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Analysis of Efficiency Sensitivity Associated with Tip Clearance in Axial Flow Compressors

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Abstract

The effects of tip clearance changes on efficiency in axial compressors are typically established experimentally. The ratio of change of efficiency with change of clearance gap varies significantly for different compressors in the published data. An analysis of this sensitivity range in terms of the blade and stage design parameters was initiated. The analysis revealed that the sensitivity range largely resulted from a derivation at constant flow of the efficiency decrement. It was also found that a generalized loss method of generating the sensitivities produced a much improved correlation of the change in efficiency with change in clearance over a variety of machines, configurations and speeds.

Nomenclature

A	area (section)
AR	blade aspect ratio
c,C	constant or chord
e/b	clearance gap/blade height
HR	hub tip ratio
L	lost work rate, P-$\Phi\Pi$
MN	power input (torque-speed,enthalpy rise)
p	pressure (total, static)
P	shaft power coefficient, $MN/\rho U_t^3 A$
R_t	tip radius
s	blade spacing or pitch
V	velocity (section axial average)
U_t	tip velocity
X	work coefficient, P/Φ
β	vector mean flow angle
σ	blade solidity
δ^*	displacement thickness
Φ	flow coefficient, V/U_t
Π	pressure rise coefficient, $\Delta p/\rho U_t^2$
η	adiabatic efficiency, $\Phi\Pi/P$

Subscripts

t	at tip radius
0	at zero clearance
p	at constant power
ϕ	at constant flow

INTRODUCTION

The effects of tip clearance changes in axial compressors have frequently been described, or modelled, in terms of the flow losses in or around the tip gap (Betz, 1926; Fickert, 1946; Rains, 1954; Vavra, 1960; Senoo and Ishida, 1986). Alternative models formulated in terms of shedding and retention of circulation (Lakshminarayana, 1970) or boundary layer mechanisms at the case wall (Smith, 1970) have also been advanced. The discussion that follows is principally related to models where a local drag loss is assumed to be the dominant mechanism associated with the clearance effect and the stage is not approaching stalled or choked conditions. Constants required to use these models are usually calibrated or empirically fitted from changes in experimental stage efficiency measured for the model evaluation or collected from other studies. These data are most frequently generated by cropping the blade or increasing the case radius in an otherwise constant stage configuration.

The observed efficiency change to normalized clearance gap change varies significantly for different compressors in the literature. Typical results for mainly single stages are shown in Figure 1. This variation in sensitivity is of some importance in a multistage machine due to the compounding effect of the stage efficiency on overall compressor efficiency. In addition to a high overall efficiency level, a low efficiency sensitivity to clearance variation is a desirable feature of a blading design for any stage.

Figure 2. shows the impact on a high pressure ratio compressor of a change of slope (sensitivity) from one to two units observed in the data of Figure 1. For a polytropic compression over a range of pressure ratios a sensitivity change of one unit is roughly equivalent to a four percent change in overall compressor efficiency at 20:1 pressure ratio. This estimate assumes that the compressor blading efficiency sensitivity and clearance gap remain constant along the gas path and that the efficiency is lowered in proportion to the reducing blade height as the gas is compressed. As aerocompressors experience a wide range of thermal and stress induced clearance changes while operating, the overall performance can be further effected in certain segments of the flight envelope. Beitler et al. (1980)

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have discussed active clearance control for core compressors in the context of adjusting the clearance to achieve the optimum clearance for different flight segments.

An analysis of the sensitivity spread in terms of the blade and stage design parameters was the initial objective of the work described in the paper. However, further examination revealed that the published data produced an improved correlation of efficiency change to clearance change over a wide range of machines if the losses were correlated against shaft power input and the efficiency change examined at constant power. This observed uniformity brought into question the complexity of many of the models which claim to account for the variation in sensitivity.

The analysis and data used to support these comments are developed after a brief review of the context of the work and its relationship to blading design.

