Analysis of low complexity max log MAP detector and MMSE detector for Interference Suppression in Correlated Fading

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Abstract—**Performance of future wireless communication systems being interference limited, researchers are focusing on interference alignment, interference mitigation and interference suppression to diminish, manage or exploit these interferers. In this paper, we carry out the performance analysis of the recently proposed low complexity max log MAP detector and linear MMSE detector for interference suppression under the realistic conditions of correlated fading in cellular environment. We assume only receive correlation as base stations (BSs) due to their extended separation are likely to be uncorrelated. However the intricacy of realizing requisite antenna spacing in the mobile station (MS) combined with the lack of scattering would instigate the individual antennas at MS to be correlated. Employing *moment generating function* (MGF)-based approach, we derive upper bounds of coded pairwise error probability (PEP) and study the degrading effect of correlation on both the detectors.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication architecture because of its inherent potential to exploit the third (spatial) dimension in addition to time and frequency dimensions has recently emerged as a new paradigm for wireless communications in rich multipath environment [1]. Employing multi-element antenna arrays (MEA) at the mobile station (MS) in addition to existing multi antenna base stations (BS) is being advocated in upcoming wireless standards as 3GPP LTE [2]. Frequency reuse factor of one in future mobile systems [2] to achieve high spectral efficiency is leading to an interference limited system. In such a scenario, the optimal use of receive diversity at the MS to either achieve multiplexing gain or to cope with the interference is still an open question. For the latter case, researchers are focusing on interference alignment, interference mitigation and interference suppression to diminish, manage or exploit these interferers. In this paper we carry out the performance analysis of the recently proposed low complexity max log MAP detector [3] and linear MMSE detector for interference suppression in the bit interleaved coded modulation (BICM) MIMO OFDM based cellular system.

For the performance analysis, naive assumption of independent and identically distributed (i.i.d.) fading channel may not be true in most of the real world scenarios [4]. Some impairments of the radio propagation channel may lead to a substantial degradation in the performance. The limitations on

the performance are primarily imposed by the limited number of multipath components or scatterers and antenna spacing. In a typical cellular scenario, the inadequate antenna separation mainly affects the MS as the components of antenna array may be separated by a distance less than half of the communication wavelength due to their size limitations. On the other hand, extended separation between BSs render them uncorrelated.

In this paper, we study the effects of correlation between the antennas of the MS combined with the strength and the rate of interference on coded pairwise error probability (PEP) of the low complexity max log MAP detector and linear MMSE detector. We consider the practical scenario of receive correlation at the MS with no transmit correlation at the BSs. Using the *moment generating function* (MGF) approach associated with the quadratic form of a complex Gaussian random variable, we derive analytical expressions for upper bounds on coded PEP of the low complexity max log MAP detector and linear MMSE detector operating over a spatially correlated fading channel in the presence of interference. We demonstrate the strength of our new analytical PEP upper bounds by simulating the performance of a MS in the presence of receive correlation and one interferer of varying alphabet size using the low complexity max log MAP detector and linear MMSE detector.

Regarding notations, we will use lowercase or uppercase letters for scalars, lowercase boldface letters for vectors and uppercase boldface letters for matrices. $(.)^T$, $(.)^*$ and $(.)^\dagger$ indicate transpose, conjugate and conjugate transpose operations respectively. $|.|$ and $\|.\|$ indicate norm of scalar and vector while $abs\,(.)$ denotes absolute value. The notation $E\,(.)$ denotes the mathematical expectation while $Q\,(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx$ denotes the Gaussian Q-function. $\mathbf{A}_{M \times N}$ indicates a matrix $\mathbf{A}$ with M rows and N columns whereas $\mathrm{vec}\,(\mathbf{A})$ denotes the vectorization operator which stacks the columns of $\mathbf{A}$. The matrix $\mathbf{I}_n$ is the $n \times n$ identity matrix and the element at the i-th row and j-th column of matrix $\mathbf{H}$ is denoted as $\mathbf{H}\,(i,j)$. $\mathbf{A} \sim \mathbf{B}$ indicates that $\mathbf{A}$ and $\mathbf{B}$ are similar matrices.

The paper is divided into six sections. In section II we define the system model while section III discusses the PEP analysis of low complexity max log MAP detector. Section IV contains the PEP analysis of MMSE detector which is followed by

simulation results and conclusions.

II. SYSTEM MODEL

We consider the downlink of a single frequency reuse cellular system with n_r antennas at the MS and 2 BSs using antenna cycling for transmission with each stream being transmitted by one antenna in any dimension. We assume that two spatial streams arrive at the MS as $\underline{\mathbf{x}}_1$ (desired stream) and $\underline{\mathbf{x}}_2$ (interference stream). x_1 is the symbol of $\underline{\mathbf{x}}_1$ over a signal set χ_1 and x_2 is the symbol of $\underline{\mathbf{x}}_2$ over signal set χ_2. During the transmission at BS-1, code sequence $\mathbf{c}_1$ is interleaved by π_1 and then is mapped onto the signal sequence $\underline{\mathbf{x}}_1 \in \chi_1$. Bit interleaver for the first stream can be modeled as $\pi_1 : k' \rightarrow (k, i)$ where k' denotes the original ordering of the coded bits $c_{k'}$ of first stream, k denotes the time ordering of the signal $x_{1,k}$ and i indicates the position of the bit $c_{k'}$ in the symbol $x_{1,k}$. Assuming an ideal OFDM system, transmission at the k-th frequency tone can be expressed as::-

$$
\begin{aligned}
\mathbf{y}_k &= \mathbf{h}_{1,k} x_{1,k} + \mathbf{h}_{2,k} x_{2,k} + \mathbf{z}_k, \qquad k = 1, 2, \cdots, T \\
&= \mathbf{H}_k \mathbf{x}_k + \mathbf{z}_k
\end{aligned} \tag{1}
$$

