Analysis of Log Periodic Dipole Array Antennas for Site Validation and Radiated Emissions Testing

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Abstract: Log periodic dipole array (LPDA) antennas are one of the most widely used antenna types for normalized site attenuation (NSA) and radiated emission (RE) testing. A thorough understanding of the possible error sources associated with the calibration and application of LPDAs for EMC tests over a ground plane is essential for evaluating and minimizing measurement uncertainties. Systematic errors are present in the standard site calibration and NSA measurement, which are independent of random effects and equipment uncertainties. In an effort to determine the effects on NSA and RE testing, these errors are examined using a numerical moment-method model and measurements. Error contributions from mutual coupling between antenna and image, active phase center variation, radiation pattern, and cross-polarization effect are investigated.

INTRODUCTION

The standard site method is commonly used for calibrating LPDAs to obtain the antenna factor (AF) [1]. This method involves height scanning of the receive antenna above a highly conducting ground plane. A previous study [2] showed the influence of a conducting ground plane on the AF of a biconical antenna. There it was shown that biconical antenna factors are influenced by the ground plane due to mutual coupling of the antennas with their ground-plane images. The effect was shown to be larger at lower frequencies and for horizontally polarized antennas. LPDA antennas for EMC emissions measurements are normally used at frequencies above 200 MHz. For antennas scanned between 1 m and 4 m in height, the image-coupling effect of the LPDAs is shown in the present work to be less pronounced at these higher frequencies, for both polarizations.

Additional issues unique to the LPDA are considered in this paper. The active phase center of an LPDA moves versus frequency [3, 4], which means the distance between the receive and transmit antennas is not constant. In a numerical study, the relative phase information can be obtained using several methods [6, 7]. Possible compensation schemes for phase center variation in the theoretical NSA and calibration procedure are presented below. In RE testing, it is not possible to completely correct for the phase center error, unless the radiation pattern of the equipment under test (EUT) is known. In most cases, the radiation property of an EUT is unknown, and this error must be treated as a contribution to the measurement uncertainty.

The phase center correction is more critical for smaller separation distances between the transmit and receive antenna, since the positioning error is a larger fraction of the total distance. Results in this paper show that in free-space, a correction for phase center position can result in an accurate free-space antenna factor for a separation distance as small as 2 m. This proves that the phase center positioning error is the dominant factor for two antennas separated by a small distance in free-space.

A logical step is to apply the phase center correction in the standard site method for antennas used above a conducting ground plane. However, results below show that the phase center corrections alone do not provide a free-space AF for calibrations performed above a conducting ground plane. This is because common EMC LPDAs have gains of 4-8 dBi, while the standard site method calibration and NSA measurements assume a point-dipole antenna pattern [1]. The numerical results show distinct differences between the LPDA and dipole antenna patterns. The dipole pattern assumption can lead to errors that can be dominant for antennas used above a ground plane.

Free-space AF was shown to be a good approximation for the different geometry-specific AFs of a biconical antenna [2], because it provides an approximate value which is close to the average over a height scan. In contrast, image coupling is not the dominant error with LPDAs, as it was for biconicals. Errors produced by pattern and phase center do not give AFs that fluctuate about the free-space value. This raises the question of which antenna factor is appropriate and under what conditions for RE or NSA testing.

Most LPDAs have opposing dipole elements on two physically-separated tubular bars or booms. The polarization of the field tends to be tilted by the over/under design of the dipoles, which induces a cross-polarized field. Over a conducting ground, the horizontally polarized field and vertically polarized field are not reflected the same. This generates a polarization-dependent uncertainty value in NSA measurements when using free-space AFs.

All of these effects in the calibration of LPDAs create systematic errors and differences due to the simplistic assumptions made in the mathematical model. These errors are independent of equipment uncertainties and other random effects. In this paper the moment method (NEC2 [5]) is employed to simulate the complete NSA and standard site measurement procedure over a conducting ground plane for specific geometries (transmit antenna height, separation distance, and receive antenna height scanning range). It is shown that the systematic errors can be dominant for certain geometries. Better theoretical NSA model and/or correction factors are necessary if lower measurement uncertainties are desired for NSA measurements.

