ANALYSIS OF MATHEMATICAL FICTION WITH GEOMETRIC THEMES

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ABSTRACT

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Analysis of mathematical fiction with geometric themes is a study that connects the genre of mathematical fiction with informal learning. This study provides an analysis of 26 sources that include novels and short stories of mathematical fiction with regard to plot, geometric theme, cultural theme, and presentation. The authors' mathematical backgrounds are presented as they relate to both geometric and cultural themes. These backgrounds range from having little mathematical training to advance graduate work culminating in a Ph.D. in mathematics. This thesis demonstrated that regardless of background, the authors could write a mathematical fiction novel or short story with a dominant geometric theme. The authors' pedagogical approaches to delivering the geometric themes are also discussed. Applications from this study involve a pedagogical component that can be used in a classroom setting.

All the sources analyzed in this study are fictional, but the geometric content is factual. Six categories of geometric topics were analyzed: plane geometry, solid geometry, projective geometry, axiomatics, topology, and the historical foundations of geometry. Geometry textbooks aligned with these categories were discussed with regard to mathematical fiction and formal learning. Cultural patterns were also analyzed for each source of mathematical fiction. There were also an analysis of the integration of cultural and geometric themes in the 26 sources of mathematical fiction; some of the cultural patterns discussed are gender bias, art, music, academia, mysticism, and social issues. On the basis of this discussion, recommendations for future studies involving the use of mathematical fiction were made.

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Chapter I

INTRODUCTION

Need for the Study

Informal knowledge stems from everyday experiences in which the knowledge gained is predominantly unintended (Hewitt, 2006). Mack (1990) defines informal knowledge as student-constructed knowledge related to applied, real-life situations that can be either correct or incorrect. (Mack also explains that different authors use various terms such as intuitive, prior or situated knowledge to represent informal knowledge.) Prawat (1989) reiterates the distinction between formal and informal knowledge by linking formal with instructed and informal with constructed knowledge, although he warns that this "may be an oversimplification." Studies (Baroody, 1987; Ginsburg, 1989; Hughes, 1986) have noted the importance of informal knowledge for learning "school" mathematics (Ginsburg, 1989; Mack, 2001), whereas the acquisition of formal knowledge occurs in connection with prior knowledge. Ignoring informal mathematics may create a disconnection for learners (Kim, 2002) and perpetuate the belief that mathematics is solely a formal activity that is learned inside a classroom. While any informal learning outside the classroom presents a likely channel for academic content (Bull et al., 2008), informal learning can also be used inside a mathematics college classroom. Especially when the students share their informal knowledge from the same source, the teachers can use this knowledge to enhance the learning task as a starting point for connecting with formal knowledge. According to the official National Science Foundation website, informal learning takes place outside formal school settings. Typical formal school settings imply a classroom that uses a textbook as its main source of knowledge. An innovative mathematical environment that fosters a synthesis of traditional formal materials and conventionally classified “informal” forms of learning is needed.

Mathematics learning seldom takes place without the influential role of mathematics textbooks. There are still those who agree with the former NCTM president (Willoughby, 1983) who stated that “the most important factor in determining what mathematics is taught and learned is the textbook used” (p. 50). Research has shown that mathematics instructors believe that the most successful teachers should employ their own materials and strategies or use them in conjunction with the textbook. Most often, teachers end up relying nearly exclusively on the textbook due to time constraints or other concerns (Nissen, 2000). “Teachers of mathematics in all countries rely heavily on textbooks in their day-to-day teaching, and this is perhaps more characteristic of the teaching of mathematics than of any other subject in the curriculum” (Robitaille & Travers, 1992, p. 706).

Many research studies on textbooks have been conducted over the decades because of their function in the teaching of mathematics. Focusing on geometry alone, a large body of research has examined materials to aid the intake of formal knowledge. Wilson (1959) analyzed 55 geometry textbooks that were all published between 1811 and 1899. Freeman (1932) analyzed 25 geometry textbooks from 1896 to shortly after 1925. Hawkins (1935) reviewed two “interesting textbooks in geometry” that at the time demonstrated how they impacted high school courses. Donoghue (2002) discussed geometry textbooks with respect to David Eugene Smith. Ackerberg-Hastings (2000) investigated specific experiences with geometry textbooks, and in one chapter of her dissertation, focused specifically on the geometry textbooks of Day, Farrar, and Davies. With respect to geometry, Nissen (2000) evaluated six high school geometry textbooks, one high school integrated mathematics textbook series, three middle school textbook series, and four primary school textbook series, in comparison with the *NCTM Standards*. Other media which assist the learning of formal knowledge, such as geometry software, have also been explored (Clements & Sarama, 2001; Jones, 2000; Marrades & Gutierrez, 2000). Consequently, there should be a vehicle for learning mathematical concepts in a way that fuses informal and formal learning. Although the positive outcome of children learning mathematics through literature has been documented (Wilburne & Napoli, 2008), a need exists to document that mathematical fiction can be beneficial for any reader to learn mathematics at any level.

Sources of informal knowledge in mathematics have been growing since the 19^{th} century. Over the past two decades in particular, a plethora of works of mathematical fiction, including both novels and short stories, has been published per year. Movies (e.g., *Proof*, *A Beautiful Mind*) and television programs (e.g., *Numb3rs*) have also become mainstream. These sources in which informal mathematical knowledge can be attained are critical to society's perception of mathematics. Mathematical fiction specifically provides a link between the mathematics profession and the general population in a way that demonstrates the power, beauty, humanism, and utility of mathematics, which can also create an appreciation of the subject. Mathematical fiction spans a variety of genres that provide exciting mathematical experiences and has existed for as long as ideas have been written down (Padula, 2006). This learning component will not fall into the predominant perspective that portrays mathematics as simply a list of facts and procedures (Siegel, Borasi, & Fonzi, 1998). Students in a traditional mathematics classroom have felt that their role is to memorize different rules and that school mathematics and their own thoughts are separate (Boaler, 1999; Schoenfeld, 1988). Enjoyment from reading can enhance motivation to learn more mathematics (Wilburne & Napoli, 2008) and also alleviate mathematics anxiety.

Connecting literature with mathematics can further an understanding of mathematical concepts (Bosse & Faulconer, 2008; Whitin & Whitin, 2004) that are taught formally. Many novels and short stories in the mathematical fiction genre are of high literary quality and mathematical exposition. These novels and short stories can educate and motivate as well as entertain the reader. Mathematical fiction can be used before and during formal learning from a textbook.

Although many novels and short stories in the mathematical fiction genre exist, an analysis of mathematical fiction as a vehicle for mathematics education (and for mathematics outreach) has not been brought into the mainstream. "Sources are of utmost importance for research into the history of mathematics education" (Schubring, 2006, p. 87). Readers will be learning mathematics through a seemingly informal route. Various mathematical themes in fiction also include the existing culture of that period which highlights societal concerns—for example, Abbott's (1884) *Flatland* and Haddon's (2004) *The Curious Incident of the Dog in the Night-Time*. Mathematical fiction in the form of novels and short stories is only one possible focus. Sylvia Nasar's *A Beautiful Mind* served as the content for director Ron Howard's movie of the same title. Further, David Auburn's *Proof* was highly successful as both a movie and a play. Television's *Numb3rs* is a popular series by producers Falacci and Heuton. A new audience has emerged for popular works whose theme is mathematics, and it is providing a rare opportunity for mathematics educators to capitalize on this pop-cultural phenomenon.

Interest in mathematical learning through informal knowledge is accelerating (Aslaksen, 2006). However, the matter of utilizing fictions with their mathematical content or how these fictions can influence teaching has thus far received only limited scholarly attention. Many books have been written about the use of children's literature in teaching mathematics (e.g., Braddon, Hall & Taylor, 1993; Burns, 1992; Schiro, 1997; Welchman-Tischler, 1992; Whitin & Wilde, 1995), and journals such as *Teaching Children Mathematics* have published many insightful articles over the years (Cotti & Schiro, 2004). Children's literature has been established as an effective tool which aids mathematics instruction. While children's books differ from the mathematical novels and short stories discussed in this study, a common purpose connects the two—that is, to further students' interest in reading and to facilitate their effortless, inherent mathematical development. It seems important to demonstrate a link between learning geometry and reading works of mathematical fiction. According to Aslaksen (2006), "one of my main goals in mathematical outreach is to show that the beauty of mathematics is all around us" (p. 345). Therefore, a study is needed that highlights the interplay between mathematics and fictional stories exposing the reader to diverse geometric themes. Included in these geometric themes are history, philosophy, aesthetics, music, art, and elements of popular culture from the mid-19th century to the present time.

More recently, of significant value is an awareness of the educational value of mathematical fiction at the college level (Mower, 1999). In fact, professors have extolled the virtues of connecting mathematical ideas with mathematical fiction inside the classroom—for example, Davies and Trout (course at Dartmouth) and Kasman (course at College of Charleston). While much information regarding formal sources of geometry is available, studies dealing with informal sources are not plentiful. Fletcher (1971) expressed that sources besides traditional textbooks should be taken into account. Although mathematical novels and short stories were not mentioned then, these resources are readily available and growing in number. The need for more research in mathematical fiction and other non-traditional sources that contain mathematical knowledge is evident. Ollerton (2002) backs up his support for new methods of teaching mathematics with the following sentiment: “mathematics is a creative subject which exists beyond the pages of a textbook.” Mathematics is a subject that goes beyond the content of a textbook.

Purpose of the Study

The purpose of this study was to analyze mathematical fiction with geometric themes as a component of informal learning. For this study, geometry is compartmentalized into plane and solid geometry, historical foundations of geometry, axiomatics, topology, and projective geometry. Twenty-six sources of mathematical fiction, novels and short stories, were analyzed for plot, geometric theme, cultural theme, and presentation of the mathematics contained in the story. The analysis is described using three categories that sort the fiction based on dominant theme. Three choices were available: the geometric theme was dominant (the first category); the geometric theme and the culture theme were of equal value (the second category), and the cultural theme was dominant (the third category).

This study also explored the authors’ mathematical backgrounds with regard to their mathematical fiction. Pedagogical considerations received analysis for the method used by the author to introduce the geometric theme to the reader, coupled with the prerequisite knowledge that the reader should possess. The writers’ mathematical backgrounds were examined by identifying three mutually exclusive groups, including: authors with little mathematical or scientific training; authors with backgrounds in related fields such as science and engineering; and authors, including professional mathematicians, with a strong background in mathematics.

The sources are all fictional, but the geometric content is factual. In addition to the analysis of geometric themes, the investigator provided cultural patterns contained in this mathematical fiction genre. The analysis demonstrated the intersection of mathematical fiction with geometric truth that may serve to educate and entertain the reader and provide a viable outreach vehicle. Patterns emerged with regard to the geometric themes. These topics found in the selected mathematical fiction, both novels and short stories, can be utilized as a teaching tool in the classroom.

The research questions that were answered by this study were as follows:

Question 1. What geometric topics are used in the mathematical fiction genre with regard to the selected novels and short stories?

Question 2. What are the authors' pedagogical approaches to delivering these topics? What are the authors' mathematical backgrounds?

Question 3. What cultural themes are integrated with geometric topics in these mathematical fiction stories and novels?

Procedures of the Study

The sources used in this study were selected from the database[1] of mathematical fiction compiled by Dr. Alex Kasman of the College of Charleston. Kasman has constructed a list of more than 1,000 entries of mathematical fiction, including novels, short stories, films, and plays, although both films and plays were excluded from the present study. Novels and short stories that were rated as having poor literary quality and insufficient mathematical exposition were also excluded. Each entry in the database has literary and mathematical content scores. The highest

[1]This online database is located at http://Kasman.people.cofc.edu.MATHFICT/browse.php

"Math Content Rating" available is now a 5, which is the same maximum for the "Literary Quality Rating." A newly created search engine allows its user to choose the topic, for instance, geometry, along with the preferred mathematical and literary content scores. Kasman's database is now "widely recognized" as a valuable research source of mathematical fiction (Swallow, 2006).

The entire database, for the purpose of this study, was considered to be the universal set of mathematical fiction. Each subset was arranged according to topic, such as algebra, analysis, logic, probability, and so on. This study focused on mathematical fiction with geometric themes contained in novels and short stories. The sources that were collected were recognized for both their high literary quality and mathematical content. Each mathematical fiction source was investigated if it contained the following components: a geometric theme, a literary content rating of at least 2.5 out of 5, and a mathematical content rating of at least 2.5 out of 5. To further narrow the selection, the sources were chosen to highlight the growth of mathematical fiction with geometric themes. Delimitations exist within this data collection. Only those sources in the English language were used. Each source must be a story involving a geometric theme.