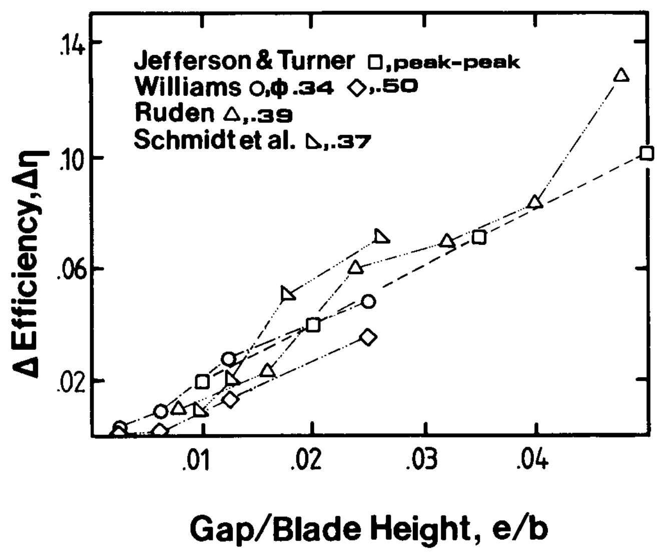

Fig. 1 Efficiency decrement at constant flow as a function of gap/blade height, derived from various studies. Flow coefficient values are indicated in the legend

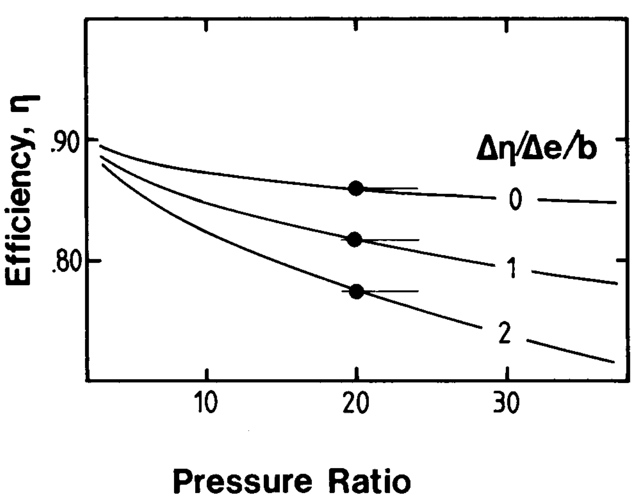

Fig. 2 Influence of stage efficiency sensitivity to clearance gap on overall compressor efficiency as a function of pressure ratio

THE CLEARANCE EFFECT AND DESIGN PROBLEM

The clearance effect is generally understood to mean the change in the compressor characteristic as the tip clearance gap is varied over a small range of the gap dimension. Typically, but not universally, the overall pressure rise and efficiency substantially decrease for a multistage compressor with increasing levels of clearance. At certain flow rates increases have been observed (Freeman, 1985). Stage test data, however, always seem to show a fall in performance on a throttle line at constant speed.

Sensitivity Data

Clearance sensitivity data are generated experimentally by measuring compressor or stage performance for a range of clearance dimensions or parameter increments and establishing the difference in efficiency between each dimension or parameter level. Direct methods of effecting the clearance change in terms of dimension have included cropping the blades (Ruden, 1937 and Williams, 1960), enlarging the case wall diameter (Inoue et al., 1986), recessing the clearance (Wisler and Beacher, 1986) and varying the blade root (Schmidt et al., 1987). Parametric changes in terms of a clearance gap (e) normalized in terms of some other dimension or feature of the flow have also been addressed by considering the boundary layer (Smith, 1970) or its thickness (Hunter and Cumpsty, 1982) or case wall roughness (Bettner and Elrod, 1982). In all these experiments the compressor (or stage) configuration was not significantly altered; that is, the change could be considered a small perturbation of the configuration primarily related to the clearance effect or local roughness.

The results generated were frequently correlated in terms of the efficiency change of the stage ($\Delta\eta$) as a function of the normalised clearance gap. The normalizing dimension most commonly used is the blade height (b), however staggered spacing, tip section chord and blade thickness have also been used. Data plotted by Lakshminarayana (1970) and subsequently by Senoo and Ishida (1986) and Schmidt et al. (1987) are shown in Figure 1 as typical examples of the data generated.

