Damage & Fracture Mechanics VI, C.A. Brebbia, A.P.S. Selvadurai, (Editors)
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# Analysis of damage tolerance to tear propagation in stressed structural fabrics

T.A.Godfrey[1] & J.N. Rossettos[2]
[1]*U.S. Army Soldier Systems Center, Natick, MA 01760, USA*
[2]*Dept.of Mechanical, Ind. & Manuf. Engineering, Northeastern University, Boston MA 02115, USA*

## Abstract

A simple micromechanical model is used to study the onset of tear propagation at slit-like damage sites (i.e., a series of consecutive aligned yarn breaks) in biaxially stressed plain weave fabrics under increasing load. The method of treating the frictional slip of yarns near the damage site is critical to the success of the model. Although the actual configuration of slipping regions and rotation of yarns is complex, the onset of tear propagation in large slits (i.e., more than say 35 breaks) is dominated by slip occurring on the first few intact yarns adjacent to the breaks. Analytical results, relating the stress concentration in the intact yarn adjacent to the yarn break at the slit tip, versus the critical applied load at the onset of tear propagation exhibit good agreement with experimental results for a variety of fabrics with initial slits of 35 and 45 breaks. A parameter, $\hat{p}_u$, that evolves from the work can provide a measure of damage tolerance for a class of such fabrics.

## 1 Introduction

Biaxially stressed woven fabrics are used in inflatable and tension structures, parachute canopies, and, increasingly, in geotextile-reinforced geotechnical structures. In end-use, fabrics are often accidentally cut or punctured by a sharp edge or by impact with projectiles. Under sufficiently high remote tension, the local damage provides the starting point for a rapidly propagating tear that results in catastrophic failure of the fabric structure.

Installed geotextiles contain holes created by accidental damage, e.g., during compaction of a landfill [1], as well as purposefully made holes to incorporate specific design features of the project. The growth of tears arising from one of these damage sites can lead to sudden failure of the structure [2]. ). Hedgepeth [3] provided the first micromechanical analysis of a damaged filamentary structure. His analysis, based on shear lag theory, has been applied to fiber/matrix composites, where the matrix transfers the load from broken to unbroken fibers by means of shear. The original work has been the basis of numerous extensions and modifications (see review by Rossettos and Godfrey,[4] ).In biaxially stressed *uncoated* woven fabrics, load transfer between yarns of a given yarn set is accomplished by rotation of the tensioned crossing yarns in the fabric plane. In this regard, the fabric acts mechanically like a remotely stressed plane pin-jointed net and the shear stiffness is stress-induced, rather than an intrinsic property of the fabric. This stress-stiffening effect has been noted by Christoffersen [5] and by Topping [6]. The ultimate transfer of load to a given yarn arises through yarn frictional contact between warp and fill yarns at the cross-over point.

The experimental results of Abbott and Skelton [7] considered slit damage introduced suddenly into uniaxially loaded fabrics, measuring the critical tension at which tear propagation from the initial slit occurs. In tests of otherwise identical coated and uncoated fabrics, critical tensions for the uncoated specimens were higher by as much as a factor of two. It appears that frictional slip acts as an important dissipative mechanism. Popova and Iliev [8] remark on the significant yarn slippage exhibited in experiments on uncoated slitted fabric specimens under uniaxial loading as compared to the behavior of the same fabrics after application of an elastomeric coating.

Godfrey and Rossettos [9] have introduced a micromechanical modeling approach that addresses the tendency of individual yarns to slip near the damage region. They identify two types of frictional slip that occur near the damage site: type 1 involves slip occurring on yarns broken in the initial slit, and type 2 involves slip occurring on intact yarns in the damage growth path near the tip of the slit. Tear propagation is assumed to occur when the maximum yarn tension in the intact yarn adjacent to the last yarn break approaches the yarn breaking load. Although the basic governing equations are simple, implementing the approach to address a damage configuration involving a practical number of breaks involves considerable difficulty in defining regions on each yarn where slip is occurring. Nonetheless, preliminary experimental results [10], demonstrate, through correlation with dimensionless parameters in the model, that the present approach captures the essential physics of the phenomenon.