NUMERICAL MODEL OF AN LPDA

The LPDA antenna analyzed in this paper is approximately 75 cm wide and 76 cm long, with 16 pairs of dipole elements. The specified frequency range is from 200 MHz to 1 GHz. To validate the numerical model, calculated results are compared with measurements. The numerical models simulate a standard site method. In the model, the transmit antenna and receive antenna are identical pairs located above an infinite perfect-electrically-conducting (PEC) ground plane. The receive antenna is scanned from 1 to 4 m in height with a step size of 0.05 m. Numerical results for horizontal and vertical

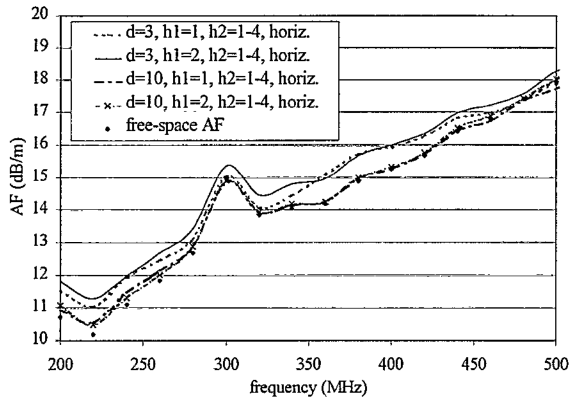

Figure 1. Modeled horizontal AFs using the standard site method.

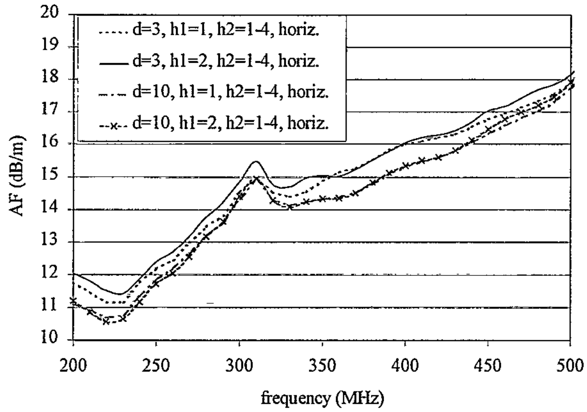

Figure 2. Measured horizontal AFs using the standard site method.

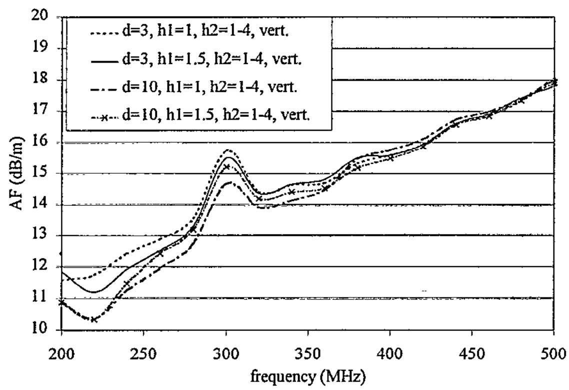

Figure 3. Modeled vertical AFs using the standard site method

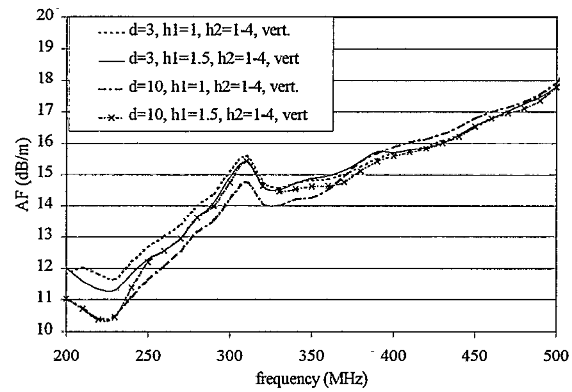

Figure 4. Measured vertical AFs using the standard site method

polarization are shown in Figures 1 and 3, respectively. Figures 2 and 4 show the measured AF for the same geometries. Good correlation is achieved between the modeled and measured data for frequencies up to 500 MHz. At higher frequencies, it becomes prohibitive to simulate the physical model due to the growth in computation time and resolution required.