where $\mathbf{H}_k = [\mathbf{h}_{1,k} \mathbf{h}_{2,k}]$ i.e. the channel at the k-th frequency tone and $\mathbf{x}_k = [x_{1,k}\ x_{2,k}]^T$. Each subcarrier corresponds to a symbol from a constellation map χ_1 for first stream and χ_2 for second stream. $\mathbf{y}_k, \mathbf{z}_k \in \mathbb{C}^{n_r}$ are the vectors of received symbols and circularly symmetric complex white Gaussian noise of double-sided power spectral density $N_0/2$ at the n_r receive antennas. $\mathbf{h}_{1,k} \in \mathbb{C}^{n_r}$ is the vector characterizing flat fading channel response from first transmitting antenna to n_r receive antennas at k-th subcarrier. The complex symbols $x_{1,k}$ and $x_{2,k}$ of the two streams are assumed to be independent with variances σ_1^2 and σ_2^2 respectively. The channels at different subcarriers are also assumed to be independent.

Correlation Structure. The entries of the channel matrix are assumed to be complex, zero-mean, circularly symmetric Gaussian random variables so their magnitudes exhibit Rayleigh distribution. $\mathbf{H}_k(i, j)$ is the complex path gain between BS j and ith antenna of MS at k-th frequency tone and has the following covariance structure:

$$
E\left[\mathbf{H}_k(p, j)\mathbf{H}_k(q, l)^*\right] = \mathbf{\Psi}_R(p, q)\,\delta(j - l) \tag{2}
$$

where $\mathbf{\Psi}_R$ is $n_r \times n_r$ receive correlation matrix (positive semi-definite) and $\delta(t) = 1$ for $t = 0$ and is zero otherwise. By Cholesky factorization we get $\mathbf{\Psi}_R = \mathbf{\Psi}_R^{\frac{1}{2}}\mathbf{\Psi}_R^{\frac{1}{2}}$ where $\mathbf{\Psi}_R^{\frac{1}{2}}$ is lower triangular matrix with real entries on the main diagonal. This fading model embodies following assumptions.

- There is no correlation between the fading from two BSs to the same receive antenna i.e. $\frac{1}{n_r}E\left[\mathbf{H}_k^\dagger \mathbf{H}_k\right] = \mathbf{I}_2$.
- The correlation between the fading from a BS to receive antenna p and to receive antenna q is $\mathbf{\Psi}_R(p, q)$ and does not depend on the base station. $\mathbf{\Psi}_R$ is equal to the correlation of $n_r \times 1$ vector channel when excited by any BS and is therefore the same for all BSs i.e. $\mathbf{\Psi}_R = \frac{1}{n_r}E\left[\mathbf{H}_k\mathbf{H}_k^\dagger\right]$

- The correlation is independent of the frequency tone.

These assumptions are usually quite accurate when antenna elements are colocated in the same physical unit at the receiver and the transmitters are far apart. $\mathbf{H}_k$ can therefore be factorized in the form $\mathbf{H}_k = (\mathbf{\Psi}_R)^{\frac{1}{2}}\mathbf{W}_k$ where the entries of $\mathbf{W}_k$ are i.i.d complex circularly symmetric Gaussian with mean 0 and variance 1. This channel matrix represents the *Kronecker correlation model* [5] since the correlation of the vectorized channel matrix can be written as the Kronecker product i.e. $\mathrm{cov}\left(\mathrm{vec}\left(\mathbf{H}_k\right)\right) = \mathbf{I}_2^T \otimes \mathbf{\Psi}_R$.

Let us now focus on the structure of correlation matrix. We consider single-parameter exponential correlation matrix model. For this model, the components of $\mathbf{\Psi}_R$ are given by

$$
\begin{aligned}
\mathbf{\Psi}_R(p, q) &= r^{abs(q-p)}, & p \leq q \\
&= \left(r^{abs(q-p)}\right)^*, & p > q
\end{aligned}
$$

where r is the (complex) correlation coefficient of neighboring receive branches with $|r| \leq 1$. Obviously, this may not be an accurate model for some real-world scenarios but it has been shown that this model can approximate the correlation in a uniform linear array under rich scattering conditions [6] with correlation decreasing with increasing distance between the antennas.

Detectors For low complexity max log MAP detection [3], the bit metric for the first stream in its full form is given as:-

$$
\lambda_1^i(\mathbf{y}_k, c_{k'}) \approx \min_{x_1 \in \chi_{1, c_{k'}}^i, x_2 \in \chi_2} \frac{1}{N_0}\|\mathbf{y}_k - \mathbf{h}_{1,k} x_1 - \mathbf{h}_{2,k} x_2\|^2
$$

where $\chi_{1, c_{k'}}^i$ denotes the subset of the signal set $x_1 \in \chi_1$ whose labels have the value $c_{k'} \in \{0, 1\}$ in the position i.

For linear MMSE detection, MMSE filter [4] for the detection of first stream is given as

$$
\mathbf{h}_{1,k}^{MMSE} = \left(\mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1}\mathbf{h}_{1,k} + \sigma_1^{-2}\right)^{-1}\mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \tag{3}
$$

where $\mathbf{R}_{2,k} = \sigma_2^2 \mathbf{h}_{2,k}\mathbf{h}_{2,k}^\dagger + N_0\mathbf{I}$. The application of this MMSE filter yields

$$
y_k = \alpha_k x_{1,k} + \beta_k x_{2,k} + \mathbf{h}_{1,k}^{MMSE}\mathbf{z}_k \tag{4}
$$
$$
= \alpha_k x_{1,k} + z_k \tag{5}
$$

where $\alpha_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{h}_{1,k}$ and $\beta_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{h}_{2,k}$. Based on the Gaussian assumption of post detection interference, the bit metric for the $c_{k'}$ bit on first stream is given as

$$
\lambda_1^i(\mathbf{y}_k, c_{k'}) \approx \min_{x_1 \in \chi_{1, c_{k'}}^i}\left[\frac{1}{N_k}|y_k - \alpha_k x_1|^2\right] \tag{6}
$$

where $N_k = \mathbf{h}_{1,k}^{MMSE}\mathbf{R}_{2,k}\mathbf{h}_{1,k}^{MMSE\dagger}$.

