

THE BALTIC JOURNAL OF ROAD AND BRIDGE ENGINEERING

> 2008 3(3): 145–151

ANALYSIS OF DEFLECTIONS OF BRIDGE GIRDERS STRENGTHENED BY CARBON FIBRE REINFORCEMENT

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Abstract. The paper deals with experimental and theoretical investigations in reinforced concrete structures strengthened with carbon fibre sheets. Four stages in the behaviour of concrete structures strengthened with the carbon fibre reinforced polymer (CFRP) are distinguished. A method for calculating the deflections of such structures is presented. The design procedure for defining the strength of the structures evaluates the stiffness of the contact between the carbon fibre and the concrete. Experimental investigations with different fastening methods of the CFRP to the concrete were performed. In experimental investigations deflections of the strengthened members have been examined. Results of the calculations of deflections for experimental beams according to the proposed method are presented. A comparison of experimental and theoretical deflections is presented in the paper.

Keywords: external carbon fibre reinforcement, strengthening, bridge beams, deflections.

1. Introduction

Most of the infrastructure such as bridges and buildings are in need of continued maintenance and repair because of the stiffness and strength degradation caused by aggressive environmental conditions. The salt water solution, humidity, alkali solution cause cracking of concrete and corrosion of steel reinforcement. Repair and retrofitting of damaged bridge members is complicated. These damaged members can be replaced or strengthened using techniques like externally bonded steel plates, steel or concrete jackets and external tensioning. However, traditional retrofitting methods using steel do not always are the best solutions. For strengthening bridge girders, carbon fibre sheets, plates and bars can be used (Gribniak *et al.* 2008; Kamaitis 2006; Kaminski, Trapko 2006; Valivonis 2006; Van Den Einde *et al.* 2003).

The carbon fibre is lighter, more durable and has a higher strength to weight ratios than the traditional reinforcing materials such as steel. Conducted researches show that the same result of strengthening can be reached with carbon fibre sheet (thickness 1–2 mm) and steel plate (thickness 6 mm). Research results received by many authors (Hag-Elsafi *et al.* 2004; Labossiere *et al.* 2000; Ramos *et al.* 2004) show that the stiffness and the strength in reinforced concrete beams with external carbon fibre reinforcement may be increased by 80% and 100%, respectively. An integrated work of the carbon fibre and the strengthened member may ensure an effective use of the fibre (Hag-Elsafi *et al.* 2001; Hassanen, Raoof 2001; Oehlers 2001; Marchukaitis *et al.* 2007; Valivonis, Skuturna 2007). The bond between external reinforcement and concrete is influenced by several variables, such as measurements of concrete members and fibre, properties of concrete and adhesive, methods of anchoring carbon fibre.

Different methods of anchoring external reinforcement do not help avoid appearence of shear strains between concrete and fibre (Buyukozturk *et al.* 2004; Colotti *et al.* 2004; Xiong *et al.* 2007). Due to shear strains, carbon fibre may move in respect to the concrete. Many authors do not pay attention to this, but an analysis of their experimental results shows that shear strains between concrete and fibre appear.

Most often the strength of the strengthened bridges girders is investigated. However, stiffness of strengthened concrete members with external fibre reinforcement is an essential condition for performance parameters too and it mostly depends on the integrated work of the carbon fibre and concrete.

2. Analysis of the behaviour of members strengthened with carbon fibre

Investigations in the deflection of reinforced concrete structures indicate that in relationship to bending moment-curvature 3 stages of behaviour can be distinguished. Their character depends on the type of reinforcement. Moment–curvature diagram for a member reinforced with reinforcement, which has a definite yield point is shown in Fig. 1a, and for a member reinforced with reinforcement without a definite yield point, this diagram obtains a different shape as indicated in Fig. 1b. The three stages of behaviour for structures strengthened with carbon fibre are researched in (Charkas *et al.* 2003; Faruqi *et al.* 2003).

