PAPER

Analysis of Large-Scale Periodic Array Antennas by CG-FFT Combined with Equivalent Sub-Array Preconditioner

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SUMMARY This paper presents method that offers the fast and accurate analysis of large-scale periodic array antennas by conjugate-gradient fast Fourier transform (CG-FFT) combined with an equivalent sub-array preconditioner. Method of moments (MoM) is used to discretize the electric field integral equation (EFIE) and form the impedance matrix equation. By properly dividing a large array into equivalent sub-blocks level by level, the impedance matrix becomes a structure of Three-level Block Toeplitz Matrices. The Three-level Block Toeplitz Matrices are further transformed to Circulant Matrix, whose multiplication with a vector can be rapidly implemented by one-dimension (1-D) fast Fourier transform (FFT). Thus, the conjugate-gradient fast Fourier transform (CG-FFT) is successfully applied to the analysis of a large-scale periodic dipole array by speeding up the matrix-vector multiplication in the iterative solver. Furthermore, an equivalent sub-array preconditioner is proposed to combine with the CG-FFT analysis to reduce iterative steps and the whole CPU-time of the iteration. Some numerical results are given to illustrate the high efficiency and accuracy of the present method.

key words: *large-scale periodic phased arrays, method of moments (MoM), block Toeplitz matrices, conjugate-gradient fast Fourier transform (CG-FFT), preconditioner of iterative method*

1. Introduction

An urgent requirement is to obtain a fast and accurate analysis of the electromagnetic performance of a large-scale array antenna which is used in the solar power satellite (SPS) systems [1]. The method of moments (MoM) is one of the effective methods to analyze antennas [2]. However, the computational complexity of the conventional direct methods and iterative methods require order of $O(N^2)$ up to $O(N^3)$ [3], [4] and $O(N^2)$ of computer memory to solve the dense matrix equation appearing in the MoM analysis, where N is the number of unknowns. Even though powerful computers with large memories are available, it is still very difficult to accurately analyze an extremely large-scale electromagnetic problem with millions of unknowns.

Many efforts have been made to accelerate the MoM computation by simplifying the computational complexity of a matrix-vector multiplication, and reducing the memory requirements in the conjugate gradient (CG) iterative solver [5], [6]. The fast multipole method (FMM) [7], [8] and multilevel fast multipole algorithm (MLFMA) [9] have been proved to be effective in reducing the computational complexity for arbitrarily shaped surface problems. With the use of MLFMA, the computational complexity of a matrix-vector multiplication has been reduced from $O(N^2)$ to $CN\log(N)$, but the mutual impedance calculated in the MLFMA causes some errors inevitably due to the truncation error from the finite series of the Green's function for far field calculation. Furthermore, when the considered object has a very large size, MLFMA is still time consuming due to the constant C is usually very large [10]. Recently, the adaptive integral method (AIM) has also received much attention for the ability of reducing computational complexity through fast far-zone's matrix-vector multiplication by fast Fourier transform (FFT) [11]. Though its computational complexity can be reduced to $O(N^{1.5}\log N)$ or less, large error may be inevitably introduced in the process of non-uniform grids' transform to uniform grids. Conjugate gradient fast Fourier transform (CG-FFT) algorithm [12], [13] is also very effective to reduce the computational complexity to $C_0 N\log N$ with a relatively small constant C_0 compared with that of MLFMA. In recent years, some researches have been carried out on the effective analysis of periodic structures, and some effective methods for the practical electromagnetic problems have been proposed in [10], [14], [15].

Many other fast algorithms have also been developed for the analysis models with periodic structure. The iterative method based on the Gauss-Seidel scheme has been proposed to give a fast analysis of the large-scale array antennas [16]. An iterative method based on space decomposition technique [17] has also been employed to give a fast analysis of the large-scale phased array antennas. Though the iterative steps have been effectively reduced in these two methods, the computational complexity is still near to $O(N^2)$. The Forward-backward method (FBM) and its some extensions have been developed for the fast analysis of electromagnetic problems, with computational complexity of near to $O(N)$, but it has also a problem in accuracy because of the error caused in filling the impedance matrix [18], [19]. Therefore, how to obtain a fast and accurate analysis of large-scale periodic array antennas is still an interesting and challenging object.

