



Analysis of Graphical Approach for Cam Profile Determination

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ABSTRACT: A novel approach based on graphical approach of cam profile determination is put forth in this study. The objective of the study is to have a high accuracy independent platform from Computer Aided Design (CAD) systems and to eliminate determination parameters of planar cams like derivatives, pressure angle and curvature. A brief description of graphical method is represented. Equations and algorithms of the proposed approach for flat face and roller translating follower mechanisms are provided. Results are compared with results of the conventional analytical approach and graphical CAD systems which indicated that the approach provides precisely correct results.

Keywords – *Cam profile, Cam mechanism, Graphical approach, Camshaft*

1. Introduction

High speed cams control position of piston valves in internal combustion engines (ICE). Cams directly affect efficiency of ICE since the valves control the flow of the air/fuel mixture intake and exhaust gases. Even though the design of displacement curve for follower of camshaft depends on adiabatic concepts such as ICE design, production process of camshafts is separately considered from curve design process and needs production expertise. Cam profile design, Numerical Control (NC) code generation, hardness treatment etc. dictate the independency of curve and camshaft design processes. There are two common cam profile determination methods, namely analytical and graphical design. Analytical method is the one that is most frequently used and requires continuous functions of displacement curve of the follower. These continuous functions are linear, trapezoidal, quadratic, cubic, harmonic, cycloid, polynomial etc. functions or piecewise combinations of these. Displacement and velocity, which is a derivative of displacement, are used to determine cam profile in the analytical method (Chen, 1982, Norton, 2009, Rothbart, 2004, Söylemez, 2010). If displacement function is determined or obtained as a data set, the derivative of the data set can be computed via numerical approaches which results in numerical approximation errors (Gordon, 2012). Interval of errors does not assure for the demands of high precision production of camshaft especially for high speed camshafts. Thus, the displacement data of cam follower should be given as a continuous function in the analytical approach. The other cam profile determination approach known as the graphical approach or geometric layout is suitable for low speed cams (Chen, 1982, Rothbart, 2004, Söylemez, 2010). Due to the difficulty of dividing displacement data to precision intervals graphically, the graphical approach is applicable only with Computer Aided Design (CAD) and requires troublesome processes (Rothbart, 2004, Söylemez,

2010). Some publications represent numerical approaches for profile determination of cams (Chen, 1972, Biswas et al. 2004). Parameters such as acceleration (Chen, 1972), pressure angle and curvature (Biswas et al. 2004) are needed in these approaches. The profile determination is thus included in the cam profile design process. Also, another work determines cam profile using CAD system despite numerical differentiation errors (Liang and Quinn, 1991). In this study, we distinctively propose a programmable iterative numerical approach independent of pressure angles, curvatures, derivatives and numerical differentiation errors. Purpose of this study is to lead off coding the graphical approach in any programming language independent of CAD software and to obtain cam profile in situations for which the follower function is not a continuous function e.g. discrete function. Acquired abilities can allow optimization of a greater number of parameters with less data in production. Thus, preprocessing stage of cam profile production is simplified and possible faults due to data operating are decreased. Translating flat face follower and centric roller follower type of high speed planar cams used in ICE were exemplified in the paper. Since the mechanisms are designed as centric in both Computer Numeric Control (CNC) grinding and CNC milling machines and measurement devices of camshafts, the eccentric follower layouts are not considered. Graphical approach which forms the basis for the proposed theory has been briefly explained in this study. The equations of the proposed approach are obtained for two types of followers which are flat face and roller. Algorithm schemes of the approach are presented for the types of followers. Results are compared with analytical approach results and graphical approach CAD system results.

Nomenclature:

θ : Angle data or x axis of displacement data or graph

θ_i : Angle value of i^{th} point

S : Displacement data of cam follower

S_i : Stroke value of i^{th} point

v : Linear velocity of follower

a : Acceleration of follower

j : Jerk of follower

r : Radius

R_b : Radius of base circle of cam

R_f : Radius of follower

R : Rotational matrix

L : Line

2. Graphical approach

Graphical transfer method is applied to S vs. θ data for the purpose of determining cam profile in graphical approach. As can be seen in Figure 1, $S_a, S_b, S_c \dots S_n$ values are transferred onto lines which rotated as angles of $\theta_a, \theta_b, \theta_c \dots \theta_n$. The envelope of follower-profile contact is generated from marked up points like A, B, C, N=A in flat face follower mechanism. Envelope of roller follower mechanism is generated from A, B, C, N=A centered followers. There are two envelopes, namely the inner and outer envelopes in the roller type and cam profile is formed by the inner envelope. Cam profile coordinates are located on curves which are tangential to the envelopes. The reference bottom levels of the S graphs vary according to follower types which are flat face and roller. As can be seen in Figure 1, the reference bottom level is the contact point of the follower as well as the base circle of cam and so the length of R_b is on the vertical axis. It can also be seen in Figure 2

that the reference bottom level is at the center of follower which is in contact with the circle of radius R_b so that the length of R_b+R_f is on the vertical axis. In CAD systems cam profile is drawn by a combination of curves tangent to envelope. Types of curves are generally spline supported by CAD software. Although the cam rotates, it is assumed for a simple explanation of the process that the follower rotates and that the cam stands in conventional approach. The challenge with CAD systems is to draw the followers one by one for all angles. The number of drawings increases with decreasing sampling angle. For instance, there should be 361 follower drawings including 0 for precision of one degree θ versus S data. Then cam profile is drawn by these 361 drawings and tangential curves.