III. PEP ANALYSIS - MAX LOG MAP DETECTOR

The conditional PEP i.e. $P\left(\mathbf{c}_1 \to \hat{\mathbf{c}}_1 | \overline{\overline{\mathbf{H}}}\right) = \mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\overline{\mathbf{H}}}}$ of low complexity max log MAP detector [3] is given as:-

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\overline{\mathbf{H}}}} = P\left(\sum_{k'} \min_{x_1 \in \chi^i_{1,c_{k'}}, x_2 \in \chi_2} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2 \geq\right.$$
$$\left.\sum_{k'} \min_{x_1 \in \chi^i_{1,\hat{c}_{k'}}, x_2 \in \chi_2} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2\right) \quad (7)$$

where $\overline{\overline{\mathbf{H}}} = [\mathbf{H}_1 \cdots \mathbf{H}_N]$ i.e. the complete channel for the transmission of the codeword $\mathbf{c}_1$. For the worst case scenario once $d(\mathbf{c}_1 - \hat{\mathbf{c}}_1) = d_{free}$, the inequality on the right hand side of (7) shares the same terms on all but d_{free} summation points for which $\hat{c}_{k'} = \bar{c}_{k'}$ where $\bar{(.)}$ denotes the binary complement. Let

$$\tilde{x}_{1,k}, \tilde{x}_{2,k} = \arg \min_{x_1 \in \chi^i_{1,c_{k'}}, x_2 \in \chi_2} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2$$

$$\hat{x}_{1,k}, \hat{x}_{2,k} = \arg \min_{x_1 \in \chi^i_{1,\bar{c}_{k'}}, x_2 \in \chi_2} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}x_1 - \mathbf{h}_{2,k}x_2\|^2 \quad (8)$$

As $x_{1,k}$ and $x_{2,k}$ are the transmitted symbols so $\|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\|^2 \geq \|\mathbf{y}_k - \mathbf{h}_{1,k}\tilde{x}_{1,k} - \mathbf{h}_{2,k}\tilde{x}_{2,k}\|^2$. The conditional PEP is given as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} \leq P\left(\sum_{k,d_{free}} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}x_{1,k} - \mathbf{h}_{2,k}x_{2,k}\|^2 \geq\right.$$
$$\left.\sum_{k,d_{free}} \frac{1}{N_0} \|\mathbf{y}_k - \mathbf{h}_{1,k}\hat{x}_{1,k} - \mathbf{h}_{2,k}\hat{x}_{2,k}\|^2\right)$$
$$= P\left(\sum_{k,d_{free}} \frac{1}{N_0} 2\Re\left(\mathbf{z}_k^\dagger \mathbf{H}_k(\hat{\mathbf{x}}_k - \mathbf{x}_k)\right) \geq\right.$$
$$\left.\sum_{k,d_{free}} \frac{1}{N_0} \|\mathbf{H}_k\mathbf{x}_k - \mathbf{H}_k\hat{\mathbf{x}}_k\|^2\right)$$
$$= Q\left(\sqrt{\sum_{k,d_{free}} \frac{1}{2N_0} \|\mathbf{H}_k(\hat{\mathbf{x}}_k - \mathbf{x}_k)\|^2}\right)$$
$$= Q\left(\sqrt{\frac{1}{2N_0} \text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \mathbf{\Delta} \text{vec}\left(\overline{\mathbf{H}}^\dagger\right)}\right) \quad (9)$$

where $\mathbf{\Delta} = \mathbf{I}_{n_r} \otimes \mathbf{DD}^\dagger$ while $\mathbf{D}_{2K \times 2K} = \text{diag}\left\{\hat{\mathbf{x}}_1 - \mathbf{x}_1, \hat{\mathbf{x}}_2 - \mathbf{x}_2, \cdots, \hat{\mathbf{x}}_{k,dfree} - \mathbf{x}_{k,dfree}\right\}$. Note that $\overline{\mathbf{H}} = [\mathbf{H}_1 \cdots \mathbf{H}_K]$ where $K = d_{free}$. Using the Chernoff bound $Q(x) \leq \frac{1}{2}\exp\left(\frac{-x^2}{2}\right)$, the conditional PEP can be written as:-

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} \leq \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \mathbf{\Delta}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)\right) \quad (10)$$

Note that $\overline{\mathbf{H}} = \mathbf{\Psi}_R^{\frac{1}{2}}[\mathbf{W}_1 \cdots \mathbf{W}_K] = \mathbf{\Psi}_R^{\frac{1}{2}}\overline{\mathbf{W}}_{n_r \times 2K}$. Using the Kronecker product identity for the product of three matrices $\text{vec}(\mathbf{AXB}) = (\mathbf{B}^T \otimes \mathbf{A})\text{vec}(\mathbf{X})$, it can be shown that $\text{vec}(\mathbf{A}_{M \times N}\mathbf{X}_{N \times L}) = (\mathbf{X}^T \otimes \mathbf{I}_M)\text{vec}(\mathbf{A})$. So $\text{vec}\left(\overline{\mathbf{H}}^\dagger\right) = \text{vec}\left(\overline{\mathbf{W}}^\dagger \mathbf{\Psi}_R^{\frac{1}{2}}\right) = \left(\mathbf{\Psi}_R^{\frac{*}{2}} \otimes \mathbf{I}_{2K}\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)$. Developing (10) further on the lines of [7]:-

