Analysis of Load Frequency Control Performance Assessment Criteria

George Gross, *Fellow, IEEE* and Jeong Woo Lee

***Abstract*—This paper presents the development and application of an analytic framework for the formulation and evaluation of control performance criteria in load frequency control (LFC). The framework is constructed so as to explicitly represent the uncertainty in the measured variables in LFC and to use metrics that are meaningful for the structure of the problem. The framework makes effective use of probability and random processes concepts to develop rather general criteria for LFC performance assessment. In fact, the NERC criteria *CPS1* and *CPS2* are special cases of the criteria of the framework. The paper thus provides an analytic rationale for the NERC control performance criteria. Analysis of the *CPS1* and *CPS2* criteria shows that, under conditions that are typically in effect in North American interconnections, the two criteria are redundant. Consequently, there is good analytical basis for not requiring the application of *CPS2* once *CPS1* is satisfied. Numerical results with four interconnections are given to illustrate the analytic results. The framework is a powerful construct that may be used to construct new criteria for LFC performance assessment.**

***Index Terms*—AGC, control performance assessment criteria, IOS, LFC, random processes, unbundled ancillary service.**

I. Introduction

THIS paper presents the development and application of an analytic framework for the formulation and evaluation of control performance criteria in load frequency control (LFC). Such criteria play an increasingly important role in the provision of LFC as an unbundled service in the new open access transmission regimes. These criteria need to be appropriately formulated to be meaningful in the new environment; in particular, they impact considerably the monitoring and metering requirements for LFC [1].

We start with a brief review of the way LFC is performed. In actual power system operations, the load is changing continuously and randomly. As the ability of the generation to track the changing load is limited due to physical/technical considerations, there results an imbalance between the actual and the scheduled generation quantities. This imbalance leads to a frequency error—the difference between the actual and the synchronous frequency. The magnitude of the frequency error is an indication of how well the power system is capable to balance the actual and the scheduled generation. The presence of an actual-scheduled generation imbalance gives rise initially to system frequency excursions in accordance to the sign of the imbalance. Then, the governor responses take effect and act to reduce the magnitude of the actual-scheduled generation imbalance. Within a few seconds, this so-called *primary speed control* [2] serves to *damp out* the frequency excursions and to *stabilize* the frequency at a new value, which is different than the synchronous frequency. The LFC function [2] is then deployed as the *secondary control process* to maintain the frequency error within an acceptable bound. The LFC is performed by the automatic generation control (AGC) by adjusting load reference set points of governors of selected units in the *control area* and then adjusting their outputs [2]–[4]. Each control area measures the actual frequency and the actual net interchange, typically, every 2–4 seconds. These measurements are used to evaluate the frequency and the net interchange errors. The net interchange error is defined as the difference between the net actual and the net scheduled interchange with the connected control areas. The area control error (ACE) [5] is then computed by taking into account the effects of frequency bias; it is the basis for the control signals sent by the control area to the generators participating in AGC.

In an interconnection, there are many control areas, each of which performs its AGC with the objective of maintaining the magnitude of ACE "sufficiently close to 0" using various criteria. In order to maintain the frequency sufficiently close to its synchronous value over the entire interconnection, the coordination of the control areas' actions is required. As each control area shares in the responsibility for LFC, effective means are needed for monitoring and assessing each area's performance of its appropriate share in LFC. This requirement, in turn, brings about the need for meaningful metrics and criteria for LFC performance assessment. The focus of this paper is on the analysis and construction of appropriate metrics and control performance criteria in LFC.

For many decades, the LFC performance of an area was assessed by the widely-adopted ***A1*** and ***A2*** criteria. These criteria, based on engineering judgment, had no analytical basis. The pioneering work in [6], [7] resulted in the adoption of new and more sophisticated criteria. These so-called ***CPS1*** and ***CPS2*** criteria required considerably more measurement and data collection. These new criteria still lacked an analytic basis but brought good engineering and power system operations insight. In 1997, NERC adopted ***CPS1*** and ***CPS2*** in place of the old ***A1*** and ***A2*** criteria.

The objective of this paper is to provide a solid analytic basis for the formulation, analysis and evaluation of LFC performance criteria. We construct an analytic framework that explicitly represents the uncertainty in the measured variables in LFC and that uses metrics that are meaningful for the structure of the problem. The framework makes effective use of probability and random

Manuscript received February 15, 2000. This paper was supported by the Grainger Foundation and the University of Illinois Power Affiliates Program.

The authors are with the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, Urbana, IL 61801.

Publisher Item Identifier S 0885-8950(01)06075-8.

processes concepts to develop rather general criteria for LFC performance assessment. In fact, the NERC criteria ***CPS1*** and ***CPS2*** are special cases of the criteria of the framework. The paper thus provides an analytic rationale for the NERC control performance criteria. Analysis of the ***CPS1*** and ***CPS2*** criteria shows that, under conditions that are typically in effect in North American interconnections, the two criteria are redundant. Consequently, there is good analytical basis for not requiring the application of ***CPS2*** once ***CPS1*** is satisfied. Numerical results with four interconnections are given to illustrate the analytic results. The framework is a powerful construct that may be used to construct new criteria for LFC performance assessment.