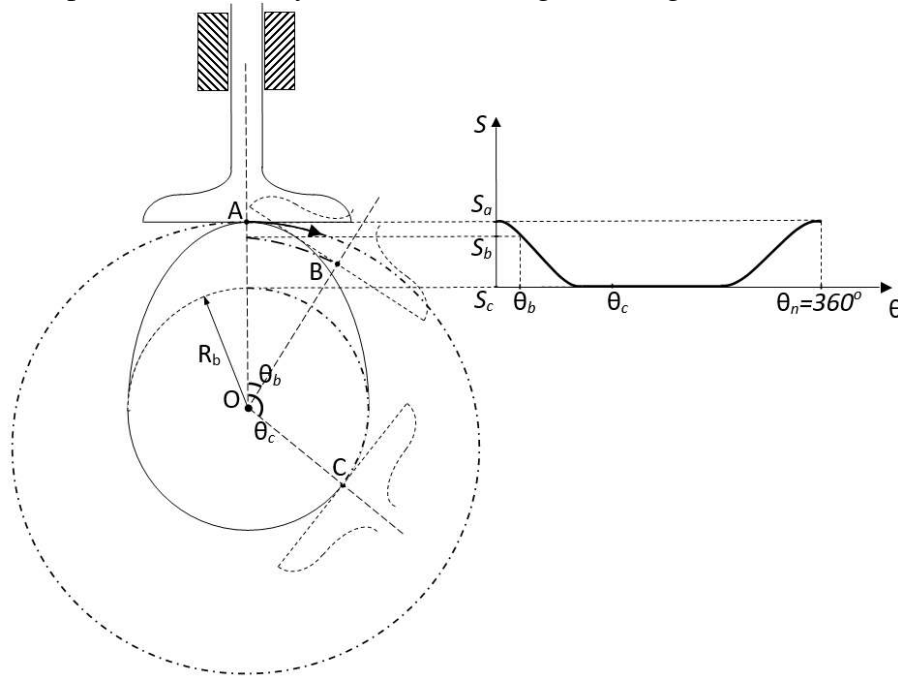


Figure 1: Graphical approach for flat face follower cam

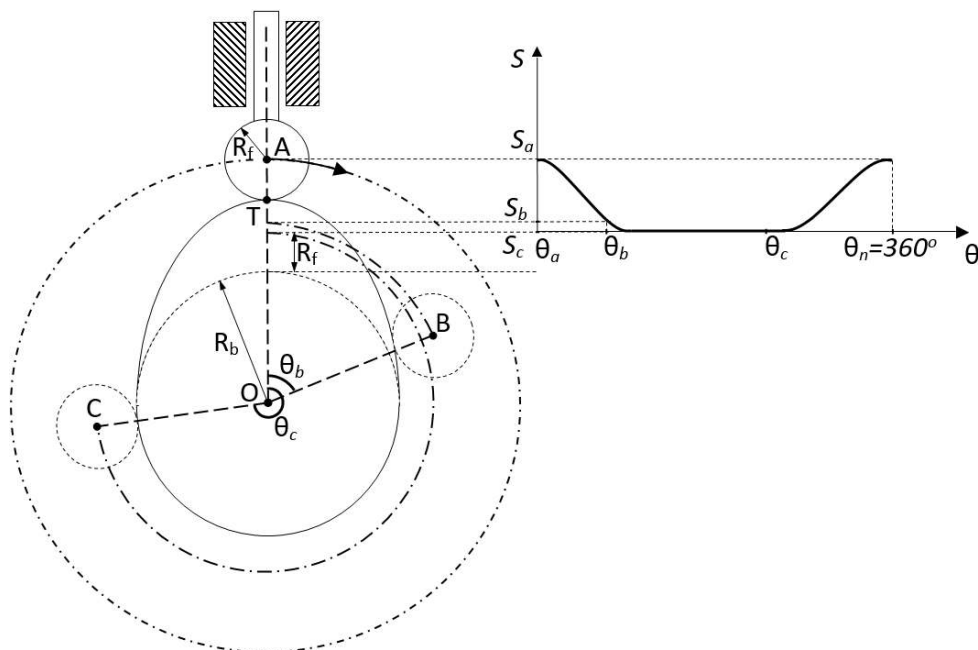


Figure 2: Graphical approach for roller follower cam

3. Analysis of Graphical Approach

Geometric objects such as lines, points, circles, distances and intersections in Figure 1 and Figure 2 can be represented as equations thus analysis of the graphical approach can be generated. A circle with a radius of $|OA|$ in flat face follower mechanism and a circle with a radius of $|OT|$ in roller follower mechanism are expressed as the outer circle which is tangent to the follower at the maximum stroke position. The set of sampled points on the outer circle is expressed as a general set. The follower is displaced in sequence and the general set is rotated as a unit angle similar to that in the real mechanism. Lines from general set to origin are used to specify new points of the general set. The actual general set consists of intersection point(s) of the follower in every iteration including the origin lines of the general set together with nonintersected points. The cam profile will in general be set at the end of one cycle, namely a 360 degree turn. The general set is rotated and recalculated in every iteration. The rotation matrix given in Eq. 1 is used for the rotation operation (Goldstein et al. 2002). Cartesian axes and rotation center of cam are placed on the O centered plane i.e. the axis of the follower motion $|OA|$ and y axis overlap each other.

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

3.1 Analysis of Flat Face Follower Mechanism

Let L_1 line be shaped by the flat face and L_2 line be shaped by any point of the general set and center, for example $|OA|$. Since the aim is to find connection point of follower and cam, it is mandatory to find intersection of follower and cam surface, in other words intersection of L_1 and L_2 . Intersection point of L_1 and L_2 can be found analytically (Goldman, 1990). Coordinates of the lines are defined as follows:

$$L_1 = \begin{bmatrix} x_{11} & y_{11} \\ x_{12} & y_{12} \end{bmatrix} \tag{2}$$

$$L_2 = \begin{bmatrix} x_{21} & y_{21} \\ x_{22} & y_{22} \end{bmatrix} \tag{3}$$