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1|\overline{\mathbf{H}}} \leq \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)^\dagger \mathbf{\Delta}\text{vec}\left(\overline{\mathbf{H}}^\dagger\right)\right)$$
$$= \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\mathbf{\Psi}_R^{\frac{T}{2}} \otimes \mathbf{I}_{2K}\right)\left(\mathbf{I}_{n_r} \otimes \mathbf{DD}^\dagger\right)\right.$$
$$\left.\times \left(\mathbf{\Psi}_R^{\frac{*}{2}} \otimes \mathbf{I}_{2K}\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)\right)$$
$$= \frac{1}{2}\exp\left(\frac{-1}{4N_0}\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\mathbf{\Psi}_R^{\frac{T}{2}} \otimes \mathbf{I}_{2K}\right)\left(\mathbf{\Psi}_R^{\frac{*}{2}} \otimes \mathbf{DD}^\dagger\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)\right)$$
$$= \frac{1}{2}\exp\left(\frac{-1}{4N_0}\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\left(\mathbf{\Psi}_R^{\frac{1}{2}}\mathbf{\Psi}_R^{\frac{1}{2}}\right)^T \otimes \mathbf{DD}^\dagger\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)\right)$$
$$= \frac{1}{2}\exp\left(-\frac{1}{4N_0}\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\tilde{\mathbf{\Psi}}_R \otimes \mathbf{DD}^\dagger\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)\right) \quad (11)$$

where we have used the identities $(A \otimes B)^\dagger = (A^\dagger \otimes B^\dagger)$ and $(A \otimes C)(B \otimes D) = AB \otimes CD$. Note that $\tilde{\mathbf{\Psi}}_R \sim \mathbf{\Psi}_R$ and $\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)^\dagger \left(\tilde{\mathbf{\Psi}}_R \otimes \mathbf{DD}^\dagger\right)\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)$ is a quadratic form of a Gaussian random variable since $\text{vec}\left(\overline{\mathbf{W}}^\dagger\right)$ is a random Gaussian vector and $\left(\tilde{\mathbf{\Psi}}_R \otimes \mathbf{DD}^\dagger\right)$ is a Hermitian fixed matrix. For a Hermitian quadratic form in complex Gaussian random variable $q = \mathbf{m}^\dagger \mathbf{Am}$ where $\mathbf{A}$ is a Hermitian matrix and column vector $\mathbf{m}$ is a circularly symmetric complex Gaussian vector i.e. $\mathbf{m} \sim \mathcal{NC}\left(\boldsymbol{\mu}, \sum\right)$ with $\boldsymbol{\mu} = E[\mathbf{m}]$ and $\sum = E\left[\mathbf{mm}^\dagger\right] - \boldsymbol{\mu}\boldsymbol{\mu}^\dagger$, the MGF is

$$E\left[\exp\left(-t\mathbf{m}^\dagger \mathbf{Am}\right)\right] = \frac{\exp\left[-t\boldsymbol{\mu}^\dagger \mathbf{A}\left(\mathbf{I} + t\sum\mathbf{A}\right)^{-1}\boldsymbol{\mu}\right]}{\det\left(\mathbf{I} + t\sum\mathbf{A}\right)} \quad (12)$$

Using the MGF, PEP is upper bounded as

$$\mathcal{P}^{\hat{\mathbf{c}}_1}_{\mathbf{c}_1} \leq \frac{1}{2\det\left(\mathbf{I} + \frac{1}{4N_0}\left(\tilde{\mathbf{\Psi}}_R \otimes \mathbf{DD}^\dagger\right)\right)}$$
$$= \frac{1}{2\prod_{k=1}^{2d_{free}}\prod_{l=1}^{n_r}\left(1 + \frac{1}{4N_0}\lambda_l\mu_k\right)} \quad (13)$$

where λ_l are the eigenvalues of $\mathbf{\Psi}_R$ (as $\mathbf{\Psi}_R \sim \tilde{\mathbf{\Psi}}_R$) and μ_k are the eigenvalues of $\mathbf{DD}^\dagger$. Here we have used the identity that for square matrices $\mathbf{A}$ ans $\mathbf{B}$ of size n and q respectively with the eigenvalues $\lambda_1, \cdots, \lambda_n$ and $\mu_1, \cdots, \mu_q$ of $\mathbf{A}$ and $\mathbf{B}$ respectively then the eigenvalues of $\mathbf{A} \otimes \mathbf{B}$ are

$$\lambda_i\mu_j \qquad i = 1, \cdots, n, \ j = 1, \cdots, q. \quad (14)$$

$(\mathbf{DD}^\dagger)_{2K \times 2K}$ is a square block diagonal matrix and its eigenvalues are

$$\mu_k(\mathbf{DD}^\dagger) = \begin{cases} \|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 & \text{for} \quad k = 1, \cdots, d_{free} \\ 0 & \text{for} \quad k = d_{free}+1, \cdots, 2d_{free} \end{cases}$$

So the PEP is upperbounded as

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2 \prod_{k=1}^{d_{free}} \prod_{l=1}^{n_r} \left(1 + \frac{1}{4N_0} \lambda_l \|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2\right)}$$

$$\leq \frac{1}{2} \prod_{k=1}^{d_{free}} \prod_{l=1}^{\kappa} \frac{4N_0}{\lambda_l \|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2} \quad (15)$$

where $\kappa = \operatorname{rank}(\mathbf{\Psi}_R)$. $\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 \geq d_{1,\min}^2 + d_{2,\min}^2$ if $\hat{x}_{2,k} \neq x_{2,k}$ and $\|\hat{\mathbf{x}}_k - \mathbf{x}_k\|^2 \geq d_{1,\min}^2$ if $\hat{x}_{2,k} = x_{2,k}$. So PEP is upperbounded as