The paper has four additional sections. Section II presents the development of the probabilistic framework for the LFC performance assessment. In Section III, the relationship between the NERC ***CPS1*** and ***CPS2*** criteria is analyzed using the proposed framework. Numerical results are given in Section IV. The concluding section evaluates the contributions of the paper.

II. The LFC Performance Criteria Framework

The load at each time instant is a random variable (r.v.) because its value is uncertain at each point of time. The sources of randomness are many; they include the weather, the operating conditions, economic conditions in the area, the work-and-rest pattern of the population and other sundry factors. The randomness in the load results in the randomness in the variables of the power system, such as the interconnection frequency and the flows on the tie lines. We use bold letters to represent r.v.'s in this paper.

A. Formulation of the Framework

Consider a control area i in a multi-area interconnection and some time interval $[t_0, t_f]$. The flows on the tie lines connecting area i to its neighboring areas are measured every $\Delta\tau$seconds. These measurements are used to compute the actual net area i interchange $\boldsymbol{T}_i^a(t_l)$ at each sampling instant $t_l \triangleq t_0 + l \cdot \Delta\tau$, $l = 0, 1, 2, \ldots$ within $[t_0, t_f]$. The interconnection frequency, which we assume to be uniform throughout the interconnection, is also measured every $\Delta\tau$seconds and is denoted by $\boldsymbol{F}^a(t_l)$. We compute the interconnection frequency error $\boldsymbol{F}(t_l) \triangleq \boldsymbol{F}^a(t_l) - F^s$ and the net interchange error $\boldsymbol{T}_i(t_l) \triangleq \boldsymbol{T}_i^a(t_l) - T_i^s(t_l)$, where $F^s[T_i^s(t_l)]$ is the synchronous interconnection frequency [the scheduled net interchange] at t_l. These errors, in turn, are used to evaluate the area i control error $\boldsymbol{ACE}_i(t_l) \triangleq \boldsymbol{T}_i(t_l) - 10B_i \cdot \boldsymbol{F}(t_l)$ at t_l, where B_i is the area i *frequency bias coefficient* which is a negative constant. We adopt the view in [8] and consider $\boldsymbol{ACE}_i(t_l)$ to equal the area actual generation minus the area scheduled generation at t_l. The area scheduled generation is the sum of the area i load, the losses, the scheduled net interchange and the area's share of support for interconnection frequency [8].

The LFC performance assessment is based on the assumption that the power system is in steady state at each instant of $[t_0, t_f]$. Under this assumption, any disturbance in the load simply transitions the power system from its current steady state to some other steady state. Then, $\{\boldsymbol{X}(t_l)\colon\ l = 0, 1, 2, \ldots\}$, where, $\boldsymbol{X}(t_l)$ represents any one of the r.v.'s $\boldsymbol{F}(t_l)$, $\boldsymbol{T}_i(t_l)$ and $\boldsymbol{ACE}_i(t_l)$ at the measurement instant t_l, is a collection of steady state r.v.'s. Moreover, we assume that the probability distribution of each r.v. is independent of time. Since the steady state r.v. at one instant has no effect on the r.v. at any other instant and the distribution of each steady state r.v. does not depend on the particular instant, the collection $\{\boldsymbol{X}(t_l)\colon\ l = 0, 1, 2, \ldots\}$ is a set of independent and identically distributed (i.i.d.) r.v.'s. We denote the time independent mean and variance of each r.v. $\boldsymbol{X}(t_l)$ by $\mu_{\boldsymbol{X}}$ and $\sigma_{\boldsymbol{X}}^2$, respectively.

The fast fluctuations in the load bring about fast fluctuations in the scheduled generation; these cannot be matched by the actual generation due to the generators' much slower response. Thus, fluctuating imbalances between the actual and scheduled generation exist at each instant. Each of the variables denoted by $\boldsymbol{X}(t_l)$ is therefore a function of the actual-scheduled generation imbalance at t_l. The objective of LFC is to drive the possible values of the variable $\boldsymbol{F}(t_l)$ at each t_l to lie within a desired range. However, such a control performance would be overly tight and it would in fact overstress the generators. So, in practice, rather than monitoring $\boldsymbol{F}(t_l)$, we develop a control performance assessment scheme capable of ensuring the effectiveness of the control in some statistically meaningful way. For this purpose, we introduce the smoothed variables that are the averages of the r.v.'s $\boldsymbol{X}(t_l)$ over windows of length W with the windows being nonoverlapping and covering the entire interval $[t_0, t_f]$. In this way, the fast fluctuations in these variables are filtered out. To make the assessment statistically meaningful, we need a large number N of W windows. Corresponding to window n, we define the subinterval $I^n = [t_0 + (n-1) \cdot W, t_0 + n \cdot W)$, $n = 1, 2, \ldots, N$. We associate the measurement time subset $\mathcal{T}^n \triangleq \{t_k\colon\ t_k$ is a measurement point in I^n, $k = 1, 2, \ldots, K\}$ with the subinterval I^n. The selection of the window length W is based on the generator response time and the volatility of the values of $\boldsymbol{X}(t_l)$. In addition, W is selected so that the cardinality K of each $\mathcal{T}^n$ is sufficiently large so as to permit the application of the Central Limit Theorem [9].