Coordinate components of intersection of L_1 and L_2 are defined as follows:

$$X = \frac{\begin{vmatrix} |L_1| & x_{11} & 1 \\ & x_{12} & 1 \\ |L_2| & x_{21} & 1 \\ & x_{22} & 1 \end{vmatrix}}{\begin{vmatrix} x_{11} & 1 & y_{11} & 1 \\ x_{12} & 1 & y_{12} & 1 \\ x_{21} & 1 & y_{21} & 1 \\ x_{22} & 1 & y_{22} & 1 \end{vmatrix}} \tag{4}$$

$$Y = \frac{\begin{vmatrix} |L_1| & y_{11} & 1 \\ & y_{12} & 1 \\ |L_2| & y_{21} & 1 \\ & y_{22} & 1 \end{vmatrix}}{\begin{vmatrix} x_{11} & 1 & y_{11} & 1 \\ x_{12} & 1 & y_{12} & 1 \\ x_{21} & 1 & y_{21} & 1 \\ x_{22} & 1 & y_{22} & 1 \end{vmatrix}} \tag{5}$$

The boundary conditions have been considered according to the aforementioned basic level equations (Thomas and Finney, 1998). y_{11} and y_{12} are equal since L_1 is parallel to the x axis and also x_{11} and x_{12} are constant. Since the starting point of L_2 is the origin and x_{21} and y_{21} are equal to 0. Under these conditions, the coordinate components are obtained as follows after simplification.

$$X = y_{11} \frac{x_{22}}{y_{22}} \quad (y_{22} \neq 0) \tag{6}$$

$$Y = y_{11} \tag{7}$$

Nature of cam mechanism prevents infinities. Infinity situation where y_{22} is equal to 0 is valid for lines on the x axis of the general set. Since the follower never leaves this position, the infinity does not really occur. There should be a boundary condition to prevent fault in the numerical process. In the intersection calculation, Y values of points of the general set should be bigger than follower's y value as the boundary condition. Thus, the procedure works only for the lines of interest. Hence, the general equation is:

$$(X_i, Y_i) = \left(y_{i1} \frac{x_{k2}}{y_{k2}}, y_{i1} \right), (y_{k2} \neq 0) \tag{8}$$

Index of i is remarked for lines of follower and iterations. Index of k is remarked for lines of general set. Quantities of i and k are equal. Rotation procedure is implemented after every trimming procedure loop.

$$(X_{i+1}, Y_{i+1}) = R \begin{bmatrix} X_i \\ Y_i \end{bmatrix} \tag{9}$$

Thus, the system is prepared for the next profile data point calculation.

3.2 Analysis of Roller Follower Mechanism

Circle-line intersection and distance between two points are considered in the roller follower mechanism as distinct from the intersection of two lines in flat face follower mechanism. It is assumed that the roller follower contacts or entries to the profile (general set) when the distance from the center of the roller follower to any point on the general set is less than or equal to R_f . Intersection points of the roller follower circle and lines which enter the follower circle region are calculated at this stage. It is assumed that any line in the general set, for example |OT| is called L_3 ; thus the organization of the line is:

$$L_3 = \begin{bmatrix} x_{31} & y_{31} \\ x_{32} & y_{32} \end{bmatrix} \tag{10}$$

$$(x_{31}, y_{31}) = (0,0) \tag{11}$$

$$L_3: Y = \frac{y_{32}}{x_{32}} X, (x_{32}) \neq 0 \tag{12}$$

$$L_3: X = \frac{x_{32}}{y_{32}} Y, (y_{32}) \neq 0 \tag{13}$$

Since the follower is on the y axis, the circle equation is,

$$X^2 + (Y - y_4)^2 = r^2, (r > 0) \tag{14}$$

From Eq. 12, 13 and 14 two quadratic equations are

$$X^2 + \left(\frac{y_{32}}{x_{32}} X - y_4 \right)^2 = r^2 \tag{15}$$

$$\left(\frac{x_{32}}{y_{32}} Y \right)^2 + (Y - y_4)^2 = r^2 \tag{16}$$

Then the intersection is,

$$X_{1,2} = \frac{y_4 \frac{y_{32}}{x_{32}} \pm \sqrt{\left(\left(\frac{y_{32}}{x_{32}} \right)^2 + 1 \right) r^2 - y_4^2}}{\left(\frac{y_{32}}{x_{32}} \right)^2 + 1} \tag{17}$$

$$Y_{1,2} = \frac{y_4 \pm \sqrt{y_4^2 - \left(\left(\frac{x_{32}}{y_{32}}\right)^2 + 1\right)(y_4^2 - r^2)}}{\left(\frac{x_{32}}{y_{32}}\right)^2 + 1} \tag{18}$$

However there are two infinity situations where y_{32} is equal to 0 and also x_{32} is equal to 0, nature of cam mechanism prevents infinities. First y_{32} case is available for lines of general set on the x axis as in the flat face of the follower mechanism. Since roller follower never reaches that position the infinity doesn't occur. There should be a boundary condition to prevent fault in numerical process. In the intersection calculation, end points of lines should be in follower circle as the boundary condition. Second x_{32} case is available for lines of a general set on the y axis. Intersection point is $(0, S_\theta)$ position where the follower intersects the y axis. Since the line intersects two points, the follower circle result consists of two points. To solve this issue, it is sufficient to use maximum values of X and minimum values of Y. Thus, the general equation is,

$$\begin{pmatrix} X_i \\ Y_i \end{pmatrix} = \begin{pmatrix} \frac{y_i \frac{y_{k2}}{x_{k2}} + \sqrt{\left(\left(\frac{y_{k2}}{x_{k2}}\right)^2 + 1\right)r^2 - y_i^2}}{\left(\frac{y_{k2}}{x_{k2}}\right)^2 + 1}}{y_i - \frac{\sqrt{y_i^2 - \left(\left(\frac{x_{k2}}{y_{k2}}\right)^2 + 1\right)(y_i^2 - r^2)}}{\left(\frac{x_{k2}}{y_{k2}}\right)^2 + 1}} \end{pmatrix} \tag{19}$$

Index of i is remarked for iterations and centers of the follower. Index of k is remarked for lines of the general set too as is the case for the flat face. Rotation procedure in Eq. 19 is implemented after every trimming procedure loop.