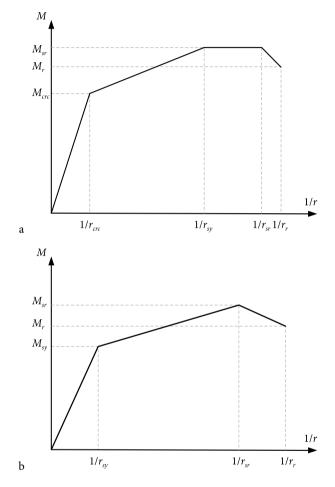


Fig. 1. Bending moment–curvature relationship when reinforcement is with definite yield point (a) and without definite yield point (b)

The analysis shows that in the behaviour of reinforced concrete members, strengthened with carbon fibre reinforcement, 4 stages can be distinguished: 1 – behaviour of the member until its cracking $(0-M_{crc})$; 2 – from the opening of the first cracks until the appearance of the yielding stress in the steel bar reinforcement ($M_{crc} - M_{sy}$); 3 – from the yielding stress in the steel bar's reinforcement until the appearance of the strength stress value in the bar reinforcement; 4 – with exceeded strength stress value in the bar reinforcement until the fracture of the additional reinforcement ($M_{sr}-M_r$). Stages of behaviour of flexural members strengthened with a carbon fibre are shown in Fig. 2.

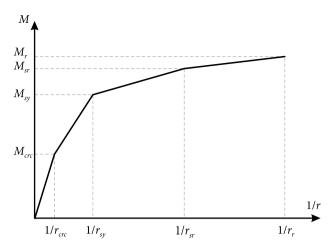


Fig. 2. Bending moment–curvature relationship for reinforced concrete members strengthened with carbon fibre

In the first stage (up to the opening of the first crack) the structure behaves elastically. The carbon fibre in the tension zone substantially restrains the strains of the concrete that is subjected to tension. Therefore, limit strains of the concrete in tension increase. Results of experimental investigations performed by the authors and others ($Ky\pi\pi\etayc\kappaac$, Hotkyc 1987) indicate that in this case the tension strain in the concrete can reach $30-40 \times 10^{-5}$. This means that the composite structure with a restrained tension zone – the bending moment value at the appearance of the crack increases substantially.

After the opening of cracks, the structure enters into the second stage of its behaviour. In this stage, the cracks develop, but the carbon fibre, covering the whole surface of the tension zone, restrains development of the cracks. The cracks increase neither in width, nor in their height. Consequently, cross-sectional stiffness decreases insubstantially. This means that stiffness of structures strengthened with the carbon fibre is greater than that of the common reinforced concrete structures.

Since the strength of the carbon fibre normally is greater than the steel strength in the tension zone, then the steel yield stress in the tension zone is reached. As the yield stress in the steel reinforcement is reached, the structure enters the third stage of its behaviour $(M_{sy}-M_{sr})$. At this stage the major part of the stress increment is taken by the carbon fibre reinforcement.

In the fourth stage, when the yield stress of the steel reinforcement is reached, the whole increment of stress is resisted by the carbon fibre. At this stage the stress in the steel reinforcement does not increase.

The carbon fibre is fastened to the reinforced concrete member with glue (generally, on the basis of epoxy). Since the moduli of elasticity and shear for the glue are 3–10 times less than these moduli for the concrete, shear strains between the reinforced concrete member and the carbon fibre develop. In this case the carbon fibre slips in relation to the concrete (Fig. 3). The increase in deflection due to the carbon fibre slip depends on the thickness of the glue layer and its mechanical characteristics, also on the way of anchorage of the carbon fibre.

Slip of the carbon fibre in relation to the reinforced concrete member normally takes place when the bending moment exceeds the cracking moment (Fig. 3). In this figure influence of the slip value of the carbon fibre in relation to the reinforced concrete member is evaluated by the zone between lines 1 and 2 (shaded). Width of the zone depends on the composition and properties of the glue and on the type of anchorage.

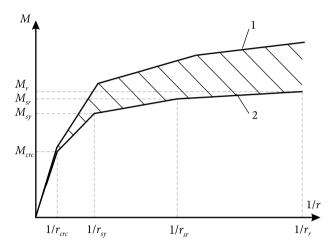


Fig. 3. Relationship between deflection and bending moment: 1 – no slip; 2 – a slip between concrete and carbon fibre

In case of an absolutely stiff connection between the carbon fibre and the concrete, curves 1 and 2 coincide. Width of the zone between curves 1 and 2 determines the effectiveness of carbon fibre use for strengthening.