In this paper, a fast and accurate analysis for a large-scale periodic array antenna is presented. Firstly, the MoM

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is used to discretize the electric field integral equation and form impedance matrix equation. Then by properly dividing a large array into equivalent sub-blocks level by level, the impedance matrix becomes a structure of Three-level Block Toeplitz Matrices. The Three-level Block Toeplitz Matrices are further transformed to a form of Circulant Matrix, whose multiplication with a vector can be fast implemented by 1-D FFT. Numerical analysis shows that the computational complexity for each iterative step of the presented analysis can be reduced to $O(N\log(N))$ and computer memory requirement can be reduced to $O(N)$, where N represents the whole number of unknowns. Thus the whole CPU-time of the present method is the order of $AN + BN\log(N)$. As a result, the conjugate-gradient fast Fourier transform (CG-FFT) is successfully applied to the analysis for large-scale periodic dipole array by speeding up the matrix-vector multiplication in the iterative solver. Because there is no approximation involved in filling the impedance matrix, a high accuracy is ensured in the present method.

Though the computational complexity (or CPU-time) of each iterative step has been greatly reduced, it is also important to improve the iterative convergence to effectively reduce the whole iterative steps. Based on the characteristics of the periodic structure of the analysis model, an equivalent sub-array preconditioner is proposed to enhance convergence of the CG-FFT analysis in order to reduce required iterative steps and the whole CPU-time. Finally some large-scale periodic dipole arrays are calculated to illustrate the high efficiency and accuracy of the present method.

2. Formulation

The analysis model of two-dimensional (2-D) periodic array antennas including $N_{total} = N_x N_y$ dipoles, is shown in Fig. 1. The length of each dipole is $2l$. The array spacing along x and y directions are d_x and d_y, respectively.

2.1 MoM Analysis

The electric field integral equation (EFIE) for all line electric currents $\mathbf{J}_n(y)$ (n=1, 2, $\cdots$, N_{total}) can be written as

$$\hat{t}\cdot\left[i\omega\mu_0\sum_{n=1}^{N_{total}}\int_{L_n}\overline{\mathbf{G}}(\mathbf{r},y)\cdot\mathbf{J}_n(y)dy\right] = -\hat{t}\cdot\mathbf{E}(\mathbf{r}), \tag{1}$$

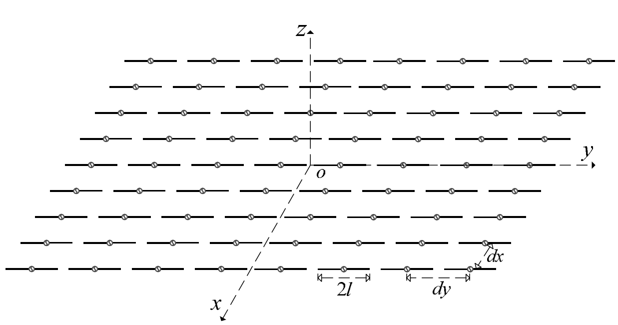

Fig. 1 Analysis model of periodic array antenna.

where $\hat{t}$ is unit tangential vector of the line antenna, $\mathbf{J}_n(y)$ is the line electric currents, $\mathbf{E}(\mathbf{r})$is the exciting electric field. $\overline{\mathbf{G}}(\mathbf{r}, y)$ denotes electric field dyadic Green's function in free space [20]. The current distribution $\mathbf{J}_n(y)$ on the nth line antenna is expanded as

$$\mathbf{J}_n(y) = \sum_{i=1}^{M} I_i\mathbf{f}_i(y), \tag{2}$$

where I_i is corresponding expansion coefficient and the basis function $\mathbf{f}_i$ is chosen as sinusoidal basis functions [21], which is given by

$$f_i(y) = \begin{cases} \dfrac{\sin k(y-y_{n-1})}{\sin k(y_n-y_{n-1})} & y_{n-1}\le y\le y_n \\ \dfrac{\sin k(y_{n+1}-y)}{\sin k(y_{n+1}-y_n)} & y_n\le y\le y_{n+1} \\ 0 & elsewhere \end{cases}. \tag{3}$$