4. Algorithms

Algorithms are identified for the purpose of preventing confusion in the coding of obtained equations for two follower mechanisms. Algorithm specified for the flat face follower mechanism is shown in Figure 3 and that for the roller follower mechanism is shown in Figure 4.

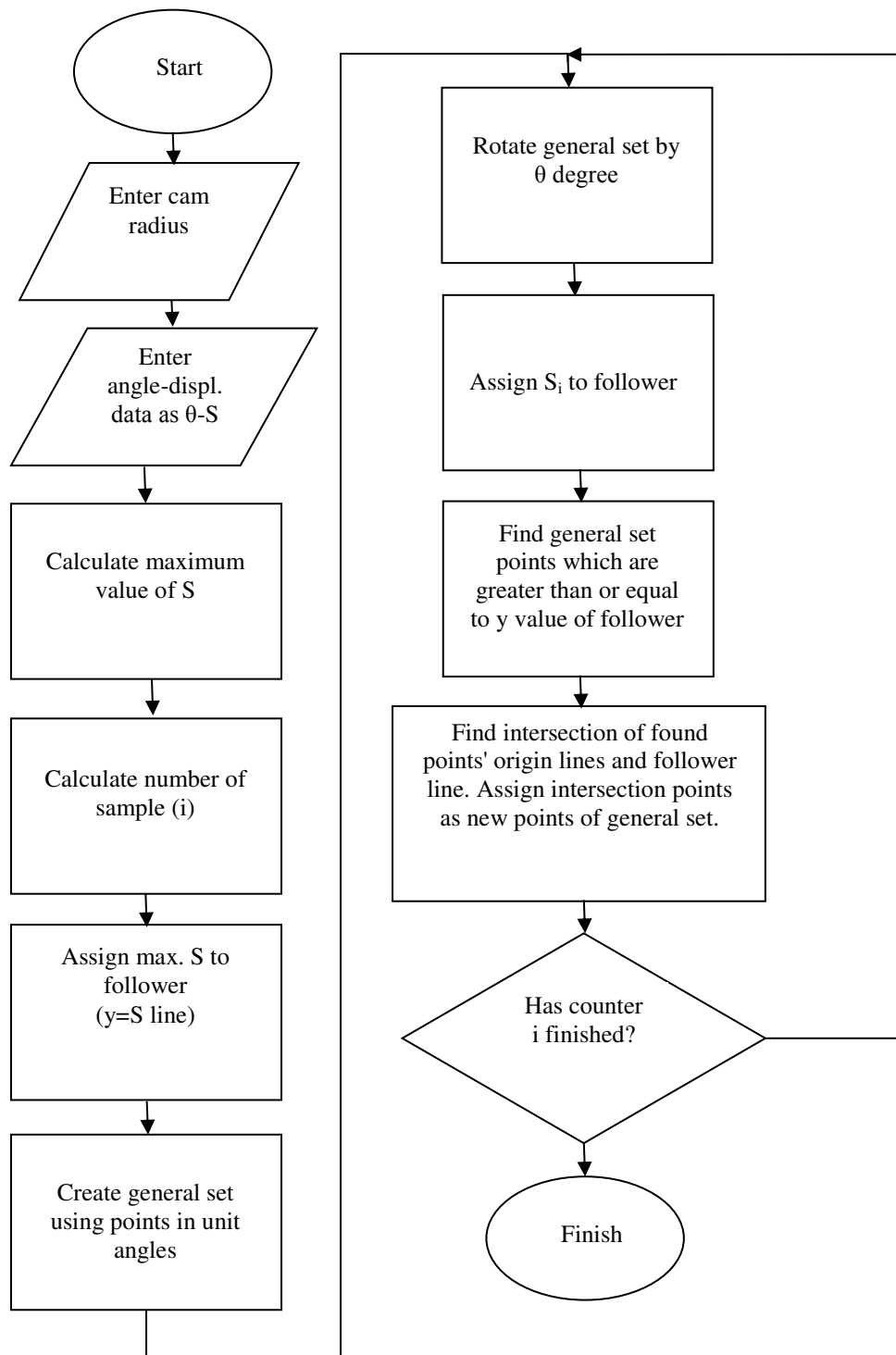


Figure 3. Graphical approach algorithm for flat face follower cam mechanism

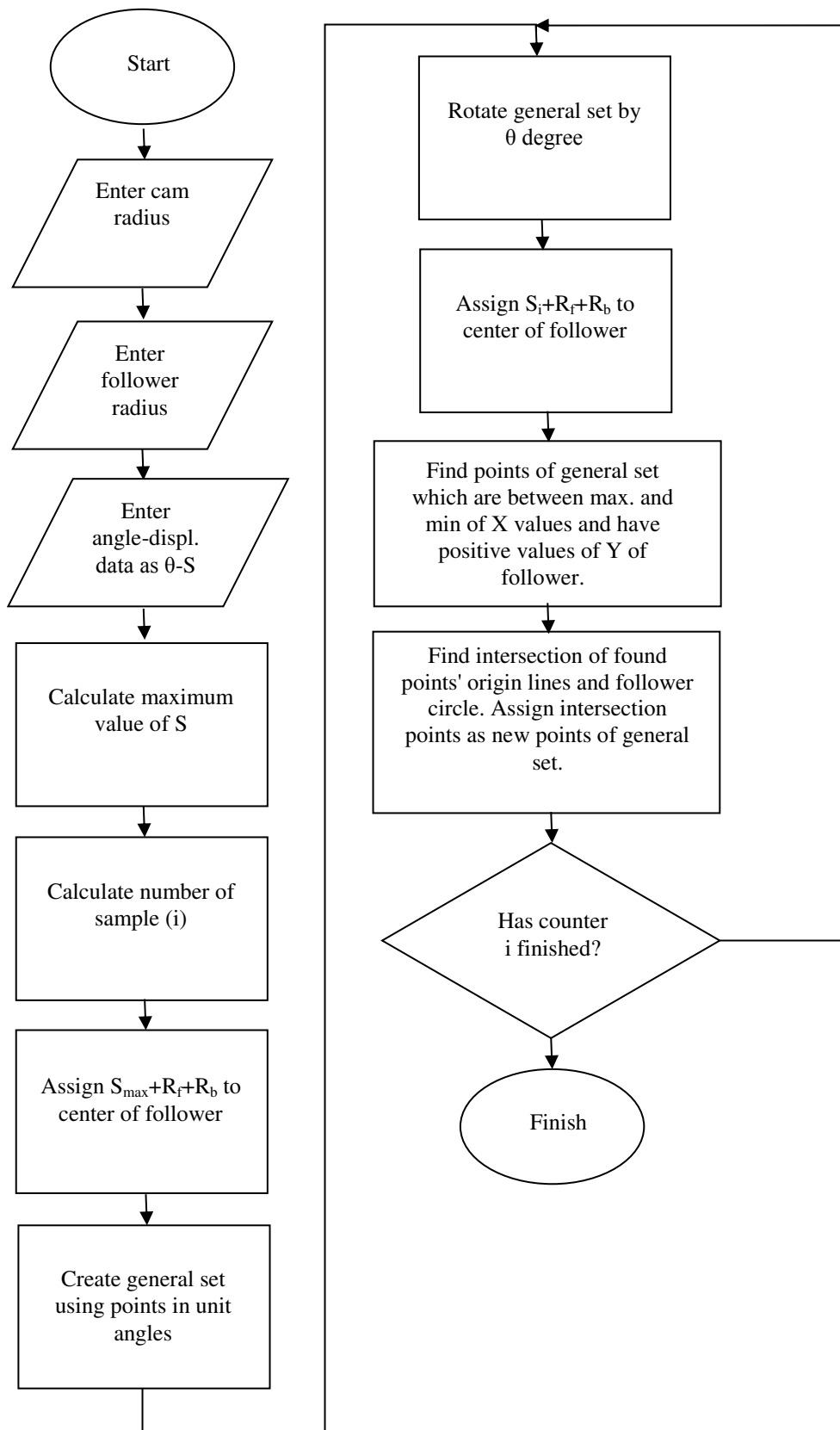


Figure 4. Graphical approach algorithm for roller follower cam mechanism

5. Verification and comparison of results

Novel and base algorithms of developed analytical approach and conventional analytical approach (Chen, 1982; Norton, 2009; Rothbart, 2004; Söylemez, 2010) coded in MATLAB® environment were compared using plot function of MATLAB® which uses linear interpolation method. Plotting operations were repeated by zooming in to the areas of interest and marked lines to prove differences between the two approaches. Values which show differences between data points were calculated and plotted as bars. These values numerically signify the velocity of the profile surface at constant radial velocity. A comparison of the developed approach and CAD system based graphical approach was implemented. It has been put forth according to the results that the max. error was $7 \cdot 10^{-5}$ based on the precision of data production. All errors were decreased to zero since the values in the developed approach were rounded to the nearest ten thousandth. Decades of samples from ESTAS Camshaft Corporation's customer orders were applied to the algorithm and results were also overlapped with CAD system results. The cycloid