The primary feature of interest in the data is the range of variation in sensitivity and its impact on performance as shown in Figure 2. The sensitivity (slope) magnitude typically varies from 0.75 to 2.25 around the one percent level in (e/b) for near design flow. The data suggest certain bladings have lower sensitivity than others. From a design perspective it would be desirable to know how to produce the lowest sensitivity for any stage configuration. The aerodynamic design of the blading, particularly the tip local geometry and tip loading, is thought to be relevant to the design optimization problem. Reducing end wall losses associated with the clearance is discussed by Wisler (1977) in terms of the tip loading of several test rotors.

Analytical Correlations

Predictive methods for the observed sensitivity have been developed by several investigators over a period of time. The most common approach has been to develop an expression for the efficiency decrement from a model of the tip flow and blade geometry and then empirically correlate parameters in the expression with test data. The functional dependence of some of the correlation formulae are set out in Table 1.

Table 1.

Correlations of Efficiency Change with Tip Clearance Parameters in Axial Compressors.

Fickert (1946)	$\Delta\eta = f(e/R_t, \Phi, X, HR)$
Rains (1954)	$\Delta\eta = f(e/R_t, \Phi, X, \cos\beta, \sigma, e/\delta^*)$
Vavra (1960)	$\Delta\eta = f(e/b, \Phi, X, \cos\beta, \sigma, b/s)$
Laksh'a (1970)	$\Delta\eta = f(e/b, \Phi, X, \cos\beta, AR)$
Senoo (1986)	$\Delta\eta = f(e/b, \Phi, X, \cos\beta, \cot\text{an}\beta, \sigma, HR, [\Pi_0, X_0, \eta_0, c_1\text{-}c_4, \Delta\Pi_h])$

Senoo's model is by far the most complex formulation. It requires conditions at zero clearance (Π_0, X_0, η_0) and also $\Delta\Pi_h$, *the pressure loss coefficient due to causes other than tip clearance*, to be known. In addition four coefficients (c_1-c_4) which account for contraction factor in the gap, pressure recovery of the gap, blockage factor of the passage and a *blockage multiplier due to clearance* are required to be known or estimated beforehand in order to predict the effect of a gap change on efficiency. The resulting curves are linear with e/b, so the model is primarily correlating the sensitivity.

The functional dependence of the correlations of Table 1 are seen to be similar, however, they vary substantially in algebraic form. This is due to their different treatment of lift coefficient (C_L) near the tip, which introduces powers of the Φ, X, $\cos\beta$, and σ terms through the relationship ($C_L = X\cos\beta/\Phi\sigma$) for moderately staggered blades. The models and their differences are discussed by Senoo (1987).

The Design Problem

It is apparent that geometric rather than flow variables, that could be locally varied in the blade tip and wall design, are desirable variables to be included in a model. The variables of particular interest are axial chord, staggered spacing, stagger angle, lean and blade camber and thickness distribution. These quantities can be varied locally within the overall design constraints of throughflow and velocity diagrams. At a more detailed level the shape of the tip gap and wall under the blade may also be varied with a wide variety of treatments.

ANALYSIS OF SENSITIVITY DATA

The initial intent of the analysis was to address the extended set of geometric parameters (discussed above) and attempt to correlate the sensitivity spread observed in the literature with design variables that might be optimized. The first task was to gather the details of known test stages and their tip geometry. In so doing, it was noted that the sensitivity varied with flow rate or stage flow coefficient, Φ. It was also noted that certain sensitivities were defined in terms of decrease in peak efficiency while others were derived at constant flow rate. A more general definition of the sensitivity was then sought to apply to all the data available.

Definition based on Losses

The primary effect of a clearance variation is a change in stage pressure rise at any flow rate. As this process can be thermodynamically characterised in terms of the shaft power converted into flow work per unit time and the power lost to inefficiencies, a general definition of the losses is given by:

$$L = P - \Phi\Pi$$

where the lost work rate (L) plus the flow work rate ($\Phi\Pi$) equals the power input (P). The aggregate lost work rate can be related to the integral spanwise passage average loss coefficents and in turn to the blade element loss coefficients by appropriate manipulation. For any change of compressor geometry from one configuration to another, for example from clearance 1 to clearance 2, the change in losses is given by:

$$L_1 - L_2 = P_1 - P_2 - \Phi\Pi_1 + \Phi\Pi_2$$

and if the efficiency is introduced as:

$$\eta = \Phi\Pi/P$$

then the efficiency to loss correlation is:

$$L_1 - L_2 = P_1 \cdot (1 - P_2/P_1 - \eta_1 + \eta_2 \cdot (P_2/P_1))$$

It is apparent from this formulation that the change in efficiency is not generally equivalent to the lost work rate change between two configurations. Alternatively, a change in efficiency only directly reflects a change in losses between two configurations if the efficiency change is determined at the same input power level, ie. if $P_1 = P_2 = P$ then;

$$(L_2 - L_1)/P = \eta_1 - \eta_2$$

This expression indicates that *if* the efficiency decrement at a constant flow coefficient is used to determine the clearance sensitivity of a stage, it does not solely reflect the change in losses over the blading but also includes any shift in the stage power characteristic that may occur. This point is significant if the flow mechanisms creating the loss are of primary interest or are being modelled.

Efficiency Change with Shaft Power

The composite effects are shown schematically in Figure 3 for throttle lines at constant speed. Flow work ($\Phi\Pi$) is plotted versus power input (P). A straight line passing through the origin of this $\Phi\Pi$ vs. P plane is a line of constant efficiency. By plotting the compressor characteristic on these coordinates, at two different clearance levels, and linking points of constant flow coefficient (A-C), the distinction

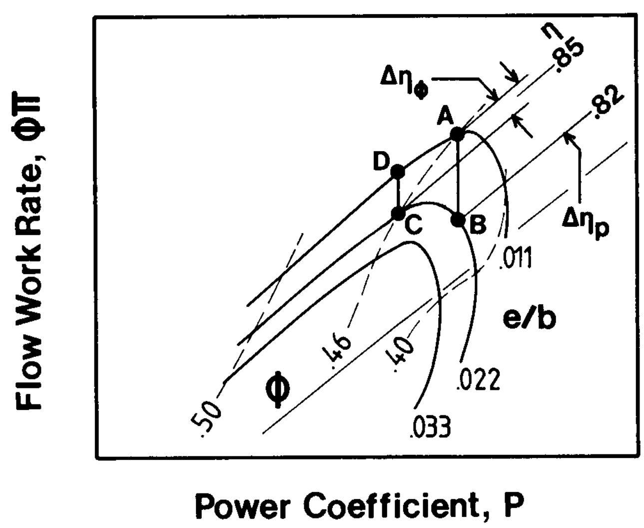

Fig. 3 Schematic flow work vs. power input characteristic, for constant speed throttle lines, illustrating methods of efficiency decrement determination. Based on data from Inoue et al. (1986)

between the increase in losses and efficiency change at constant flow rate and at constant power is clearly demonstrated. The lost work rate differs substantially between the two clearance levels as the power increases over the range shown, ie. (A-B) compared to (D-C). However, the change in efficiency at constant flow ($\Delta\eta_\phi$), spanning the two power levels (A-C), is seen to be relatively small compared to $\Delta\eta_p$, the change at constant power. The efficiency correlations with gap/blade height of Fig. 1 have been established by this method of subtraction of efficiency values (at constant flow) over a range of clearance increments. This mode of derivation of an efficiency to clearance effect tends to produce a linear correlation with a low slope due to the constant flow lines' oblique angle to the constant efficiency lines over much of the range. This is most evident at higher flow rates where the stage characteristics also follow the constant efficiency lines and yield $\Delta\eta$ vs. e/b curves similar to those shown in Fig. 1. As flow rate is reduced, the slope of the constant flow efficiency curve tend to increase then flatten out, as shown by the Φ = 0.4 line in Fig. 3. This trend has been observed in the data by Schmidt et al. (1987), however, the reason for the trend was not addressed.

It should also be noted that changes in peak efficiency more closely relate to constant power efficiency decrements (and hence losses) than decrements at constant flow. This can be seen as (C and D) on Fig 3. are moved in the direction of increasing flow toward the peak efficiency point of each throttle line. In order to standardize the sensitivity data available in the literature, several sets of experimental data were plotted in this $\Phi\Pi$ vs. P graphical format and the efficiency decrements and clearance sensitivities determined at constant power.