This study consists of over two dozen sources dating from 1852 to 2009. A qualitative analysis was used in the form of a non-random sampling technique, called purposeful sampling, based on source characteristics relevant to the research questions contained in the study (Patton, 2001). For example, the sample consisting of mathematical fiction with geometric themes may be broken down into sub-samples (subsets) such as axiomatics, projective geometry, plane geometry, historical foundations, and so forth. In purposeful sampling, sub-samples are selected sharing a common attribute. In this study, the novels and short stories in each sample have a common characteristic—the topic in geometry that they possess. Consequently, the analysis of a particular geometric theme is demonstrated. This qualitative analysis was flexible so that multiple connections could be established between the fictions and diverse geometric themes.

For each of the selected fictions, this report includes the mathematical background of the author, an overall plot summary, the major geometric theme, and the cultural context of the story. A system of coding was developed to organize the findings. Later, a grounded theory approach was employed for purposes of analysis. A grounded theory methodology was considered suitable because the literature contains no model or empirical theory focusing on the analysis of mathematical fiction with regard to geometry. The most important themes and approaches in these mathematical fictions were identified in this way.

In answering the first and second research questions, the investigator made use of the aforementioned database. Using a developed system of coding and grounded theory, the major geometrical themes and topics were identified. The same approach was employed in analyzing the authors' pedagogy.

In responding to the third question, the investigator gathered data from all selected mathematical fictions that reflect cultural trends inherent in each story. Their role and connections with the mathematical topic covered were discussed. Again, the system of coding and grounded theory approach was employed.

Chapter II

LITERATURE REVIEW

Overview

This literature review covers a span of research related to mathematical fiction, including research on informal and formal learning, linking literature and mathematics, and the current usage of mathematical fiction. It also includes definitions necessary for understanding this dissertation. Additionally, an introduction to mathematical fiction is presented in this chapter, as this topic is still developing and needs further notoriety. The purpose of this chapter is to present the relevant literature that supported this study so that the educational value of mathematical fiction becomes meaningful as a teaching and learning tool. The order of presenting this literature review of current research begins with the informal learning of mathematics via reading books other than texts. The literature and mathematics connection paves the way for mathematical fiction. Accordingly, the literature is organized into three main categories: 1) informal and formal learning; 2) linking literature and mathematics; and 3) mathematical fiction: research and usage with an introduction to mathematical fiction.

Mathematical fiction is not the standard source of classroom learning. For this reason, a look at informal learning was considered necessary. Many everyday aspects can be broken down into being informal or formal—namely, learning. Informal learning can supply individuals with mathematical knowledge. It has been recently understood that mathematical knowledge can be attained outside of the classroom and mathematics educators may consider the significant connotations of this realization (Nunes, 1992). This section also discusses informal and formal geometry.

According to Draper (2002), the mathematics classroom has changed greatly from a traditional setting. Students can learn the same information in different ways, depending on what materials have been selected for the teachers to use. Draper explains: “teaching practices that are typical of the school mathematics tradition have been recommended by and for math teachers as the most efficient and effective way to deliver instruction” (p. 521). There already is a trend in using literature and mathematics in the classroom, especially the use of books to teach mathematics. The section titled Linking Literature and Mathematics provides a brief glance at the connection between literature and mathematics and how pieces of literature are being employed in the classroom. Given that the topic of this dissertation involves mathematical fiction, the study of previous directions of using literature in teaching mathematics will be presented in this section.

Part of the section titled Mathematical Fiction: Research and Usage includes an introduction to mathematical fiction, which contains a definition and explanation of mathematical fiction, with its various uses, pedagogical considerations, and educational aims. It includes mention of the authors of mathematical fiction and ways in which mathematical fiction can be beneficial to society. This section also presents recent research and studies in mathematical fiction. Focus is placed on those studies connected to college courses, given that the mathematical fiction used in this thesis is predominately for the adult learner. Current approaches of using mathematical fiction are featured.

Informal and Formal Learning

Although the most central part of education is carried out within classrooms through the use of textbooks and other educational materials, learning also happens outside of this formal setting (e.g., Cross, 2006). Even for college students, learning takes place inside and outside the classroom, and teachers should especially encourage the latter because it enhances student learning (Kuh et al., 1994). Many activities can be part of informal learning. Researchers have been interested in informal learning and the resulting informal knowledge gained. This knowledge acquired by the individual has been investigated inside classrooms to see how teachers can utilize it for teaching mathematics (Kim, 2002).[2]

Livingstone (1999) defines informal learning as “any activity involving the pursuit of understanding, knowledge or skill which occurs outside the curricula of institutions providing educational programs, courses or workshops” (p. 51). Livingstone was interested in the informal learning activities of adults in Canada. In 1998, he conducted the first extensive survey of Canadian adults gaining information on individuals’ intentional informal learning and any activity that furthered their education.

Livingstone and Myers (2006) write that it is likely that the majority of adults continuously engage in various forms of learning. Despite this, the only type of further education that has gained full acceptance is the schooling sanctioned by the state**.** Mathematics knowledge has not been sufficiently researched and should be further examined for older students. Livingstone and Myers go on to say that “it is clear that both adults’ informal education/training and their self-directed informal learning have been relatively little explored to date and warrant

[2]Kim references Brown, Collins, and Duguid (1989), Carpenter and Fennema (1988), Green (1986), and Nunes, Schliemann, and Carraher (1993) as researchers who have investigated the role of previous knowledge in learning and of teacher preparation.

much fuller attention from those interested in comprehending the nature and extent of adult learning" (p. 205).

Fisher (2003) took note of Livingstone's findings and also worked with Canadian adults and informal learning, although her interest was only in seniors (older adults). During interviews, the most commonly reported resource was "print," which included books, magazines, journals, and newspapers. The seniors also included people, computers (and the Internet), TV, videos, radio, talks, lectures, and more. Fisher wrote: "Learning is not limited to classrooms; it is woven into the fabric of daily lives and connections" (n.p.). She also believed that as informal learning becomes more recognized and valued, learning at all ages will be given higher value and not be confined by formal educational contexts.

Davey and Tatnall (2007) followed the method used in Livingstone's study when they carried out interviews (across 11 countries) with academics. They found that the participants did not value formal learning as much as informal learning. Davey and Tatnall included the following introduction to their participants:

> Every academic does some informal learning outside of formal classes or organised programs. You may spend a little time or a lot of time at it. It includes anything you do to gain knowledge, skill or understanding from learning about your health or hobbies, household tasks or paid work, or anything else that interests you. Please begin to think about any informal learning you have done during the last year outside of formal or organised courses. First, let's talk about any informal learning activities outside of courses that have some connection with your current or possible future paid employment. This could be any learning you did on your own or in groups with colleagues, that is, any informal learning you consider to be related to your employment. (p. 244)

Gurganus (2007) notes that informal knowledge is important with older students as well. The older the individual is, the more challenging accessing informal knowledge can be, and the more difficult it will be for the individual to remember the knowledge correctly. Misconceptions may have been stored for years and, if so, are firmly implanted in the individuals. Gurganus goes on to explain that these misconceptions are usually built on by inadequate concept development during formal instruction or by inadequate informal experiences.

The National Science Foundation (NSF) (2001) states that every individual partakes in informal learning as he or she acquires information. This information is gained from casual interactions and from the use of available resources. In a document titled *Elementary, Secondary, and Informal Education*,[3] the NSF (2001) make an announcement about an informal education program and offer information backed by research. This document states that "People of all ages learn science, technology and mathematics from experiences in science museums, from watching films and other media, and from participating in community activities." The technology around us shapes what should be learned, where learning takes place, and how learning occurs. The Division of Elementary, Secondary, and Informal Education (ESIE) wants to look beyond the traditional textbook standard. One of ESIE's many objectives is to offer motivating opportunities of learning outside of school.

To achieve its objectives, ESIE provides stimulating opportunities outside of school to promote appreciation, interest, and understanding of science, mathematics, and technology (SMT) for youth and adults through its Informal Science Education (ISE) program; supports the development of high-quality course and curriculum materials for all students through its Instructional Materials Development (IMD) program; and strengthens teachers' content knowledge and pedagogical skills and creates an infrastructure of professional educators, educational researchers, and administrators to support SMT education reform through its Teacher Enhancement (TE) program.

[3]This document can be viewed through the following link: http://www.nsf.gov/pubs/2001/nsf0160/nsf0160.txt (Quotes are not given a referenced page as there are no page numbers in this document due to the .txt format.)

Many researchers mention that individuals tend to separate formal and informal learning. They do not often recognize the informal learning and informal knowledge gained. Bishop (1988) concludes that individuals of all ages can be unaware that they are partaking in mathematical activities. Kim (2002) writes that based on the argument that learning is actively constructed by connecting new information to informal knowledge,[4] "the connection between informal and formal mathematical knowledge could well be made for instruction" (p. 35). Creating situations that cause interaction between this dichotomous knowledge can be beneficial. Gravemeijer and Doorman (1999) comment that many approaches fail to bind informal and formal knowledge, and in an ideal situation, formal knowledge naturally flows from students engaging in an activity.

In *Children's Mathematical Thinking,* Baroody (1987) writes, "informal mathematics, then, is the foundation for 'mastering the basics' and successfully tackling more advanced mathematics" (p. 35). This comes after Baroody indicates that according to cognitive theory, students are not blank slates; rather, they enter school with informal mathematical knowledge. One of the major educational implications of this is "formal instruction should build on children's informal mathematical knowledge" and "gaps between informal knowledge and formal instruction frequently account for learning difficulties." Baroody writes that it is imperative for this informal knowledge to be a factor during educational planning. It can also make formal instruction more exciting and meaningful. When formal instruction disregards informal knowledge, students tend to memorize facts and not create meaningful connections.

Although informal mathematics may create a valuable foundation for learning and building the mathematics taught in school (e.g., Baroody, 1987; Bull et al., 2008; Ginsburg,

[4]There are many synonymous labels for informal knowledge throughout the literature such as: intuitive knowledge, everyday mathematics, practical mathematics, street mathematics, situated knowledge, and home mathematics.

1989; Kim, 2002; Mack, 1990), many teachers do not reference this informal knowledge due to its potential hindrance, and thus teachers usually ignore it instead of realizing its potential strength (O'Toole, 2006). Teachers must be cautioned when tapping into students' informal knowledge, such as during the integration of formal instruction with fractions, since there may be some initial discord (Mack, 1990). O'Toole (2006) writes that teachers first must acknowledge informal knowledge, then incorporate this knowledge into formal mathematics.

Informal learning should help individuals recognize that mathematics is not only for classrooms. "Many students come to believe that school mathematics consists of mastering formal procedures that are completely divorced from real life, from discovery, and from problem solving" (Schoenfeld, 1987, p. 197). Glass (2002) comments on traditional instruction and its propensity for students to absorb knowledge. While dealing with informal mathematical knowledge, the learning process should not feel forced. Mathematical truths are being learned but in a different way. Boaler and Greeno (2000) discuss that social practices can add a context for learning mathematics, and there is even a view that "participation in social practices is what learning mathematics is" (p. 172).

While knowledge can be classified as either informal or formal, Hewitt (2006) contends that this system of knowledge is complex. Learning accurate knowledge in just one field is an important task; one needs to be careful when tying informal and formal knowledge together. Teachers should try and anticipate what informal knowledge their students possess to help plan for successful teaching. More studies are recommended, especially in higher education, regarding effective teaching by assessing informal mathematical knowledge. Artigue (1999) points out that not much effort has been given to research on teaching at the university level, although Calculus has received some attention.[5] Lastly, Livingstone's (1999) findings involving Canadian adults encourage others to pay more attention to adults' informal learning, which should have a role "in shaping educational, economic and other social policies; adult educators should take this detectable informal learning into greater account to develop more responsive further education opportunities" (n.p.).

The exact definition of informal learning varies, but researchers continuously mention certain underlying sources. Informal learning can occur through conversations, conferences, books, the Internet, and other daily occurrences (e.g., Davey & Tatnall, 2007; Fisher, 2003). Researchers point out that it is difficult at times to distinguish when informal learning takes place (e.g., Burns & Schaefer, 2003; Fisher, 2003; Schugurensky, 2000). It is generally agreed upon, though, that informal learning does not take place in a formal classroom setting (e.g., Burns & Schaefer, 2003). Therefore, knowledge that has not been fully developed inside an institution of learning is classified as informal.