function used to produce displacement data necessary for profile determination that is appropriate to the scenario has been given in Table 1. Dataset of S is given in Table 2 and kinematic graphs of S are shown in Figure5. Displacement calculations were performed using the following Eq. 20;

$$S = \frac{H}{\pi} \left[\frac{\pi\theta}{\beta} - \frac{1}{2} \sin \left(\frac{2\pi\theta}{\beta} \right) \right] \tag{20}$$

Where S is displacement, H is $\max(S)$, θ is current angle and β is difference between $\max(\theta)$ and $\min(\theta)$.

Table 1: Instructions of displacement (S) scenarios

Follower Type -R	$\theta=0^\circ \dots\dots 70^\circ$	$\theta=71^\circ \dots\dots 289^\circ$	$\theta=290^\circ \dots\dots 360^\circ$	Cam Radius
Flat Face Follower - 0	8 0 mm	dwell	0 8 mm	27 mm
Roller Follower - 12 mm	8 0 mm	dwell	0 8 mm	27 mm

Table 2: Displacement data

θ (°)	Displacement S (mm)		θ (°)	Displacement S (mm)		θ (°)	Displacement S (mm)	
	FALL	RISE		FALL	RISE		FALL	RISE
0	8.0000	8.0000	24	6.3198	6.3198	48	1.3435	1.3435
1	7.9998	7.9998	25	6.1383	6.1383	49	1.1891	1.1891
2	7.9988	7.9988	26	5.9489	5.9489	50	1.0444	1.0444
3	7.9959	7.9959	27	5.7520	5.7520	51	0.9097	0.9097
4	7.9902	7.9902	28	5.5484	5.5484	52	0.7852	0.7852
5	7.9810	7.9810	29	5.3388	5.3388	53	0.6709	0.6709
6	7.9673	7.9673	30	5.1239	5.1239	54	0.5669	0.5669
7	7.9484	7.9484	31	4.9045	4.9045	55	0.4730	0.4730
8	7.9234	7.9234	32	4.6816	4.6816	56	0.3891	0.3891
9	7.8917	7.8917	33	4.4559	4.4559	57	0.3149	0.3149
10	7.8526	7.8526	34	4.2284	4.2284	58	0.2502	0.2502
11	7.8055	7.8055	35	4.0000	4.0000	59	0.1945	0.1945
12	7.7498	7.7498	36	3.7716	3.7716	60	0.1474	0.1474
13	7.6851	7.6851	37	3.5441	3.5441	61	0.1083	0.1083