Low Speed Cropped and Recessed Data

As the flow work vs. power characteristic was entirely general in terms of the losses indicated, comparisons could be made between the effects of clearance changes by cropping the blades, increasing the casewall diameter, increasing hub diameter and recessing the clearance. Provided input shaft power had been measured in the experiment, data from different tip and wall perturbation experiments could be compared. Data from four low speed compressor experimental programs are shown plotted on Figures 4 and 5. The data are all for air compressors except Williams' stage, which was for water.

The figures show, respectively, the data of Ruden (1937) for a cropped blading in an isolated stage without guide vanes and Williams' (1960) cropped data for stage a with inlet vanes. The data are carpet plotted on Fig. 4 and the incremental change in loss with gap increments can be clearly seen. There is a trend towards larger efficiency improvements as the clearance is decreased. This sensitivity trend is indicated by a horizontal bar chart on the figure beside the constant power line. One division corresponds a unit of slope, ie., a bar two units wide represents a change of two percent in efficiency for a one percent change in clearance gap. Ruden's data shows the strongest improvement as the clearance is reduced. This trend is generally consistent with peak pressure rise data shown schematically by Koch (1981) and is *not* consistent with the trends shown in Figure 1.

Inoue's (1986) data for an essentially isolated rotor with no preswirl vanes and Wisler and Beacher's (1986) data for a four stage (averaged) compressor at two clearance levels with different degrees of recessing of the clearance are shown on Fig. 5. The data from Wisler and Beacher show the losses to be largely insensitive to the gap location (ie. either in or out of the wall) but strongly dependent on the gap dimension. In terms of effecting changes in performance with local profile variations, Wisler and Beacher's data were not encouraging.

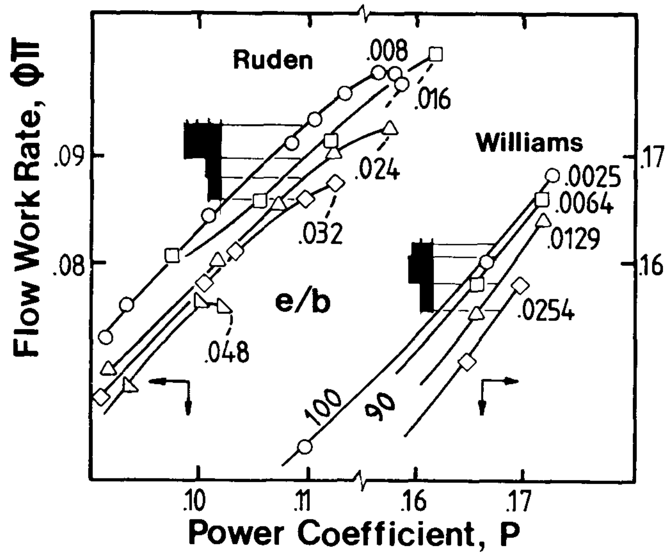

Fig. 4 Flow work vs. power input characteristic for cropped blades tested by Ruden (1937) at constant speed and Williams (1960) data with a small speed change on one line

This effect notwithstanding, the striking feature of the sample data is the almost identical effect of the clearance change on the characteristics and the *decrease* of the constant power sensitivity with increasing clearance. The trend is consistent for three of the four substantially different configurations, bladings and power levels, as well as, two fluids. Wisler and Beacher's data did not provide a trend.

The *decrease* in constant power sensitivity with increasing clearance is the opposite of the constant flow sensitivity *increase* shown by Lakshminarayana (1970). Although the sensitivities were different in terms of derivation a similar trend was expected to be discernable.