Using this structure, we define for each time subset $\mathcal{T}^n$ the *smoothed* r.v.

$$\overline{\boldsymbol{X}}^n \triangleq \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{X}(t_k), \qquad n = 1, 2, \ldots, N. \tag{1}$$

Since $\{\boldsymbol{X}(t_l)\colon\ l = 0, 1, 2, \ldots\}$ is a set of i.i.d. r.v.'s for the collection of elements in $\mathcal{T} \triangleq {}^1\mathcal{T} \cup \mathcal{T}^2 \cup \mathcal{T} \cdots \cup \mathcal{T}^N$, it follows that $\mu_{\overline{\boldsymbol{X}}^n} = \mu_{\boldsymbol{X}}$ and $\sigma^2_{\overline{\boldsymbol{X}}^n} = (1/K)\sigma^2_{\boldsymbol{X}}$. Furthermore, for each $n = 1, 2, \ldots, N$, the Central Limit Theorem establishes that $\overline{\boldsymbol{X}}^n$ is approximately normally distributed with $\mathcal{N}(\mu_{\boldsymbol{X}}, (1/\sqrt{K})\sigma_{\boldsymbol{X}})$. Consequently, the collection of smoothed r.v.'s $\{\overline{\boldsymbol{X}}^n\colon n = 1, 2, \ldots, N\}$ is also a set of i.i.d. r.v.'s. Since each r.v. $\overline{\boldsymbol{X}}^n$ is approximately normally distributed, its distribution is fully known once the estimated mean and variance are determined.

The use of smoothed variables involves less volatility and is more practical for the LFC performance assessment. We use therefore the r.v.'s $\overline{\boldsymbol{X}}^n$, $n = 1, 2, \ldots, N$, and consider no further the r.v.'s $\boldsymbol{X}(t_l)$ for assessing LFC performance. Accordingly, the objective of LFC is modified to maintain for each

n, the values of the *smoothed variable* $\overline{\boldsymbol{F}}^n$ within a desired range. For the probabilistic structure, a natural metric is the mean square value $E\{[\overline{\boldsymbol{F}}^n]^2\}$ to measure how closely the values of $\overline{\boldsymbol{F}}^n$ are bounded within the desired range. We, thus, propose $E\{[\overline{\boldsymbol{F}}^n]^2\}$ as the interconnection LFC performance measure. We specify its bound to be ε^2 and formulate the interconnection LFC performance criterion $\boldsymbol{C}$ as

$$\boldsymbol{C}: \quad E\left\{\left[\overline{\boldsymbol{F}}^n\right]^2\right\} \le \varepsilon^2 \qquad n = 1, 2, \ldots, N. \tag{2}$$

Such a metric, however, is not practical for several reasons. One is that criterion $\boldsymbol{C}$ does not consider the contribution of the various control areas to the interconnection performance. Another is that the criterion fails to use much of the information in the monitored measurements. In actual interconnected power system operations, each control area shares in the responsibility for LFC. Since some areas may have undesirable control performance, we need to construct for all the areas specific LFC performance metrics and criteria to ensure that each area is performing its appropriate share in LFC.

B. Area Control Performance Criteria

As the first step in monitoring each area's performance, we need to know how each area responds to the interconnection frequency error based on available measurements. To assess how the behavior of two r.v.'s is related, we use their correlation as an indicator. A positive correlation indicates that the two r.v.'s *move* in similar direction, while a negative correlation indicates the opposite [10]. We propose to monitor the correlation of $\overline{\boldsymbol{F}}^n$ and $\overline{\boldsymbol{ACE}}_i^n$. The frequency error $\overline{\boldsymbol{F}}^n$ is a function of the interconnection's actual-scheduled generation imbalance [2] and $\overline{\boldsymbol{ACE}}_i^n$ is the actual-scheduled generation imbalance of area i. Consider the case when $\overline{\boldsymbol{F}}^n$ has a positive value. In such a case, there is overgeneration throughout the interconnection, i.e., total generation exceeds the total load in the interconnection. If $\overline{\boldsymbol{ACE}}_i^n$ has a positive value, this area has more actual than scheduled generation and, consequently, it does not help the interconnection to reduce the interconnection overgeneration and restore the frequency to the synchronous value. Then, $\overline{\boldsymbol{ACE}}_i^n$ has a positive correlation with $\overline{\boldsymbol{F}}^n$, i.e., $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\} > 0$, and area is control performance does not help to restore the frequency to its scheduled value. On the other hand, if $\overline{\boldsymbol{ACE}}_i^n$ has a negative value, this area has lower actual than scheduled generation. Then, $\overline{\boldsymbol{ACE}}_i^n$ has a negative correlation, i.e., $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\} < 0$, and this situation helps the interconnection to reduce the overgeneration and restore the frequency to the synchronous value. Then, positive [negative] $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\}$ indicates the area i's *undesirable [desirable]* control performance. A symmetric situation occurs for the case of $\overline{\boldsymbol{F}}^n < 0$. We propose to use the correlation $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\}$ for each n as a metric of each area's control performance. To ensure that the positive correlation is not excessive, we need to impose an upper bound on $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\}$, whereas we need not care about the negative correlation since it indicates a *desirable* performance.