14	7.6109	7.6109	38	3.3184	3.3184	62	0.0766	0.0766
15	7.5270	7.5270	39	3.0955	3.0955	63	0.0516	0.0516
16	7.4331	7.4331	40	2.8761	2.8761	64	0.0328	0.0328
17	7.3291	7.3291	41	2.6612	2.6612	65	0.0190	0.0190
18	7.2148	7.2148	42	2.4516	2.4516	66	0.0098	0.0098
19	7.0903	7.0903	43	2.2480	2.2480	67	0.0041	0.0041
20	6.9556	6.9556	44	2.0511	2.0511	68	0.0012	0.0012
21	6.8109	6.8109	45	1.8617	1.8617	69	0.0002	0.0002
22	6.6565	6.6565	46	1.6803	1.6803	70	0.0000	0.0000
23	6.4926	6.4926	47	1.5074	1.5074	71	0.0000	0.0000

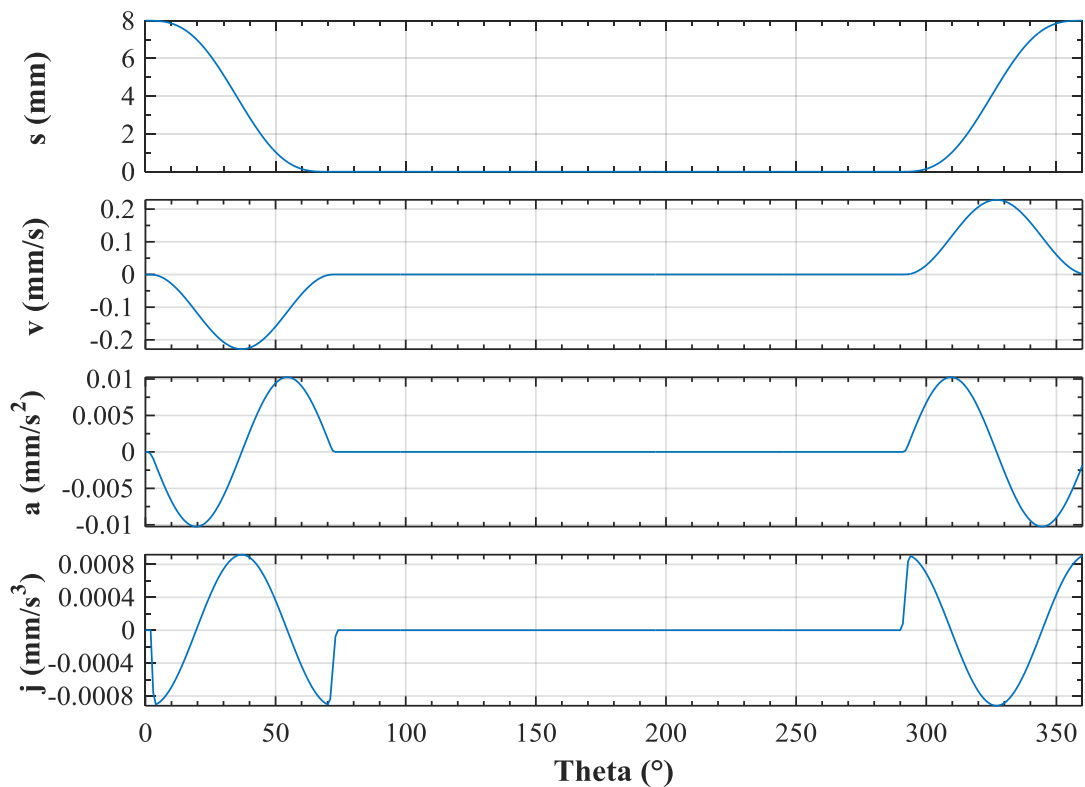


Figure 5. Kinematic graphs of used displacement data

5.1. Results of Flat Face Follower Cam Profile

A cam follower displacement scenario which starts to fall at 0° from 8 mm height and reaches a height of 0 mm at 70° dwelling between 71° and 290° and rising from 0 mm height to 8 mm height between 291° and 360° was applied to obtain S data. Base circle of cam has a radius of 27 mm. A comparison was made between the results of this scenario developed via graphical and conventional analytical approaches the results of which are given in Figure 6. At first glance, data of two profiles overlap seamlessly. When we zoom in towards the top region of the profile as in Figure 7, it is obvious that the data are produced more frequently in the analytical approach in comparison with the graphical approach. When zoomed to the base circle as in Figure 8, it can be seen that the data are produced more scattered in the analytical approach in comparison with the graphical approach. Frequencies of data are equal for the base circle. Graphical approach provides profile data that has a smaller variation. It means that the profile surface has a more stable

velocity at constant angular velocity. The variation data of the two approaches were compared in Figure 9. Constant intervals of variation occur in the base circle region.

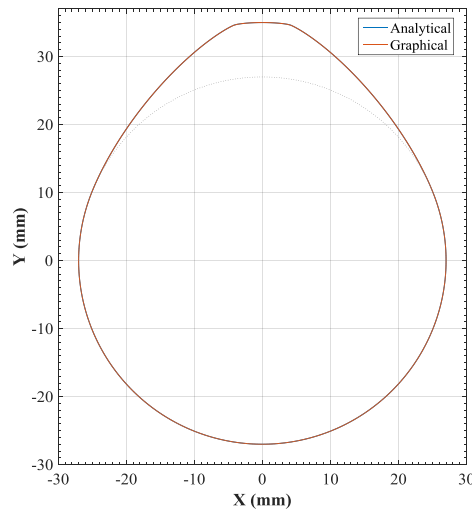


Figure 6. Overlapping of analytical and graphical approaches - flat face follower cam profile