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2} \prod_{k=1}^{d_{free}} \prod_{l=1}^{\kappa}$$

$$\times \left[\frac{4N_0}{\lambda_l \left\{ \left(d_{1,\min}^2 + d_{2,\min}^2\right) P\left(\hat{x}_{2,k} \neq x_{2,k}\right) + d_{1,\min}^2 \left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right) \right\}} \right] \quad (16)$$

where $P(\hat{x}_{2,k} \neq x_{2,k})$ is the uncoded probability that the output of max log MAP detector $\hat{x}_{2,k}$ is not equal to the actual transmitted symbol $x_{2,k}$ and has been derived in the Appendix. PEP can be rewritten as

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2} \left(\frac{4N_0}{d_{1,\min}^2}\right)^{\kappa \times d_{free}} \prod_{k=1}^{d_{free}} \prod_{l=1}^{\kappa}$$

$$\times \left[\frac{1}{\lambda_l \left\{ \left(1 + \frac{d_{2,\min}^2}{d_{1,\min}^2}\right) P\left(\hat{x}_{2,k} \neq x_{2,k}\right) + \left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right) \right\}} \right]$$

$$= \frac{1}{2} \left(\frac{4N_0}{\sigma_1^2 \breve{d}_{1,\min}^2}\right)^{\kappa \times d_{free}} \prod_{l=1}^{\kappa} \frac{1}{\lambda_l^{d_{free}}} \prod_{k=1}^{d_{free}}$$

$$\times \left[\frac{1}{\left\{ \left(1 + \frac{\sigma_2^2 \breve{d}_{2,\min}^2}{\sigma_1^2 \breve{d}_{1,\min}^2}\right) P\left(\hat{x}_{2,k} \neq x_{2,k}\right) + \left(1 - P\left(\hat{x}_{2,k} \neq x_{2,k}\right)\right) \right\}} \right] \quad (17)$$

where $d_{j,\min}^2 = \sigma_j^2 \breve{d}_{j,\min}^2$ with $\breve{d}_{j,\min}^2$ being the normalized minimum distance of the constellation χ_j for $j = \{1,2\}$. (17) shows full diversity ($n_r \times d_{free}$) of max log MAP detector in the case of correlation matrix being full rank. The coding gain increases as the interference gets stronger relative to the desired stream i.e. $\sigma_2^2 > \sigma_1^2$ or the rate of interference decreases relative to the desired stream i.e. $\breve{d}_{2,\min}^2 > \breve{d}_{1,\min}^2$. It has been shown in the Appendix that $P(\hat{x}_{2,k} \neq x_{2,k})$ depends on the eigenvalues of correlation matrix in addition to its dependence on the strength of desired signal and interference. Analyzing (17) combined with (27) gives more insights into the dependence of the performance of max log MAP detector on the strength and rate of interference relative to the desired stream and the eigenvalues of the correlation matrix. It shows that the degrading effect of correlation on the performance of max log MAP detector reduces as the rate of interference decreases or its strength increases relative to the desired stream.

IV. PEP ANALYSIS - MMSE DETECTOR

Conditional PEP for MMSE basing on Gaussian assumption of post detection interference is given as

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} = P\left(\sum_{k'} \min_{x_1 \in \chi_{1,c_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k} \geq \sum_{k'} \min_{x_1 \in \chi_{1,\hat{c}_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k} \right) \quad (18)$$

Let

$$\tilde{x}_{1,k} = \arg \min_{x_1 \in \chi_{1,c_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k}, \quad \hat{x}_{1,k} = \arg \min_{x_1 \in \chi_{1,\hat{c}_{k'}}^i} \frac{|y_k - \alpha_k x_1|^2}{N_k}$$

Considering the worst case scenario $d(\mathbf{c}_1 - \hat{\mathbf{c}}_1) = d_{free}$ and using the fact that $\frac{1}{N_k}|y_k - \alpha_k x_{1,k}|^2 \geq \frac{1}{N_k}|y_k - \alpha_k \tilde{x}_{1,k}|^2$, the conditional PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} \leq Q\left(\sqrt{\sum_{k,d_{free}} \frac{\alpha_k^2}{2N_k} |\hat{x}_{1,k} - x_{1,k}|^2} \right) \quad (19)$$

Bounding $|\hat{x}_{1,k} - x_{1,k}|^2 \geq d_{1,\min}^2$ and using the Chernoff bound, we get

$$\mathcal{P}_{\mathbf{c}_1|\mathbf{H}}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2} \exp\left(-\frac{d_{1,\min}^2}{4} \sum_{k,d_{free}} \mathbf{h}_{1,k}^\dagger \mathbf{R}_{2,k}^{-1} \mathbf{h}_{1,k} \right) \quad (20)$$

where the summation in (20) can be written as

$$= \left[\mathbf{h}_{1,1}^\dagger, \cdots, \mathbf{h}_{1,d_{free}}^\dagger\right] \operatorname{diag}\left[\mathbf{R}_{2,1}^{-1}, \cdots, \mathbf{R}_{2,d_{free}}^{-1}\right] \left[\mathbf{h}_{1,1}^T, \cdots, \mathbf{h}_{1,d_{free}}^T\right]^T$$

$$= \left[\mathbf{w}_{1,1}^\dagger, \cdots, \mathbf{w}_{1,d_{free}}^\dagger\right] \left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{\dagger}{2}}\right) \operatorname{diag}\left[\mathbf{R}_{2,1}^{-1}, \cdots, \mathbf{R}_{2,d_{free}}^{-1}\right]$$

$$\times \left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{1}{2}}\right) \left[\mathbf{w}_{1,1}^T, \cdots, \mathbf{w}_{1,d_{free}}^T\right]^T \quad (21)$$