In this work, we use a micromechanical modeling approach [9], which is adapted here for large slits, and where type 2 slip is assumed to occur on the first few intact yarns at the tip of the slit. For clarity of exposition, some of the original development given in [9], will be summarized here and extended. Experimental results for the onset of tear propagation at 35 and 45 break slits in a variety of biaxially stressed plain weave fabrics are presented and exhibit good agreement with the analytical predictions.

## 2 Micromechanical Model

In this section, we summarize the main features of a micromechanics-based

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mathematical model. Consider a plain weave fabric with damage consisting of a slit-like series of consecutive yarn breaks arrayed parallel to the $x_2$ coordinate direction at $x_1$=0, where the $x_1$ $x_2$ coordinate system is aligned with the yarn directions. The microstructural geometry and nomenclature pertaining to the damaged fabric is indicated in Fig.1. The slit interrupts only number one (# 1) yarns, referring to yarns parallel to the $x_1$ and $x_2$ directions as # 1 and number two (# 2) yarns, respectively. The plain weave unit cell dimensions are $y_{01}$ along the $x_1$ axis (the spacing of the # 2 yarns) and $y_{02}$ along the $x_2$ axis (the spacing of the # 1 yarns). The remote biaxial stress is such that # 2 yarns are under constant remote tensions $F_2$* and # 1 yarns are under quasistatically increasing remote tensions $p$. As $x_1$ direction loading increases, the # 1 yarns exhibit displacements in the $x_1$ direction and the # 2 yarns exhibit $x_1$ direction displacements and small rotations in the fabric plane (Fig. 1). With continued increasing loading, the # 1 and # 2 yarns may be observed to slip at cross-over points in a region near the breaks (Fig. 2). When $p$ reaches a critical value $p_c$, the initial slit damage propagates through rupture of the intact # 1 yarns on either side of the breaks roughly along the line $x_1$=0.

Appropriate differential equations that describe the equilibrium of yarns in regions where slip at cross-over points occurs, and in the region where slip does not occur, can easily be derived [9]. For instance, in the region where cross-over point slip does not occur, equilibrium of the # 1 yarns can be derived by taking into account the load transfer to the # 1 yarns that occurs due to the rotated tensioned # 2 yarns in the fabric plane. For small rotations, the angles are indicated in Fig. 1(b). The component of the # 2 yarn tension along the # 1 yarn can then be written. Introduce $u_n^j$ as the $x_1$ displacement of the $j$ th cross-over point on the $n$ th # 1 yarn, where the reference for displacements are the positions of points on an otherwise identical stressed fabric *without damage*. For sufficiently high values of the loading, $p$, the # 1 yarns are assumed to be in a nearly straightened out condition, and display an effective constant axial stiffness property $(EA)_{eff}$ having the dimension of force. Rotation of the # 2 yarns is represented by relative displacements at points on adjacent #1 yarns. Considering the cross-over point unit cell as a free body, Fig. 1(b), the component of force in the $x_1$ direction acting on the # 2 yarn entry and exit boundaries will be $F_2^*(u^j_{n-1}-u^j_n)/y_{02}$ and $-F_2^*(u^j_n-u^j_{n+1})/y_{02}$. Therefore, equilibrium of the $j^{th}$ cross-over unit cell in the $x_1$ direction is written as

$$F_{1n}^{j+1} - F_{1n}^{j} + \frac{u_{n-1}^{j} - 2u_n^j + u_{n+1}^j}{y_{02}} F_2^* = 0 \tag{1}$$

Replace the $j^{th}$ and $j$+1$^{th}$ $F_{1n}$ terms with differences of cross-over point displacements $(EA)_{eff}(u^j_n-u^{j-1}_n)/y_{01}$ and $(EA)_{eff}(u^{j+1}_n-u^j_n)/y_{01}$, respectively. These terms are strains on either side of cross-over point $j$ multiplied by $(EA)_{eff}$. Eqn (1) can now be written as

$$\frac{(EA)_{eff}}{y_{01}}(u_n^{j-1} - 2u_n^j + u_n^{j+1}) + \frac{F_2^*}{y_{02}}(u_{n-1}^j - 2u_n^j + u_{n+1}^j) = 0 \tag{2}$$