The following impedance matrix equation is obtained by using Galerkin's procedure

$$ZI = V. \tag{4}$$

2.2 Application of CG-FFT to Periodic Array Antennas

Firstly, the array antenna is divided into several sub-blocks. If N_x dipoles along x axis are assumed to be one sub-block, the array is composed of N_y equivalent sub-blocks. Thus Z matrix in (4) can be described as the following Block Toeplitz Matrices [4], [22], [23]

$$Z = \begin{bmatrix} T_0 & T_{-1} & \cdots & T_{1-N_y} \\ T_1 & T_0 & \cdots & T_{2-N_y} \\ \vdots & \vdots & \ddots & \vdots \\ T_{N_y-1} & T_{N_y-2} & \cdots & T_0 \end{bmatrix}. \tag{5}$$

Secondly, each sub-block T is further divided into N_x smaller sub-blocks, each containing one dipole antenna. Thus the T matrix in (5) will be written as the following another Block Toeplitz Matrices

$$T = \begin{bmatrix} TT_0 & TT_{-1} & \cdots & TT_{1-N_x} \\ TT_1 & TT_0 & \cdots & TT_{2-N_x} \\ \vdots & \vdots & \ddots & \vdots \\ TT_{N_x-1} & TT_{N_x-2} & \cdots & TT_0 \end{bmatrix}. \tag{6}$$

Finally, because each dipole is divided into M basis functions (2), TT matrix can also be Toeplitz Matrix

$$TT = \begin{bmatrix} ttt_0 & ttt_{-1} & \cdots & ttt_{1-M} \\ ttt_1 & ttt_0 & \cdots & ttt_{2-M} \\ \vdots & \vdots & \ddots & \vdots \\ ttt_{M-1} & ttt_{M-2} & \cdots & ttt_0 \end{bmatrix}. \tag{7}$$

Therefore the Three-level Block Toeplitz Matrices is formed by making use of array geometry and division of the basis function.

In order to implement fast matrix-vector multiplication for solving (4) in iterative solver, CG-FFT is employed. It is necessary to transform the Three-level Block Toeplitz Matrices to a Circulant Matrix. For the lowest level matrix TT, the following Circulant Matrix can be constructed as

$$K_{2M} = \begin{bmatrix} TT & BB \\ BB & TT \end{bmatrix}, \tag{8}$$

where the matrix BB can be written as

$$BB = \begin{bmatrix} 0 & ttt_{M-1} & \cdots & ttt_1 \\ ttt_{1-M} & 0 & \cdots & ttt_2 \\ \vdots & \vdots & \ddots & \vdots \\ ttt_{-1} & ttt_{-2} & \cdots & 0 \end{bmatrix}. \tag{9}$$

According to three same implementations of transform from the Toeplitz Matrix to the Circulant Matrix, the whole Circulant Matrix C can be attained by

$$C = \begin{bmatrix} c_0 & c_{8N_y*N_x*M-1} & \cdots & c_1 \\ c_1 & c_0 & \cdots & c_2 \\ \vdots & \vdots & \ddots & \vdots \\ c_{8N_y*N_x*M-1} & c_{8N_y*N_x*M-2} & \cdots & c_0 \end{bmatrix}. \tag{10}$$