LPDA COUPLING WITH THE GROUND PLANE

To show the AF variation with height, it is necessary to generate a plane wave to illuminate the receive antenna. This is done by placing a short dipole antenna a large distance (100 m) away from the receive LPDA. The transmit dipole is chosen to be 2 cm long from tip to tip. It was shown in [2] that for both horizontal and vertical polarization at large distances, the fields above a ground plane at the receiving antenna positions (at 1 to 4 m heights) are uniform. Since NEC2 gives the empty E-field values at these receiving positions, AF is simply the direct solution of

$$AF = E / V , \quad (1)$$

where V is the induced voltage across a 50 ohm load connected to the antenna feed. Figure 5 shows less than a 0.3 dB variation in AF versus height for a horizontally-polarized LPDA antenna. For a vertical LPDA, AF was found to vary by less than 0.2 dB versus height. These results prove that coupling of the antenna with its image as a function of height causes minimal variations in the LPDA AF.

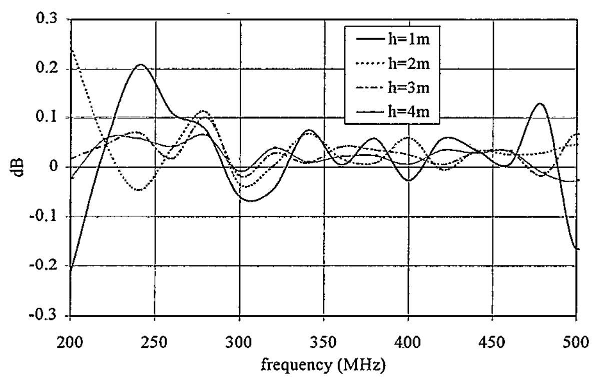

Figure 5. AF variation versus height for a horizontal LPDA.

PHASE CENTER VARIATION WITH FREQUENCY

Antenna phase center (*PC*) is defined as the location from which radiation is considered to emanate. The LPDA active elements move during a frequency sweep, which causes the *PC* to change with frequency. This leads to an undefined actual separation distance between the radiation and reception points. However, the calculation of the AF from site attenuation requires knowledge of the exact separation distance. Choosing a fixed separation distance introduces some amount of error. This is the case for the ANSI C63.5, where the separation distance is specified to be from the center of each antenna boom. On the other hand, if the phase center is measured or predicted, the correct distance can be used in the NSA calculation.

OBTAINING THE PC A popular method for determining phase center is referred to as the "center of minimum phase variation" [6]. The *PC* is determined by rotating the transmit antenna and locating a single point on the antenna boresight axis which results in a minimum variation in the far-field phase response. Best [6] proposed an alternative method to obtain the *PC* from the far-field radiation pattern. In his method, two tangent lines on an equiphase contour line are found, and the interception point of two lines perpendicular to the tangential lines is the *PC*. However, the method requires a highly accurate far-field phase response. It has also been proposed that *PC* be computed by using a field-amplitude fall-off method [7]. This method is also prone to large deviations due to small measurement or calculation errors.

In this paper, the *PC* is calculated using a variation of the "center of minimum phase variation" method. The coordinate system is illustrated in Figure 6. If points A and B are equidistant from the *PC*, the *PC* is given by

$$PC = \frac{x_0^2 + y_0^2 - R^2}{2(R - x_0)}. \qquad (2)$$

Point A is at boresight in the far field. Point B is obtained by moving the observation position a small distance y_0 off-axis, and then moving horizontally (parallel to the *x*-axis) towards the antenna until the first position is found where the E-field has the same phase as at point A.

The resulting *PC* as calculated by NEC2 is plotted in Figure 7. The dashed line shows the positions where the dipoles are at half-wavelength dimensions. *PC*s normally occur in front of the half-wave positions. For the LPDA considered here, it is found that the numerical *PC*s are roughly located at dipole elements that are 90% of the length of a half-wave dipole. However, at about 300 MHz, the *PC* position shows an unexpected trend. This can be explained by looking at the LPDA current distribution. Figure 8 shows the current at the center of the dipole elements along the LPDA boom. Normally three consecutive elements are active at any given frequency, but at about 300 MHz, there are actually two active regions. For the "compressed" log antennas used in EMC (small relative spacing for a given gain), it is not uncommon to have more than one active region [8]. This also contributes to the AF peak around 300 MHz.