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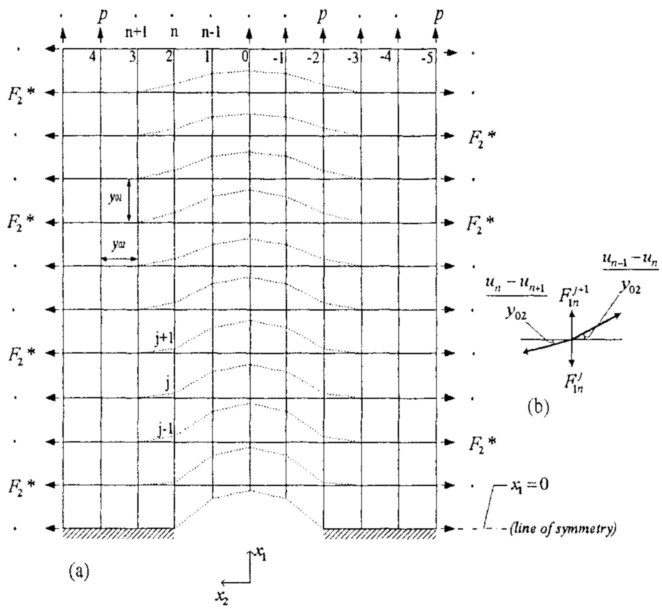


Figure 1: (a ) Geometry of damaged fabric indicating the elastic (non-slipping) deformation. Breaks in # 1 yarns, dashed lines represent deformed # 2 yarns. (b) Equilibrium of the j th cross-over point unit cell.

Smearing out the interaction with the # 2 yarns and regarding $u_n$ as a continuous function of position $x_1$, this equilibrium equation, for non-slipping yarns, can then be written as

$$\frac{d^2 u_n}{dx^2{}_1} + \frac{F_2 *}{(EA)_{eff}\, y_{01} y_{02}} (u_{n-1} - 2u_n + u_{n+1}) = 0 \tag{3}$$

Eqn (3) is written in dimensionless form as

$$U_n'' + U_{n-1} - 2U_n + U_{n+1} = 0 \tag{4}$$

using the following nondimensionalization

$$x_1 = \sqrt{\frac{(EA)_{eff}\, y_{01} y_{02}}{F_2 *}}\ \xi, \quad u_n = p\sqrt{\frac{y_{01} y_{02}}{(EA)_{eff} F_2 *}}\ U_n \tag{5}$$

where primes denote differentiation with respect to ξ.

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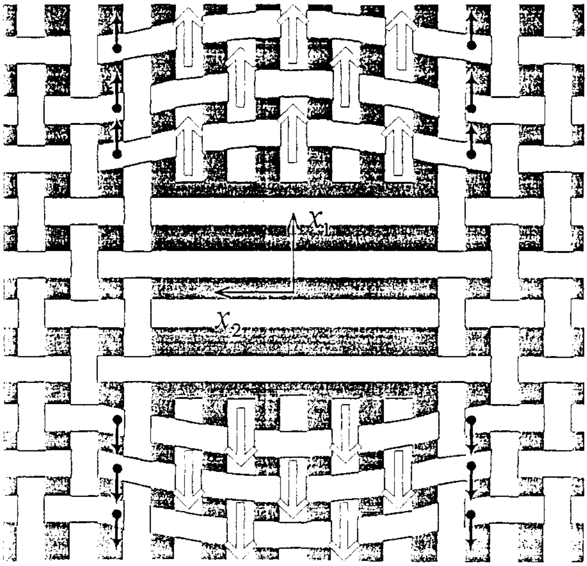


Figure: 2 Yarn slip and deformation pattern near slit. White arrows indicate motion of broken # 1 yarns in type 1 slip. Black arrows indicate motion of # 2 yarns in type 2 slip. Highly schematic.

Cross-over point slip is assumed to occur in a region near the breaks, $0 < x_1 < l_n$, where $l_n$ denotes the extent of the slip region on the $n^{th}$ # 1 yarn. In the slipping region, the yarn is assumed to experience a periodic array of frictional tractions $f$ of constant value (simple slip assumption [9]), the period corresponding to the cross-over point spacing. The equilibrium equation can be written as

$$\frac{d^2u_n}{dx_1^{\,2}} \mp \frac{f}{(EA)_{eff}\, y_{01}} = 0 \tag{6}$$

where the minus sign applies for slip of a broken yarn and the plus sign applies for slip occurring on an intact yarn. It is noted that yarns that are broken (# 1 yarns) will slip in the positive $x_1$ direction relative to the crossing # 2 yarns (Fig. 2), thereby incurring a friction force in the negative $x_1$ direction (type 1 slip: minus sign). In the slip that occurs on an intact yarn, the # 2 yarns slip in the positive $x_1$ direction (Fig. 2), applying a friction force on the # 1 yarn in the positive $x_1$ direction (type 2 slip: plus sign). Eq. (6) is written in dimensionless form as

$$U_n'' \mp \hat{f} = 0 \tag{7}$$

where slip occurs in a region $0 < \xi < \hat{l}_n$ and a dimensionless loading parameter and dimensionless extent of the slip region are introduced