Informal and Formal Geometry

This dissertation focuses on only one of the various mathematical fields—namely, geometry. The sheer multitude of works of mathematical fiction required the selection of a concentrated field in order to explore it. Geometry is a field of study that is required of all individuals going through the school system and is taught at multiple grade levels. According to Ringenberg (1967):

> Geometry is a branch of knowledge with origins in antiquity. It includes our great heritage of knowledge regarding those properties of space and physical objects which have to do with the form, shape, and size of things. Geometry has its roots in man's

[5] Artigue (1999) goes on to explain that Calculus receives the most attention because it is the mathematical area with the majority of undergraduate failures.

> experiences with physical objects. Modern geometries are abstract structures created by man in his efforts to organize geometrical facts and to create more geometry. (p. 1)

Historically, geometry since Euclid was considered to be an example of a strictly formal and deductive field. Since the early part of the 20th century, the need and importance of informal geometry has been recognized (Hamlin, 2006). As Hamlin explains, "In the United States informal geometry has been considered, for the last century, the study of geometric concepts and reasoning without the formality of proof" (p. 29). What may be the earliest reference to informal geometry in the United States took place at the Committee of Fifteen in 1921, where the importance of teaching informal geometry to young students was one topic of discussion (p. 30).

"The basis of informal geometry is the physical world. Informal geometry is developed through intuition, experimentation, observation and inductive reasoning" (Ringenberg, 1967, p. 3). Formal geometry includes knowledge and usage of terms, postulates, definitions, and theorems. Ringenberg designed his text, *Informal Geometry,* into separate chapters according to this division. Informal geometry has also been recognized as a vehicle to interest students in geometry without beginning with a "formal treatment of the subject" (González & Herbst, 2006).

Many courses are entitled "Informal Geometry" in various school systems. The "informal" in the title implies that the course will not focus on formal proofs, but rather on other pedagogical methods such as letting students conduct experimentations and design their own conjectures. "Formal geometry, also called demonstrative geometry, is therefore concerned with proof" (Pandiscio, 2005, p. 331).

The Fifth Yearbook of the National Council of Teachers of Mathematics (NCTM), entitled *The Teaching of Geometry* (Reeve, 1930), presents a rationalization for both formal and informal geometry, as certain students can find one type more advantageous than the other. Informal geometry should make learning geometry more entertaining (Peterson, 1973). Informal geometry also focuses on connecting new ideas to experiences and examples instead of connecting new ideas to more abstract mathematics (Aichele & Wolfe, 2008).

Billstein, Libeskind, and Lott (1999) write:

> From knowing that the world is flat, to measurement between planets, to measurement between cells to Mandelbrot's work with fractals, geometry has changed as our knowledge base has expanded. In schools, too, geometry has changed. We no longer consider geometry in isolation from the rest of mathematics. (p. 462)

The authors express that as a result of the aforementioned changes, what constitutes geometry has changed as well.

Linking Literature and Mathematics

Relating mathematics to other subject areas (not only with science) in the curriculum is a challenging feat (Griffiths & Clyne, 1991). A way to assist people in delving into mathematical concepts is to create comfort by providing special contexts and familiarity (Griffiths & Clyne, 1991; Whitin, 1994). Educators continuously look for effective teaching methods to unite mathematics with reading, writing, and oral language due to current curricular trends. A popular and successful method has been using children's literature in mathematics classrooms (e.g., Welchman-Tischler, 1992).

Wilburne and Napoli (2008) note that "Literacy scholars advocate that reading is an interactive constructive process that allows students to interpret and comprehend content subjects such as mathematics while also promoting thinking skills" (p. 2). Aside from the evident value of reading, literature can be seen as a tool to inspire and achieve mathematical understanding and as an aid to teaching mathematics.

The foreword for *New Visions for Linking Literature and Mathematics* (Whitin & Whitin, 2004) claims that although teachers are gradually more aware that children's literature can add to the teaching of mathematics, acting on this requires more than some teachers are willing to give, which is the reason for the book—to let the authors help teachers use literature for teaching mathematics. Also mentioned is how literature and mathematics link together, specifically within this book; e.g., through the selected math-related books, through suggestions given by the authors that will engage students with literature, and through compliance with Standards from English language arts and mathematics.

New mathematical knowledge and skills can be linked with meaningful contexts if students can connect with a story or find enjoyment in a character, setting or storyline (Kolstad, Briggs, & Whalen, 1996). Literature can change the view of students and non-students alike regarding their perception of mathematics and their willingness to learn the subject. Individuals who are reading a novel or short story are acquiring tangible mathematical knowledge. This knowledge can then be developed further, given the resources for the individuals and their willingness to explore the mathematical topic in more depth.

Integrating Literature and Mathematics

The connection between mathematics and literature is not a new concept; its growth and interest have been documented over the last few decades. Sriraman and Beckmann (2007) mention that "Mathematics and literature have an ancient affinity as seen in the writings of the Greek philosophers, medieval theologians and natural philosophers of the post Renaissance period."[6] According to the Mathematical Fiction website,[7] the earliest work available of mathematical fiction can be traced back to the year 414 B.C. with Aristophanes' classic Greek

[6]This quote is taken from Sriraman and Beckmann's abstract during the *Proceedings of the 9th International History, Philosophy and Science Teaching Group (IHPST)* in Calgary, which took place June 22-26, 2007.

[7]Alex Kasman of the College of Charleston has compiled a list of mathematical fiction at the site http://kasmana.people.cofc.edu/MATHFICT/ which has been reviewed and continues to be a go-to source of mathematical fiction for mathematics educators, writers, and avid readers.

play entitled *The Birds*. One particular scene in this play contains geometrical references. It should be noted that the amount of mathematics and its role in the various pieces of mathematical fiction differ greatly. In this particular play, mathematics is not a central theme. Yet the play makes a reference to a famous Greek geometer, Thales, and a geometric construction using a straightedge and compass to inscribe a square within a circle.[8] While these works were excluded from this dissertation, they may serve as a springboard for further mathematical discussion and learning.

According to Bernstein et al. (2008), “Many teachers, like their students, still think of math as a totally separate subject from language arts” (p. 1). Mathematics educators need to demonstrate the connections between mathematics and other subjects. Integrating language arts with mathematics has been a documented instructional goal with the Standards of the National Council of Teachers of Mathematics (NCTM) since 1989. Various studies shed light on the connection between literature and mathematics (e.g., Haury, 2001; Murphy, 2000; Smith, 1999; Usnick & McCarthy, 1998; Welchman-Tischler, 1992). These studies propose how literature is beneficial to the learning of mathematics, whether by generating interest or by helping readers relate to a cultural context or fascinating plot. The findings suggest that scenes from a particular novel can be used as a catalyst for further mathematical discussion.

Wilburne and Napoli (2008) identify various studies in favor of connecting literature and mathematics, showing that “there is a strong correlation between learning mathematics content by listening to and interacting with mathematical stories” **(p.1).** A number of reasons suggest why the integration of these areas is important, including the strengthening of real-world connections and the intention to motivate readers in mathematics with this unique platform.

[8]“METON: With the straight ruler I set to work to inscribe a square within this circle; in its centre will be the market-place, into which all the straight streets will lead, converging to this centre like a star, which, although only orbicular, sends forth its rays in a straight line from all sides.”

Another aspect of the importance of literature and mathematics comes from a simple notion: mathematics requires more than just manipulating numbers. Balas (1997) comments how mathematics goes beyond numbers, much like how reading requires more skills that identifying letters. Working with mathematics can involve word problems, discussions, presentations, reading, reports, and much more. Fogelberg et al. (2008) have a similar view that “Doing math is more than just arithmetic; it also includes understanding and solving math problems, and reading and understanding the math book” (p. 67). Those wishing to partake in mathematical activities need to have such skills and develop them.

The integration of literature and mathematics also increases communication. Moyer (2000) discusses the benefits of children’s literature, which include helping children with communicating mathematical concepts. Lake (2008) reports that “Math-related literature provides a real-world feeling to mathematics” (p. 26) and books are a great vehicle for fostering mathematical-communication skills. Lake also writes, “There are many ways to use books to ensure that children are afforded as many opportunities as possible to do math both inside and outside the classroom” (p. 8). If mathematical learning can occur outside the classroom, mathematics might not seem so difficult and as an isolated subject learned only in school with no real-world relevance.

Readers of mathematical fiction can connect mathematical ideas into a significant context with their stories. Teachers can also help students recognize the mathematical value inherent in these novels and short stories. This, of course, is not a simple task for many teachers. Blintz and Moore (2002) comment on this situation and offer advice to resolve the dilemma that teachers of varying experience have while using literature to teach geometry and measurement in the classroom. Some comments from teachers include being uncomfortable with the idea of using literature to teach mathematics because they lack training and experience with literature.

During this section, a variety of important reasons are given for integrating literature and mathematics. Besides those views already mentioned, Lake (2008) states, "Math-related literature encourages integration with other subject areas" (p. 23). The increased visibility between mathematics and different fields can help individuals understand how mathematics is not just an isolated subject studied in school. Zemelman, Daniels, and Hyde (1998) state that "Mathematics is not a set of isolated topics, but rather, a science of patterns and relationships" (p. 90). They explain that students need to see the connections between mathematics and other areas.

Using Literature to Teach Mathematics

Extra consideration has been given to an integrated curriculum over the past two decades. This integration has mainly focused on connecting literature to teaching mathematics. Bishop (1988) describes reading as a way of learning and this extends to learning mathematics. Recently, many instructors are recognizing children's literature as an effective way to teach mathematics (Nolan, 1997). Mathematical fiction courses have even begun to appear in colleges and universities.[9]

There are different areas of research on using literature to teach mathematics, and some of these subsets are presented in this section. The specific areas that are briefly examined include

[9]These institutions include, but are not limited to, the College of Charleston, Dartmouth, and Westfield State University. At the College of Charleston, Kasman taught an Honors Seminar on "Mathematics and Fiction." A link to the course webpage can be found here: http://kasmana.people.cofc.edu/HONS380/. Dartmouth College offered a course called Mathematics and Science Fiction taught by Davies and Trout. Their course page address is: http://www.math.dartmouth.edu/archive/c18w99/public_html/. At Westfield State University is a listing for the course "Studies in the Literature of Mathematics," available at the site for all course descriptions in the department at http://www.westfield.ma.edu/math/courselist.asp#216.

comparisons between textbooks; curricular and educational goals; the role of mathematics anxiety, and the perception of mathematics. There is also mention of some potential pitfalls in using literature to teach mathematics.

Mathematical fiction and children's literature are favorable to mathematics textbooks in many aspects. Siegel, Borasi, and Smith (1989) state that "reading mathematics, in particular, presents a challenge due to the unique qualities of mathematics textbooks" (p. 1). Lamme and Ledbetter (1990) point out that textbooks will not tug on the emotions of their readers. Students do not tend to form connections with their textbooks as they can with stories. According to Nolan (1997), "While textbooks provide a comprehensive sequential presentation of their content which serve as blueprint for students, they often fail to provide the motivation many students need in order to learn" (p. 19). Nolan continues to discuss how children's literature tends to "excite and enlighten readers," while textbooks do neither simultaneously. Balas (1997) asserts that mathematics learners derive motivation, along with context, from reading. Textbooks may provide a short blurb about the context of a certain topic, but the way a novel or short story provides more than just a blurb is unparalleled. In the following section titled Mathematical Fiction: Research and Usage, more comparisons between textbooks and mathematical fiction are discussed in reference to informal and formal learning.

Haury (2001) states that a method for linking life outside of the classroom to the mathematics taught inside schools is to consider everyday literature, which contains mathematics. Mathematics from a source connecting to those beyond the classroom can "reveal the mathematics inherent in human thinking and communication about life experiences" (n.p.). Fletcher (1971) believes that

> In particular a geometry course which envisages making a number of optional studies available on an individualised basis will certainly need reading material, and reading

> material of different kinds suited to the problems in hand. Thus one must consider not only textbooks of a traditional kind but also a range of books going from highly didactic programmed material to freer reference books which cover historical background or further applications of mathematics, and place the work of the course in a wider setting. (p. 67)

Siegel, Borasi, and Smith (1989) explain how views on mathematics education and goals for the subject will determine what types of reading material are suitable and beneficial for acquiring mathematical knowledge. The authors mention that what is being read in the classroom is determined by the view of those creating the curriculum in use. “A curriculum grounded in a view of mathematics as a ‘way of knowing’ would welcome other kinds of reading materials in addition to textbooks” (p. 4). Textbooks tend to keep students in the mindset that mathematics is an activity done inside the classroom. (While this dissertation solely considers novels and short stories, other non-textbooks can also be used in teaching mathematics. Siegel, Borasi, and Smith [1989] list historical essays, philosophical arguments, stories, and poems as sources that can demonstrate the beauty and “value-laden aspects of mathematics” [p. 4].)

