sin\phi - \rho'\sin\phi')(3 + 3jkR - k^2R^2)] \\
&+ (a^2 \sin\phi \sin\phi' + a\rho \cos\phi \sin\phi' + a\rho' \sin\phi \cos\phi' + \rho\rho' \cos\phi \cos\phi') \\
&\cdot [(\rho\sin\phi - \rho'\sin\phi')^2 \cdot (3 + 3jkR - k^2R^2) - (1 + jkR) \cdot R^2 + k^2R^4] \Big\} \\
&\cdot \left(\frac{e^{-jkR}}{4\pi R^5}\right),
\end{aligned} \tag{10}$$

where $R = \sqrt{\rho'^2 + \rho^2 - 2\rho\rho'\cos(\phi - \phi') + a_e^2}$, $a_e = W/4$. Here we adopt the concept of equivalent radius. Observe from (10) that $R \neq 0$ and, thus, the singularity has been avoided. It is worth pointing out that we easily obtain the expression of Green's function G_T^f by replacing k in (10) with $k_0 (= k/\sqrt{\varepsilon_d})$. Following Leung's same procedure [1], we have

$$\begin{aligned}
G_T^{dH} &= \left(\frac{-1}{\sqrt{a^2+\rho'^2}\sqrt{a^2+\rho^2}}\right)\left\{\frac{1}{4\pi\omega\mu_0 k}\cdot\left(\frac{a^2}{\rho^2\rho'^2}\right)\right. \\
&\quad \cdot\sum_{n=1}^{\infty} b_n n(n+1)(2n+1)P_n[\cos(\phi-\phi')]\hat{J}_n(k\rho')\hat{J}_n(k\rho) \\
&\quad +\frac{1}{4\pi\omega\mu_0}\cdot\left(\frac{a}{\rho^2}\right)\sum_{n=1}^{\infty} b_n(2n+1)\frac{\partial}{\partial\phi'}P_n[\cos(\phi-\phi')]\hat{J}_n'(k\rho')\hat{J}_n(k\rho) \\
&\quad +\frac{1}{4\pi\omega\mu_0}\cdot\left(\frac{a}{\rho'^2}\right)\sum_{n=1}^{\infty} b_n(2n+1)\frac{\partial}{\partial\phi}P_n[\cos(\phi-\phi')]\hat{J}_n(k\rho')\hat{J}_n'(k\rho) \\
&\quad +\frac{k}{4\pi\omega\mu_0}\sum_{n=1}^{\infty} e_n\frac{(2n+1)}{n(n+1)}P_n'[\cos(\phi-\phi')]\hat{J}_n(k\rho')\hat{J}_n(k\rho) \\
&\quad \left.+\frac{k}{4\pi\omega\mu_0}\sum_{n=1}^{\infty} b_n\frac{(2n+1)}{n(n+1)}\frac{\partial^2}{\partial\phi\partial\phi'}P_n[\cos(\phi-\phi')]\hat{J}_n'(k\rho')\hat{J}_n'(k\rho)\right\},
\end{aligned} \tag{11}$$

where

$$\begin{aligned}
b_n &= -\frac{1}{\Delta_n^{TE}}\left[\hat{H}_n^{(2)}(ka_r)\hat{H}_n^{(2)'}(k_0a_r) - \frac{k}{k_0}\hat{H}_n^{(2)'}(ka_r)\hat{H}_n^{(2)}(k_0a_r)\right], \\
e_n &= -\frac{1}{\Delta_n^{TM}}\left[\hat{H}_n^{(2)'}(ka_r)\hat{H}_n^{(2)}(k_0a_r) - \frac{k}{k_0}\hat{H}_n^{(2)}(ka_r)\hat{H}_n^{(2)'}(k_0a_r)\right],
\end{aligned}$$

and

$$\begin{aligned}
\Delta_n^{TE} &= \hat{J}_n(ka_r)\hat{H}_n^{(2)'}(k_0a_r) - \frac{k}{k_0}\hat{J}_n'(ka_r)\hat{H}_n^{(2)}(k_0a_r), \\
\Delta_n^{TM} &= \hat{J}_n'(ka_r)\hat{H}_n^{(2)}(k_0a_r) - \frac{k}{k_0}\hat{J}_n(ka_r)\hat{H}_n^{(2)'}(k_0a_r).
\end{aligned}$$

In (11), $P_n^m(x)$ is the associated Legendre function of the first kind with order m and degree n, and $\hat{J}_n(x)$ and $\hat{H}_n^{(2)}(x)$ are, respectively, spherical Bessel function of the first kind (Schelkunoff type) and spherical Hankel function (Schelkunoff type) of the second kind, both of order n. All other symbols have the usual meanings. It is possible to evaluate directly the homogeneous parts because the integral of G_T^{dH} converge in a small number of modal terms. Therefore, Y_{mn}^d and Y_{mn}^f can be calculated in a straightforward manner. The factors $1/\left(\sqrt{a^2+\rho'^2}\sqrt{a^2+\rho^2}\right)$ in Green's functions can be reduced with the factor $\sqrt{a^2+(a\phi'+\Delta)^2}\sqrt{a^2+(a\phi+\Delta)^2}$ in the integrand in (8). Of course, care has to be taken in performing the numerical integration around $\phi = \phi'$, where the integrand as a very sharp (but finite) amplitude due to the factor of $1/R^5$.