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$$f = p\sqrt{\frac{F_2 * y_{01}}{(EA)_{eff}\ y_{02}}}\ \hat{f}\ ,\quad l_n = \sqrt{\frac{(EA)_{eff}\ y_{01}y_{02}}{F_2 *}}\ \hat{l}_n \tag{8}$$

## 2.1 Application to Large Slits—Type 2 Slip

The essential mechanisms of the model are depicted by the equilibrium equations, *i.e.*, (4) and (7), and appropriate boundary and continuity conditions [9]. For predicting the onset of tear propagation, our interest is in the stress concentration in the first intact yarn at the tip of the slit. It turns out that for large slits (i.e., 35+ broken yarns) type 2 slip at the slit tip dominates in determining the stress concentration factor (SCF). ). The post-test configuration of a 45 break specimen, after removal of external loads, is exhibited in Fig. 3. This test ended in tear propagation, as evidenced by the frayed looking yarn ends resulting from the rupture of the initially intact # 1 yarns along the line $x_1 \cong 0$. Cross-over points on any particular # 2 yarn, where it is interlaced with the 45 initially broken # 1 yarns, are seen to lie at greater distances from the line $x_1 \cong 0$ than cross-over points where the # 2 yarn is interlaced with the initially intact # 1 yarns (except in the region of complicated deformation very near the cut ends). This pattern can be seen in the entire field of view of Fig. 3. The occurrence of type 2 slip along several intact # 1 yarns at the tips of the slit provides a plausible explanation of the observed deformation pattern. As will be seen, using the type 2 slip mechanism in the analysis, gives results which compare well with experiment.

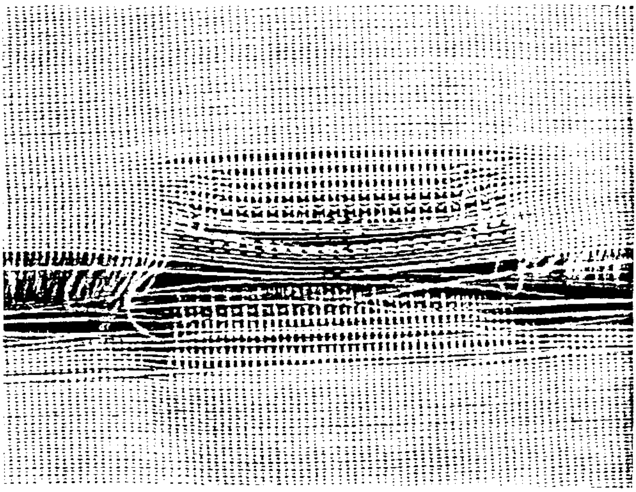

Figure 3: Post-test permanent deformation pattern in 45 break specimen, polyester fabric.

We consider a finite width configuration of *2q+1* # 1 yarns with *2N+1* consecutive aligned breaks (slit) centered at the zero[th] yarn. Yarns in the

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positive $x_2$ half-plane are numbered so that $n$ equals 1 to $q$. Yarns in the negative $x_2$ half-plane are numbered -1 to -$q$. The series of breaks is symmetrical about the center yarn, so that, concerning ourselves only with nonnegative $n$, yarns numbered $0 \le n \le N$ are broken and yarns numbered $n > N$ are intact. Because of symmetry, we need only consider equations for yarns $n$ = 0 to $n = q$.