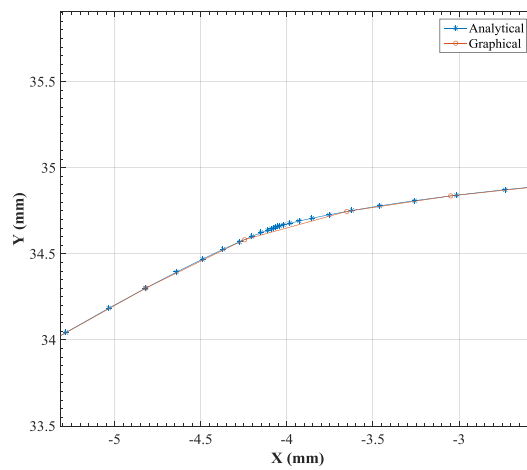


Figure 7. Left top region view of two profile data produced by analytical and graphical approaches - flat face follower cam profile

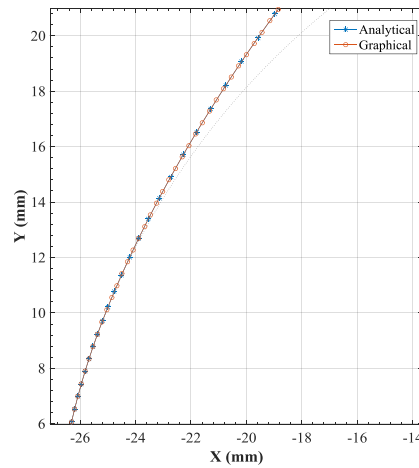


Figure 8. Left middle region view of two profile data produced by analytical and graphical approaches - flat face follower cam profile

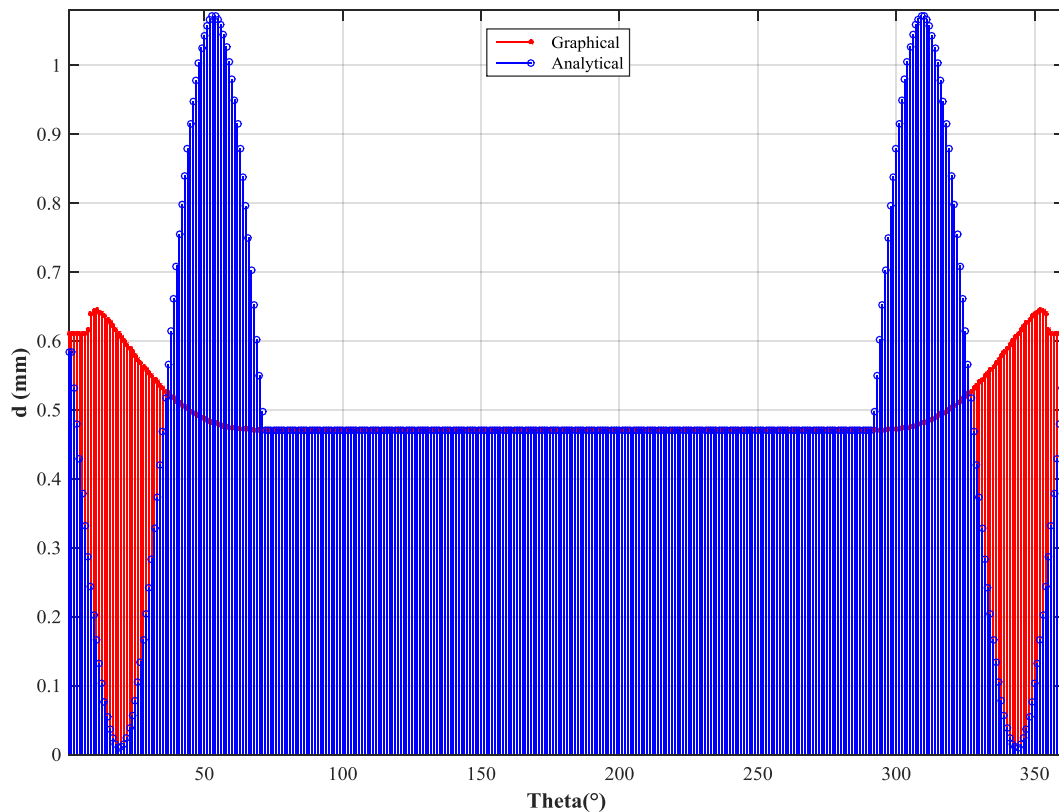


Figure 9: Comparison of data point difference variations of analytical and graphical approaches - flat face follower cam profile

5.2 Results of Roller Follower Profile

The scenario of flat face follower profile was applied to obtain the S data of the roller follower. The radius of the base circle was 27 mm and the radius of the roller follower was 12 mm. A comparison was made between the results of the scenario obtained via graphical and conventional analytical approaches which has been shown in Figure 10. Local zoomed comparisons are shown in Figure 11 and Figure 12. In comparison of the graphical approach and conventional approach it is seen that produced data were closer together near the top region of the profile and vice versa near the base circle. Frequencies of data were equal at the base circle. Even though differences of varying of data is not big as much as flat face follower cam profile, graphical approach produces less varying profile data. The variation data of the two approaches were compared in Figure 13.

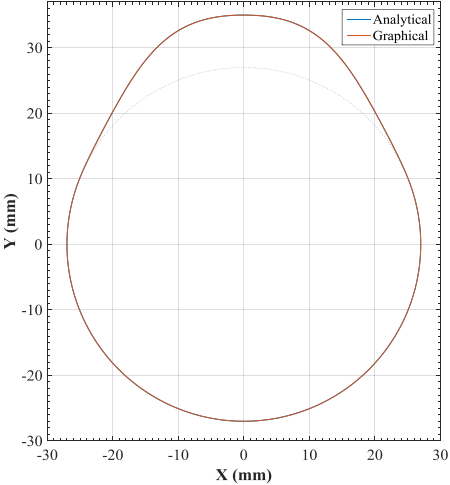


Figure 10. Overlapping of conventional and graphical approaches - roller follower cam profile