A further consideration is that too wide of an excursion of the ACE from zero is not desirable in every single area. We again use a probabilistic measure to assess how well each control area can contain the value of ACE within a specified range. Given a specified tolerance γ_i for area i, let Ξ be the event that $\overline{\boldsymbol{ACE}}_i^n$ has an absolute value less than or equal to γ_i. Then, $P\{|\overline{\boldsymbol{ACE}}_i^n| \le \gamma_i\}$ is the confidence level that the event Ξ happens [9]. The closer the confidence level is to 1, the more likely is the event Ξ to occur. We propose $P\{|\overline{\boldsymbol{ACE}}_i^n| \le \gamma_i\}$ as an additional metric of each area's control performance. We want this value to exceed a specified confidence level.

Using the proposed metrics, we construct for each area's control performance criteria $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$, where,

$$\boldsymbol{C}_1: \quad E\left\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\right\} \le -10B_i \cdot \varepsilon^2, \qquad n = 1, 2, \ldots, N \tag{3}$$

$$\boldsymbol{C}_2: \quad P\left\{\left|\overline{\boldsymbol{ACE}}_i^n\right| \le \gamma_i\right\} \ge 0.9, \qquad n = 1, 2, \ldots, N. \tag{4}$$

We show that the criterion $\boldsymbol{C}_1$ is a sufficient condition for the criterion $\boldsymbol{C}$. In addition, we prove that under assumptions that generally hold for windows whose length $W \ge 10$ minutes, the criterion $\boldsymbol{C}_2$ is equivalent to the criterion $\boldsymbol{C}_1$. The proofs are given in the Appendix. We used the development in [6] to motivate the formulation of the criteria in equations (3) and (4).

Consider the discrete time random process (d.r.p.) $\{\overline{\boldsymbol{X}}^n: n = 1, 2, \ldots, N\}$ obtained for the time set $\mathcal{T}$. This is a set of i.i.d. r.v.'s. For large N, the d.r.p. $\{\overline{\boldsymbol{X}}^n: n = 1, 2, \ldots, N\}$ is ergodic [9]. The mean, the variance and the mean square value of $\overline{\boldsymbol{X}}^n$ may be estimated from any sample path of the process [9]. For example, the mean $E\{\overline{\boldsymbol{X}}^n\}$ of $\overline{\boldsymbol{X}}^n$ may be estimated by $m \triangleq (1/N)\sum_{n=1}^{N} \overline{x}^n$, where $\overline{x}^n$ is the observed value of $\overline{\boldsymbol{X}}^n$ from any sample path [9]. This characterization of $\overline{\boldsymbol{X}}^n$ consequently applies to the r.v.'s $\overline{\boldsymbol{F}}^n$, $\overline{\boldsymbol{T}}_i^n$, $\overline{\boldsymbol{ACE}}_i^n$. We make use of this important characteristic of ergodic processes in the application of the proposed criteria.

The framework we constructed in this section is based on the application of basic notions of r.v.'s and random processes (r.p.'s) to LFC. The judicious use of the measurement set and smoothing allows the specification of implementable criteria to assess the performance of each area in discharging its LFC responsibility. The proposed criteria make use of natural probability measures such as correlation and confidence level of r.v.'s and take advantage of the ergodic d.r.p. characteristics. We use this framework to analyze the control performance assessment criteria in the section below.

III. Relationship of the Proposed and NERC Criteria

The proposed criteria $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are formulated to be rather general criteria for control performance assessment. In this section we show that, in particular, we can derive from them the NERC ***CPS1*** and ***CPS2*** criteria [5] as special cases. In addition, we derive an important new result showing that under certain conditions that generally hold, the satisfaction of ***CPS1*** implies that ***CPS2*** is also satisfied. As such, ***CPS2*** is a redundant criterion. Once ***CPS1*** is satisfied, it becomes unnecessary to check CPS2.

The starting point of our analysis is the framework that was constructed in Section II. Note that the elements of the d.r.p.'s $\{\overline{\boldsymbol{F}}^n: n = 1, 2, \ldots, N\}$ and $\{\overline{\boldsymbol{ACE}}_i^n: n = 1, 2, \ldots, N\}$

are the i.i.d. r.v.'s $\overline{\boldsymbol{F}}^n$ and $\overline{\boldsymbol{ACE}}_i^n$, respectively. Moreover, for $n \neq m$, $\overline{\boldsymbol{F}}^n$ and $\overline{\boldsymbol{ACE}}_i^m$ are statistically independent. Consider the r.v.'s $\boldsymbol{Z}_i^n \triangleq \overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n$, $n = 1, 2, \ldots, N$. It follows that $\{\boldsymbol{Z}_i^n: n = 1, 2, \ldots, N\}$ is a set of i.i.d. r.v.'s and consequently is an ergodic d.r.p. For each n, $E\{\boldsymbol{Z}_i^n\}$ may be estimated from any sample path of the d.r.p. [9]. Let $\{z_i^n: n = 1, 2, \ldots, N\}$ be an arbitrary sample path. For sufficiently large N, $E\{\boldsymbol{Z}_i^n\}$ may be estimated for each n by $(1/N)\sum_{n=1}^N z_i^n$ [9]. Then, criterion $\boldsymbol{C}_1$ is satisfied if and only if

$$\frac{1}{N}\sum_{n=1}^{N} z_i^n \leq -10B_i \cdot \varepsilon^2. \tag{5}$$

Let the window length W be 1 minute and the interval length $I \triangleq t_f - t_0$ be 1 year, then the number N of 1-minute windows within the one-year interval is sufficiently large. Let $\varepsilon = \varepsilon_1$, the specified tolerance in the NERC ***CPS1*** [5]. The expression in equation (5), then, is the same as the NERC ***CPS1*** criterion. We adopt the notation $\boldsymbol{C}_q|_{W,\,I,\,\varepsilon}$ for $q = 1, 2$, where W is expressed in minutes, I is given in years and ε is the tolerance in mHz. We use this notation also for $\boldsymbol{C}$. Then, ***CPS1*** may be expressed as $\boldsymbol{C}_1|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}$.