Through deducing, the whole factorization of C has the form of

$$\begin{aligned} C = & (\hat{F}_{2M}^{2N_x \cdot 2N_y})^H \cdot P_1 \cdot (\hat{F}_{2N_x}^{2N_y \cdot 2M})^H \\ & \cdot P_2 \cdot (\hat{F}_{2N_y}^{2N_x \cdot 2M})^H \cdot diag(\hat{F}_{2N_y}^{2N_x \cdot 2M} d) \\ & \cdot \hat{F}_{2N_y}^{2N_x \cdot 2M} \cdot P_2 \cdot \hat{F}_{2N_x}^{2N_y \cdot 2M} \cdot P_1 \cdot \hat{F}_{2M}^{2N_x \cdot 2N_y}, \end{aligned} \tag{11}$$

where

$$\hat{F}_i^j = \begin{bmatrix} F_i & & & \\ & F_i & & \\ & & \ddots & \\ & & & F_i \end{bmatrix}, \tag{12}$$

where F_i is the 1-D fast Fourier transform for i elements and $(F_i)^H$ is the 1-D inverse fast Fourier transform for i elements. The whole vector is divided into j groups uniformly, and each group includes i elements. Suppose permutation matrix $PP(m,n)$, and it has the following property.

$$PP(m,n) \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}, \tag{13}$$

where $A_\alpha = [a_{\alpha,1}, a_{\alpha,2}, \cdots, a_{\alpha,n}]^T$ and $B_\beta = [a_{1,\beta}, a_{2,\beta}, \cdots, a_{m,\beta}]^T$. For example, when $m = 3$, $n = 2$, $PP(m,n)$ will be the following permutation matrix

$$PP(3,2) = \begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \end{bmatrix}. \tag{14}$$

So the general form of permutation matrix P_1 in (11) can be expressed by

$$P_1 = \overbrace{\begin{bmatrix} PP(m_1,n_1) & & \\ & \ddots & \\ & & PP(m_1,n_1) \end{bmatrix}}^{k_1}, \tag{15}$$

and the general form of permutation matrix P_2 in (11) can be directly denoted by

$$P_2 = PP(m_2, n_2), \tag{16}$$

where these parameters $k_1 = 2N_y, m_1 = 2N_x, n_1 = 2M$ and $m_2 = 2N_y, n_2 = 2N_x \cdot 2M$ are for 1-D fast Fourier transform procedure in (11); $k_1 = 2N_y, m_1 = 2M, n_1 = 2N_x$ and $m_2 = 2N_x \cdot 2M, n_2 = 2N_y$ are for 1-D inverse fast Fourier transform procedure in (11). If the first column vector in (10) is denoted by vector C_1, then the vector d_2 can be described by

$$d_2 = P_1 \begin{bmatrix} F_{2M} v_0 \\ F_{2M} v_1 \\ \vdots \\ F_{2M} v_{4N_y*N_x-1} \end{bmatrix}, \tag{17}$$

where $v_i = [c_{i*2M+0}, c_{i*2M+1}, \cdots, c_{i*2M+(2M-1)}]^T$, $C_1 = [c_0, c_1, \cdots, c_{8N_y*N_x*M-1}]^T$, $k_1 = 2N_y, m_1 = 2N_x, n_1 = 2M$ for permutation matrix P_1. Thus vector d_1 can be written by

$$d_1 = P_2 \begin{bmatrix} F_{2N_x} \omega_0 \\ F_{2N_x} \omega_1 \\ \vdots \\ F_{2N_x} \omega_{4N_y*M-1} \end{bmatrix}, \tag{18}$$

where $\omega_i = [\psi_{i*2N_x+0}, \psi_{i*2N_x+1}, \cdots, \psi_{i*2N_x+(2N_x-1)}]^T$, $d_2 = [\psi_0, \psi_1, \cdots, \psi_{8N_y*N_x*M-1}]^T$, $m_2 = 2N_y, n_2 = 2N_x \cdot 2M$ for permutation matrix P_2. ψ_i is the ith element of the vector which has be denoted in Eq. (17). Therefore vector d in (11) can be expressed by

$$d = \begin{bmatrix} F_{2N_y} \xi_0 \\ F_{2N_y} \xi_1 \\ \vdots \\ F_{2N_y} \xi_{4N_x*M-1} \end{bmatrix}, \tag{19}$$