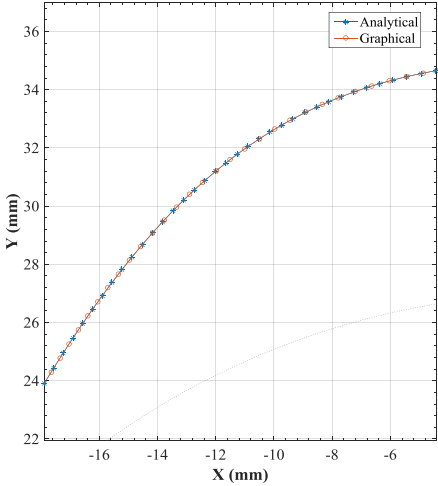


Figure 11. Left top region view of two profile data produced by analytical and graphical approaches - roller follower cam profile

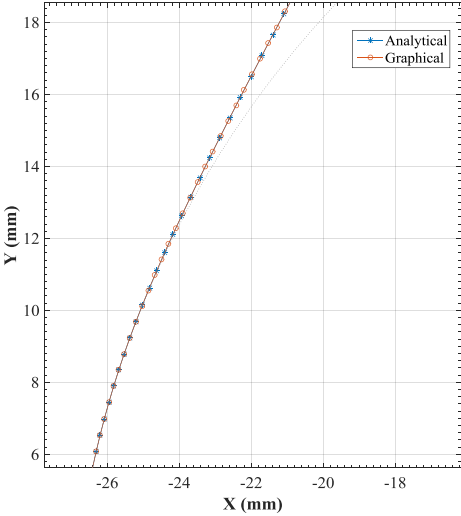


Figure 12. Left middle region view of two profile data produced by analytical and graphical approaches - roller follower cam profile

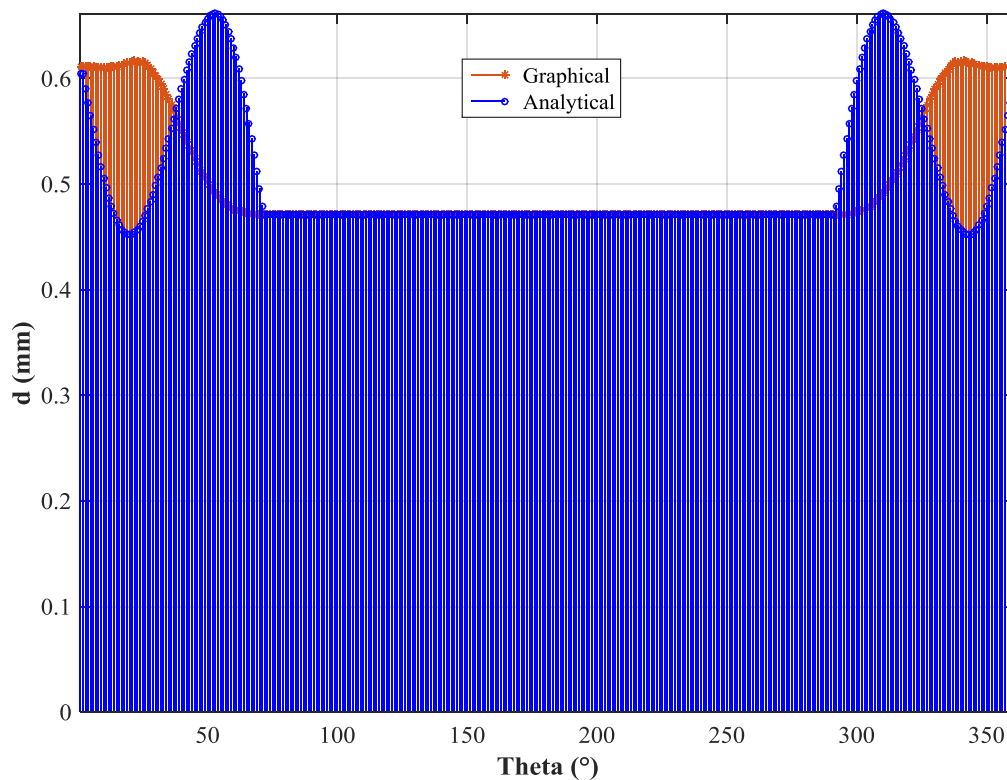


Figure 13. Comparison of data point difference variations of analytical and graphical approaches - roller follower cam profile

6. Conclusion

Graphical approach was analyzed in the study as the calculation method of profiles of radial cams which can be performed only in CAD systems. Equations and algorithms of the approach were developed for centric flat face and roller translating follower types of spatial cams. The equations can thus be generated via computers using any programming language. Written scripts of developed graphical and conventional analytical approaches were executed. Differences between two approaches were investigated by plotting the obtained results. Graphs show that the novel graphical approach and the conventional analytical approach overlap each other but only for sampling frequencies. Graphical approach has less variable e.g. more stable intervals in comparison with the analytical approach. Indeed, thus variations do not affect the theoretical kinematics of the follower mechanism but cause the production of extra NC codes in high frequency regions or defective NC codes in low frequency regions. High frequency codes may cause time consumption whereas low frequency codes disrupt the surface roughness in the process of tool path interpolation and/or evaluation. Variation of velocity of profile surface is a complication of NC case. Evaluating stable velocity is easier and cheaper for control and machine dynamics in comparison with variable velocity. The approach developed is also suitable for inverse engineering which uses measurement data of any produced camshaft. In addition, the production process of camshafts can be performed independently from design processes by applying the approach used in the study without the need for the continuous functions of the analytical approach and with only the need for displacement data and base

dimensions of the follower and cam. Therefore, the graphical approach is cost effective according to the analytical approach and is suitable for discrete types of displacement data.

7. Acknowledgment

This study was designed according to a project which is cooperated with ESTAS and Cumhuriyet University. The project was supported by Republic of Turkey, Ministry of Science, Industry and Technology and ESTAS Corporation (0402.STZ.2013-2). This study is not official view of the Ministry.

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