3. Theoretical analysis of deflections

Deflections of the beams strengthened with external carbon fibre reinforcement can be estimated by applying the theory of built–up bar (Ржаницын 1986). The design scheme for calculations is provided in Fig. 4.

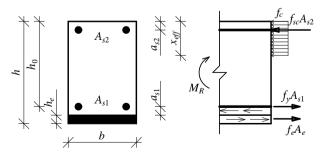


Fig. 4. The design scheme for deflection calculation

The max deflection for a simply supported beam loaded with two concentrated forces is received by formula (1):

$$f = M \left(\frac{l^2}{8E_{eff} I_{eff}} + \frac{1}{D} \left(\frac{ch(0.5\lambda l) - 1}{\lambda^2 ch(0.5\lambda l)} \right) \right), \tag{1}$$

where l – the beam length, m; M – bending moment for the action in which the deflection is to be determined, kNm; λ – the value describing the stiffness of the contact.

$$E_{eff} I_{eff} = E_{cm} I_{c,eff} + \frac{E_{cm} A_{c,eff} E_e A_e z^2}{E_{cm} A_{c,eff} + E_e A_e},$$
 (2)

$$\frac{1}{D} = \frac{1}{E_{cm}I_{c,eff}} - \frac{1}{E_{eff}I_{eff}},$$
(3)

where E_{cm} – moduli of elasticity of concrete, GPa; E_e – moduli of elasticity of carbon fibre, GPa; $A_{c,eff}$ – the area of effective reinforced concrete cross-section, m²; $I_{c,eff}$ – the moment of inertia of a concrete cross-section, cm⁴; A_e – the area of external reinforcement, m²; z – the distance from the weight centre of the element and the carbon fibre centre, m.

The value λ , assessing the stiffness of the contact is calculated by the formula (4):

$$\lambda = \sqrt{\alpha \gamma}, \qquad (4)$$

$$\alpha = \frac{bG_{w,eff}}{z},\tag{5}$$

$$\gamma = \frac{1}{E_{cm}A_{eff}} + \frac{1}{E_{e}A_{e}} + \frac{z^{2}}{E_{cm}I_{c,eff}}.$$
 (6)

The characteristics $G_{w,eff}$ of the stiffness of the contact in respect to the shear was identified from experimental research results and can be calculated by applying the formula:

$$G_{w,eff} = 0.001 K E_{cm},\tag{7}$$

where K – the coefficient evaluating the method of anchoring external reinforcement (1 – CFRP is not anchored; 1.5 – CFRP anchored with keys; 2 – CFRP anchored with carbon fibre strips; 37 – CFRP under the supports); E_{cm} – moduli of elasticity of concrete, GPa.

The moment of inertia of a concrete cross-section with cracks:

$$I_{c,eff} = \frac{bx_{eff}^3}{3},$$
(8)

where b – the beam width, m; x_{eff} – the compression zone height of the reinforced concrete beam, m.

The compression zone height of reinforced concrete beam is calculated by the formulae:

$$x_{eff} = \beta h_0 \left(\sqrt{\mu^2 + \frac{\mu h}{h_0}} - \mu \right), \tag{9}$$

$$\mu = \mu_{s1} \alpha_{s1} + \mu_{s2} \alpha_{s2} + \mu_e \alpha_e, \qquad (10)$$

$$\mu h = 2 \left(\mu_{s1} \alpha_{s1} h_0 + \mu_{s2} \alpha_{s2} a_{s2} + \mu_e \alpha_e h_{0e} \right), \qquad (11)$$

where α_{s1} , α_{s2} , α_e – ratio of the elasticity modulus of reinforcement in tension, compression and external reinforcement to the concrete elasticity modulus; μ_{s1} μ_{s2} μ_e – reinforcement ratios; a_{s2} – distance from centroid of compressive steel to extreme compressive fibre, m; β – the coefficient for evaluating an equivalent compression zone depth variation with a value of stress in the compression zone; h_0 – the effective depth of reinforced concrete beam, m; h_{0e} – the effective depth of reinforced concrete beam with external reinforcement, m.