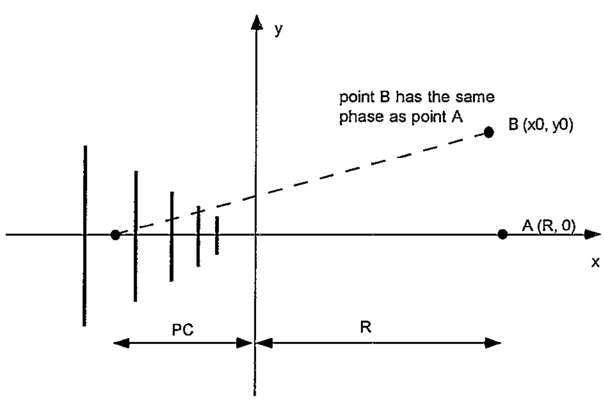

Figure 6. Geometry for phase center calculation.

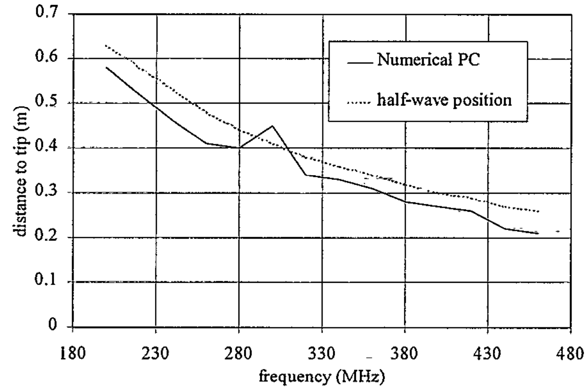

Figure 7. Numerical *PC* as a function of frequency.

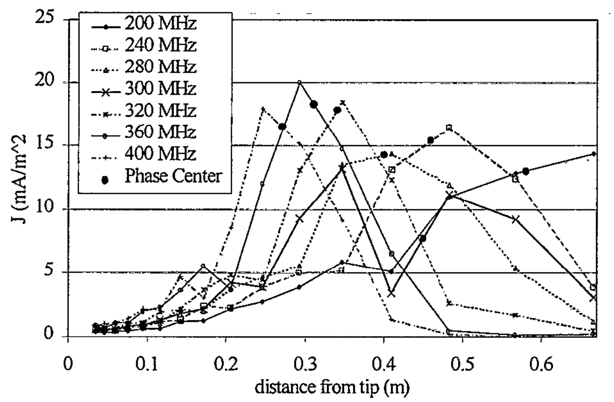

Figure 8. Current distribution at the centers of the dipole elements along the LPDA.

APPLYING PC CORRECTIONS IN FREE-SPACE Once the *PC* positions are known, the distance between the transmit and receive antennas is determined. Figure 9 shows the AFs obtained for transmit and receive antenna pointing line-of-sight in free-space. Other than the solid line, all AFs are calculated based on fixed distances measured between the centers of the transmit and receive antennas. For a 100 m separation distance, the LPDA length and *PC* error are negligible, and AF is equal to the free-space value. Figure 9 illustrates that the errors introduced by using a fixed distance are the largest at small separation distances. However, if the frequency-

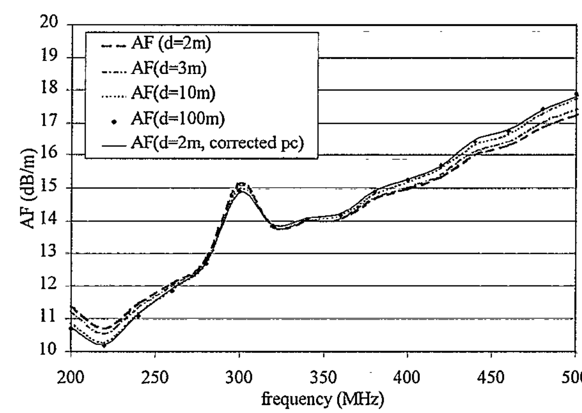

Figure 9. Antenna factors in free-space with transmit and receive antennas in-line with each other.

dependent distance between the *PC*s is used, the free-space AF is obtained even at 2 m distance. This shows that in a free-space environment, the dominant influence on the AF is from the definition of the *PC* positions.

APPLYING PC CORRECTIONS OVER A CONDUCTING GROUND The same *PC* correction can be used in the standard site method where the antennas are over an infinite PEC ground plane. Figure 10 shows the AFs determined by using the variable distances based on *PC* positions. In contrast to the free-space case, the corrected geometry-specific AFs do not align with the free-space values. It was shown in a previous section that ground plane coupling is small, so it can be conjectured that a different error mechanism causes geometry-specific ground-plane AFs to differ from the free-space value. This error can be mostly attributed to the pattern effect as described in the next section.