where $\xi_i = [\gamma_{i*2N_y+0}, \gamma_{i*2N_y+1}, \cdots, \gamma_{i*2N_y+(2N_y-1)}]^T$, $d_1 = [\gamma_0, \gamma_1, \cdots, \gamma_{8N_y*N_x*M-1}]^T$. Suppose current vector I in (4) is represented by

$$I = \begin{bmatrix} I_1^{(1)} \\ I_2^{(1)} \\ \vdots \\ I_{N_y}^{(1)} \end{bmatrix}, \tag{20}$$

where, $I_i^{(1)} = [I_{i,1}^{(2)}, I_{i,2}^{(2)}, \cdots, I_{i,N_x}^{(2)}]^T$, and $I_{i,j}^{(2)} = [\kappa_{i,j,1}, \kappa_{i,j,2}, \cdots, \kappa_{i,j,M}]^T$. Therefore, ZI in (4) can be easily calculated by multiplying C with I', where vector I' is expressed by

$$I' = [J', 0]^T, \tag{21}$$

where, $J' = [J_1'^{(1)}, J_2'^{(1)}, \cdots, J_{N_y}'^{(1)}]^T$, and $J_i'^{(1)} = [J_i^{(1)}, 0]^T$, and $J_i^{(1)} = [J_{i,1}'^{(2)}, J_{i,2}'^{(2)}, \cdots, J_{i,N_x}'^{(2)}]^T$, $J_{i,j}'^{(2)} = [J_{i,j}^{(2)}, 0]^T$, and $J_{i,j}^{(2)} = [\kappa_{i,j,1}, \kappa_{i,j,2}, \cdots, \kappa_{i,j,M}]^T$.

From the following numerical examples, it is shown that the computational complexity of the present method can be reduced to $O(N\log(N))$. Because only the first row matrix elements is required to be stored to implement the fast matrix-vector multiplication, the memory requirement can be reduced to $O(N)$. Here N represents the whole number of unknowns, i.e. $N = N_x N_y M$. It is also should be noted that since there is no approximation introduced in the present method, high accuracy is expected to be achieved.

2.3 An Equivalent Sub-Array Preconditioner

The CG-FFT is very effective in reducing the computational complexity of each iterative step. However, the whole CPU-time is proportional to the number of iterative steps to achieve desired accuracy. And the whole CPU-time is estimated by $N_{iter}C_0N\log(N)$, where N_{iter} is the whole iterative steps and C_0 is a constant. In the application of CG-FFT to large-scale periodic array antennas, the number of iterative steps N_{iter} is often a very large number. Therefore, it is also very practical and important to reduce the number of iterative steps.

The preconditioning technique is a very direct method to reduce the number of iterative steps [15], [24]–[26]. To make use of the preconditioning technique, it is required to find a preconditioner matrix P so that the number of the iterative number of the iterative steps required for obtaining the solution $PZI = PV$ is reduced compared with that required for the original matrix equation $ZI = V$.

Let P be a $N \times N$ preconditioner matrix. Considering the characteristic of the periodic structure of the periodic structure of the analysis model, an equivalent sub-array preconditioner matrix P is determined in the following.

For an example of a $N_x \times N_y = 9 \times 4$ array in Fig. 2, an equivalent sub-array can be $n_x \times n_y = 3 \times 1$ as a choice.

The whole preconditioning matrix P can be formed by

$$P = \begin{bmatrix} S_r & 0 & 0 & \cdots & 0 \\ 0 & S_r & 0 & \cdots & 0 \\ 0 & 0 & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & S_r & 0 \\ 0 & 0 & \cdots & 0 & S_r \end{bmatrix}, \tag{22}$$

where S_r is given by

$$S_r = Z_s^{-1}, \tag{23}$$

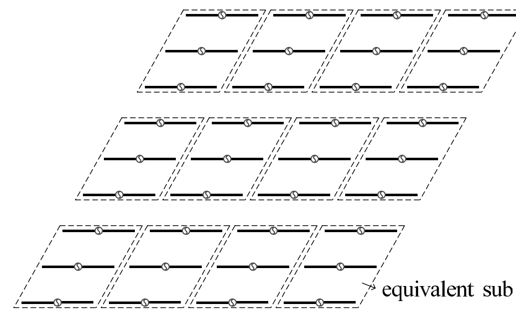

Fig. 2 An equivalent (or similar) sub-array preconditioner.