The coefficient β can be calculated by applying the formula:

$$\beta = \varphi - 0.04 \frac{E_e M}{E_{cm} M_{cr}},$$
(12)

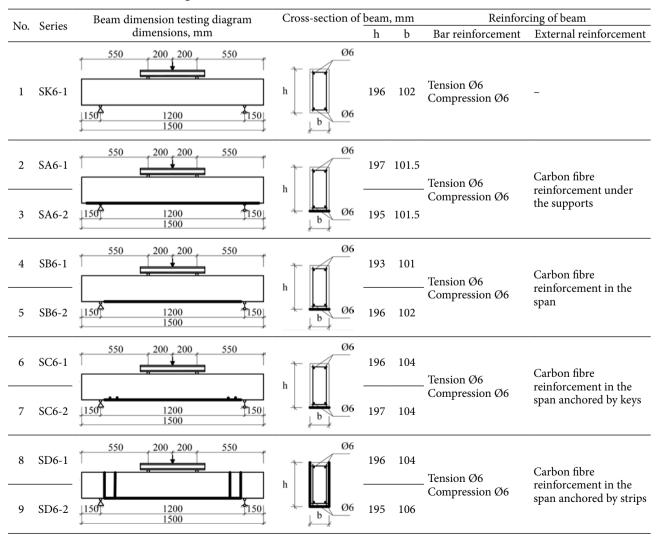
where M_{cr} – a cracking moment, kNm; E_{cm} – moduli of elasticity of concrete, GPa; E_e – moduli of elasticity of carbon fibre, GPa; φ – the coefficient evaluating the slip of the carbon fibre in relation to the concrete.

Table 1. Main characteristics of strengthened beams

Taking into account the results of experimental investigations, it is concluded that deflections of reinforced concrete beams strengthened with carbon fibre are significantly influenced by shear deformations, emerging in the contact between the carbon fibre and the concrete. Therefore, in the analysis of deflections of such structures it is necessary to evaluate the slip of the carbon fibre in relation to the concrete emerging in their contact. In the method proposed by the authors it is taken into account by applying coefficient ϕ (2.4 – fibre is not anchored; 2.6 – CFRP anchored with keys; 2.45 – fibre anchored with carbon fibre strips; 2.5 – fibre under the supports).

4. Experimental investigations

The composite structures consist of concrete, steel reinforcement, carbon fibre and glue. Normal weight concrete was used for the manufacture of reinforced concrete beams. Concrete was made from Portland cement, quartz sand and crushed gravel. For testing 9 beams were produced. The properties of materials were determined by testing samples of concrete, steel, fibre and glue. The mean compressive strength of the concrete at time of beams



test was $f_c = 33 \text{ N/mm}^2$ ($E_{cm} = 32 \times 10^3 \text{ N/mm}^2$) and $f_c = 38 \text{ N/mm}^2$ ($E_{cm} = 34 \times 10^3 \text{ N/mm}^2$). Beams had Ø6 mm steel reinforcement, the mean yield stress of which $f_y = 358 \text{ N/mm}^2$ and elasticity modulus $E_s = 205 \times 10^3 \text{ N/mm}^2$, ultimate stress $f_u = 460 \text{ N/mm}$ and deformation modulus (when the yield stress is exceeded) $E_{st} = 1500 \text{ N/mm}^2$. The beams were strengthened with carbon fibre, the strength of which was $f_e = 3800 \text{ N/mm}^2$ and elasticity modulus $E_e = 231 \times 10^3 \text{ N/mm}^2$. Compression strength of the glue was $f_{gc} = 90 \text{ N/mm}^2$, tension strength $f_{gt} = 32 \text{ N/mm}^2$, elasticity modulus $E_g = 5100 \text{ N/mm}^2$.