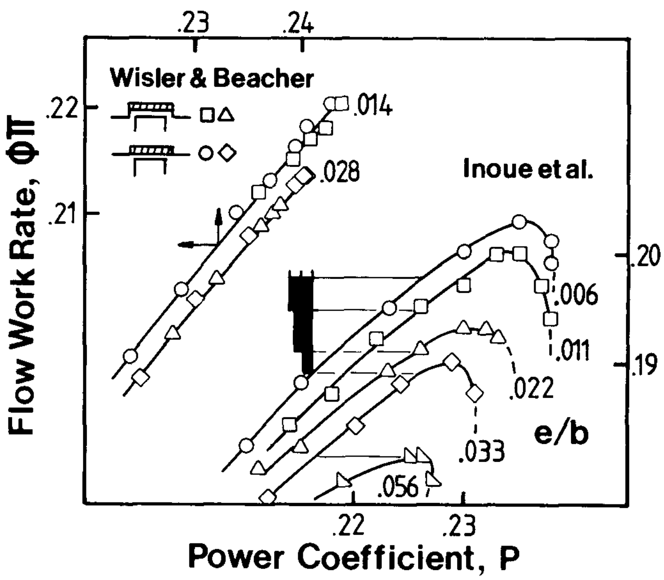

Fig. 5 Flow work vs. power input characteristic for case wall diameter changes by Inoue (1985) and recessing by Wisler and Beacher (1986). Data at constant speed

Lakshminarayana's correlation at constant flow seemed to provide an excellent fit to Williams' (1960) data, so the data points plotted were verified by the author to see why the same data gave different trends. Williams' $\Phi = 0.343$, $e/b = 0.0129$ point, as plotted by Laksminarayana, is in error by about 0.7 percent (low). In addition, Ruden's $\Phi = 0.388$, $e/b = 0.032$, 0.040, 0.048 points were positioned one percent low. When positioned correctly, the constant flow data are consistent with the decreasing sensitivity trend of the constant power plots, as the clearance is increased. The corrected data points are plotted in Figure 1, which is a composite of (essentially the same) data plotted by Lakshminarayana, Senoo and Schmidt. Schmidt's data also follow the trend discussed quite closely and unfortunately Schmidt et al. attempted to reconcile the disagreement of their data with Lakshminarayana's model and plots. Actually their data are in general agreement with other tests.

High Speed Cropped and Recessed Data

As the clearance effect in terms of losses seemed to be consistent between low speed stages, high speed data were also correlated on a flow work vs. power characteristic. Data on high speed stages are not as abundant as at the low speed and fewer test conditions are reported. Testing by Holman and Kidwell (1975), Moore and Osborne (1977) and Moore (1982) have included clearance variations and efficiency measurements.

The data of Moore (1982) included rotor efficiency variation with clearance and power and has been plotted in Figure 6. Although the data are sparse, the general trend of the low speed data is evident and sensitivity to clearance could be generated. The 100% and 70% speed conditions had sufficient test points and these data have been included on the figure. The 70% speed case showed lower sensitivity overall. This was consistent with less blade growth at the lower speed and, hence, a larger clearance level. Both of Moore's experiments employed casewall inserts of increasing radius to alter the clearance over similar stages with different rotor to stator matching. The power levels were similar for the two stages. Holman and Kidwell cropped the rotor tips of a single stage and measured at two speeds.

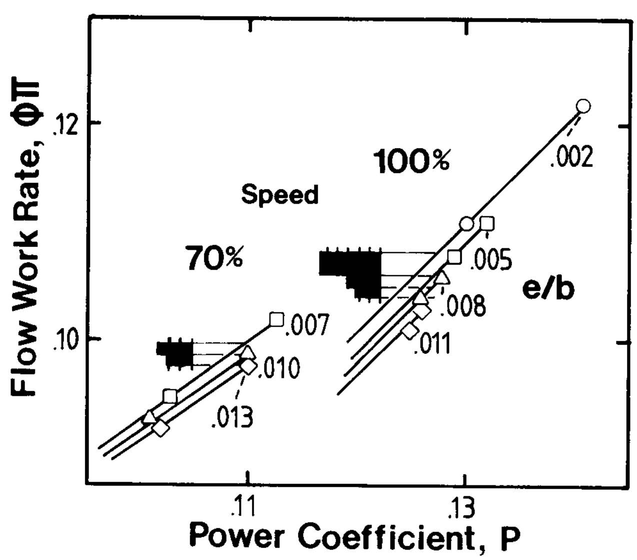

Fig. 6 Flow work vs. power input characteristic for a transonic rotor at two constant speeds tested by Moore (1982) with wall diameter variations