The fabric is divided into regions I, $0 < \xi < \hat{l}$, where type 2 slip is occurring on yarns $n = N+1$ to $n = N+s$, and II, $\xi \ge \hat{l}$, where no slip is occurring. In the analysis, we will arbitrarily take $s$ to be four, for definiteness.Varying the value of $s$ from four to ten was shown to have an insignificant effect on the stress concentration in the first intact yarn[11].The value of $q$ is assumed to be sufficiently large such that the behavior of the finite width configuration closely approximates that of an infinite fabric with an isolated slit. Studies described in [11] have shown exponential decay in the width direction, so the displacements of yarn $q$ are taken to be those of the undamaged reference, *i.e.*, $U_q(\xi) = 0$.

In region II (non-slipping region), the equilibrium equations have the form of (4), where symmetry about the center yarn and the above assumption regarding the $q^{\text{th}}$ yarn, lead to the following special forms for yarns 0 and $q$-1, written as

$$U_0'' - 2U_0 + 2U_1 = 0 \tag{9}$$

$$U_{q-1}'' + U_{q-2} - 2U_{q-1} = 0 \tag{10}$$

In region I, (4), and the special forms (9) and (10), hold everywhere except for yarns $n = N$ through $n = N+5$. For the slipping yarns, $N+1 \le n \le N+4$, the equilibrium equations are (7) where the plus sign is taken for type 2 slip. Yarns $N$ and $N$+5 require special equations derived from consideration of the equilibrium and deformation [11] of that portion of a crossing (# 2) yarn that spans the cross-over points on # 1 yarns from yarn $N$ to yarn $N$+5. The equations for the $N^{\text{th}}$ and $N+5^{\text{th}}$ # 1 yarn are derived to be

$$U_N'' + U_{N-1} - \frac{6}{5}U_N + \frac{1}{5}U_{N+5} = 2\hat{f} \tag{11}$$

$$U_{N+5}'' + \frac{1}{5}U_N - \frac{6}{5}U_{N+5} + U_{N+6} = 2\hat{f} \tag{12}$$

Since the broken yarn ends are stress free at the slit, the boundary condition on the broken yarns at $\xi = 0$ is $U_n' = -1$. The -1 is due to the fact that the sum of the reference state strain and the additional strain must vanish and the reference state strain is $dU_R/d\xi=1$. The boundary conditions may be written for intact and broken yarns as

$$U_n(0) = 0, \quad n \ge N+1; \qquad U_n'(0) = -1, \quad n \le N; \tag{13}$$

at $\xi = 0$ and as

$$U_n'(\infty) = 0 \tag{14}$$

for all yarns, $0 \le n \le q$-1, at $\xi = \infty$. Since all yarns are continuous at $\xi = \hat{l}$, the following continuity conditions hold, where roman numeral subscripts I and II refer to the solution in regions I and II, respectively

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$$U_{In}(\hat{l}) = U_{IIn}(\hat{l})\,; \qquad U'_{In}(\hat{l}) = U'_{IIn}(\hat{l}) \tag{15}$$

An additional continuity condition arises from the assumption that slipping is approached in a continuous fashion. This may be illustrated by considering a point at $x_1 = a$ on yarn $n = N+1$. As the remote load $p$ is increased, a value of $p$ is reached that just starts slip with the extent $l = 0^+$. As the load is increased further, the slip extent $l$ increases, but, as long as $l < a$, no slip occurs at the point $x_1 = a$. During this time, the frictional forces at the non-slipping cross-over points in the neighborhood of $x_1 = a$ increase continuously, until they reach a maximum value of $f$ (on the verge of slip) as the slip extent $l$ approaches $a$. Therefore, the frictional force on yarn $N+1$ (proportional to $U''_{N+1}$) is taken to be continuous at $x_1 = l$, which may be stated in the dimensionless variables, using (4) and (7), as

$$\{U_N - 2U_{N+1} + U_{N+2}\}_{II}\big|_{\xi=\hat{l}} = \hat{f} \tag{16}$$

A large number of #1 yarns are modeled to represent an isolated slit in an essentially infinite fabric. It is assumed that slip occurs in the region $0 < \xi < \hat{l}$ on a small number of intact #1 yarns at the slit tip. A system of equations is formulated from eqns (4), (9), (10) in the region $\xi \geq \hat{l}$. Another system of equations is developed from eqns (4),(7) and (9) to (12) in the region $0 < \xi < \hat{l}$. A solution is obtained by an eigenvector expansion technique [12] for the boundary value problem, which is defined by selected values of $\hat{l}$. In the solution, the value of $\hat{f}$ corresponding to the selected $\hat{l}$ is obtained and indicates the intensity of a dimensionless applied load, $\hat{p} = \hat{f}^{-1}$. The value of the *SCF*, the ratio of the maximum yarn tension to the remote applied $p$, is calculated for increasing $\hat{p}$. The stress concentration factor (SCF) is defined here as the ratio of the maximum tension in the intact yarn adjacent to the yarn break at the tip of the slit to the remote applied load, $p_{\max}/p$. Using the displacement reference and the nondimensionalization scheme, it is straightforward to show that the SCF can be written as $SCF = U'_{N+1}(0) + 1$.