where Z_s denotes the self-impedance matrix of selected sub-array. Because the sub-array block is usually much smaller than the total array, i.e. unknown r is a small integer compared with N. The LU decomposition (LUD) can be used to compute the S_r.

Since all sub-arrays are equivalent and small, the implementation of the present preconditioner costs little extra memory and CPU time in each matrix-vector multiply. The high efficiency of introducing the precondtioner is demonstrated in the following examples.

3. Numerical Results and Discussions

In the first example, a $\lambda/2$ -dipole array ($2l = \lambda/2$, $N_x \times N_y = 32 \times 32$) with array spacing of $d_x = 0.5\lambda$, $d_y = \lambda$ is analyzed by the present method to demonstrate the accuracy. The phase difference of the exciting voltage to the array elements is $jkd_x \sin\theta_0$ between the neighboring elements along x direction, while it has the same phase in y direction so that the radiation of the array is directed to θ_0 in xz plane. In this example, θ_0 equals 30^0. Each dipole is expanded by 9 overlapped sinusoidal functions expressed in (3). The dipole number of the equivalent sub-array preconditioner is 8, the number of dipoles along x and y direction is 8 and 1 respectively, so the unknown r in sub-array is 72. The residual error is defined as $\prod_L = \|ZI - V\|/\|V\|$. The residual error is 10^{-8} in following cases, which means that the iterative procedure continues until the residual error is less than 10^{-8}.

Figure 3 gives the comparisons of input impedances of 1st row dipoles along x axis between the results using the LU decomposition method (LUD) and using the present method. The corresponding actual gain with load impedance $Z_{load} = 50\,\Omega$ is also shown in Fig. 4. The excellent agreements between two methods are achieved.

In order to show the performance of the present method in dealing with large-scale problems, the array dimension is changed from 16×16 to 256×256 corresponding to the number of unknowns (N) from 2,304 to 589,824. The dimension of sub-array preconditioner is 8×1 (unknowns are 72) in all cases. The memory requirements and CPU time per iteration are plotted as function of the number of unknowns in Fig. 5. The CPU time per iteration with preconditioner and that without preconditioner are nearly same from Fig. 5. The value of CPU-time shown was measured by us-

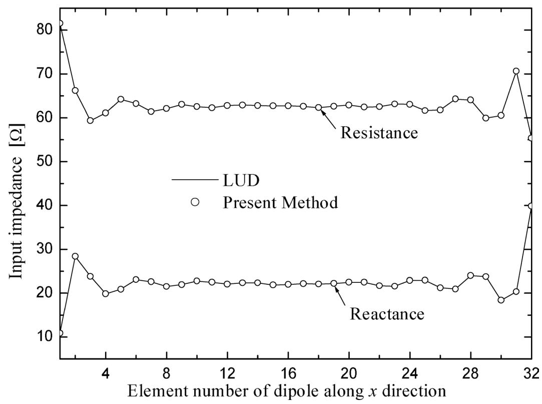

Fig. 3 Input impedance of 1st row ($Row_y = 1$).

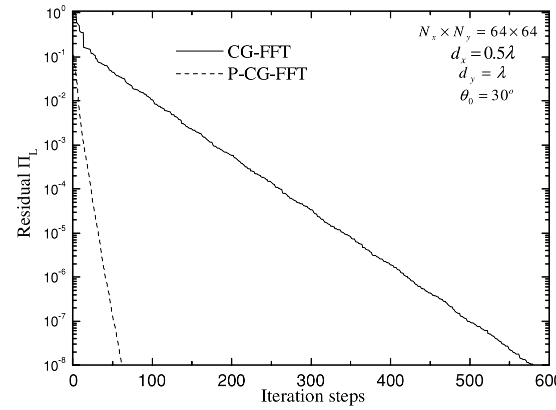

Fig. 6 Convergence comparison of two methods.