Educational goals, such as those promoted by the National Council of Teachers of Mathematics (NCTM), should find their way into the mathematics curriculum. These goals, such as communication and an integrated curriculum, go hand in hand with using literature in the mathematics classroom. This natural occurrence enhances language arts skills along with mathematical skills. According to Whitin (1994), children’s literature is the idyllic medium for teaching mathematics in terms of meeting these goals. Gates (1992) calls for using creative tactics to help students acquire mathematical knowledge that will connect them to the subject in a meaningful way.

The NCTM (2000) includes the campaign of mathematical understanding through forms of written communication in the Principles and Standards for School Mathematics. Borasi, Siegel, Fonzi, and Smith (1998) reference that the NCTM (1989), along with the National Research Council (NRC, 1989), call for a rethinking of curricular goals in mathematics due to advances in technology and the prevalent negative disposition towards mathematics.

Reading and writing across the curriculum are significant goals in classrooms today. Using literature provides a natural connection between mathematics and reading that has a widespread appeal. According to Fogelberg et al. (2008), “Teachers have the awesome responsibility of providing a challenging curriculum and powerful instruction that engages students and enables them to be successful in school and in life” (p. 18). Reading is undoubtedly an integral part of today’s society, both inside and outside of the classroom. “We are not only using language arts as tools in the study of mathematics, but are also helping students to meet standards in both disciplines” (p. 18). Combining literature with mathematics enhances these two areas of study. Flick et al. (2002) discuss another valuable relationship between reading and learning science and mathematics that require higher level thinking. Using science and mathematics as the backdrop for reading comprehension may address sophisticated learning goals.

Teachers can motivate their students while advancing their knowledge base. Promoting activities that necessitate higher-level thinking in the mathematics classrooms can replace activities that involve students performing disconnected tasks and rote computations. Hunsader (2004) writes, “Engagement with literature provides a natural way for students to connect the abstract language of mathematics to their personal world” (p. 618). Furthermore, “The most striking stories reveal mathematical situations in unexpected circumstances” (Swallow, 2006, p. 762).

Kenney (2005) states that “Mathematics is truly a foreign language for most students: it is learned almost entirely at school and is not spoken at home. Mathematics is not a ‘first language’; it does not originate as a spoken language, except for the naming of small whole numbers.”[10] According to Fogelberg et al. (2008), literature can comfort students by providing some familiarity (i.e., having a story) while presenting new mathematical vocabulary and ideas. Although the example provided by the authors involved read-alouds, novels and short stories might also benefit adults in a similar way. As mentioned, involving interdisciplinary approaches to teaching that take this notion of mathematics as a foreign language has obvious value and can help decrease students’ fear of and anxiety about mathematics.

Using literature, including mathematical fiction, can alleviate the epidemic of mathematics anxiety that is especially prevalent in the United States. One of the first definitions of mathematics anxiety from Richardson and Suinn (1972) explains that it involves “feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations” (p. 551). More currently, Sousa (2007) believes that “Students at all grade levels often develop a fear (or phobia) of mathematics because of negative experiences in their past or current mathematics class or have a simple lack of self-confidence with numbers” (p. 171). Ways to combat this serious issue include furthering student confidence and allocating appropriate, non-traditional tasks that will interest students. Mathematics anxiety affects most students throughout their academic careers, not just younger students. Currently, most college students experience some form of mathematics anxiety (Perry, 2004). To address this epidemic, teachers can create a non-rigid classroom environment while providing tasks that are motivating and less stress-inducing.

[10]This excerpt by Joan M. Kenney can be viewed at http://www.ascd.org/publications/books/105137/chapters/Mathematics-as-Language.aspx.

Olness (2007) advocates the use of literature to prevent mathematics phobia. She explains that it is important, especially for girls—who experience the most cases of mathematics anxiety—to see mathematics as an everyday activity. Olness gives her own account:

> I was one of those girls who was convinced I could not excel in mathematics. But I continue to see connections when I share good books with children and am often surprised when I am able to understand mathematical concepts and see relationships between mathematics and everyday activities. I truly believe that if the current trade books had been available when I was in elementary school, I could have avoided years of math phobia! (p. 107)

This same idea may hold true for all good books, whether for children or adults.

Using literature in the classroom does not guarantee a success story. Teachers' knowledge of literature containing mathematics should be assessed before he or she begins using a novel or short story in the classroom. Carpenter, Fennema, and Franke (1997) reinforce this by stating that "teachers' actions are governed to no small degree by the knowledge they bring to a situation" (p. 3). Teachers' knowledge is thus a deciding factor in teaching. Those implementing mathematics instruction through mathematical fiction or children's literature need to have a complete understanding of the material and how to effectively teach in this non-traditional way. Teacher education programs should prepare its preservice teachers with knowledge on how to use literature to teach mathematics (Wilburne & Napoli, 2008). Teachers, independently, can explore and become educated in using literature with the various resources available (e.g., Thiessen & Smith, 1998). These resources provide annotated bibliographies of children's books with notes on the content and its appropriateness along with information such as activities related to the story, the writing style of the author, and more. The January-March 2007 issue of the journal *Problems, Resources and Issues in Mathematics Undergraduate Studies* (PRIMUS) features ways to use mathematical fiction in higher education. If literature is to have an impact and boost mathematical learning, the selection of appropriate literary material is an important consideration.

Welchman-Tischler (1992) mentions various ways to use children's literature to enhance the learning process for mathematics students, including establishing a creative mathematical interaction and using literature as a precursor to a mathematical topic. This is not to say that all traditional tasks lack creativity, but the general perception is that many traditional mathematical activities do not result in many successful outcomes. Literature used to teach mathematics is a non-traditional task that seems to fit into the category of a "rich task."

In *How the Brain Learns Mathematics,* Sousa (2007) provides a table comparing traditional with rich tasks. Below is a verbatim list of those tasks he describes as "rich" that relate to using literature to teach mathematics:

- prepare for success outside of school
- address learning outcomes in mathematics and other subject areas
- provide an opportunity to use broad range of skills in an integrated and creative fashion
- are authentic and in context
- encourage thinking, reflection, and imagination
- allow for demonstration of a wide range of performance
- provide enrichment within the task
- encourage the use of a wide variety of teaching and learning strategies
- encourage greater engagement of students and teachers in the task (p. 177)

Using mathematical fiction in the classroom can be another way to create a non-traditional mathematical environment in school. Given the proper environment, classrooms can create unique experiences for mathematical learning in relation to reading. Siegel, Borasi, and Fonzi (1998) mention, in regard to engaging students in inquiry, that "at present, however, there is very little in the mathematics education literature that can help teachers with this challenging task" (p. 381). Further research should be done at all grade levels, including college, to demonstrate that mathematical fiction is of beneficial use in this regard. Novels or short stories might guide mathematical inquiry and investigation in an underused but structural way.

Mathematical texts are different and more challenging than other texts students read (Bernstein et al., 2008). Flick (2002) reports that mathematics texts have a unique organization which requires special skills, and emphasizes that novels are read in a much different way than learners who read mathematics textbooks. Westbury (1990) states,

> It is a truism that textbooks are the central tools and the central objects of attention in all modern forms of schooling. As givens in particular situations, the textbooks teachers have are the most significant resource for their teaching and often the most significant limiting force they face as they seek to accomplish their purposes. (p. 1)

Giving students a non-textbook that contains mathematics has many benefits. As previously mentioned, textbooks are very different from novels and short stories. Allowing students to be introduced to mathematical topics in a relaxing, familiar way can make learning mathematics more enjoyable. It can also facilitate mathematical conversations. Communication is an important educational aim endorsed by the NCTM.

Linking literature and mathematics can yield a number of benefits. Integrating these two areas will serve to educate the public and enhance the public perception of mathematics in a positive light, whether as the reader's conscious effort or not. Books can be influential tools for language, along with mathematical concepts (Whitin & Whitin, 2004). Literature can be one tool used to teach mathematics, regardless of the focus of the course.

Wilburne and Napoli (2008) believe that teacher education programs should look into conducting similar studies since the preservice teachers who took part had positive changes in the way they viewed, felt, and enjoyed using literature to teach mathematics. There are many areas to explore within this category of using literature to teach mathematics. Although children's literature has gained much popularity in teaching mathematics, the use of novels and short stories to teach mathematical subjects in college courses is rare.

The Professional Standards for Teaching Mathematics from the NCTM (1991) explain that teachers are responsible for altering how mathematics is taught and learned in schools. Teachers play an important role in the recommended shift "towards connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures." Teachers need to be aware of different and creative ways to integrate mathematics with other subject areas and to show that mathematics is an integral part of the everyday world. There seems to be a current discrepancy in the use of literature to teach mathematics. The literature mostly discusses the utility and power that using mathematical texts has for children. Children's literature specifically designed to teach mathematics is readily available while this is not the case for higher education.

Using literature to teach mathematics has changed dramatically over the past 20 years. The NCTM, technological advances, and the sheer number of classroom studies have been factors in the growing significance of literature as a teaching tool for mathematics learning. Implementing books (in both primary and secondary school) is no longer as daunting a task for teachers given the current and ever-growing number of books, articles, online websites, and various studies devoted to this area. Now, the use of literature in conjunction with teaching has been acknowledged as a creative and motivational tool when utilized properly by well-informed instructors and parents.

An entire issue of the *Journal of the British Society for the History of Mathematics* (BSHM: V.25:2, 2010) is devoted to mathematical fiction. One such article in the BSHM concerns some of the uses of mathematical fiction at the college level. This article titled "From Sylvia Plath's The Bell Jar to the Bad Sex Award: A Partial Account of the Uses of Mathematics in Fiction" cites many examples of mathematical fiction that may alter the perception of mathematics in society (Mann, 2010). Dedicating his article to Martin Gardner, Tony Mann writes about *Flatland* as a "canonical Victorian mathematical novel." Mention is also made of Mark Haddon's *The Curious Incident of the Dog in the Night-Time.* Both of these books were selected for discussion in the present study.

Among the uses of mathematical fiction in Mann's article are "fiction expounding mathematics, fiction about real mathematicians and fictions about mathematical ideas" (p. 60). The role of mathematics in plot development is highlighted with references to historical figures like Gödel, B. Russell, Newton, Ramanujan, and Pythagoras.

Pythagorean Crimes by T. Michaelides (2006) and *Pythagoras' Revenge* by A. Sangalli (2009) are discussed by Mann, and again both works appear in the present study. Mann expresses the belief that mathematical fiction is a viable vehicle for mathematical outreach. On page 64, Mann writes, "I find the incidental appearance of characters who just happen to be mathematicians an encouraging sign that mathematics is no longer universally regarded as a strange, unwholesome activity but is regaining its place as a natural human activity." Mann concludes that mathematics and fiction are not mutually exclusive as creative literary expression and that mathematical fiction may alter the popular perception of mathematics.

Another article in the BSHM dealing with mathematical fiction is “Let Us Put on the Shade of Newton: Isaac Newton on Stage, 1829-2006” (Wardhaugh, 2010). This article discusses the fictionalization of Newton through many staged plays from 1829 to 2006. George Bernard Shaw’s play *Good King Charles* (1939) and *Newton’s Hook* by David Pisner are discussed with six other plays about Newton. “Without Newton, there would be no play” (p. 79) is the theme of this article.

Connecting mathematics to poetry and literature is considered in “Mathematics, Poetry, Fiction: The Adventure of the Oulipo” (Bellows, 2010). The Oulipo, which stands for *Ouvoir de Literature Potentielle,* was formed in Paris in 1960 and consists of writers and scholars whose aim is to investigate “what mathematics could do for literature” (p. 105). The members of Oulipo construct algorithms for poems using combinatorics. With regard to novels, Oulipo applied Knight’s tours, tree diagrams, and Latin Squares to arrange the chapters and settings.

Mathematical Fiction: Research and Usage

An Introduction to Mathematical Fiction

The term “mathematical fiction” is used to describe the genre of fictional works that contain mathematics. Media of mathematical fiction vary and include but are not limited to novels, short stories, movies, plays and poems. Much as mathematics is not limited to one topic, neither are the subjects considered in works of mathematical fiction. Numerous mathematical fields are represented in mathematical fiction, such as number theory, algebra, geometry, and analysis. For the present study, only mathematical fiction regarding geometry was selected. Mathematical fiction can serve multiple purposes, including as an instructional tool used in all walks of academia or as a leisurely activity for any individual who enjoys reading.