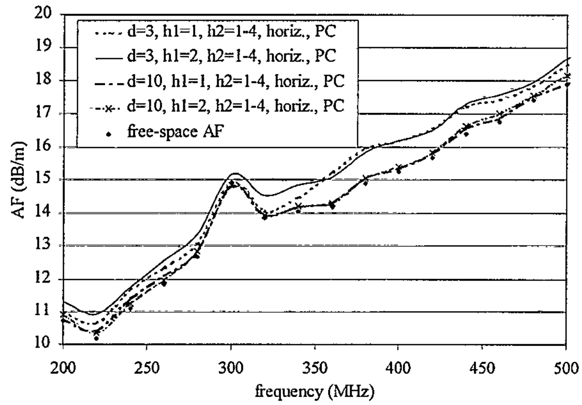

Figure 10. AF using *PC* positions by standard site method.

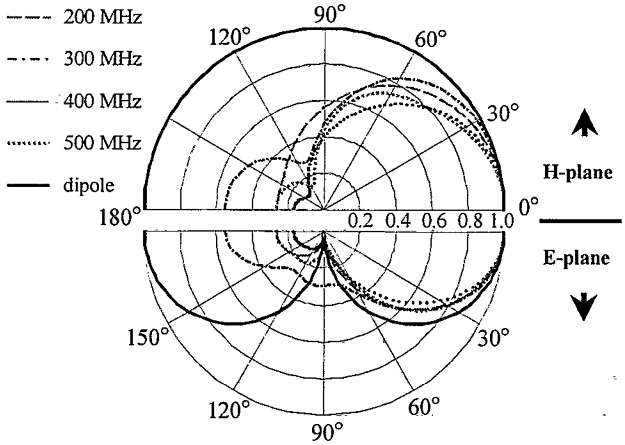

Figure 11. Antenna patterns for the LPDA. The top half is the H-plane pattern, and the bottom half is the E-plane pattern. The other halves are not drawn due to symmetry.

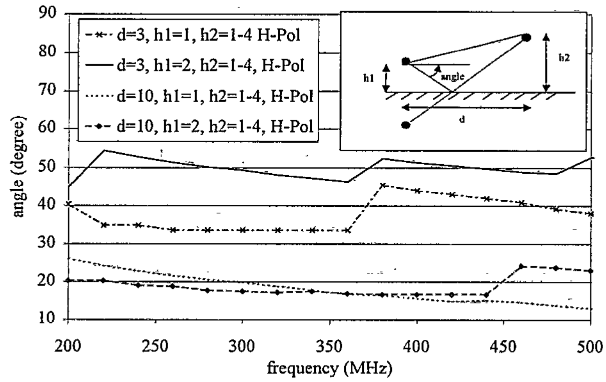

Figure 12. Angle of the reflected ray for the maximum reception in a standard site method.

ANTENNA PATTERN EFFECTS AND CORRECTIONS

ANTENNA PATTERN EFFECTS ON AF A basic assumption of the standard site method is that both transmit and receive antennas have point dipole radiation patterns . This assumption is valid for dipole and biconical antennas at lower frequencies, but it is worth re-examining for an LPDA whose gain normally ranges from 4 to 8 dBi.

Figure 11 shows the LPDA free-space far-field radiation pattern at 200, 300, 400, and 500 MHz. The patterns are in a linear scale normalized to the boresight (0°) value. The point dipole pattern is plotted as a solid thick line. The top half of the diagram is the H-plane, where the dipole pattern is a circle (omni-directional). The bottom half is the E-plane, where the dipole pattern is a sine function of observation angle. The LPDA patterns are close to the dipole pattern at small angles. As the observation angle increases away from boresight, the LPDA patterns deviate from the dipole.