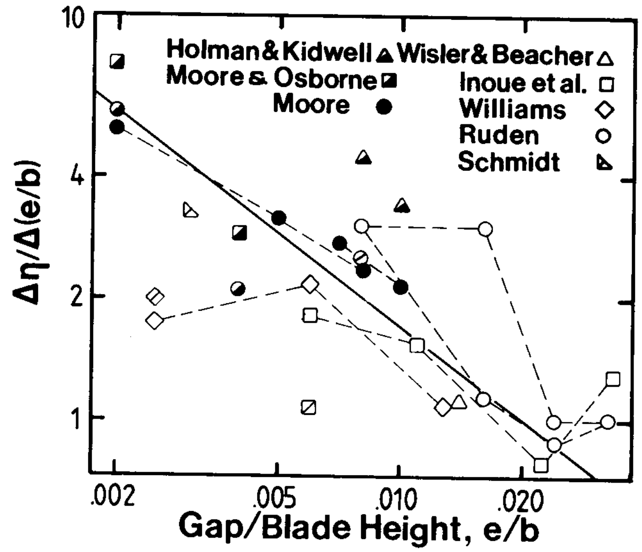

Fig. 7 Log-log correlation of constant power and peak-to-peak efficiency sensitivity to gap/blade height for high (solid symbols) and low speed (open symbols) stages. Peak-to-peak data is slashed diagonally

Low vs. High Speed Sensitivity

Because the test points were sparse and power was not extractable from all of the high speed test data, changes in peak efficiency were used as a substitute for determining constant power sensitivity. In comparing the low and high speed data, the sensitivity derived from Figures 4 to 6 and Table 2 were set out on Figure 7 in a log-log plot of sensitivity ($\Delta\eta/\Delta e/b$) vs. gap/blade height (e/b). It is apparent from this figure that the high and low speed data fall in a consistent band. The high speed data shows the larg est sensitivity at clearances less than one percent. The log sensitivity ($\Delta\eta/\Delta e/b$) can be roughly correlated with log (e/b) by a straight line of significant slope. Data for each set of low and high speed stage data also maintain the nominal slope indicated for the whole sample. The slope is obviously far from the horizontal line which would be consistent with the linear $\Delta\eta$ vs. e/b correlations of Fig. 1.

Ruden's data appears to show a distinct transition about the 2.5% (e/b) level. However, they are not detailed enough to isolate a change of flow character, which might be suspected if the gap dimension exceeds the passage boundary layer displacement thickness. The data show significant scatter between different stages but a consistency of slope between all the machines. The slope uniformity implied a relationship of the form;

$$\Delta\eta/\Delta e/b = C\,(e/b)^{-a}$$

could be applied to the data, where (C) varies from compressor to compressor and (a) is generally constant for all the data. The (C) generates most of the scatter observed in the data.

In addition to the graphically derived constant power sensitivities, the stage peak-to-peak efficiency sensitivities tabulated were log-log plotted on Figure 7 at the lower clearance of the $\Delta e/b$ range.

Table 2.

Characteristics of the Stages and Experimental Compressor Test Data used to develop the Efficiency Sensitivity. ($\Delta\eta$ = Stage Peak Efficiency Change)

	R_t (m)	U_t (m/s)	rpm (%)	e/b	Δe/b	$\Delta\eta$	$\frac{\Delta\eta}{\Delta e/b}$
Low Speed Stages							
Ruden (1937)	.250	?	100	.008	.040	.010	2.5
Williams (1960)	.178	3.7	100	.002	.023	.045	2.0
Inoue et al (1985)	.225	?	100	.006	.050	.055	1.1
Wisler & Beacher (1986)	.762	70.8	100	.014	.014	.015	1.1
Schmidt et al. (1987)	.254	39.4	100	.003	.022	.075	3.3
High Speed Stages							
Holman & Kidwell (1975)	.059	474	100	.008	.014	.060	4.4
		427	090	.010	.014	.045	3.3
Moore & Osborne (1977)	.250	423	100	.002	.009	.071	7.5
		296	070	.004	.009	.027	2.9
Moore (1982)	.250	423	100	.002	.009	.054	5.7
		296	070	.004	.009	.019	2.0