Let $\mathbf{R}_2^{-1} = \operatorname{diag}\left[\mathbf{R}_{2,1}^{-1}, \cdots, \mathbf{R}_{2,d_{free}}^{-1}\right]$ Using the MGF (12), PEP conditioned on $\underline{\mathbf{h}}_2 = \left[\mathbf{h}_{2,1}, \cdots, \mathbf{h}_{2,d_{free}}\right]$ is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1|\underline{\mathbf{h}}_2}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2 \det\left(\mathbf{I}_{n_r d_{free}} + \frac{d_{1,\min}^2}{4}\left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{\dagger}{2}}\right) \mathbf{R}_2^{-1} \left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{1}{2}}\right)\right)}$$

$$= \frac{1}{2 \det\left(\mathbf{I}_{n_r d_{free}} + \frac{d_{1,\min}^2}{4}\left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{1}{2}}\right)\left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R^{\frac{\dagger}{2}}\right) \mathbf{R}_2^{-1}\right)}$$

$$= \frac{1}{2 \det\left(\mathbf{I}_{n_r d_{free}} + \frac{d_{1,\min}^2}{4}\left(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R\right) \operatorname{diag}\left[\mathbf{R}_{2,1}^{-1}, \cdots, \mathbf{R}_{2,d_{free}}^{-1}\right]\right)}$$

$$= \frac{1}{2 \prod_{i=1}^{n_r d_{free}} \left(1 + \frac{d_{1,\min}^2}{4} \Upsilon_i\right)}$$

$$\leq \frac{1}{2 \prod_{i=1}^{n_r d_{free}} \left(\frac{d_{1,\min}^2}{4} \Upsilon_i\right)}$$

where we have used the identity $\det(\mathbf{I}_m + \mathbf{AB}) = \det(\mathbf{I}_n + \mathbf{BA})$. Note that Υ_is are the eigenvalues of the matrix $(\mathbf{I}_{d_{free}} \otimes \mathbf{\Psi}_R) \operatorname{diag}\left[\mathbf{R}_{2,1}^{-1}, \cdots, \mathbf{R}_{2,d_{free}}^{-1}\right]$. Here we assume that the correlation matrix $\mathbf{\Psi}_R$ has full rank. Using the
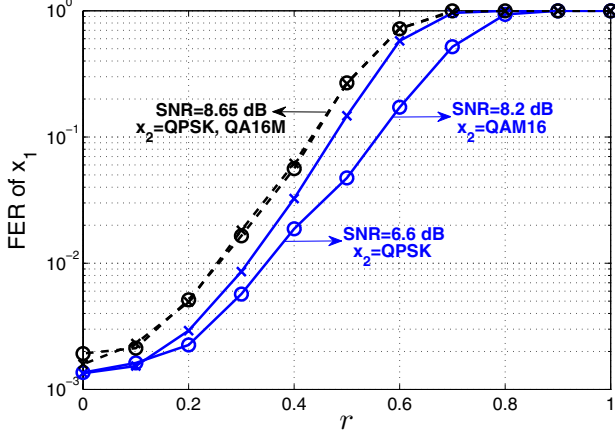
Fig. 1. Desired stream x_1 is QAM16 while interference stream x_2 is QPSK and QAM16. Continuous lines indicate low complexity max log MAP detection while dashed lines indicate linear MMSE detection. LTE turbo code is used with maximum of 5 decoding iterations using max log MAP decoder.
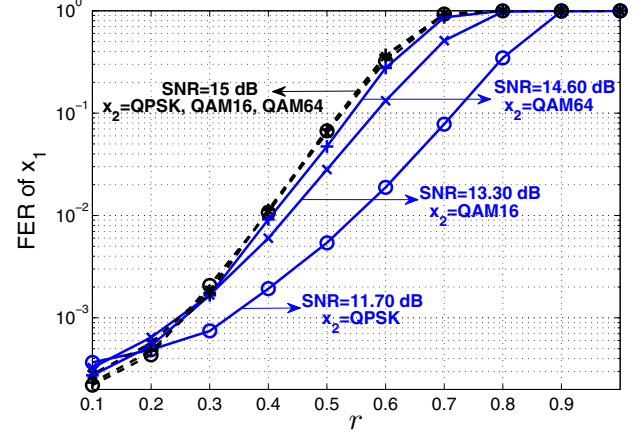

Fig. 2. Desired stream x_1 is QAM64 while interference stream x_2 is QPSK, QAM16 and QAM64. Continuous lines indicate low complexity max log MAP detection while dashed lines indicate linear MMSE detection. LTE turbo code is used with maximum of 5 decoding iterations using max log MAP decoder.

identities that $\det(\mathbf{AB}) = \det\mathbf{A}\det\mathbf{B}$ and $\text{rank}(\mathbf{AB}) \leq \min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}))$, we can decompose

$$\prod_{i=1}^{n_r d_{free}} \frac{1}{\Upsilon_i} = N_0^{d_{free}(n_r-1)} \prod_{l=1}^{n_r} \frac{1}{\lambda_l^{d_{free}}} \prod_{k=1}^{d_{free}} \left(\sigma_2^2\|\mathbf{h}_{2,k}\|^2 + N_0\right) \quad (22)$$

where λ_l are the eigenvalues of $\boldsymbol{\Psi}_R$ while the eigenvalues of $\mathbf{R}_{2,k}^{-1}$ are

$$\eta_j = \begin{cases} \left(\sigma_2^2\|\mathbf{h}_{2,k}\|^2 + N_0\right)^{-1}, & j = 1 \\ N_0^{-1}, & j = 2, \cdots, n_r \end{cases} \quad (23)$$