Main characteristics of the beams are shown in Table 1. Cross-sectional depth of all beams equals 193–197 mm, and cross-sectional width of beams is 101–106 mm. The length of the beams is 1500 mm. The tension zone and the compression zone of beams was reinforced with 206 mm steel bars. Supporting zones of the beams were reinforced with 206 mm stirrups spaced at 100 mm along the beam.

The beams were reinforced with a fabric of carbon fibre, the cross-sectional area of which was the same in all strengthened beams. The carbon fibre was pasted to reinforced concrete beams with epoxy glue. Before pasting, the surfaces of all beams were cleaned with steel brushes and degreased. The width of the carbon fibre strip was 100 mm. Its cross-sectional area was 16.8 mm². For an assessing of the influence of the carbon fibre anchorage on deflections of strengthened beams, a different way of anchorage of the carbon fibre at supports was used. Two beams (SA6-1 and SA6-2) were strengthened by overlapping the carbon fibre behind the supports. Two beams (SB6-1 and SB6-2) were strengthened with the carbon fibre and overlapped up to the supports. In beams SC6-1 and SC6-2 recesses were made before strengthening (the keys were shaped), but the carbon fibre was lapped up to the supports. In beams SD6-1 and SD6-2 carbon fibre hoops were installed besides the supports. One beam was used as control without carbon fibre.

The beams were tested with two concentrated loads (Fig. 5). During testing load was increased in steps. Values of the load, deflections and deformations were recorded with the automated recording system ALMEMD 5590.

5. Results and analysis

During testing, deflection of beam axis in the midspan and deformations in the concrete and the carbon fibre in the cross-section of the beam were measured. It was determined that deflections of reinforced concrete members, strengthened with carbon fibre under the action of forces of the same value, are substantially less (Figs 9–11). Deflections of the beams strengthened with carbon fibre were reduced in comparison with that of control beam by 53% (Figs 6–9). Experimental investigations showed that deflection of strengthened beams also depend on the way of anchorage of the carbon fibre.

Deflection of beams (SA6–1 and SA6–2) with carbon fibre overlapped behind the supports was the smallest (Fig. 6), while deflection of beams (SB6–1 and SB6–2) with the carbon fibre, in the middle of the beam, was the greatest (Fig. 7).

This is also confirmed by assumptions in the analysis of behaviour of the members strengthened by the carbon fibre presented in Section 2 of this paper. Experimental investigations indicated that in the contact between the carbon fibre and the concrete shear deformations occur. The

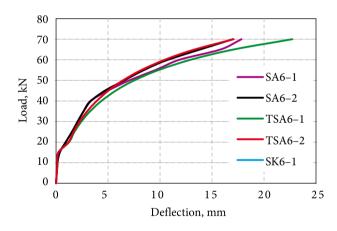


Fig. 6. Experimental (SA6) and theoretical (TSA6) deflections of beams and experimental deflection of control beam SK6–1

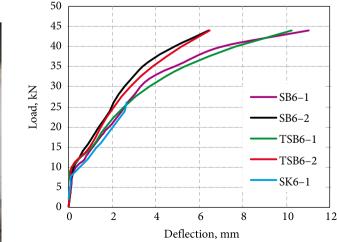


Fig. 7. Experimental (SB6) and theoretical (TSB6) deflections of beams and experimental deflection of control beam SK6–1



Fig. 5. Overview of test set up

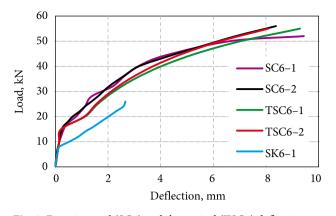


Fig. 8. Experimental (SC6) and theoretical (TSC6) deflections of beams and experimental deflection of control beam SK6–1

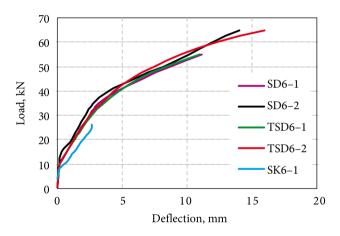


Fig. 9. Experimental (SD6) and theoretical (TSD6) deflections of beams and experimental deflection of control beam SK6–1

value of these deformations depends on the way of the carbon fibre anchorage.