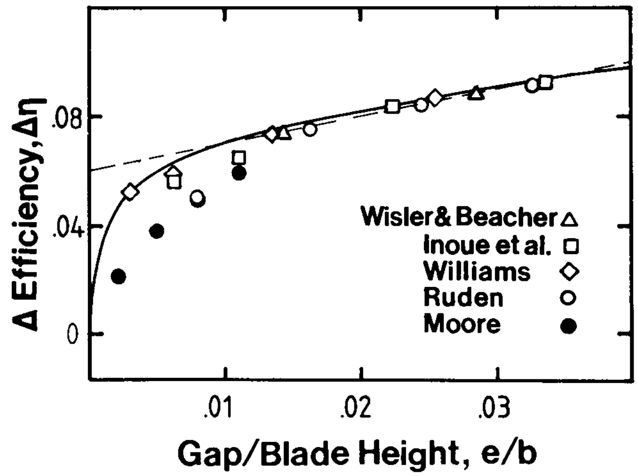

Fig. 8 Efficiency decrement correlation with gap/blade height developed by integration of the approximation to the data of Fig. 7 crossplotted with test data

Correlation of the Sensitivity at Constant Power

The approximate straight line dependence of the log sensitivity data to log e/b shown on Fig. 7 can be integrated to yield the variation of efficiency decrement with clearance at constant power. By selecting a constant of integration to normalize the function at the 7% $\Delta\eta$ and the 1% e/b point, the curve of Figure 8 can be obtained. The curve shows an initially rapid increase in the efficiency decrement associated with the clearance very close to the wall and a gradual tapering-off of the decrement toward the limit of a 1% efficiency decrease for a 1% clearance increase, at clearances greater than one percent. This limiting curve is shown on the figure intersecting the $\Delta\eta$ axis at 0.06. The constant power efficiency curve strictly represents the lost work rate associated with a clearance perturbation and is roughly representative of the peak efficiency variation with clearance change. The expression for the approximate curve shown in Fig. 8 is;

$$\Delta\eta = 0.214\,(e/b)^{.227} - 0.005$$

It should be *stressed* that this correlation is an approximation to the data sample addressed in the paper and it is expected it could be refined with more data over a wider range of machines and clearances. Efficiency decrement data from the experimental stages have been plotted on the figure by aligning the unity slope region of each data set at the larger clearances with the unity slope region of the derived curve. It can be seen that the low speed data then align well with the derived curve at smaller clearances. The high speed data reflect the lower slope of their sensitivities shown in Fig. 7. They tend to curve over to the limit more gradually as e/b is increased from a much larger initial slope. The form of the efficiency curve derived in this manner, and supported by the data, contrasts with the linear to slightly parabolic, increasing correlations proposed by earlier studies and is of some interest as an indication of losses.

CONCLUSION

It can be seen that the results of this analysis (ie. the general form of the constant power sensitivity correlation with gap/blade height) strongly suggest that the geometry or flow variables do not affect the loss character as greatly as might be expected from the sensitivity of Fig 1. An alternative inference in the data is that the only significant correlating parameter is the gap/blade height. The principal conclusions were as follows:

1. The clearance efficiency effect, formulated in terms of sensitivities derived at constant power, showed a well defined decreasing trend in efficiency change with increasing clearance gap. This is a trend that is contrary to some earlier published data and correlations for changes derived at constant flow.

2. The use of linear extrapolations of efficiency changes at constant flow to zero clearance does not reflect the apparent non-linear character of the loss development, especially for small clearances.

3. Relatively complex parametric models developed from loss considerations in the tip region might be simplified by use of constant power rather than constant flow efficiency sensitivities in analysing test data.

4. The slope on the log-log efficiency sensitivity vs. gap/blade height plot is similar for the stages where constant power sensitivity was calculated. Normalization of the clearance gap by any other characteristic dimension of the blade passage will not alter the slope presented or the exponent derived.

The final observation (4) indicates there is an underlying consistency in the loss generation mechanism in the data examined. This point is worthy of further exploration as it may provide the basis for an improved analytical description of the tip clearance effect or better approximation of empirically derived model constants.

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