In the standard site method, the receive antenna is scanned in height from 1 to 4 m, and the maximum pickup during the height scan is recorded at each frequency. The angles from both the direct ray and the ray reflected off the conducting ground change as a function of height. The angles of the reflected ray at the maximum reception are shown in Figure 12. It can be shown that the transmitted magnitude of each ray changes both as a function of frequency and height. For example, for the case of d=3 m, h_1=2 m, h_2=1-4 m, the angle at the maximum transmission changes between 45° and 55° over the frequency range. From the H-plane pattern in Figure 11, the field can be 0.8 times the level of a dipole at those angles, or 1.9 dB down compared to a dipole. A similar conclusion regarding the angle can also be drawn for the receive antenna. The angles at both the transmit side and receive side combine to determine the reception due to the ground reflection. Angles for the 3 m separation distance are larger than those for 10 m. In fact, because of the smaller angles at 10 m, the standard site method AFs at 10m are generally closer to the free-space values than the 3 m AFs, as shown in Figures 1 to 4 (note that at 10 m phase centers positioning is also less critical).

Because the LPDA pattern differs from the dipole, the maximum receptions do not occur at the heights predicted by the point-dipole formulation of the standard site method. The height differences are much more obvious for the 3 m distance than the for 10 m distance. This is easily explained by the fact that the angles for the reflected ray are smaller for the 10 m case. The predicted dipole heights match fairly well with the LPDA heights at the 10 m distance.

The pattern differences also help in answering the small ground plane coupling of an LPDA. Due to the smaller fields in the direction of ground plane and large distances in wavelength, LPDAs ground coupling is much smaller than the biconicals.

CORRECTIONS FOR ANTENNA PATTERNS It is possible to apply the actual antenna pattern to the mathematical model during the theoretical NSA calculation if the actual antenna patterns are known.

However, this requires pattern measurements at all frequencies and polarizations of interest. The pattern measurements also need to be performed in the far-field in a free-space or fully-anechoic environment. It has been suggested to calculate the ground reflection and E^D_{max} by using numerical methods [3]. The concept seems very promising, especially with the improved accuracies now possible in numerical modeling as shown by the present work.

It would also be desirable to develop a simple analytical model that agrees with actual antenna patterns. Because the H-plane patterns are frequency-dependent, as shown in Figure 11, it would be difficult to derive a simple formula for horizontal polarization patterns versus frequency. Thus a pattern measurement or numerical calculation is necessary. For vertical polarization, the E-plane of the LPDA has a stable pattern. A curve-fit to the patterns reveals that $\sin(\theta)^{1.5}$ agrees well for the LPDA under study. Figure 13 shows AF obtained with this simple pattern correction and with the phase center correction for a pair of vertically polarized LPDAs over a PEC ground plane. Compared to the uncorrected AFs (Figure 3), AF after correction is much closer to the free-space values. Among other things, the residual 0.5 dB errors could be due to the near field patterns being more complicated than a simple sine function. Possible errors due to effects such as non-uniform incident fields, cross-polarization, and near field coupling are also neglected in these corrections. The near-field effects are worthy of further investigation in a future study. In any case, an error component of 0.5 dB in AF may be tolerable in a typical NSA measurement.

Antenna patterns depend on the design of the log antenna. Most EMC log antennas have similar gains and antenna factors, because the element scale factor (τ) and relative spacing (σ) of these antennas are very similar. However, design variations are inevitable, and a pattern correction factor of $\sin(\theta)^{1.5}$ can not be applied universally. It should be noted that the $\sin(\theta)$ dipole pattern assumption gives worse errors for most LPDAs.

Another way to reduce pattern errors is "boresighting", or tilting the antennas to ensure the direct and reflected rays impinge at small angles. The RE test method of CISPR 16-1 [9] requires boresighting for any receive antennas (complex antennas) other than dipoles. This is necessary to reduce measurement uncertainty with an LPDA for a 3 m measurement. Interestingly, even biconical antennas, which have dipole-like patterns at low frequencies, must be boresighted under the

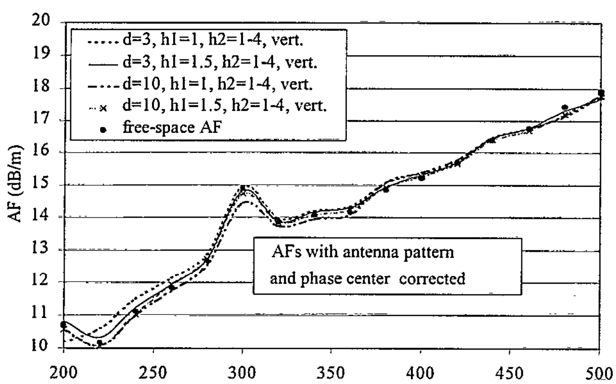

Figure 13. Modeled vertical AFs by standard site method with antenna pattern and phase center corrected.

