# Analysis of four-wave mixing between pulses in high-data-rate quasi-linear subchannel-multiplexed systems

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We study four-wave mixing between pulses in two subchannels of a quasi-linear 40-Gbit/s subchannel-multiplexed system. For a pseudorandom bit string there are resonances in the mean of the ghost pulse energy and in the jitter of the energy in the marks as functions of the subchannel frequency spacing. However, away from these resonances the effect of four-wave mixing decreases as the subchannel spacing increases, permitting propagation over longer distances. © 2002 Optical Society of America

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Intrachannel four-wave mixing has been identified as the major nonlinear effect that limits the propagation distance in long-haul quasi-linear fiber-optic communication systems operating at data rates of 40 Gbits/s and above.[1] In these systems four-wave mixing transfers energy from triples of pump pulses into the bit slots of the spaces, generating ghost pulses, and into the bit slots of the marks, inducing jitter in the energy of the marks. Studies have shown that intrachannel four-wave mixing can be reduced by use of appropriate dispersion precompensation and Raman amplification.[2–4] In this Letter we study four-wave mixing between pulses in two closely spaced frequency subchannels of a high-data-rate quasi-linear system. Our main goal is to analyze four-wave mixing in a subchannel-multiplexed system in which each 40-Gbit/s wavelength-division multiplexing channel is replaced with two 20-Gbit/s subchannels. We note that these subchannels are not separate channels, which would require them to be separately demultiplexed and detected. Instead, at the end of the transmission the pulses in the two subchannels must not overlap in the time domain. We show that four-wave mixing can be reduced by use of two subchannels for each wavelength-division multiplexing channel, provided that one chooses the subchannel spacing to avoid resonances between the nonlinear four-wave mixing perturbations as a result of different triples of pump pulses. This method relies on trading off decreased spectral efficiency for increased propagation distance.

We study the subchannel multiplexing method by use of noise-free simulations of a single-channel 40-Gbit/s dispersion-managed system with $D_+$ and $D_-$ fiber. We compute the mean energy of the ghost pulses in the spaces and the jitter in the marks, which equals the standard deviation of the energy in the marks, in the optical domain as functions of the frequency spacing, $\Delta\Omega$, and the relative phase difference between the two subchannels. We find that there are strong resonances in the mean ghost energy and the jitter in the marks as functions of the subchannel spacing and that at the resonant frequency spacings the ghost energy and jitter vary significantly as the relative phase between the two subchannels is varied. We also present an analytical formula for the nonlinear perturbation resulting from three pump pulses with different central times and frequencies. We apply this formula to explain the physical origin of the resonance phenomenon and to obtain equations describing the nonresonant dependence of the mean ghost energy and the jitter in the marks on the subchannel frequency spacing.

We use a standard perturbation analysis to represent quasi-linear solutions of the nonlinear Schrödinger equation in the form $u + q$, where $u$ is a solution of the linear dispersive equation and $q$ represents the perturbation resulting from nonlinearity.[2,5] In this approximation the perturbation $q$ satisfies the forced linear dispersive equation

$$i\,\frac{\partial q}{\partial z} - \frac{1}{2}\,\beta''(z)\,\frac{\partial^2 q}{\partial t^2} + i\Gamma(z)q = -\gamma(z)F(u)\,, \qquad (1)$$

where $z$ is propagation distance, $t$ is retarded time, $\beta''(z)$ is the group-velocity dispersion, $\Gamma(z)$ describes the fiber attenuation, $\gamma(z)$ is the nonlinear coefficient, and $F(u) = |u|^2 u$ is the forcing function. Suppose that $u_m$, $u_n$, and $u_l$ are three pump pulses that are centered at times $t = mT$, $t = nT$, and $t = lT$, where $T$ is the bit period, and whose relative central frequencies are $\Omega_m$, $\Omega_n$, and $\Omega_l$, respectively. Suppose that these pulses are Gaussians that are initially of the form $u_k(z = 0, t) = P_0^{1/2} \exp[-(t - kT)^2/2\tau_0^2]\exp(-2\pi i\Omega_k t)$, where $P_0$ is the peak power and the pulse width is $\tau_{\mathrm{FWHM}} = 2\sqrt{\ln 2}\,\tau_0$. Let $t_* = t/\tau_0$, $T_* = T/\tau_0$, $\Omega_{k,*} = \Omega_k\tau_0$, $T_{k,*} = kT_* - i\Omega_{k,*}$, and $B_*(z) = B(z)/{\tau_0}^2$, where $B(z) = \int_0^z \beta''(s)\mathrm{d}s$ is the accumulated dispersion. Also, let $G(z)$ be the gain and loss function, where $G(0) = 1$. Then, at a distance $L$ for which