Next, we consider the criterion $\boldsymbol{C}_2$ in equation (4). We define the r.v.

$$\boldsymbol{A}_i^n \triangleq \begin{cases} 1 & \text{if } \left|\overline{\boldsymbol{ACE}}_i^n\right| \leq \gamma_i \\ 0 & \text{otherwise} \end{cases}, \qquad n = 1, 2, \ldots, N \tag{6}$$

for a given value of γ_i. Then, the mean of $\boldsymbol{A}_i^n$ is $E\{\boldsymbol{A}_i^n\} = P\{|\overline{\boldsymbol{ACE}}_i^n| \leq \gamma_i\}$ and the $\boldsymbol{C}_2$ criterion in equation (4) may be restated as

$$E\{\boldsymbol{A}_i^n\} \geq 0.9, \qquad n = 1, 2, \ldots, N. \tag{7}$$

Since the r.v.'s $\overline{\boldsymbol{ACE}}_i^n$, $n = 1, 2, \ldots, N$ are i.i.d., the r.v.'s $\boldsymbol{A}_i^n$, $n = 1, 2, \ldots, N$ are also i.i.d. and the d.r.p. $\{\boldsymbol{A}_i^n: n = 1, 2, \ldots, N\}$ is therefore ergodic. Let $\overline{ace}_i^n$ and a_i^n be values of the r.v.'s $\overline{\boldsymbol{ACE}}_i^n$ and $\boldsymbol{A}_i^n$, respectively. Then,

$$a_i^n \triangleq \begin{cases} 1 & \text{if } |\overline{ace}_i^n| \leq \gamma_i \\ 0 & \text{otherwise} \end{cases}, \qquad n = 1, 2, \ldots, N. \tag{8}$$

The mean $E\{\boldsymbol{A}_i^n\}$ is estimated by $(1/N)\sum_{n=1}^N a_i^n$. We may interpret this estimate to be the proportion of the N windows covering the interval of length I which have the absolute value of ACE smaller than or equal to γ_i. Then, criterion $\boldsymbol{C}_2$ of equation (4) is satisfied if and only if

$$\frac{1}{N}\sum_{n=1}^{N} a_i^n \geq 0.9. \tag{9}$$

Let $W = 10$, $I = 1$ and $\varepsilon = \varepsilon_{10}$, the specified tolerance value in the NERC ***CPS2*** [5]. Then, $\boldsymbol{C}_2|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}$ in equation (9) is the same as ***CPS2***.

We next consider the relationship between $\boldsymbol{C}_1|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}$ and $\boldsymbol{C}_2|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}$. The mean of $\overline{\boldsymbol{F}}^n$ for each n is independent of the window length W since

$$\mu_{\overline{\boldsymbol{F}}^n}|_W = \mu_{\boldsymbol{F}}, \qquad n = 1, 2, \ldots, N. \tag{10}$$

Furthermore, for the variance, we have

$$\sigma^2_{\overline{\boldsymbol{F}}^n}|_W = \frac{1}{K|_W}\,\sigma^2_{\boldsymbol{F}}, \qquad n = 1, 2, \ldots, N. \tag{11}$$

Here, $K|_W$ is the cardinality of the window W. We use equations (10) and (11) to determine

$$E\left\{\left[\overline{\boldsymbol{F}}^n\right]^2|_W\right\} = \sigma^2_{\overline{\boldsymbol{F}}^n}|_W + \mu^2_{\overline{\boldsymbol{F}}^n}|_W = \frac{1}{K|_W}\,\sigma^2_{\boldsymbol{F}} + \mu^2_{\boldsymbol{F}} = \mu_{(\overline{\boldsymbol{F}})^2}|_W. \tag{12}$$

Since $t_l = t_0 + l \cdot \Delta\tau$, the grid is uniform so that $K|_{W=10} = 10K|_{W=1}$ and it follows that

$$\mu_{(\overline{\boldsymbol{F}})^2}|_{W=10} = \frac{1}{10}\,\mu_{(\overline{\boldsymbol{F}})^2}|_{W=1} + \frac{9}{10}\,\mu^2_{\boldsymbol{F}}. \tag{13}$$

Consider the criterion $\boldsymbol{C}$ for two distinct windows W and tolerances ε:

$$\boldsymbol{C}|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}: \quad \mu_{(\overline{\boldsymbol{F}})^2}|_{W=1,\,I=1} \leq (\varepsilon_1)^2, \tag{14}$$

$$\boldsymbol{C}|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}: \quad \mu_{(\overline{\boldsymbol{F}})^2}|_{W=10,\,I=1} \leq (\varepsilon_{10})^2. \tag{15}$$