CISPR 16-1 criteria. This creates an unnecessary disadvantage for biconicals. For AF calibration and NSA measurement, the logistics of boresighting becomes more complex. A possible geometry is shown in Figure 14. During a height scan (varying h_2), the angles of both antennas need to be adjusted continuously to ensure that both the direct and reflected rays enter the pattern at an equal angle from the boresight direction. However, this method produces gains that are different from the 0° on-axis value. An analytical model for this arrangement can be readily derived independent of pattern, but the scheme does require a very complicated measurement procedure. It should be noted that this method actually matches the use of such an antenna in a CISPR 16-1 RE test (off-axis gain is used in that case as well).

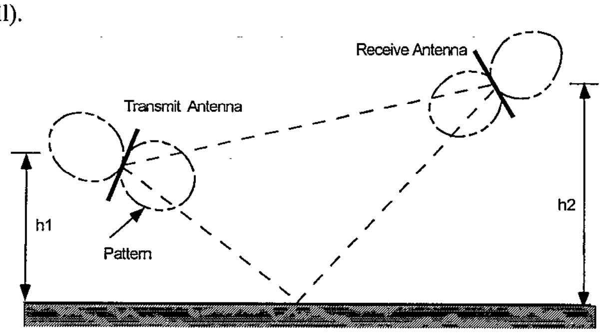

Figure 14. Boresighting for antennas with patterns which are different from a point dipole.

CROSS-POLARIZATION OF AN LPDA

The over/under element placement on the booms of most LPDA antennas can cause an impure polarization of the field. The polarization rejection of typical EMC log antennas is from 14 to more than 20 dB from 200-1000 MHz. This denotes the level of attenuation of a cross-polarized electromagnetic field compared to the co-polarized component. If antennas calibrated by the standard site method are used for an NSA test, the effect of the cross-polarization from the direct and reflected rays will cancel, since the same effect was measured in the original calibration of the pair. However, if free-space AFs are used for an NSA test, cross-polarization errors are not accounted for. The cross-polarization is a random error and must be treated as an uncertainty in this case. In addition, the ground plane does not reflect the horizontal wave the same as the vertical wave. Thus the uncertainty due to cross-polarization is dependent on both the height and polarization of the antennas when over a PEC ground plane. For example, for a 3 m separation and 2 m transmit height from 200-1000 MHz, the difference between the horizontal E^D_{max} and the vertical E^D found at the same height where the horizontal E^D_{max} occurs is as small as 5.8 dB. In comparison, the worst case for the vertical E^D_{max} is 9.5 dB. Thus a horizontal NSA measurement will generally have a slightly larger cross-polarization uncertainty than a vertical one, since the level of the cross-polarized component is closer to that of the co-polarized one. The deviation between the curves in Figure 13, which shows the free-space and standard site AFs with pattern and phase center corrections, indicates the magnitude of the possible errors due to other effects when using a free-space AF in a vertical NSA measurement. This includes errors due to cross-polarization, near field terms, non-plane-wave illumination, etc. The AF errors due to cross-polarization for the vertical case shown in Figure 13 should be small, as evidenced by the fact that the AF deviations shown in Figure 13 are larger at lower frequencies, while the cross-polarization effect is typically more severe at higher frequencies.

For a RE test, cross-polarization errors also need to be treated as uncertainties. Assuming the interfering electric field from the EUT at the receive antenna has the form of

$$\vec{E} = \hat{x}E_x e^{j\theta_x} + \hat{y}E_y e^{j\theta_y}, \quad (3)$$

where $\hat{x}$ is in the co-polar direction, and $\hat{y}$ is in the cross-polar direction, θ is the phase angle, and the antenna has a polarization rejection of

$$\vec{\rho} = \rho e^{j\theta_p} = 10^{-\frac{\rho(dB)}{20}} e^{j\theta_p}, \qquad \rho \geq 0. \quad (4)$$