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$B_*(z) = 0$ and $G(L) = 1$, the formula for the nonlinear perturbation $q_{m,n,l}$ that is due to the forcing function $F_{m,n,l} = u_m u_n \bar{u}_l$ is

$$q_{m,n,l}(t) = iP_0^{3/2} \exp(-i\phi_{m,n,l}) \times \exp[-(\Omega_{m,*}^2 + \Omega_{n,*}^2 + \Omega_{l,*}^2)/2] \times \exp(-\hat{t}^2/6)I(t), \quad (2)$$

where $\phi_{m,n,l} = T_*(m\Omega_{m,*} + n\Omega_{n,*} - l\Omega_{l,*})$, $\hat{t} = t_* - (T_{m,*} + T_{n,*} - \overline{T}_{l,*})$, and

$$I(t) = \int_0^L \frac{G(z)\gamma(z)}{[1 + 2iB_*(z) + 3B_*^2(z)]^{1/2}} \times \exp\left\{\frac{-3[2\hat{t}/3 + (T_{m,*} - \overline{T}_{l,*})][2\hat{t}/3 + (T_{n,*} - \overline{T}_{l,*})]}{1 + 3iB_*(z)}\right\} \times \exp\left[-\frac{(T_{m,*} - T_{n,*})^2}{1 + 2iB_*(z) + 3B_*^2(z)}\right]\mathrm{d}z. \quad (3)$$

Equation (2) is a simple extension of the formula in the single-channel case.[2] Numerical simulation shows that in the case of four-wave mixing $q_{m,n,l}(t)$ is centered at the phase-matching time $t = (m + n - l)T$ and is almost Gaussian in shape.

We now focus on the case of subchannel multiplexing. The subchannel-multiplexed signal is obtained by replacing each wavelength-division multiplexing channel with a pair of subchannels that are created by shifting of the central frequencies of the pulses in the even-numbered bit slots by $+\Delta\Omega/2$ and those in the odd bit slots by $-\Delta\Omega/2$. For a 40-Gbit/s signal this procedure produces two 20-Gbit/s signals whose pulse widths are appropriate for a 40-Gbit/s signal and that are spaced $\Delta\Omega$ apart in frequency with a time offset of half a 20-Gbit/s time slot. At the receiver the two subchannels are treated as though they were a single 40-Gbit/s channel. To ensure that at the end of the transmission the pulses in the two subchannels do not overlap in the time domain, we assume that the total accumulated dispersion and dispersion slope are zero.

To analyze the method we performed noiseless, single-channel simulations on a 40-Gbit/s dispersion-managed system with a dispersion map of length $L = 48$ km. The dispersion map consisted of $D_+$ fiber of length $2L/3$, followed by $D_-$ fiber of length $L/3$ and an amplifier. The $D_+$ fiber had a dispersion of $D_+ = 20$ ps/nm km and a dispersion slope of 0.06 ps/nm$^2$ km at 1550 nm. The average dispersion and dispersion slope of the map were zero. For the $D_+$ and $D_-$ fibers, the effective areas were 110 $\mu$m$^2$ and 30 $\mu$m$^2$, respectively, and the losses were 0.19 dB/km and 0.25 dB/km, respectively. The nonlinear Kerr coefficient was $2.6 \times 10^{-20}$ m$^2$/W. The input 40-Gbit/s return-to-zero signal consisted of Gaussian-shaped pulses with a carrier frequency of 1550 nm, $\tau_{\mathrm{FWHM}} = 5$ ps, and $P_0 = 4$ mW. Since intrachannel four-wave mixing is reduced by use of an accumulated dispersion function that is approximately symmetric about the zero dispersion axis,[4] we used linear dispersion precompensation of $-D_+L/3$ and linear postcompensation of $+D_+L/3$.