Since the criterion $\boldsymbol{C}_1$ is a sufficient condition for the criterion $\boldsymbol{C}$ and the ***CPS1*** is equivalent to the criterion $\boldsymbol{C}_1|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}$, it follows that the ***CPS1*** is a sufficient condition for the criterion $\boldsymbol{C}|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}$. Using equation (13) and (15), $\boldsymbol{C}|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}$ holds if and only if

$$\mu_{(\overline{\boldsymbol{F}})^2}|_{W=1,\,I=1} \leq 10(\varepsilon_{10})^2 - 9\mu^2_{\boldsymbol{F}}. \tag{16}$$

We introduce the following condition

$$|\mu_{\boldsymbol{F}}| \leq \mu_{\boldsymbol{F}\,limit}, \tag{17}$$

where,

$$\mu_{\boldsymbol{F}\,limit} \triangleq \tfrac{1}{3}\sqrt{10(\varepsilon_{10})^2 - (\varepsilon_1)^2}. \tag{18}$$

It follows that under this condition, $(\varepsilon_1)^2 \leq 10(\varepsilon_{10})^2 - 9\mu^2_{\boldsymbol{F}}$. If ***CPS1*** is satisfied, $\mu_{(\overline{\boldsymbol{F}})^2}|_{W=1,\,I=1} \leq (\varepsilon_1)^2 \leq 10(\varepsilon_{10})^2 - 9\mu^2_{\boldsymbol{F}}$, where, the second inequality is due to the condition in equation (17). Consequently, the relationship of equation (16) holds so that $\boldsymbol{C}|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}$ is also satisfied. Thus, under the condition in equation (17), if each area satisfies ***CPS1***, then $\boldsymbol{C}|_{W=1,\,I=1,\,\varepsilon=\varepsilon_1}$ and $\boldsymbol{C}|_{W=10,\,I=1,\,\varepsilon=\varepsilon_{10}}$ are also satisfied.

The ***CPS2*** criterion provides no additional assurance or insight in terms of control performance. Then, under the condition in equation (17) there is no need to monitor two separate criteria ***CPS1*** and ***CPS2*** and the monitoring of ***CPS1*** suffices.

IV. Numerical Studies

We carried out a number of numerical studies to ascertain whether the condition in equation (17) for redundancy of ***CPS2*** can be determined. We performed the studies on a number of NERC interconnections. We provide a representative sample here for the following four interconnections: Eastern Interconnection, WSCC, ERCOT and Hydro-Quebec. Using the values of the NERC-specified ε_1 and ε_{10} for each interconnection [5], we calculate the corresponding $\mu_{\boldsymbol{F}\,limit}$ values. We estimate each interconnection's expected value of frequency error $\mu_{\boldsymbol{F}}$

TABLE I
DATA FOR THE NUMERICAL STUDIES

Parameter	Eastern Interconnection	WSCC	ERCOT	Hydro Quebec
ε_1 mHz	18.0	22.8	20.0	21.2
ε_{10} mHz	5.7	7.3	7.3	12.5
$\mu_{\boldsymbol{F}\,limit}$ mHz	0.3	1.2	3.8	11.1
$\lvert\mu_{\boldsymbol{F}}\rvert$ mHz	0.05	0.3	0.3	0.5

using the frequency data obtained from sufficiently large number of time instants in each interconnection. The selected periods for the inspection are one year from May 1998 to April 1999 in the Eastern Interconnection [5], the year 1998 in WSCC, the month of January 1998 in ERCOT and the month of August 1997 in Hydro-Quebec [11]. We developed the results in the previous section under the assumption of $I = 1$ year. However, given that the frequency errors are sampled every 1–3 seconds in these interconnections, one month is sufficiently long to provide a statistically meaningful data set for the evaluation of the $\mu_{\boldsymbol{F}}$ estimate. Due to the ergodic nature of the r.p.s in the control performance criteria, the single month data can be tested effectively to ascertain the redundancy of the ***CPS2*** criterion. Table I gives the values of ε_1 and ε_{10} specified by the NERC [5], the calculated values of $\mu_{\boldsymbol{F}\,limit}$ and the estimated values of $|\mu_{\boldsymbol{F}}|$ for each interconnection.

We can see that the condition in equation (17) is satisfied by each of the four interconnections for the selected periods. In these cases, ***CPS2*** is a redundant criterion and the control areas do not need to monitor whether ***CPS2*** is satisfied. As such, the areas accomplish their objective for the LFC once ***CPS1*** is satisfied.

V. CONCLUSION

In this paper, we constructed a random process based LFC performance assessment framework. This framework is effective in dealing with the random characteristics of the variables of interest in LFC. The framework provides a solid analytic basis for the formulation, analysis and evaluation of LFC performance criteria.

We used the smoothed variables over non!overlapping windows to get more insight on the control performance assessment issues. The use of statistically meaningful metrics for the area control performance assessment led to the formulation of rather general performance criteria $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. The ergodic nature of the random processes permits the evaluation of the proposed metrics using chronological data. The proposed criteria are general and include the criteria ***CPS1*** and ***CPS2*** as special cases. Moreover, the application of the framework shows that the satisfaction of ***CPS1*** implies that ***CPS2*** is also satisfied under a certain condition. Consequently, ***CPS2*** criterion provides no additional insight in terms of control performance and the monitoring of ***CPS1*** suffices.

The framework brings considerable new analytic insights into the LFC performance assessment problem. It can be used for the formulation of new control performance criteria and for the analysis of the measurement and monitoring requirements for LFC. This is particularly important as LFC is becoming an unbundled ancillary service in the new open access transmission regimes.