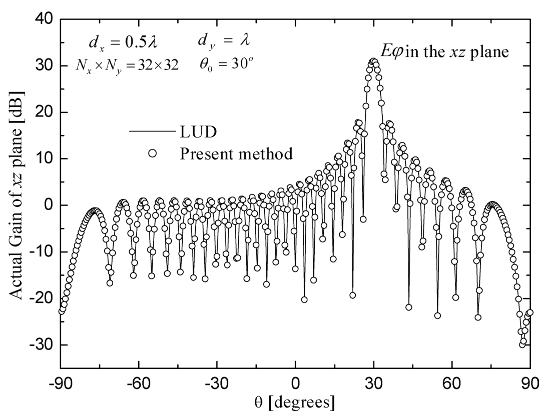

Fig. 4 Actual gain of xz plane.

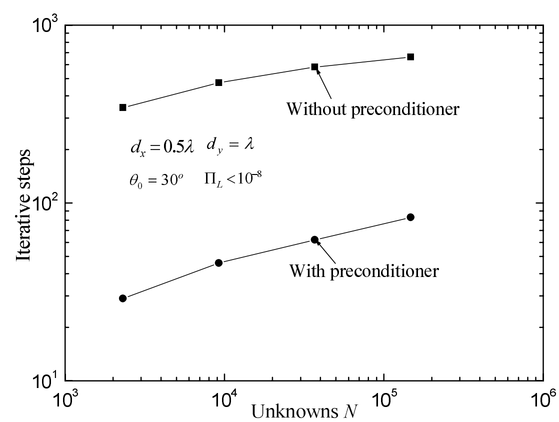

Fig. 7 Comparison of iterative steps with unknowns.

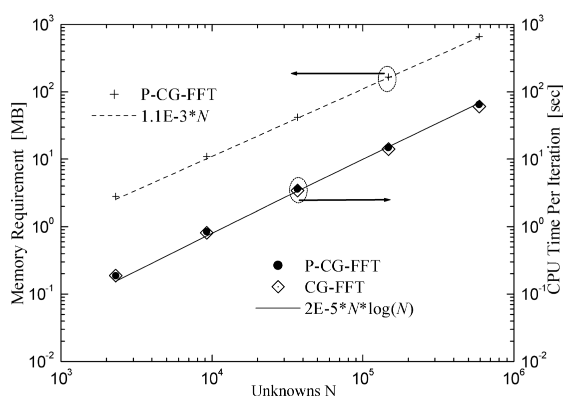

Fig. 5 Computational complexity and memory requirement.

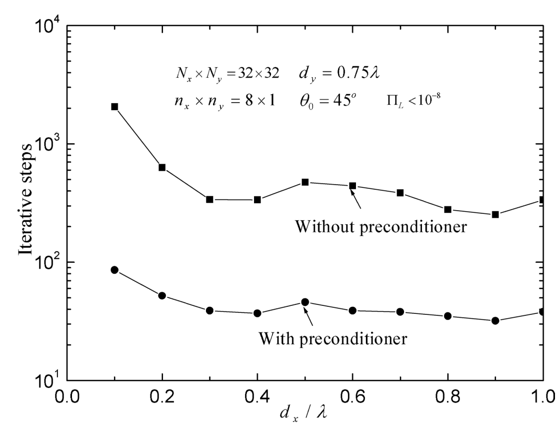

Fig. 8 Comparison of iterative steps with d_x/λ.

ing a Pentium-IV 2.6 GHz PC with 1.5G Byte of memory. Both CPU times per iteration are close to $2\times10^{-5}N\log(N)$ sec and memory requirement is close to $1.1\times10^{-3}N$ MB.