Reading and learning mathematics through mathematical fiction may accomplish two important educational goals; namely, humanizing mathematics and exploring mathematical ideas through literature. References to mathematics in fiction as major themes of the story can reflect and shape how society perceives mathematics. Enjoyment from reading mathematical fiction can enhance motivation to learn more mathematics and may alleviate math anxiety (Swallow, 2006).

Tracing the evolution of mathematical fiction as a vehicle for mathematical outreach is unique in mathematics education (Kasman, 2005). Mathematical fiction provides flexibility in the examination of mathematical ideas in general and geometric concepts in particular. Emphasis on interdisciplinary courses across the curriculum is indeed not only possible but also desirable because it combines the elements of reading, writing, and mathematical literacy. The cement that holds these components together is mathematical fiction.

Connecting mathematical fiction to geometric themes along with its cultural context is highly advantageous because it motivates and educates the reader (Storey, 1996). For example, in *Flatland: A Romance of Many Dimensions* (Abbott, 1884), rigid class structure and gender bias in Victorian England are powerful cultural themes. This societal concern enhances the mathematical theme of geometric dimensionality in *Flatland,* which was an integral part of geometry in mathematics education during Abbott's life. Mathematical fiction can be investigated by depicting its geometric theme and its corresponding cultural component.

While analyzing Abbott's *Flatland*, immediate parallels can be made between dimensions and gender bias in Victorian England. The ideas of dimension in geometry and the roles of women are intertwined by the characteristics Abbott assigns to the shapes in his story. For example, the women are a certain shape and dimension in *Flatland* for a reason. In Haddon's (2004) *A Curious Incident of the Dog in the Night-Time*, the main character might make the reader reflect on societal attitudes towards the mathematical talent of a teenager with autism and reconsider the preconceived notion that autism and mathematics are mutually exclusive (Pinter, 2007).

Many mathematics educators are concerned with how mathematics, as a subject, is being portrayed in mainstream society, along with the stereotypes assigned to those who enjoy mathematics. Representations of nerds and geeks and “mad” geniuses proliferate in popular culture (Goff & Greenwald, 2007). Being a “math geek” may imply that one can appreciate the beauty and utility of mathematics. Through mathematical fiction, such individuals are admired for their mathematical talent as well as for their excelling in other fields. Mathematical fiction dispels the negative connotation associated with being a mathematics nerd. For the present study, characters in mathematical fiction are intelligent, innovative, sensitive people who appreciate the elegance and simplicity of geometry. In this regard, the characters reflect the best intellectual and cultural aspects of society (Kasman, 2005). Credit must go to the authors of mathematical fiction who create engaging literary works containing mathematical elements that are blended together with cultural themes.

Novels and short stories dealing with the genre of mathematical fiction have connections to mathematical pedagogical considerations (Bull et al., 2008). The geometric and cultural themes noted in each story presented in the current study will perhaps both motivate and educate readers to view mathematics as a human endeavor and think of their geometric environment as ubiquitous. In each source of mathematical fiction there exists mathematical truth. While the sources are all in the mathematical fiction genre, the content nonetheless deals with geometric topics that are non-fiction. This intersection of fiction and mathematical truth serves to enlighten and entertain readers. Weaving a good story around a mathematical theme exposes readers to literature and mathematics simultaneously. Thus, this interplay between mathematics and a fictional story transports readers into such areas as history, philosophy, aesthetics, music, art, psychology, and other fields of inquiry (Bidwell, 1993).

Emphasis is on the informal learning component that was discussed in the earlier section titled Need for the Study. This component is applied through mathematical fiction that deals not only with mathematics but also cultural concepts that are not usually found in mathematics textbooks. Cultural concepts "can be a powerful method for engaging diverse audiences" (Greenwald & Nestor, 2004). According to the authors, student enjoyment of popular cultural concepts that are integrated with mathematical ideas can reduce math anxiety. Many novels and short stories in the mathematical fiction genre explore the representation of women in mathematics and some stories expose gender bias (Fennema et al., 1996). Most important is that mathematical fiction raises awareness and sparks interest in mathematical ideas as in *The DaVinci Code* (Brown, 2003).

Part of the value of mathematical fiction is its use as a pedagogical tool to educate and enlighten the reader in both mathematics and the language arts. Frequently the mathematical theme of the story is presented as a mini-lecture. Swallow (2006) writes, "Weaving mathematics and fiction together is certainly an art, and our expectations for the genre are nontrivial. Mathematical fiction must explore the intersection of the world of mathematics with the rest of humanity" (p. 762). An illustration of the above is the mini-lecture that appears in *Pythagorean Crimes,* where a mystery and the axiomatics of plane geometry are blended together (Michaelides, 2007). Swallow sums up the value of mathematical fiction in the August 2006 issue of the AMS journal as follows: "for the student new to the world of mathematics, the stories provide an accessible and entertaining look into some significant mathematics, as well as into what it means to be a mathematician" (p. 763).

Mathematical fiction includes a diverse group of authors, both male and female, with no geographic boundaries and many who are in fact noted writers. Included in the present study are two particularly famous writers: Aldous Huxley, who was a well-known author, and Martin Gardner, who was a popular writer in recreational mathematics. Beyond the story itself, mathematical fiction explores the culture at the time the story takes place. The culture transmits societal values through the characters in the story. A rich history of mathematical evolution unfolds through cultural changes which impact the mathematical content of the story. In sum, mathematical fiction is an outreach vehicle that connects mathematics to other disciplines; namely the humanities. Two seemingly diverse disciplines as mathematics and literature are bridged via mathematical fiction. The perception of a gap between the humanities and mathematics may be exaggerated because they can be fused into a new discipline that connects the qualitative aspect of writing with the quantitative aspect of mathematics (Goff & Greenwald, 2007).

Current Usage and Research of Mathematical Fiction

A previous subsection of this chapter presented an introduction to mathematical fiction. The aim of the last sections in this literature review is to provide readers with current research in this emerging field and discuss the use and future of mathematical fiction.

McCallum (1999) explains that in mathematics, only one correct answer is possible, whereas in mathematics education, there is no universal agreement on the best teaching strategies. He suggests, "in addition to listening to each other, we need to take the next step and learn to listen to voices from outside our profession" (p. 135). That is, to improve the current system, benefits can be gained from observing successful teaching and learning in other fields. As employing children's literature is becoming common practice to teach content areas in primary and secondary schools, this tool should be closely considered for higher education as well.

Ollerton writes, "Mathematics is a creative subject which exists beyond the pages of a textbook" (Gates, 1992, p. 273). Mathematics educators have been moving towards a more creative environment in the past few decades and away from a traditional classroom. Educators have distanced themselves from classrooms that involve only a chalkboard as a way to teach mathematics and, although it is still in use, Smart Boards, computer aids, manipulatives, overheads, and other tools are now available to help teachers at all grade levels.

Despite the significant changes in these teaching tools, the main physical source of knowledge students are given is a textbook. In most classrooms, this is the only source. "While textbooks provide a comprehensive sequential presentation of their content which serves as a blueprint for study, they often fail to provide the motivation many students need in order to learn" (Nolan, 1997, p. 19). Mathematical fiction is an effective, non-traditional tool for teachers to use, especially with older students, that can help inspire and motivate its readers.

The volume of work in the mathematical fiction genre has certainly increased and continues to expand, given the number of anthologies and articles being published and the number of college courses being designed. Creating an increased interest in mathematics through novels and short stories is a current cause for using mathematical fiction with students. Ollerton states the need to create imaginative approaches for students learning mathematics in order to increase motivation. While most literature connects children's literature with teaching mathematics, the present study concentrates on mathematical fiction with a more mature audience. There is an apparent need for more research to consider mathematical fiction and its adult readers, including college and university students. Mathematical fiction has a place in higher education, as will be seen in this section.

A number of college courses that have included mathematical fiction used the novels and short stories that were also investigated in the present study. Sriraman (2003) is one of many to use *Flatland* by Abbott in a course (for students as young as 13 and 14). Some outcomes listed were as follows: this book helped students investigate the norms and biases of society; students used their imagination; "the ideas in *Flatland* also exposed students to some very advanced mathematical ideas such as dimension"; using this novel provided the ideal setting for the teacher to develop broader mathematical topics; and "although the students were discussing the novel, there was a general inquisitiveness about a variety of societal issues and new mathematical ideas" (p. 30).

In *Mathematical Fiction for Senior Students and Undergraduates: Novels, Plays, and Film,* Padula (2006) mentions how these various forms can motivate students during different points in a mathematical course. Works of mathematical fiction can be assigned before the mathematics is taught in class; they can also be read after the mathematics is taught as a review and be given as an independent research study (Padula, 2006; Smith, 2003).

Pinter (2007) discusses that a seminar course at a university can provide the ideal conditions for teachers to use popular culture in relation to mathematics. During a semester of teaching,[11] Pinter used various works of mathematical fiction,[12] including *The Curious Incident of the Dog in the Night-Time.* In his discussion of an assignment, Pinter explains, "In addition to

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[11]The author, Mike Pinter, references the Fall 2005 semester at Belmont University for this First-Year Seminar course.

[12]The use of only *The Curious Incident of the Dog in the Night-Time* from his course will be considered because it was also selected for the present study. (Yann Martel's *Life of Pi*, from 2003, is another novel discussed in the article.)

the students' exposure to Mersenne primes from the websites they found, the exercise created an opportunity to talk about the current pursuit of large prime numbers" (p. 46). The student feedback Pinter received was encouraging. In the review of *The Curious Incident of the Dog in the Night-time,* Aslaksen (2006) writes,

> I am passionately involved in mathematical outreach, and I would say that Christopher is a natural at it. Many mathematicians wax lyrical about the beauty of mathematics, but when asked to share the beauty with the general public, they are apt to use on phrases like 'let *X* be a projective variety over a field of characteristic *p*'. It really bothers me that some mathematicians do not care enough about our profession and the public to try to find some link between what they do and what the public can relate to. One of my main goals in mathematical outreach is to show that the beauty of mathematics is all around us. (p. 3)

Aslaksen describes how one work of mathematical fiction can be so significant and how mathematical fiction can impart so much to society. Further, the Informal Science Education (ISE) believes in increasing public understanding of mathematics. While the ISE mentions an exhibit to gather interest, books can be the link needed to involve the general public. Novels and short stories, in particular, are available to everyone and are cost-effective.

Kasman has created a valuable link between mathematical fiction and the general public. His database[13] has brought mathematics educators and avid readers to a site where one can spend hours on browsing works of mathematical fiction. Aside from the database, Kasman is also an author of mathematical fiction, publishing an anthology of short stories in 2005 (two of these stories are analyzed in the present study).

According to Padula (2006), when mathematical fiction is used properly in the classroom or lecture hall, the novels and short stories can: "motivate students; introduce mathematical ideas in an informative context; elaborate on topics; supply imaginative applications; and help clarify

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[13]The database can be found by the following url: http://kasmana.people.cofc.edu/MATHFICT/, as previously mentioned.

mathematics…" (p. 43). Currently mathematical fiction has been used in such courses as History of Mathematics (e.g., Washburn University), a Seminar course (e.g., Belmont University), an Honors Seminar entitled "Mathematics in Fiction" (e.g., College of Charleston), and a Mathematics and Science Fiction course (e.g. Dartmouth College).[14] Although these courses explicitly involve mathematical fiction, teachers in other courses may recommend further reading for students that might also be works of mathematical fiction. Such further reading can be utilized in practically any mathematical content course, given that the range in topics in mathematical fiction is expansive. Also, in many community colleges and universities, Mathematical Ideas courses can be a platform for the inclusion of mathematical fiction. Padula (2006) writes that the genre is currently expanding and being "rediscovered." This brings renewed hope that there will be more courses involving Mathematical Fiction in the future.

In "Truth and Beauty: A Course in Math and Literature," Cohen (2010) discussed the connection between mathematics and literature with regard to teaching a college-level course. Examples of mathematics fiction included in this article are Abbott's *Flatland,* Kasman's *Reality Conditions,* and Haddon's *The Curious Incident of the Dog in the Night-Time.* These pieces of mathematics fiction are included in the present study along with 23 other works.

Summary

The literature reviewed offers a solid foundation for further research utilizing mathematical fiction. There is support for new methods of teaching mathematics. Some tools are becoming routine (such as graphing calculators), while others are now practically extinct (such as the slide rule and the abacus). Currently, children's literature is increasingly being used in

[14]The syllabus from this 1999 course by Professors Davies and Trout can be found at http://www.dartmouth.edu/archive/c18w99/public_html/. Several books listed in this syllabus were also used in the present study.

elementary mathematics classrooms (Haury, 2001). Perhaps the next trend will be using mathematical fiction in the higher grades and college courses to aid in mathematics instruction.