The graphs of the load-deflection relation for beams with the best anchorage (when carbon fibre is overlapped behind the supports) and with the worst anchorage are presented in Fig. 10.

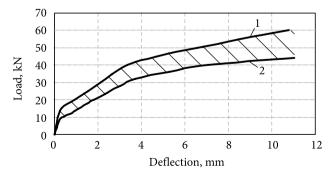


Fig. 10. Load-deflection relationship for beam SA6–1 with best anchorage (1) and for beam SB6–1 with worst anchorage (2)

Analysis of this graph shows that under the action of an external force different deformations in the contact between the carbon fibre and the concrete occur. A different influence of the shear deformations is demonstrated by greater deflections in the beams with a worse anchorage of the carbon fibre.

Analysis of graphs expressing relationships load-deflection (Figs 6-9) indicates that in the first stage of behaviour of structures caused by an external load for all beams reinforced with Ø6 mm, steel reinforcement deflections were absolutely of the same value. The normal cracks opened earlier in the beams without overlap of the carbon fibre behind the supports and without any use of additional anchorage means. Under the action of a force of the same value cracks opened, and in beams that were provided with carbon fibre strips for additional anchorage of the carbon fibre (SD6-1, SD6-2). After the appearance of normal cracks, deflections began to increase significantly, and the beams entered the second stage of behaviour (Section 2). Stiffness of the beams is reduced due to cracks in the beams. At this stage of behaviour shear deformations between the carbon fibre and the concrete emerge. Graphs in Figs 6-9 indicate that the shear deformations grow with the bending moment. When the value of stress in the steel of the bar reinforcement reaches the value of the yield stress, the load-deflection curves demonstrate the second stage of the structures and then the behaviour of the structures enters stage 3.

Experimental investigations indicate that the difference in deflection values increases for the beams with different ways of carbon fibre anchorage. This is also seen under the influence of the shear deformation appearing in the contact between the concrete and the carbon fibre. Existence of shear deformations is confirmed by distribution of longitudinal deformations over the height of the cross-section (Fig. 11).

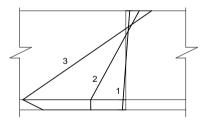


Fig. 11. Distribution of longitudinal deformations in experimental beams when $M/M_R = 0.3$ (1), when $M/M_R = 0.6$ (2) and when $M/M_R = 0.9$ (3)

When keys are installed in the support zones (the beams SC6–1 and SC6–2), then deflections are similar to those beams with overlap of the carbon fibre up to the end (under the supports) of the beams. But it was established that in the case of additional anchorage of carbon fibre with carbon fibre strips (beams SD6–1 and SD6–2) the deflections were similar to those of beams without additional anchorage (beams SB6–1 and SB6–2). The fibre strips influence was small.

Theoretical values of deflections for the experimental beams were determined by the method described in Section 3. Deflections of experimental beams tested by the authors were calculated. Theoretical and experimental deflections of beams strengthened by carbon fibre are shown in Figs 6–9. Comparison of experimental and calculated (theoretical) values of deflections pointed out that experimental and theoretical values were close to each other.

6. Conclusions

The investigations revealed that carbon fibre can be employed for an effective increase in stiffness of flexural reinforced concrete structures.

The performed analysis of behaviour of strengthened structures gave the opportunity to distinguish 4 stages of behaviour of such structures: behaviour up to cracking: from opening the first cracks up to yielding stresses in the steel bar reinforcement; from yielding stresses in the steel bar reinforcement up to reaching the stress value of strength in the bar reinforcement; when the stress value of strength is exceeded up to breaking the additional reinforcement.

Influence of different ways of fastening the carbon fibre to the concrete on behaviour of the strengthened beams subjected to the action of the external forced was investigated. The investigations showed that shear deformations occurring in the contact between the carbon fibre and the concrete depend on the anchorage of the carbon fibre, glue used and the value of the force acting on the structure.

In the method used for calculating deflections, the stiffness of the contact is taken into account. Values of theoretical deflections agree sufficiently well with experimental data obtained by the authors.

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Received 5 March 2008, accepted 22 Aug 2008