The convergence comparison of without preconditioner and with present preconditioner is shown in Fig. 6. The array has a dimension of $N_x \times N_y = 64\times64$. The high efficiency of the present preconditioner is demonstrated.

Figure 7 gives the convergence of the P-CG-FFT by increasing the unknowns N. The iterative steps increase with the increase of the unknowns in both methods, but the introduction of the present preconditioner achieves a great reduce of the iterative steps. Figure 8 gives the performance of iterative steps with the d_x/λ using two methods. It is clear that iterative steps increase when the dipoles' couplings become stronger in both methods, but the present preconditioner reduces iterative steps greatly.

Finally, a large-scale periodic array antenna with $N_x \times N_y = 256\times256$ dipoles is analyzed. The number of unknowns reaches up to 589,824. The input impedances of 1st

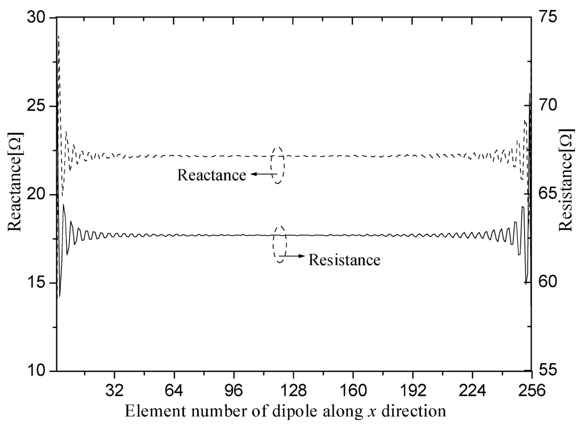

Fig. 9 Input impedance of 1st row ($Row_y = 1$).

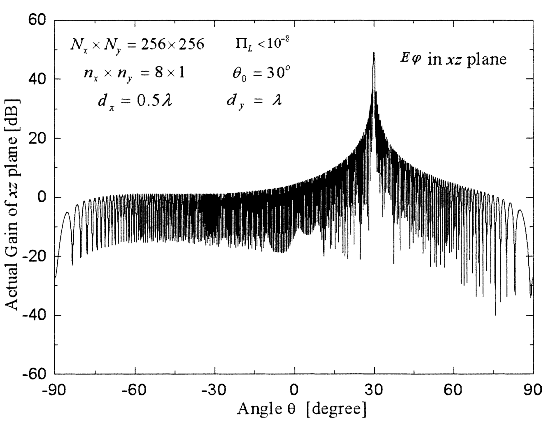

Fig. 10 Actual gain of xz plane.

row dipoles along x direction in the case of $N_y = 1$ is given in Fig. 9, and the actual gain ($Z_{load} = 50\,\Omega$) radiation pattern of dipole array calculated by P-CG-FFT is shown in Fig. 10, where the CPU time is only 6694 seconds with the residual error of 10^{-8}.

4. Conclusions

In this paper, we propose a fast and accurate analysis of large-scale periodic array antennas by CG-FFT combined with a high effective sub-array preconditioner. Three-level Block Toeplitz Matrices and corresponding Circulant Matrix are obtained according to the equivalent sub-block divisions. Conjugate-gradient fast Fourier transform (CG-FFT) has been successfully employed for the analysis of a large-scale dipole array to reduce the computational complexity and computer memory requirement to $O(N\log(N))$ and $O(N)$ respectively. Because all required impedance matrix elements are calculated without approximation, both the near field and far field can be achieved accurately. Furthermore, an equivalent sub-array preconditioner has been proposed to combine with the CG-FFT analysis to effectively reduce the iterative steps and the whole CPU-time. It should be noted that the algorithm of the CG-FFT with the preconditioner is easily parallelized on a parallel computing system. An extremely large scale problem with more than millions of unknowns is expected to be solved by the parallelized algorithm at the next stage.

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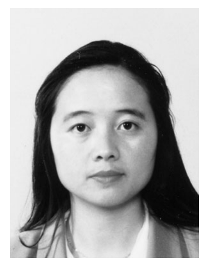

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