While children's literature has been employed by primary school teachers to promote mathematical thinking in an attractive and effective way (Padula, 2006), this same idea can be applied to adult literature and college students and can also be used in graduate courses. As previously mentioned, research at the university level is significantly less than at other educational levels. Mathematical fiction thus appears to be a currently untapped resource, but can be used as a starting point for learning almost any mathematical subject. Even if mathematical fiction is used by chance, an individual might become curious or inspired to learn more. Whether used for academic purposes or not, mathematical fiction is a vast, innovative source of informal learning.

Areas relating to mathematical fiction include the connection between mathematics and literature and informal and formal learning and knowledge. It is clear that more studies need be done in the mathematical fiction field and that mathematics teachers should be aware of the potential of using works of mathematical fiction to teach mathematics. Although the future of mathematical fiction is unknown, Ogawa (2009) writes, "With the growth of mathematical fiction, we welcome the emergence of this genre and look forward to more works that can lead to multidisciplinary dialogues among literary studies, mathematics education, and history of mathematics" (p. 76).

Chapter III

METHODOLOGY

Overview

This study analyzed selected mathematical fiction novels and short stories that contain geometric themes. Three main research questions guided this study. First, what geometric topics are used in the mathematical fiction genre with regard to the selected novels and short stories? Second, what are the authors' pedagogical approaches to delivering these topics? What are the authors' mathematical backgrounds? Third, what cultural themes are integrated with geometric topics in these mathematical fiction stories and novels?

This chapter describes the major methodological approaches and procedures used in answering the research questions. The first section addresses the issue of selecting sources for the study; the second briefly addresses grounded theory, which is the major theoretical foundation for this study; and the final section summarizes procedures used to collect and interpret the data.

This study focused on the mathematical field of geometry within these works of mathematical fiction. The selection of geometry as representative of mathematics is motivated by the history of mathematics in which geometry plays an important role. (Similar studies can be done with other fields of mathematics.) One main consideration for selecting this particular area of mathematics is that geometry is a key subject in mathematics; moreover, almost every high school student has been exposed to Euclidean geometry (Heilbron, 1998). Geometry, both formal and informal, is taught in elementary, middle, and high school. Children's mathematical fiction is also used to broaden geometric learning. Relating geometry through hands-on activities can engage students' imagination and motivate them to explore and experiment independently to arrive at geometric truth (Aichele & Wolfe, 2008). Furthermore, a variety of geometry courses is offered at the university level. Therefore, geometry has a connection to students at all levels along with perspective teachers and current classroom teachers.

Geometry, as it is referred to today, contains many branches, which are introduced in Figure 1 below (modern geometry, however, is not limited to these branches). Other branches of geometry were not considered in this study because they were not as appropriate for most students. For example, fractals and chaos theory are generally not required in geometry courses.

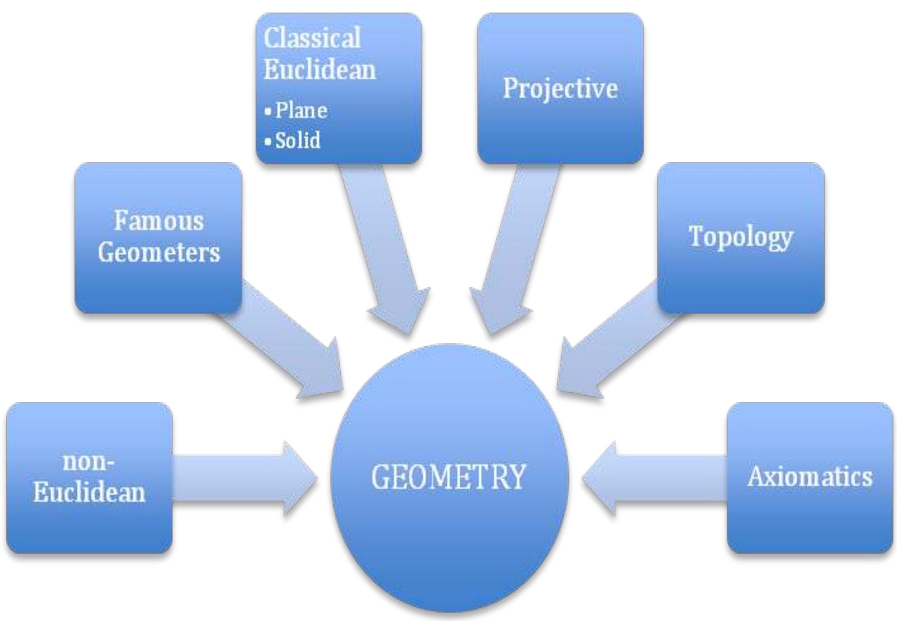

Figure 1. Branches of geometry

Sources of Data

This study utilized a type of non-probability sampling called purposive (purposeful) sampling. Its main characteristic is that it does not involve random sampling. According to Patton (2002), the sample should possess relevant knowledge and share common properties within their categories. The sources used in a qualitative study such as the present research are "information-rich cases" (p. 242). This means that the sample of fictions is used by specific selection of the relevant works from the given database to fit a category. The database used in this study is a mathematical fiction database that is considered the source for this entire genre (Swallow, 2006). This database is compiled and maintained by Dr. Alex Kasman of the College of Charleston and is updated regularly with new sources (http://Kasman.people.cofc.edu. MATHFICT/ browser.php). The sources listed represent a variety of branches of mathematics, such as:

Algebra	Geometry
Arithmetic	Trigonometry
Number theory	Mathematical finance
Calculus	Mathematical physics
Chaos/fractals	Probability
Logic	Statistics
Set theory	

Kasman's database is "widely recognized" as a research source of mathematical fiction (Swallow, 2006). All the authors are arranged alphabetically and comprise a diverse group of both male and female individuals with no geographic boundaries and having talents in various fields. This study includes the literary icon Aldous Huxley and Martin Gardner, the renowned author of popular mathematics who passed away in 2010 at age 95, together with the philosopher/logician Bertrand Russell and *The New York Times* best-selling author Mark Haddon. These authors, along with other celebrated writers of mathematical fiction, are included in the 26 novels and short stories discussed in the present study. Only English-language sources were included in this study.[15]

Also arranged in chronological order, the database begins with Aristophanes' (414 B.C.) *The Birds*, and ends (at the moment of writing the present study) with John Banville's (2010) *The Infinities*. Continuously updated, this database contains a link titled "30 most recently added/modified entries." A search engine is also available, where one can enter keywords on a particular subject or literary motif, and control the search by assigning a minimum math content and literary quality rating for a source.

The "browse the mathematical fiction" link provides a wide array of topics that can be refined by using three other components (medium, motif, genre and topic). The novels and short stories in this study were chosen only from the "select a medium" link. The "select a motif "link ranged from academia, mathematical genius, and mathematical culture to proving theorems and much more.

The relevant works were selected via the following procedures from this database of over 25 pages and more than 900 entries (as of October 2010). The sources, novels, and short stories were selected and grouped into geometric categories. The researcher focused on selecting those works of mathematical fiction whose analysis would illuminate an evolution of mathematical fiction with geometric themes. Both literary and mathematical content scores are given in the

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[15]While the sources included are all in the English language, this does not suggest that the work was originally published in English. For example, *Pythagorean Crimes* by Tefcros Michaelides was originally released in Greek.

mathematical fiction database. Many of the works selected for this study are indicated as "highly rated." The criteria shown below provided a judicious selection for each code.

Each source should have:

i. a literary quality score (LQS) of at least 2.5 out of 5.0
ii. a mathematical content score (MCS) of at least 2.5 out of 5.0
iii. a logical mathematical reason (LMR) for its inclusion within a particular code and a dominant geometric theme representative of its category
iv. a compelling reason for inclusion based upon a confluence of the geometric and cultural themes

The researcher did not create these ratings. The literary quality and mathematical content scores were provided by the database. Also, the researcher did not participate in voting on any novels and short stories read from the database. Two examples of ratings from the database are *Tangents,* which had a literary quality score (LQS) of 4 and a mathematical content score (MCS) of 4 at the time of its selection, while *The Island of Five Colors* had a LQS of 3.7 and an MCS of 5.

Clearly, it is desirable to satisfy all four criteria mentioned above; however, realistically, this is not feasible. The researcher relied on (ii) and, in general, a source was selected if it satisfied three of the four criteria. When a novel and short story had few to no ratings, reviews were read. In addition, to show potential changes within geometric categories of mathematical fiction, at least a 50-year span of works was investigated.

A total of 26 novels and short stories was chosen by this method. The dominant geometric theme in each of the 26 sources of mathematical fiction subdivides the novels and short stories into 6 groups or categories: Plane Geometry, Solid Geometry, Axiomatics, Projective Geometry, Historical Foundations of Geometry, and Topology. The following lists the category into which the source falls:

Plane Geometry Category

1884	*Flatland: A Romance of Many Dimensions*	E.A. Abbott
1885	*The Sirdar's Chess-Board*	E.W. Latimer
1962	*The Mathematician's Nightmare: The Vision of Professor Squarepunt*	Bertrand Russell
1979	*Convergent Series*	Larry Niven
2004	*The Curious Incident of the Dog in the Night-Time*	Mark Haddon
2009	*Pythagoras' Revenge: A Mathematical Mystery*	Arturo Sangalli

Solid Geometry Category

1928	*The Appendix and the Spectacles*	Miles Breuer
1986	*Tangents*	Gregory Bear
2005	*The Object*	Alex Kasman

Axiomatics Category

1954	*The Devil and Simon Flagg*	Arthur Porges
1975	*Euclid Alone*	William Orr
1998	*The Wild Numbers*	Phillibert Schogt
2006	*Pythagorean Crimes*	T. Michaelides

Projective Geometry Category

1940	*And He Built a Crooked House*	R.A. Heinlein
1989	*The Blind Geometer*	Kim S. Robinson

Historical Foundations of Geometry Category

1852	*Hypathia: New Foes with an Old Face*	Charles Kingsley
1924	*Young Archimedes*	Aldous Huxley
1940	*The Death of Archimedes*	Karel Capek
1988	*Thomas Gray: Philosopher Cat*	Philip J. Davis
1989	*The Eight*	Katherine Neville
2005	*Murder She Conjectured*	Alex Kasman

Topology Category

1950	*A Subway Named Möbius*	A.S. Deutsch
1952	*The Island of Five Colors*	Martin Gardner
1974	*No-Sided Professor*	Martin Gardner
2008	*Perelman's Song*	Tina Chang

Grounded Theory

The data interpretation was based on the grounded theory approach, which allows one to organize the data without bringing any theoretical approach in advance. This methodology developed by Glaser and Strauss (1967) generates theory which naturally arises from the collected data. This study used Strauss' development, where the protocol is a step-by-step approach (Glaser & Strauss, 1967). First, the researcher carefully examined the mathematical fiction database with respect to geometry. After deciding on the geometric categories, the aforementioned criteria were utilized for selecting the individual novels and short stories. By analyzing each piece of mathematical fiction, the dominant cultural theme also became known and was identified.

This grounded theory methodology was used when identifying the cultural and social themes with which geometrical discussions are connected. For example, gender bias and the class structure prevalent in Victorian England is the dominant cultural theme in *Flatland: A Romance of Many Dimensions.* This societal issue is intertwined with the geometric theme of dimension in *Flatland.* There is an evident reason that the women are a certain shape and dimension in *Flatland.* Mathematical fiction can be analyzed not solely for its geometric theme, but also for its corresponding cultural component. In accordance with this methodology, concepts emerged during the analysis of the data (Glaser & Strauss, 1967); then, groups of similar themes could be identified and analyzed.

For each work of mathematical fiction, a geometric theme was coupled with a cultural context. The cultural context can provide a backdrop for the story, which sharpens the focus of the geometric theme. The cultural theme can even be the major idea within the story and can be told through geometry. It should be noted that in some works of mathematical fiction, the cultural theme or the geometric theme may be dominant, or both may be of equal importance. In *Flatland: An Edition with Notes and Commentary* (Lindgren & Banchoff, 2010), a cultural connection is made with regard to the treatment of women in 19th-century England and the representation of women as needles in Abbott's *Flatland* (1884) and the doctrine of coverture pertaining to the exclusive rights of husbands (p. 35). These cultural and societal themes were uncovered within each work of mathematical fiction. They are diverse and present a wide range of societal concerns during the time in which the story takes place. Some of the cultural themes connect geometry to art, music, warfare, gender bias, magic and superstition, chess, academia, and so forth.