So the PEP is upper bounded as

$$\mathcal{P}_{\mathbf{c}_1|\underline{\mathbf{h}}_2}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{d_{1,\min}^2}\right)^{d_{free}(n_r-1)}\left(\frac{4}{d_{1,\min}^2}\right)^{d_{free}n_r}\prod_{l=1}^{n_r}\frac{1}{\lambda_l^{d_{free}}}\prod_{k=1}^{d_{free}}\left(\sigma_2^2\|\mathbf{h}_{2,k}\|^2 + N_0\right)$$

Channel independence at each subcarrier yields

$$\mathcal{P}_{\mathbf{c}_1}^{\hat{\mathbf{c}}_1} \leq \frac{1}{2}\left(\frac{4N_0}{d_{1,\min}^2}\right)^{d_{free}(n_r-1)}\left(\frac{4}{d_{1,\min}^2}\right)^{d_{free}}\left(n_r\sigma_2^2 + N_0\right)^{d_{free}}\prod_{l=1}^{n_r}\frac{1}{\lambda_l^{d_{free}}}$$

$$= \frac{1}{2}\left(\frac{4N_0}{\sigma_1^2\breve{d}_{1,\min}^2}\right)^{d_{free}(n_r-1)}\left(\frac{4}{\sigma_1^2\breve{d}_{1,\min}^2}\right)^{d_{free}}\left(n_r\sigma_2^2 + N_0\right)^{d_{free}}\prod_{l=1}^{n_r}\frac{1}{\lambda_l^{d_{free}}} \quad (24)$$

which not only demonstrates the well known result of the loss of one diversity order in MMSE detection in the presence of an interferer (x_2) [8] but also exhibits a coding loss as interference gets stronger. However the performance of MMSE is independent of the rate of interference (constellation size). It also shows the dependence of MMSE performance on the eigenvalues of correlation matrix.

V. SIMULATION RESULTS

We consider 2 BSs each using BICM OFDM system for downlink transmission using the punctured rate $1/2$ turbo code[1] of 3GPP LTE [2]. We consider an ideal OFDM based system (no ISI) and analyze the system in frequency domain. We assume antenna cycling at the BS and receive diversity at the MS with two antennas. The resulting SIMO channel at each sub carrier from BS to MS has correlated Gaussian matrix entries while we assume independence between different subcarriers. For the structure of correlation matrix, we consider exponential correlation matrix model. Perfect channel state information (CSI) is assumed at the MS while BSs have no CSI. Furthermore, all mappings of coded bits to QAM symbols use Gray encoding. We consider linear MMSE detector and the low complexity max log MAP detector. Figs. 1 and 2 show the frame error rates (FERs) of desired stream for the frame size of 1056 information bits. The effects of receive correlation and the rate of interference stream have been isolated in these simulations. To achieve this, the strengths of desired stream and interference stream are kept same (Cell Edge case) while FERs of the two streams have been approximately equated once there is no correlation. Then as the correlation gets stronger, the degrading effect on the performance of both the detectors in the presence of different interferences is compared. In the case of equal rate streams i.e. desired and interference streams belong to same constellation, the degrading effect of correlation on both MMSE and low complexity max log MAP detection is approximately same. However the degradation of the performance of max log MAP detector with enhanced correlation gets reduced as the rate of interference decreases relative to the desired stream. The degradation of MMSE performance with increase in correlation is independent of the

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[1]The LTE turbo decoder design was performed using the coded modulation library www.iterativesolutions.com

rate of interference which is in line with PEP analysis. The SNR gap between the max log MAP detection and MMSE detection for the same FER (at zero correlation) widens as rate of interference stream decreases relative to the desired stream. This can be attributed to the ability of max log MAP detector to partially decode interference once the lower rate or the higher strength of interference permits its partial decoding [4]. This partial decoding capability of max log MAP detector reduces with increased correlation especially once interference has comparable rate relative to the desired stream. MMSE does not benefit from exploiting interference as it is based on the attenuation of interference strength and the subsequent assumption of Gaussianity for its behavior.

VI. Conclusions

We have focused in this paper on the effects of receive correlation, interference strength and interference rate on the performance of low complexity max log MAP detector and linear MMSE detector. The degrading effect of correlation is less for max log MAP detector as compared to MMSE detector especially in the cases when interference because of its relative rate or strength allows its partial decoding. However interestingly, the degrading effect of correlation gets more pronounced in max log MAP detector as the rate of interfering stream increases or its strength decreases. On the other hand, degradation of MMSE performance with enhanced correlation is independent of the rate and the strength of interference.

Acknowledgments

Appendix

$\mathbf{P}(\hat{x}_{2,k} \neq x_{2,k})$

Considering the definition of $\hat{x}_{2,k}$ in (8), it can be expanded as:-

$$\hat{x}_{2,k} = \arg \min_{x_1 \in \chi_{1,\check{c}_k}^i, x_2 \in \chi_2} \left[\frac{1}{N_0} \left\{ \|\mathbf{h}_{1,k}(x_{1,k}-x_1)+\mathbf{z}_k\|^2 + \|\mathbf{h}_{2,k}(x_{2,k}-x_2)\|^2 + \right. \right.$$

$$\left. \left. 2\Re\left(\mathbf{h}_{1,k}(x_{1,k}-x_1)+\mathbf{z}_k\right)^\dagger \mathbf{h}_{2,k}(x_{2,k}-x_2) \right\} \right]$$