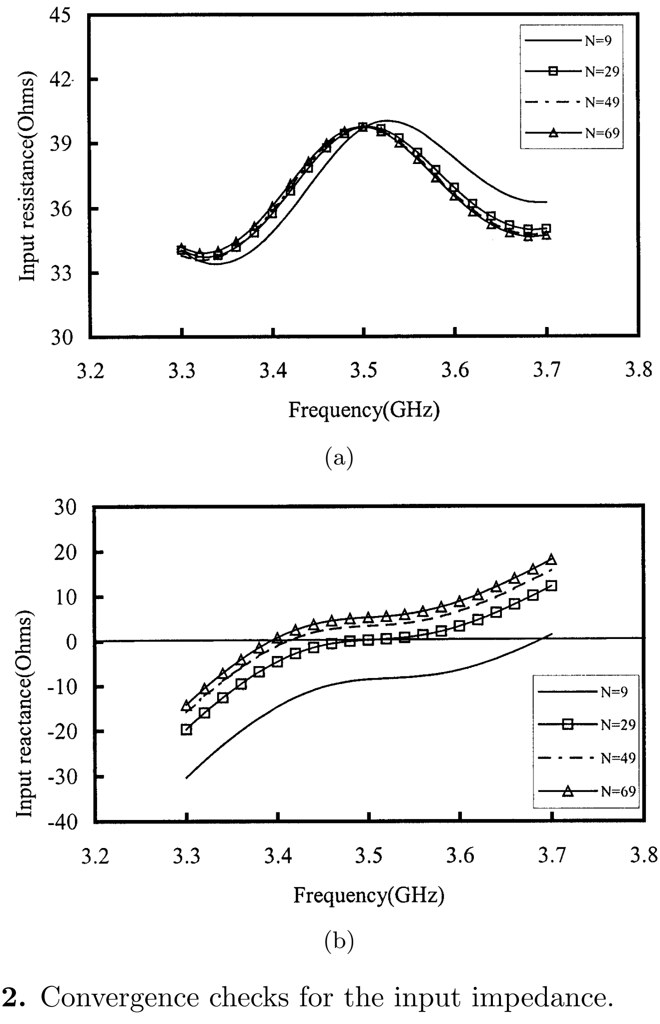

Figure 2. Convergence checks for the input impedance.

3. NUMERICAL RESULTS AND DISCUSSION

In this numerical analysis, the DRA has radius $a_r = 12.5\,\text{mm}$ and dielectric constant $\varepsilon_d = 9.5$. Other original parameters include $\phi_S = 5.5\pi$, $\phi_E = 7.5\pi$, $\phi_0 = 6.0\pi$, $a = 0.33\,\text{mm/rad}$, $\Delta = 0.18\,\text{mm/rad}$, $W = 0.8\,\text{mm}$. The configuration of the spiral slot for radiation of a circularly polarized wave is optimized by an arm truncation technique. The convergence checks of the MoM solution were first carried out for the input impedance. It is found that 6 modal terms are enough to ensure reasonable convergence in the homogeneous part of Green's function G_T^{dH}. As shown in Figure 2, the numerical results of the input resistance and reactance are investigated for the different numbers of expansion functions. It can be found that the convergence is observed when 69 expansion functions are used. Therefore, $N = 69$ was used in the following calculation. As given in Figure 3, the calculated return loss is plotted versus the operating frequencies. It can be seen that the minimum return loss reaches $-17.8\,\text{dB}$ at $3.5\,\text{GHz}$. The axial ratio is $0.35\,\text{dB}$ at this frequency point and the frequency response of axial ratio is plotted in Figure 4. It is suitable for engineering applications since the impedance matching and axial ratio are both optimal at the same operating frequency $3.5\,\text{GHz}$. Finally, we calculate the antenna radiation pattern at $3.5\,\text{GHz}$. The left- and right-handed electric fields are indicated in Figure 5 by using the solid line and dot line, respectively. Our analysis demonstrates that it is feasible to excite the circularly polarized hemispherical DRA by using a spiral slot.