If $\rho = 0$, the antenna only measures field in the $\hat{x}$ (co-) direction, or responds only to $E' = E_x e^{j\theta_x}$. Otherwise, the field in the $\hat{y}$ direction can be picked up by the antenna, and the equivalent field is

$$E'' = E_x e^{j\theta_x} + E_y e^{j\theta_y} \rho e^{j\theta_p}. \quad (5)$$

The ratio of $|E''/E'|$ is the uncertainty, or

$$\left|\frac{E''}{E'}\right| = \left|1 + \frac{E_y}{E_x}\rho e^{j(\theta_y - \theta_x + \theta_p)}\right| = \sqrt{\left(1 + \frac{E_y}{E_x}\rho\cos\Theta\right)^2 + \left(\frac{E_y}{E_x}\rho\sin\Theta\right)^2}, \quad (6)$$

where $\Theta = \theta_y - \theta_x + \theta_p$. The ratio between the cross- to co-polarized signal (E_y/E_x) and the relative phase (Θ) are unknown. Traditionally, the uncertainty has been based on $E_y/E_x = 1$ (if this ratio is greater than one, the measurement is done in the other polarization) and the value of Θ which maximizes or minimizes $|E''/E'|$ to give the upper and lower bounds of ($1+\rho$) and ($1-\rho$) respectively. A rectangular distribution is then applied. If we can assume E_y, E_x, and Θ are random, a rectangular distribution may not be the most appropriate description based on equation (6), and it may be worthy of further studies.

Another option may be to determine the worst case cross-polarization error on a case-by-case basis, and use the measured cross- to co-polar signal ratio. The field values measured with LPDAs include both co- and cross-polarized signals. For a given measurement, the maximum cross- to co-polarization ratio can be obtained by

$$\frac{E_y}{E_x} \leq \frac{\widetilde{E}_y + \widetilde{E}_x^{\max}\rho}{\widetilde{E}_x^{\max} - \widetilde{E}_y\rho}, \text{ or } 1\text{, whichever is smaller.} \quad (7)$$

$\widetilde{E}_x^{\max}$ is the measured maximum field during a height scan, and $\widetilde{E}_y$ is measured with antenna polarized in the $\hat{y}$ direction at the heights where $\widetilde{E}_x^{\max}$ occurs. If $\widetilde{E}_y$ is not available, the maximum value obtained during a height scan can be used, but the resulting uncertainty estimate may be larger than necessary. Note that since a RE test is required at both polarizations, no additional measurement is needed.

SUMMARY AND CONCLUSION

The excellent agreement between the measured and modeled data allows a confident numerical study of different error sources involved in using an LPDA antenna for EMC testing in free-space and over a ground plane. It was shown that LPDA coupling with a ground plane causes less than a 0.3 dB change in AF. However, it is shown that AFs obtained by the standard site method can differ from the free-space AF by up to 1.5 dB for a 3 m separation distance and 0.6 dB for a 10 m separation distance. This is mainly due to the active phase center variation and antenna pattern differences. The active phase center can be accurately predicted. For the LPDA under study, it is seen that the phase centers are normally at the element positions which are 90% of a half-wavelength in length. Possible exceptions occur due to multiple active regions in an LPDA, which induce non-linear variations in AF. The other main error source in the standard site method is the assumption that the LPDA radiation pattern is like a dipole. Correcting for the phase center and antenna pattern brings standard-site-method AFs to within 0.5 dB of the free-space AFs. Boresighting can be used to reduce pattern-related uncertainty. Cross-polarization rejection of an LPDA can have an impact on NSA and RE testing. To account for this, it may be beneficial that the measured ratio of the cross- to co-polarization from a regular RE test over a ground plane be used to assess the cross-polarization uncertainty.

Because a 1.5 dB difference exists between geometry-specific and free-space AFs, the free-space AF is not suitable for use in geometry-specific ANSI C63.4-1992 site validation tests (the NSA difference due to the AF is twice this, or 3 dB). The improvements described in this paper and/or the incorporation of numerical simulation into the current theoretical NSA model are warranted. The alternative is to perform the antenna calibrations in the same geometry as in the NSA measurement. Since the NSA measurement and the antenna calibration use the exact same site attenuation measurement technique, the systematic errors in the theoretical NSA model are canceled. However, any errors due to defects in the reference site are transferred to the resulting NSA by way of antenna factor errors, so this is the equivalent of site inter-comparison, which requires that the reference site be close to a theoretically ideal site.

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