Summary

To answer the first two research questions, the researcher began with the database of mathematical fiction provided by Kasman. The sources of mathematical fiction were selected as described in this chapter. In answering the question, *What geometric topics are used in the mathematical fiction genre with regard to the selected novels and short stories?,* the dominant mathematical theme was identified. The geometric topic found in the works of mathematical fiction was then grouped into the categories: Plane Geometry, Solid Geometry, Axiomatics, Projective Geometry, Historical Foundations of Geometry, and Topology. Comparisons were also made across categories.

To answer the next part of the question, *What are the authors' pedagogical approaches to delivering these topics?,* three main factors were considered:

a) Is the presentation formal or informal?

b) What is the reader's prerequisite knowledge?

c) How is the geometric theme presented?

The researcher used a few methods to identify *What are the authors' mathematical backgrounds?* While some of the authors are well-known writers or renowned mathematicians, other writers were unknown. If the works of mathematical fiction did not provide the relevant information on the author, Internet searches were conducted. These searches aimed to find the homepages of the author at their university, their curriculum vitae, and book reviews or articles that contained academic information about the individual. Based on this background research, this study endeavored to identify whether the goal of the author was to provide a mathematical fiction story or a work of fiction that happens to mention a bit of mathematics.

Answering the third question involved identifying groups of relevant themes and analyzing these themes versus their geometric content. In particular, attempts were made to understand the motivation to use this content in each particular case or the motivation to associate the specific theme with the content. The authors' backgrounds were analyzed in this connection. The third research question, *What cultural themes are integrated with geometric topics in these mathematical fiction stories and novels?,* makes use of these 26 selected samples. This question uncovers how the cultural and geometrical themes in these works have connections. The researcher gathered data from all selected mathematical fictions that reflect cultural trends inherent in each story. For example, in Abbott's *Flatland* (1884), immediate parallels were drawn between dimensions and gender bias in Victorian England. The ideas of dimension in geometry and the roles of women are intertwined by the characteristics Abbott assigns to the shapes in his story. The cultural themes in this story were gender bias and class structure and these themes were identified and coded. "Open coding," the term used to identify and name each category (Strauss & Corbin, 1990), was done for all novels and short stories. The cultural themes were generally unknown before the piece of mathematical fiction was read. There also were overlapping cultural themes, which emerged through grounded theory.

Chapter IV

ANALYSIS OF MATHEMATICAL FICTION:

TWENTY-SIX NOVELS AND SHORT STORIES WITH GEOMETRIC THEMES

Overview

Geometry has many "faces" and mathematical fiction provides multifaceted sources illustrated by a diversity of topics. Connected with each geometric topic is a cultural context implicit in each source of mathematical fiction (Goff & Greenwald, 2007). The sources are presented chronologically with respect to the following six geometric topics, as discussed in previous chapters: Historical Foundations of Geometry[16] [HF/G], Plane Geometry [P/G], Solid Geometry [S/G], Projective Geometry [Pj/G], Axiomatics [Ax], and Topology [Tp].

The Three Categories of Mathematical Fiction with Geometric Themes

Three categories for the 26 pieces of mathematical fiction with geometric themes were identified. Category I signified geometry as the dominant theme, Category II represented the novels and short stories in which both geometry and a cultural theme were of equal value to the story line of the source, and Category III dealt with mathematical fiction composed of a dominant theme other than geometry which plays a subordinate role. These categories were examined separately. The sources in each category will be presented chronologically. Each

[16]This category (Historical Foundations of Geometry) denotes the contributions of famous mathematicians to geometry.

source has been analyzed with respect to the following four components: (i) plot, (ii) geometric theme, (iii) cultural theme, and (iv) presentation of the geometry.

Category I

1852 *Hypatia: New Foes with an Old Face* Charles Kingsley [HF/G]

Reverend Charles Kingsley has written a detailed account of the life and times of Hypatia. Many monks believed that women in general and Hypatia in particular were "the authors of all evil," which explains why the conflict between Hypatia and religious zealots was the plot of this novel. As a leader of the Neoplatomist philosophy, Hypatia defended paganism, which was viewed as anti-Christian and led to her barbarous death (XXIX, pp. 360-371).

Attracting a large and loyal following, Hypatia lectured on the geometry of Euclid and Apollonius at the famous Alexandrian Museum, the center of Greek learning for many centuries. Both Hypatia and her father Theon wrote commentaries on Euclid's *Element*s and the *Conic Sections* of Apollonius, who was regarded as the greatest geometer of antiquity. On page 172 of Kingsley's novel, Hypatia in responding to a student's question on conic sections states, "In what does my commentary differ from the original text of Apollonius, on which I have so faithfully based it."

Religious fanaticism and gender bias are two cultural themes connected to the life and times of Hypatia. Dominating Athenian culture was the unquestionable power of the church led by Cyril of Alexandria. His religious fanaticism was manifested in anti-Semitism and misogyny: "Down with the Heathens, Jews and Barbarians! Down with the favorite of Hypatia" (p. 195).

Through Hypatia's lectures, geometry is presented in both a historical and philosophical context. Hypatia's views of mathematics in general and geometry in particular are filtered through the lens of her philosophy. Being a Neoplatonist, she lectures on abstract structures and forms that were well suited to the geometry of both Euclid and Apollonius. During one of her lectures on conic sections, Hypatia states that the "circle holds infinite perfection akin to the human soul" (p. 174).

1924 *Young Archimedes* Aldous Huxley **[HF/G]**

While vacationing in Italy, a couple and their young son, Robin, meet Guido, an Italian peasant with a talent for mathematics and music. A theme echoed throughout the story is the strong relationship between music and geometry. Woven into this story is how Guido's insatiable appetite for mathematics is satisfied when he receives a gift of Euclid's *Elements* from Robin's father and how he expresses the similarities between music and geometry through patterns.

The Pythagorean Theorem and its geometric proofs are discussed in this story. One of the proofs described by Huxley is the classic proof of Euclid (Heath, 1956) about which Guido exclaims, "it's so beautiful" (p. 24). Guido offers yet another proof (a dissection proof) of this celebrated theorem.

A cultural theme echoed throughout the story is the strong relationship between music and geometry. This connection between music and geometry is illustrated in this story by a child prodigy who excels in both disciplines. According to Huxley, child prodigies usually are musical and mathematical (p. 25); as examples, the author reminds the reader that Mozart's talent in music was evident at age 4 and Pascal's important works of mathematics were created when he was a teenager.

Three proofs of the Pythagorean theorem are discussed in this story. One proof (Guido is only 7 years old) of the celebrated Pythagorean theorem, $a^2 + b^2 = c^2$, is demonstrated using a burnt stick on smooth paving stones (p. 24). Guido discovers another proof of the Pythagorean Theorem based on similar triangles and the high school theorem that states in a right triangle, the altitude upon the hypotenuse is the mean proportional between the segments of the hypotenuse, and either arm is the mean proportional between the hypotenuse and the adjacent segment.

1940: *And he Built a Crooked House* Robert A. Heinlein **[Pj/G]**

Mr. Bailey hires an architect to design his dream house in Los Angeles. The eccentric architect, Quintus Teals, has a vision of designing an extraordinary house in the shape of an unfolded tesseract or hypercube (p. 70). The hypercube contains eight cubes and the plot of the story is centered on the stability of this geometric structure.

The geometry of a hypercube is described architecturally. Quintus has plans for an eight-room house built on a "one-room foundation," the foundation being a cube (p. 74). Unfortunately, the house collapses because of a mild earthquake (recall that the house is being built in southern California) and Quintus' explanation is that in three dimensions, the house is perfectly stable, but it becomes unstable in four dimensions (p. 81).

Applications of the hypercube are described in architecture and art. The modern design of a "dream house" is inspired by taking geometry to a new level—the fourth dimension. In art, Salvador Dalí used the unfolded hypercube to represent a crucifix in his famous painting titled "Christus Hypercubus" or "Corpus Hypercubus" (see Appendix).

Four-dimensional geometry, or hyperspace, is described through the hypercube and illustrated in architecture. The design of the house is an inverted cross; that is, it is comprised of four cubes that "stick out" in four directions. The foundation is the ground-floor room and the upper cube may serve as a study. Other cubes may be used as living rooms, bedrooms, dining rooms, dens or kitchens. Quintus mentions that his design is "a hypercube unfolded in three dimensions" (p. 71).

[1950] *A Subway Named Möbius* Armin Deutsch **[Tp]**

The plot of this quirky story is the disappearance of a subway car which is "lost" in a transit system that has 227 trains operating on 7 lines of tracks (p. 222). Each line is capable of switching to another track so that a subway train can travel from any one station to any other station throughout the entire system. Given the complex topology of the system, train number 86 vanishes with 350 passengers (together with the motorman and the conductor).

The geometry contained in this story is explained by a Harvard mathematics professor named Roger Tulepo. Using mathematical analysis, Tulepo conjectures that the train "hit a node" which he defines as a singularity. Although the actual topology is very complicated, Tupelo explains the mathematics in terms of a Möbius band (p. 223).

A cultural theme in this story is the connection with a pop song of the Kingston Trio whose musical genre was folk songs. One of their most popular songs was the "MTA" that dealt with the story of a man who "never returned" from the Boston subway system. Connecting Deutsch's short story published in 1950 with the "MTA" song written in 1948 by Jacqueline Steiner and Bess Lomax Hawes, one readily notes the song's connection with Deutsch's mathematical fiction.

The geometry is presented by describing the properties of a Möbius strip and a Klein bottle. During a meeting with city officials, Tulepo suggests that the missing train is still in the system (closed network) but cannot offer an explanation for how to locate it. On page 233, Tulepo states the possibility that the system has "an infinite number of infinite discontinuities."

[1958] *The Island of Five Colors* Martin Gardner **[Tp]**

A mathematician and a cultural anthropologist (p. 197) wish to draw up a map of an island which is divided into five regions. Each region is inhabited by a tribe with unique characteristics that distinguish it from the other tribes and these tribes share common borders. Dr. Alma Bush, the anthropologist, claims that five colors would be needed to color the map of the island. On the other hand, the mathematician emphatically states that only four colors are necessary since "it's a theorem in topology."

Basically, four colors will suffice when coloring a map so that no two bordering regions share the same color. In the case where two regions touch at a single point, then they may share a common color. Martin Gardner's narrative intuitively explains the geometry involved in the famous four-color problem.

In *The Island of Five Colors,* Gardner assumes that only four colors suffice so that adjacent tribal territories have different colors. It should be noted that before 1976, the four-color theorem was appropriately called the four-color conjecture since no proof was offered and accepted. It was not until 1976 that Appel and Haken of the University of Illinois proved the theorem with the aid of a computer.

Gardner explains the geometry by giving an example of a coloring problem. Coloring the five regions of the island is to be accomplished by "spraying colored paint" (p. 203) from a low-flying plane and then taking a color photograph from an aerial view. Twenty thousand gallons of the most inexpensive paint in five colors were ordered. As the story progresses, only four colors are used and arrangements are made to take aerial photographs as confirmation of the four-color theorem.

[1954] *The Devil and Simon Flagg* Arthur Porges **[Ax]**

There are two main characters in this story: Simon Flagg, a professor of mathematics, and the devil himself. At the start of the story, Professor Flagg makes a strange bargain with the devil

(p. 63). One hundred thousand dollars is wagered against Flagg's soul. The professor will win the bargain if the devil cannot solve a mathematical problem (chosen by Flagg) within 24 hours.

The problem is to solve Fermat's Last Theorem (FLT). After Professor Flagg states FLT, he gives examples of Pythagorean triples. Illustrating a Pythagorean triple, Simon assures his wife that it is an easy problem for $n = 2$, "but when you try to find two cubes that add up to a cube, or higher powers that work similarly, there don't seem to be any" (p. 66).

The mathematical culture of proof is illustrated in this short story. First, an intellectual engagement exists between the solver and the problem (theorem). Being challenged to prove a theorem involves both pleasure and frustration. Often, an obsessive interest in solving the problem develops.

The discussion of FLT is reader-friendly. Porges assumes that the reader is familiar with Pythagorean triples. He points out the difficulty of FLT and shows that collaboration in problem-solving is helpful, but not in the case of FLT.

[1974] *No-Sided Professor* Martin Gardner **[Tp]**

This story takes place at a banquet for the Möbius Society, where the annual lecture is attended by mathematicians whose specialty is topology. Hailed as the world's leading topologist, Professor Slapenarski amazes his fellow topologists by announcing that he will demonstrate the existence of a surface with no sides—that is, a "nonlateral surface." Although the audience is highly skeptical of the existence of a surface with no sides, the professor claims that he will construct such a surface. The plot of this story centers on this claim.