To examine how the ghost pulse energy in the spaces and the jitter in the marks depend on the subchannel spacing we performed simulations in which the spacing between the two subchannels was increased from 0 to 200 GHz in 1-GHz increments. Since the ghost energy grows quadraticaly and the jitter in the marks grows linearly with the number of map periods,[5] it was sufficient to use a single map period. If all three pump pulses belong to the same subchannel we call $q_{m,n,l}$ an intrasubchannel perturbation; otherwise, we call it an intersubchannel perturbation. First, we consider the ghost pulse that is due to the bit pattern 1 1 0 1, where the ghost is centered at time $t = 62.5$ ps in the third bit slot. This ghost pulse is the sum of three intersubchannel perturbations. In Fig. 1 we plot the relative ghost pulse energy versus the subchannel spacing in gigahertz. The thick solid curve is the energy of the total ghost pulse computed with Eq. (2). The circles show the same total energy computed with the nonlinear Schrödinger equation with $\Delta\Omega$ increments of 20 GHz, where the bit pattern was 1 1 0 1 0 0 ... and the time window was 25,600 ps. The thin solid curve is the sum of the energies of the three intersubchannel perturbations, and all ghost energies in the figure are given relative to the value of this sum when $\Delta\Omega = 0$. The sum of the energies is well approximated by the dashed curve given by the formula $E_{\mathrm{inter}}(\Delta\Omega) = E_{\mathrm{inter}}(0)\exp[-2/3\,(2\pi\Delta\Omega\,\tau_0)^2]$. This approximation of the energy of an intersubchannel perturbation is obtained from Eq. (2) under the assumption that the integral $I(t)$ is independent of the subchannel spacing. Consequently, $E_{\mathrm{inter}}(\Delta\Omega)/E_{\mathrm{inter}}(0)$ depends only on the subchannel spacing relative to the spectral bandwidth of a single-channel 40-Gbit/s signal and is independent of the dispersion and power maps. The reason for the large oscillations in the total energy is that the three perturbations that contribute to the ghost pulse move in and out of phase with each other as $\Delta\Omega$ increases.

Next, we performed simulations based on the nonlinear Schrödinger equation, using a pseudorandom

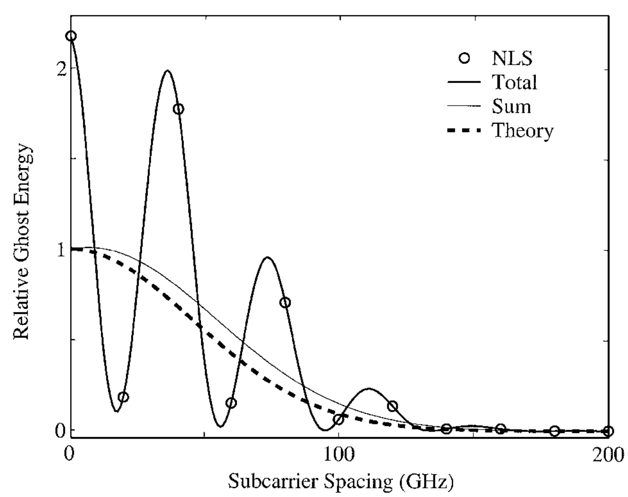


Fig. 1. Ghost pulse energy for the bit pattern 1 1 0 1.

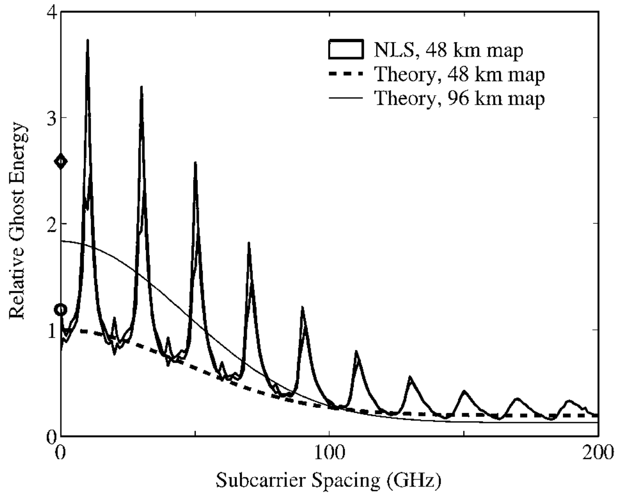


Fig. 2. Mean ghost energy for a pseudorandom bit sequence.