The last two terms will be zero if $\hat{x}_{2,k} = x_{2,k}$. Conditioning it on x_1, the probability $P(\hat{x}_{2,k} \neq x_{2,k}|\mathbf{h}_{1,k}, \mathbf{h}_{2,k}, x_1) = \mathcal{P}_{x_2|\mathbf{H}_k, x_1}^{\hat{x}_2}$ is given as:-

$$= P\left(-2\Re\left((\mathbf{h}_{1,k}\mathrm{X}_{1,k}+\mathbf{z}_k)^\dagger \mathbf{h}_{2,k}\mathrm{X}_{2,k}\right) \geq \|\mathbf{h}_{2,k}\mathrm{X}_{2,k}\|^2 |\mathbf{H}_k, x_{1,k}\right)$$

$$= Q\left(\sqrt{\frac{\|\mathbf{h}_{2,k}\mathrm{X}_{2,k}\|^2}{2N_0}} + \sqrt{\frac{2}{N_0}}\Re\left(\frac{(\mathbf{h}_{1,k}\mathrm{X}_{1,k})^\dagger \mathbf{h}_{2,k}\mathrm{X}_{2,k}}{\sqrt{\|\mathbf{h}_{2,k}\mathrm{X}_{2,k}\|^2}}\right)\right)$$

where $\mathrm{X}_{j,k}$ denotes $(x_{j,k}-x_j)$. Using the relation $Q(a+b) \leq Q(a_{min}-|b_{max}|)$ and $\Re\left(\mathbf{a}^\dagger \hat{\mathbf{b}}\right) \leq \|\mathbf{a}\|$ where $\hat{\mathbf{b}}$ is the unit vector

we get

$$\mathcal{P}_{x_2|\mathbf{H}_k}^{\hat{x}_2} \leq \frac{1}{2}\exp\left(-\frac{\|\mathbf{h}_{2,k}\|^2 d_{2,min}^2}{4N_0} - \frac{\|\mathbf{h}_{1,k}\|^2 d_{1,max}^2}{N_0}\right.$$

$$\left. + \frac{\|\mathbf{h}_{2,k}\|\|\mathbf{h}_{1,k}\| d_{2,min}d_{1,max}}{N_0}\right) \qquad (25)$$

Considering the norms of $\mathbf{h}_{1,k}$ and $\mathbf{h}_{2,k}$ we make two non-overlapping regions as $(\|\mathbf{h}_{2,k}\| \geq \|\mathbf{h}_{1,k}\|)$ and $(\|\mathbf{h}_{2,k}\| < \|\mathbf{h}_{1,k}\|)$ with the corresponding probabilities as $\mathcal{P}_{\mathbf{h}_1}^<$ and $\mathcal{P}_{\mathbf{h}_1}^>$. Note that in the first region $\|\mathbf{h}_{2,k}\|\|\mathbf{h}_{1,k}\| \leq \|\mathbf{h}_{2,k}\|^2$ while for second region $\|\mathbf{h}_{2,k}\|\|\mathbf{h}_{1,k}\| < \|\mathbf{h}_{1,k}\|^2$. So

$$\mathcal{P}_{x_2|\mathbf{H}_k}^{\hat{x}_2} \leq$$

$$\frac{1}{2}\left[\exp\left(-\|\mathbf{h}_{2,k}\|^2 \frac{d_{2,min}^2 - 4d_{2,min}d_{1,max}}{4N_0}\right)\exp\left(-\frac{\|\mathbf{h}_{1,k}\|^2 d_{1,max}^2}{N_0}\right)\mathcal{P}_{\mathbf{h}_1}^<\right.$$

$$\left. + \exp\left(-\frac{\|\mathbf{h}_{2,k}\|^2 d_{2,min}^2}{4N_0}\right)\exp\left(-\|\mathbf{h}_{1,k}\|^2 \frac{d_{1,max}^2 - d_{2,min}d_{1,max}}{N_0}\right)\mathcal{P}_{\mathbf{h}_1}^>\right]$$

$$(26)$$

We upperbound both the probabilities i.e. $\mathcal{P}_{\mathbf{h}_1}^<$ and $\mathcal{P}_{\mathbf{h}_1}^>$ by 1. Taking expectation over $\mathbf{h}_{2,k}$ conditioned on $\mathbf{h}_{1,k}$ and then subsequently taking expectation over $\mathbf{h}_{1,k}$ yields:-

$$\mathcal{P}_{x_2}^{\hat{x}_2} \leq \frac{1}{2}E_{\mathbf{h}_1}\left[\left(\frac{4N_0}{d_{2,min}^2 - 4d_{2,min}d_{1,max}}\right)^\kappa \exp\left(-\frac{\|\mathbf{h}_{1,k}\|^2 d_{1,max}^2}{N_0}\right)\prod_{l=1}^\kappa \frac{1}{\lambda_l}\right.$$

$$\left. + \left(\frac{4N_0}{d_{2,min}^2}\right)^\kappa \exp\left(-\|\mathbf{h}_{1,k}\|^2 \frac{d_{1,max}^2 - d_{2,min}d_{1,max}}{N_0}\right)\prod_{l=1}^\kappa \frac{1}{\lambda_l}\right]$$

$$\leq \frac{1}{2}\left(\frac{4N_0}{\sigma_2^2 \check{d}_{2,min}^2}\right)^\kappa \left(\frac{N_0}{\sigma_1^2 \check{d}_{1,max}^2}\right)^\kappa \prod_{l=1}^\kappa \frac{1}{\lambda_l^2} \times$$

$$\left(\frac{1}{\left(1 - \frac{4\sigma_1 \check{d}_{1,max}}{\sigma_2 \check{d}_{2,min}}\right)^\kappa} + \frac{1}{\left(1 - \frac{\sigma_2 \check{d}_{2,min}}{\sigma_1 \check{d}_{1,max}}\right)^\kappa}\right) \qquad (27)$$

where we have used the MGF of (12) while writing $\|\mathbf{h}_{j,k}\|^2 = \mathbf{h}_{j,k}^\dagger \mathbf{I}_{n_r}\mathbf{h}_{j,k}$ where $\mathbf{h}_{j,k} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Psi}_R)$. This expression shows the dependence of $P(\hat{x}_{2,k} \neq x_{2,k})$ on the interference strength, SNR and the correlation.

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