### 2.1.1 Experiments and damage tolerance parameter

In experiments (apparatus is described in[10]) on biaxially stressed cotton and polyester fabrics with 35 and 45 break slits, the critical remote yarn load $p_c$ (at tear propagation) was measured. It is assumed that the onset of tear propagation occurs when the maximum yarn tension in the fabric, $p_{max}$, in the $N+1$ th (first intact) yarn attains the value of the *in situ* ultimate breaking load of the # 1 yarn, $p_u$. The SCF in a particular fabric, under specific crossing yarn tension $F_2$*, for a slit consisting of a given number of # 1 yarn breaks, takes on a special value at the onset of tear propagation, which we denote $SCF_{tp}$. Since the applied load has the value $p_c$ ( *p critical*) at the instant of tear propagation, we may write $SCF_{tp}$ as $SCF_{tp} = p_u / p_c$. The data is presented through $SCF_{tp}$ versus $\hat{p}_c$ ( the

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value of the dimensionless applied load at the onset of tear propagation, written, using (8), as

$$\hat{p}_c = \frac{p_c}{f}\sqrt{\frac{F_2 * y_{01}}{(EA)_{eff}\ y_{02}}} \tag{17}$$

In Fig. 4, the experimental results are exhibited alongside analytical predictions made using the large-slit approximation. Agreement is seen to be quite good.

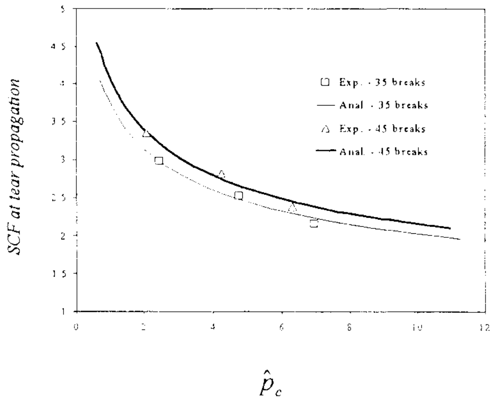


Figure 4:Comparison of analytical and experimental results for 35 and 45 break slits.

The parameter $\hat{p}_c$ is not useful as a predictive tool, since its evaluation requires actual damage growth tests (i.e., finding $p_c$ ). We suggest a related parameter,

$$\hat{p}_u = \mathrm{SCF}_{tp}\ \hat{p}_c = \frac{p_u}{f}\sqrt{\frac{F_2^* y_{01}}{(EA)_{eff}\, y_{02}}} \tag{18}$$

It is seen that the contents of the parameter, $\hat{p}_u$, include microstructural properties of the fabric (e.g., yarn stiffness, yarn strength, warp and fill direction crossover point spacings and crossover point slip frictional forces).These quantities are evaluated (measured) for several fabric configurations involving cotton, polyester and nylon yarns. The experimental data in [10] indicate that increasing values of $\hat{p}_u$ correlate with decreasing values of $\mathrm{SCF}_{tp}$, so that fabrics with high values of $\hat{p}_u$ can withstand higher tearing loads.

## 3 Conclusions

A simple micromechanical model is used to predict the onset of tear propagation at slit-like damage sites in biaxially stressed plain weave fabrics under increasing loading perpendicular to the line of breaks. The model is adapted to large slits parallel to a yarn direction, where it is assumed that slip

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occurs at cross-over points in a region along the first few intact yarns near the tip of the slit. The simplified slipping configuration applies to slits with greater than, say, 35 breaks, where it can be shown *a posteriori* that the slip mechanism assumed in the analysis is the one that dominates for remote load values of interest. Experimental results for the onset of tear propagation in a variety of stressed cotton and polyester fabrics containing 35 and 45 break slits agree well with predictions made using the present model..

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