APPENDIX

We prove the fact that the satisfaction of the area control performance criterion $\boldsymbol{C}_1$ implies the satisfaction of the criterion $\boldsymbol{C}$. In addition, we show the criterion $\boldsymbol{C}_2$ to be equivalent to $\boldsymbol{C}_1$ under assumptions that generally hold for window length $W \geq 10$.

We start with the definition of the average ACE variables for each W window:

$$\overline{\boldsymbol{ACE}}_i^n \triangleq \overline{\boldsymbol{T}}_i^n - 10B_i \cdot \overline{\boldsymbol{F}}^n, \qquad n = 1, 2, \ldots, N. \quad \text{(A.1)}$$

Let α be the number of control areas in the interconnection. Since the sum of the net interchange errors over the interconnection is zero,

$$\sum_{i=1}^{\alpha} \overline{\boldsymbol{ACE}}_i^n = -10 \sum_{i=1}^{\alpha} B_i \cdot \overline{\boldsymbol{F}}^n = -10B_T \cdot \overline{\boldsymbol{F}}^n, \quad \text{(A.2)}$$

with $B_T = \sum_{i=1}^{\alpha} B_i$.

Suppose each area i satisfies the criterion $\boldsymbol{C}_1$ of equation (3), i.e., $E\{\overline{\boldsymbol{F}}^n \cdot \overline{\boldsymbol{ACE}}_i^n\} \leq -10B_i \cdot \varepsilon^2$, $i = 1, \ldots, \alpha$. Then, for all the α areas

$$\begin{aligned} E\left\{\overline{\boldsymbol{F}}^n \cdot \sum_{i=1}^{\alpha} \overline{\boldsymbol{ACE}}_i^n\right\} &= E\left\{\overline{\boldsymbol{F}}^n \cdot \left[-10B_T \cdot \overline{\boldsymbol{F}}^n\right]\right\} \\ &= -10B_T \cdot E\left\{\left[\overline{\boldsymbol{F}}^n\right]^2\right\} \\ &\leq \varepsilon^2 \cdot \sum_{i=1}^{\alpha}(-10B_i) \\ &= -10B_T \cdot \varepsilon^2. \end{aligned} \quad \text{(A.3)}$$

It follows that $E\{[\overline{\boldsymbol{F}}^n]^2\} \leq \varepsilon^2$ and so the $\boldsymbol{C}$ criterion is satisfied. Consequently, the $\boldsymbol{C}_1$ criterion is a sufficient condition for the $\boldsymbol{C}$ criterion.

Next, we show that the $\boldsymbol{C}_2$ criterion is equivalent to the $\boldsymbol{C}_1$ criterion under a set of assumptions. We introduce the following two assumptions for window length $W \geq 10$:

i) The r.v.'s $\overline{\boldsymbol{ACE}}_i^n$ and $\overline{\boldsymbol{ACE}}_j^n$ of two different control areas $i \neq j$ are independent; and,
ii) $E\{\overline{\boldsymbol{ACE}}_i^n\} = 0$, $n = 1, 2, \ldots, N$.

These assumptions are reasonable for the interconnections of interest. In fact, analysis of actual data shows that for the control areas in North America, for windows $W \geq 10$, $\overline{\boldsymbol{ACE}}_i^n$ and $\overline{\boldsymbol{ACE}}_j^n$ of two different control areas $i \neq j$ are independent r.v.'s [6, pp. 6–7]. It is also shown in [6] that $E\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{ACE}}_i^m\}$, $n \neq m$, is sufficiently small for window length $W \geq$

10 so as to be negligible. Consequently, for such window length W the assumption

$$E\left\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{ACE}}_i^m\right\} = 0, \qquad n \neq m, \tag{A.4}$$

introduces no loss of generality. Let $\boldsymbol{M} \triangleq (1/N)\sum_{n=1}^{N} \overline{\boldsymbol{ACE}}_i^n$. Then,

$$\begin{aligned} E\{\boldsymbol{M}^2\} &= \frac{1}{N^2} E\left\{\left[\sum_{n=1}^{N} \overline{\boldsymbol{ACE}}_i^n\right]^2\right\} \\ &= \frac{1}{N^2}\sum_{n=1}^{N} E\left\{\left[\overline{\boldsymbol{ACE}}_i^n\right]^2\right\} \\ &\quad + \frac{1}{N^2}\sum_{n=1}^{N}\sum_{\substack{m=1\\ m\neq n}}^{N} E\left\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{ACE}}_i^m\right\} \\ &= \frac{1}{N} E\left\{\left[\overline{\boldsymbol{ACE}}_i^n\right]^2\right\}. \end{aligned} \tag{A.5}$$

Here we used equation (A.4) and the i.i.d. property of $\overline{\boldsymbol{ACE}}_i^n$. We also determine the mean of $\boldsymbol{M}$ as

$$E\{\boldsymbol{M}\} = \frac{1}{N}\sum_{n=1}^{N} E\left\{\overline{\boldsymbol{ACE}}_i^n\right\} = E\left\{\overline{\boldsymbol{ACE}}_i^n\right\}. \tag{A.6}$$

Now, as N becomes sufficiently large, $E\{\boldsymbol{M}^2\}$ in equation (A.5) tends to zero. Since var$\{\boldsymbol{M}\}$ and $[E\{\boldsymbol{M}\}]^2$ are positive and $E\{\boldsymbol{M}^2\} = \text{var}\{\boldsymbol{M}\} + [E\{\boldsymbol{M}\}]^2$, the fact that $E\{\boldsymbol{M}^2\}$ tends to zero implies that necessarily both var$\{\boldsymbol{M}\}$ and $[E\{\boldsymbol{M}\}]^2$ tend to zero. It follows from equation (A.6) that $E\{\overline{\boldsymbol{ACE}}_i^n\}$ also tends to zero. Therefore, the introduction of assumption ii) does not lead to any loss of generality.