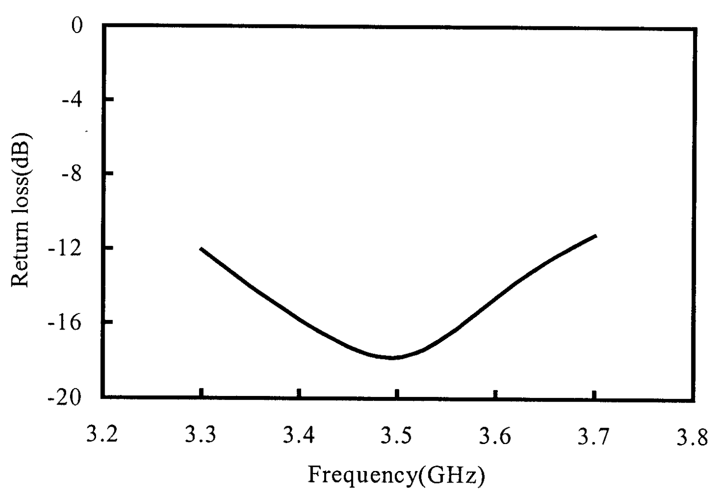

Figure 3. Calculated return loss as a function of frequency.

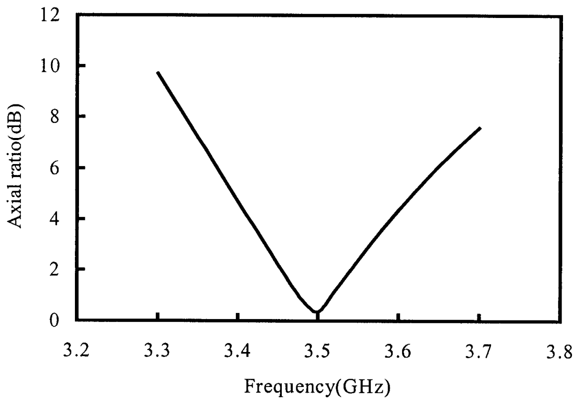

Figure 4. Frequency response of the axial ratio.

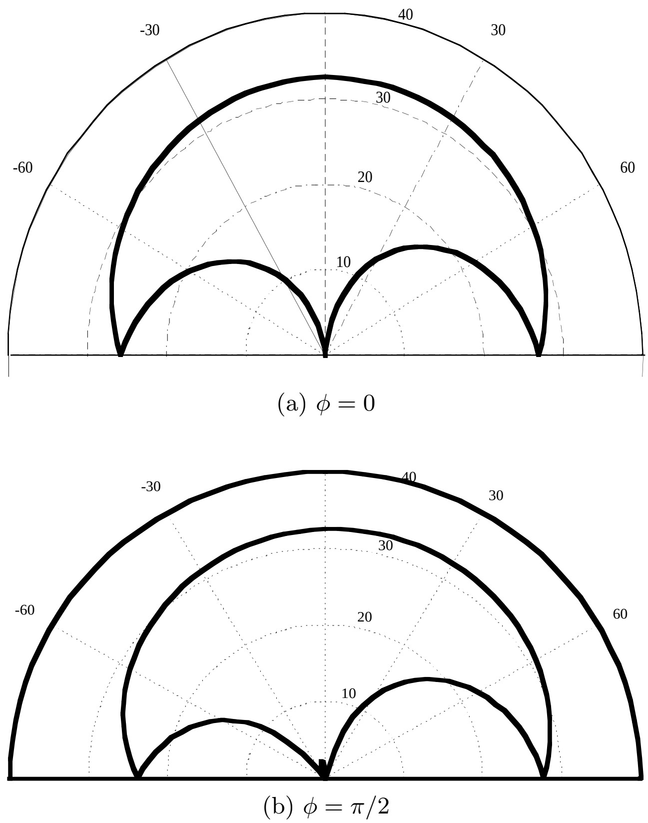

Figure 5. Calculated antenna radiation pattern at 3.5 GHz.

4. CONCLUSIONS

The hemispherical DRA excited by a single-arm spiral slot is studied theoretically. The Green's function technique is employed to analyze the problem, with the equivalent magnetic current expanded by a set of basis functions using the moment method and the analysis is efficient. Good impedance matching and axial ratio can be obtained at the resonating frequency of DRA.

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REFERENCES

1. Leung, K. W., "Rigorous analysis of dielectric resonator antenna using the method of moments," Ph.D. Dissertation, the Chinese University of Hong Kong, May 1993.
2. Khamas, S. K. and G. G. Cook, "Moment method analysis of printed eccentric spiral antennas using curved segmentation," *IEE Proc.-Microw. Antennas Propag.*, Vol. 146, No. 6, 407–410, Dec. 1999.
3. Khamas, S. K. and G. G. Cook, "Moment-method analysis of printed wire spirals using curved piecewise sinusoidal subdomain basis and testing functions," *IEEE Trans. Antennas Propagat.*, Vol. 45, No. 6, 1016–1022, June 1997.
4. Nalcano, H., K. Nakayama, H. Mimaki, J. Yamauchi, and K. Hirose, "Single-arm spiral slot antenna fed by a triplate transmission line," *Electronics Letters*, Vol. 28, No. 22, 2088–2090, Oct. 1992.
5. Chen, C., W. E. McKinzie III, and N. G. Alexopoulos, "Stripline-fed arbitrarily shaped printed-aperture antennas," *IEEE Trans. Antennas Propagat.*, Vol. 45, No. 7, 1186–1198, July 1997.

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