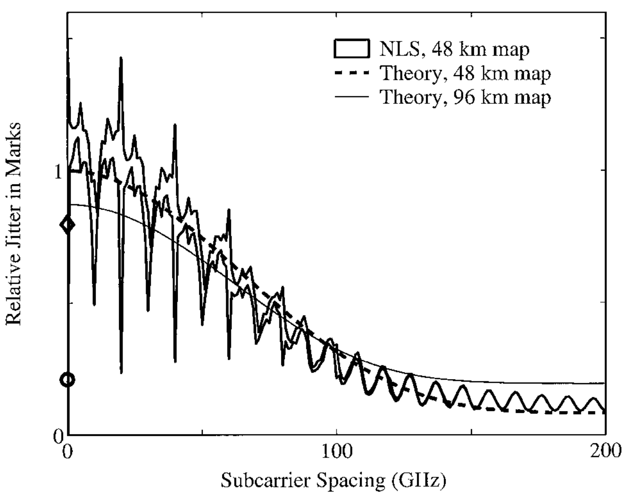


Fig. 3. Jitter in the marks for a pseudorandom bit sequence.

bit string of length $2^{10}$. We computed the mean ghost energy in the spaces as a function of both the subchannel frequency spacing and the relative phase difference between the two subchannels. In Fig. 2, the two thick solid curves are the minimum and maximum of this ghost energy as functions of $\Delta\Omega$, where the optima are taken with respect to the phase difference between the two subchannels. All ghost energies in Fig. 2 are given relative to the phase-averaged mean ghost energy when $\Delta\Omega = 0$. There are strong resonances in the ghost energy when $\Delta\Omega = 10, 30, \ldots, 190$ GHz, and at these resonances there is a large variation in the ghost energy as a function of the phase difference between the subchannels. The behavior of the ghost energy away from the resonances is well approximated by the dashed curve given by the formula $E(\Delta\Omega)/E(0) = R + (1 - R)\exp[-2/3\,(2\pi\Delta\Omega\tau_0)^2]$, where $R$ is the ratio of the mean ghost energy for a signal consisting of only one of the 20-Gbit/s subchannels relative to the phase-averaged mean ghost energy at $\Delta\Omega = 0$. This formula was derived under the assumption that, for all subsets of perturbations with approximately the same energy, the phases of these perturbations are uniformly distributed. The thin solid curve is the nonresonant energy, $E(\Delta\Omega)$, for a dispersion map of length $L = 96$ km, normalized with respect to the 48-km map. The circle and diamond are the mean ghost energies for the 48-km and 96-km maps, respectively, at $\Delta\Omega = 0$ when there is no phase difference between the two subchannels. The resonances are ~50% weaker for the 96-km map than for the 48-km map and occur at different $\Delta\Omega$. In Fig. 3 we show analogous results for the jitter in the energy of the marks.

To investigate the physical origin of the resonances we compared the energies of all nonlinear perturbations that could potentially contribute to a given ghost pulse by performing simulations for the isolated space in the bit sequence ...1110111..., using Eq. (2). For the 48-km map the two largest contributions to the ghost energy are $q_{\pm1,\pm1,\pm2}$, which have energies that are 26% of the phase-averaged mean ghost energy in Fig. 2. By contrast, for the 96-km map, the 18 largest contributions each have energies that are 3–5% of the phase-averaged energy. Moreover, for both maps these large perturbations are all intersubchannel perturbations whose energies decrease smoothly as $\Delta\Omega$ increases. Consequently, we conclude that the strong resonances for the 48-km map occur at those $\Delta\Omega$ for which the largest two possible perturbations $q_{\pm1,\pm1,\pm2}$ in each space are in phase with each other, whereas the resonances are weaker for the 96-km map because of the greater amount of the pulse overlap.

In conclusion, we have studied four-wave mixing in a quasi-linear 40-Gbit/s subchannel-multiplexed system and analyzed the dependence of the mean ghost pulse energy and the jitter in the marks on the frequency spacing between the two subchannels. We found that it is possible to increase the propagation distance by use of subchannel multiplexing as long as special resonance frequencies are avoided.

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