We first rewrite the $\boldsymbol{C}_2$ criterion as

$$P\left\{\left|\overline{\boldsymbol{ACE}}_i^n\right| \leq 1.65\varepsilon \cdot \sqrt{(-10B_i)\cdot(-10B_T)}\right\} \geq 0.9, \tag{A.7}$$

where, we substitute $\gamma_i = 1.65\varepsilon \cdot \sqrt{(-10B_i)\cdot(-10B_T)}$ in equation (4). Let

$$\boldsymbol{Y}_i^n \triangleq \overline{\boldsymbol{ACE}}_i^n / \sigma_{\overline{\boldsymbol{ACE}}_i^n}, \qquad n = 1, 2, \ldots, N. \tag{A.8}$$

Then, $\boldsymbol{Y}_i^n$, $n = 1, 2, \ldots, N$ are i.i.d. r.v.'s with standard normal distribution, i.e., $\boldsymbol{Y}_i^n \sim \mathcal{N}(0, 1)$ [8]. We rewrite relationship (A.7) in terms of $\boldsymbol{Y}_i^n$ as

$$P\left\{|\boldsymbol{Y}_i^n| \leq 1.65\,\frac{\varepsilon \cdot \sqrt{(-10B_i)\cdot(-10B_T)}}{\sigma_{\overline{\boldsymbol{ACE}}_i^n}}\right\} \geq 0.9. \tag{A.9}$$

Since $P\{|\boldsymbol{Y}_i^n| \leq 1.65\} = 0.9$ for the standard normal distribution [8], the relationship (A.9) holds if and only if

$$\frac{\varepsilon \cdot \sqrt{(-10B_i)\cdot(-10B_T)}}{\sigma_{\overline{\boldsymbol{ACE}}_i^n}} \geq 1, \tag{A.10}$$

or, equivalently if and only if

$$\sigma^2_{\overline{\boldsymbol{ACE}}_i^n} \leq \varepsilon^2 \cdot (-10B_i)\cdot(-10B_T). \tag{A.11}$$

It follows from assumption ii) that $\sigma^2_{\overline{\boldsymbol{ACE}}_i^n} = E\{[\overline{\boldsymbol{ACE}}_i^n]^2\}$ so that the relationship (A.9) holds if and only if

$$E\left\{\left[\overline{\boldsymbol{ACE}}_i^n\right]^2\right\} \leq \varepsilon^2 \cdot (-10B_i)\cdot(-10B_T). \tag{A.12}$$

Since the assumptions i) and ii) imply that $E\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{ACE}}_j^n\} = E\{\overline{\boldsymbol{ACE}}_i^n\}\cdot E\{\overline{\boldsymbol{ACE}}_j^n\} = 0$ for $i \neq j$, it follows that

$$\begin{aligned} E\left\{\left[\overline{\boldsymbol{ACE}}_i^n\right]^2\right\} &= E\left\{\overline{\boldsymbol{ACE}}_i^n \cdot \sum_{j=1}^{\alpha} \overline{\boldsymbol{ACE}}_j^n\right\} \\ &= -10B_T \cdot E\left\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{F}}^n\right\} \end{aligned}$$

using equation (A.2) to obtain the last equality. Thus, the relationship (A.12) holds if and only if

$$E\left\{\overline{\boldsymbol{ACE}}_i^n \cdot \overline{\boldsymbol{F}}^n\right\} \leq -10B_i \cdot \varepsilon^2, \tag{A.13}$$

which is the same as the $\boldsymbol{C}_1$ criterion.

Consequently, the $\boldsymbol{C}_2$ criterion is equivalent to the $\boldsymbol{C}_1$ criterion under the two assumptions introduced above. Since the $\boldsymbol{C}_1$ criterion is a sufficient condition for the $\boldsymbol{C}$ criterion as shown above, the $\boldsymbol{C}_2$ criterion is also a sufficient condition for the $\boldsymbol{C}$ criterion under these two assumptions.

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George Gross (F'88) is Professor of Electrical and Computer Engineering and Professor, Institute of Government and Public Affairs, at the University of Illinois at Urbana-Champaign. His current research and teaching activities are in the areas of power system analysis, planning, economics and operations and utility regulatory policy and industry restructuring. His undergraduate work was completed at McGill University, and he earned his graduate degrees from the University of California, Berkeley. He was previously employed by Pacific Gas and Electric Company in various technical, policy and management positions.

Jeong Woo Lee received the B.S. and M.S. degrees in electrical engineering from Seoul National University in 1994 and 1996, respectively. He is currently working toward the Ph.D. degree in University of Illinois at Urbana-Champaign.