Making the distinction between a torus (a "doughnut") and a sphere, the reader is presented with historical information about the German topologist August Ferdinand Möbius

(1790-1868). Giving a strip of paper a single half-twist and pasting the ends together results in a surface with one side called a Möbius strip.

Paper folding is often considered to be part of recreational mathematics. Nevertheless, paper folding may be both recreational and educational with regard to studying properties of geometric figures. Paper folding is very popular in different cultures, but the Japanese have popularized it into an art called origami.

Gardner offers the reader instructions on constructing a Möbius strip as well as historical information on August Ferdinand Möbius (1790-1868). In addition, Gardner directs the interested reader to two popular textbooks containing chapters on elementary topology (Courant & Robbins, 1996; Kasner & Newman, 1940).

[1975] *Euclid Alone* William F. Orr **[Ax]**

The plot of this story focuses on the construction of a geometric proof by a non-tenured mathematics professor named David. An experienced and scholarly mathematician is refereeing his article. After spending countless and agonizing hours reviewing the proof, the reviewer is convinced that the author's reasoning contains a flaw and is not worthy of publication, which will negatively impact David's tenure.

The geometric topic of this story deals with the consistency of the five postulates of Euclidean geometry, and includes a discussion of the fifth postulate, called the parallel postulate. In particular, there is reference to Lobachevskian geometry in which the parallel postulate is denied, yet a consistent geometry emerges.

This instructive work of mathematical fiction contains two main cultural themes. The first is the connection of art to mathematics and the second is the pressure of "publish or perish" that is part of the culture of academia. Since David's article was rejected for publication, he was not granted tenure and was forced to abandon his college teaching position.

The mathematics of this story is presented through a careful discussion of consistency of a postulational system. The referee of David's article was having nightmares about the possibility that Euclid's five postulates are inconsistent, and his dreams were of "being trapped in Konigsberg, forever recrossing those bridges, while Euler stands by the river and laughs" (p. 190). The set of Euclid's five postulates is indeed consistent because no contradictory statement can be implied by this set. The author informs his readers that consistency is the most fundamental property of a set of postulates.

[2005] *The Object* Alex Kasman **[S/G]**

Alice Wu is a computer geek who is very talented in mathematics, although she dropped out of MIT to start her own biotech consulting business. Alice has discovered an algorithm that models molecular structures; together with her partner Sophia, Alice constructs plastic models of three-dimensional geometric configurations. Sophia is puzzled when her computer screen shows a regular polyhedron containing 37 vertices (which is mathematically impossible)!

The geometry of regular polyhedra is presented by Alice with the aid of Euler's formula. Alice writes down Euler's formula; $V - E + F = 2$, where V, E, and F represent the number of vertices, edges, and faces of the polyhedron. Alice and Sophia both decide to examine this strange "object" on the computer screen from different perspectives and conclude that it almost resembles a regular polyhedron.

Mathematical experimentation with different solids is a theme of Kasman's short story. Through experimentation, geometric patterns emerge that may lead to mathematical discovery.

This experimentation is applied to the five regular solids (cube, tetrahedron, octahedron, icosahedron, and dodecahedron) with regard to verification using Euler's formula.

Educating the readers of this story, Alice patiently explains the impossibility of the existence of such a polyhedron with 37 vertices. First, she explains that regular polygons must have equal sides and equal angles such as equilateral triangles, squares, regular pentagons, and so forth. She goes on to tell Sophia about regular polyhedra. Clearly, regular polyhedra have "faces each of which is a copy of the same regular polygon" (p. 195).

[2005] *Murder She Conjectured* Alex Kasman **[HF/G]**

In Kasman's short story, an unsolved murder takes place in Cambridge, England around 1870. The victim is Heather Blaine Clifton, wife of the "mathematician" Graene Clifton, whose theorem on manifolds bears his name. It appears that the murder of Heather is part of a series of serial murders, of which she was victim number five.

Historical information dealing with female mathematicians whose contributions took place during 19th-century England is presented within the plot of this mystery. Charlotte Angas Scott (1858-1930) and Emmy Noether (1882-1935) were both gifted mathematicians who experienced a great deal of gender bias; on page 13, Beth notes "…it must have been pretty hard to be a woman mathematician in those days." Consequently, in this fictional murder story, the reader learns about two non-fictional women mathematicians: Scott and Noether.

The main cultural theme in this short story is gender bias during 19th-century England. Two non-fictional women mathematicians, Scott and Noether, who were both mathematically talented women, were denied opportunities because of their gender. Scott was denied graduation in England and obtained her degrees in America.

Attributable to Kasman's knowledge of the history of mathematics, the narrative brims with suspense and the reader gains a valuable understanding of the obstacles that female mathematicians faced during this time period. *In Murder She Conjectured*, gender bias is vividly described in relation to Heather Blaine Clifton, a gifted female mathematician who specialized in geometry and published her research under her husband's name.

[2006] *Pythagorean Crimes* Tefcros Michaelides **[Ax]**

In *Pythagorean Crimes*, the reader is presented with a story within a story; both involving the murder of two mathematicians. Twenty-five hundred years ago, the geometer and Pythagorean Hippasus of Metapontium (ca 500 B.C.) was found drowned at sea, possibly for revealing the irrationality of two. This is the first murder story in *Pythagorean Crimes*. The second murder took place in 1929; the body of the mathematician Stefanos Kantartzis is discovered and his best friend Michael Igerinos, another mathematician, is the chief suspect.

Essentially, the geometric content of this story deals with the study of axiomatics as it relates to Euclidean Geometry. The properties discussed are that the axiom set should be consistent, independent, complete, and categorical. To the author's immense credit, each property is carefully explained in his novel and it is the property of completeness that is of major importance in Hilbert's second problem.

Many cultural components in *Pythagorean Crimes* serve to educate the reader in such diverse areas as art, literature, and architecture. For example, a summary of E.A. Abbott's *Flatland*, discussed above, is described as "a clever little book that aspired to satirize the morals of Victorian England by means of geometry" (p. 43). Cubism is discussed by Picasso, who summarizes his interest in geometry by stating, "I'm interested in anything that can help me understand space with my mind" (p. 115).

The specific mathematical content is due to David Hilbert, who was perhaps the most influential mathematician of his time. In the novel, Hilbert challenges the mathematical community to find an algorithm that decides whether an axiom system is complete. At the Second International Congress of Mathematics, held in Paris on August 8, 1900, Hilbert gives a list of 23 problems which serves as a blueprint for mathematical research. The second problem of Hilbert is the focus of this novel.

[2008] *Perelman's Song* Tina Chang **[Tp]**

Chang uses a dialogue between a god and a goddess to describe the Poincare Conjecture. In this story, a god and a goddess ponder the question of whether the "blob" is in fact a sphere. The goddess replies, "Any simply connected closed three manifold is a sphere."

The geometry of this short story deals with, perhaps, the most famous theorem of topology. From the point of view of a topologist, a football and a sphere are topologically equivalent since a football can be "squished" to form a sphere, and the converse is also true. On the other hand, this transformation cannot be accomplished with a doughnut.

The theme of Chang's work is presented using a poetic metaphor in dealing with Perelman's solution of the Poincare Conjecture. This point in the story is considerably joyous as the god and goddess make millions of infinitely small spheres. Now they "meld them back together" to form a three-space sphere.

In layman's terms, Professor Christina Sormani clarifies the essence of the famous conjecture by her intuitive and highly descriptive passage, which is given below:

> Hey, you've got this alien blob that can ooze its way out of the hold of any lasso you have around it. Then that blob is just an "out-of-shape ball." Perelman and Hamilton prove this fact by heating the blob up, making it sing, stretching it like hot mozzarella and chopping it into a million pieces. (then reassemble the pieces to form a sphere). In short, the alien ain't no bagel you can swing around with a string through its hole. (http://comet.lehman.cuny.edu/sormani/others/perelman/introperelman.html).

[2009] *Pythagoras' Revenge* Arturo Sangalli **[P/G]**

The absence of any of Pythagoras' manuscripts provides a mysterious theme in Arturo Sangalli's novel. Suppose for a moment that Pythagoras did leave a mathematical manuscript (papyrus). First, what were the contents of this hypothetical manuscript? Second, where would the manuscript be located? The story is woven around these two questions.

Sangalli provides the reader with historical background information on Pythagoras and the Pythagoreans. The geometric information presented in the novel is historically accurate when compared to leading texts on the history of mathematics. The geometry contained in the fictional papyrus is a method for constructing right triangles.

The transmission of Euclid's *Elements* is part of the cultural theme of *Pythagoras' Revenge.* Translations by the Neo-Platonist Proclus and the English monk Adelard of Bath are mentioned in this novel on page 28: "The 1482 Latin edition up for sale was based on a medieval translation from Arabic by the English monk Adelard of Bath."

Pythagorean mathematics is presented in this novel as an educational component. For example, Sangalli discusses the mathematical pedagogy using the Socratic method that was mentioned in Plato's *Meno*. Sangalli writes about Euclid's *Elements* as a model for mathematical rigor. The importance of axioms, first principles, and the structure of mathematical proof is explained by the author. According to the famous British mathematician Sir Michael Atiyah, "proof is the glue that holds mathematics together" (p. 109).

[2009] *Pythagoras' Darkest Hour* Colin Adams **[P/G]**

This short story is a comical interpretation of how Pythagoras discovered his celebrated theorem. At the start of the story, Pythagoras is found in a bar with his trusted servant Triangulus. The initial story line was that Pythagoras thought that $x^2 + y^2 = z^2$ related to a circle.

Examining an isosceles right triangle, each of whose sides is 1, they note that the hypotenuse would have a length whose square is 2. Puzzled at this strange-looking number, they wonder what fraction when squared yields the number 2. The culture of the Pythagoreans would never acknowledge the existence of such a number because it would upset their entire cosmology.

The geometry is presented using a trial-and-error approach. For example, playing with triples such as 3, 4, and 5, they note that indeed $3^2 + 4^2 = 5^2$. They also discover that multiples of the triple work. Pythagoras tells Triangulus that a theorem cannot be concluded based on one example, so Pythagoras and his servant try other triples such as the 5, 12, 13 right triangle. They now conclude that given a right triangle, with sides x, y and z, where z is the hypotenuse, then $x^2 + y^2 = z^2$.

Category II

In this section, the novels and short stories possess themes generated by both geometric and cultural considerations that constitute the core of this study. In Category II, the geometric and cultural themes play a balanced role in the development of the story line. Both the geometric and cultural themes are dominant. The geometric theme does not overpower the cultural themes and vice versa, yet they both complement each other in the development of the mathematical fiction sources. The researcher now presents those sources that belong to Category II in the same format that was used in the previous section.

[1884] *Flatland: A Romance of Many Dimensions* E.A. Abbott **[P/G]**

Abbott's classic work of mathematical fiction is a fantasy adventure where geometric figures have human attributes and geometric dimensionality plays a key role in the story line. The narrator of *Flatland* is A. Square who travels between Lineland and Spaceland, thus being exposed to both two and three dimensions. When a sphere visits Flatland, A. Square is so overwhelmed that he tells his fellow Flatlanders about the properties of 3—space, but the ruling priesthood renounces him for these troubling accusations.

One of the geometric themes in this novel is the resistance to new geometric ideas. Another theme is how hierarchies of plane geometric regular polygons correlate to one's social class. Denial of the third dimension is prevalent, but A. Square correctly concludes that a cube is composed of six planes (faces) and "eight terminal points called solid angles" (vertices)—namely a solid (pp. 86-87).

Victorian culture during Abbott's time did not hold women in high regard. Consequently, the main cultural theme is gender bias, which is blatant throughout the story. On page 12, "if a soldier is a wedge a woman is a needle (line); being so to speak, *all* point, at least at the two extremities."

The geometry of this story is presented by A. Square, who can distinguish between a square and a cube as well as a circle and a sphere. His idea of a solid in Flatland society falls on deaf ears (pp. 86-87). On page 101, A. Square "exhorted all my heavens to divest themselves of prejudice and to become believers in the Third Dimension." Sadly, he is thrown in prison for his geometric views so that his knowledge of higher dimensions will be kept secret from the Flatlanders.

[1949] *The Death* of *Archimedes* Karel Capek **[HF/G]**

Karel Capek has written a fictional account of the last day of Archimedes' life. It is written as a dialogue between Lucius (the Roman soldier) and Archimedes (287-212 B.C.), at the end of the Second Punic War (214-212 B.C.) between Syracuse and Rome. During this dialogue, it is clear that Syracuse has lost the war after resisting the Roman siege for three years in spite of Archimedes' "war machines."

Geometric applications of weapons invented by Archimedes are part of the story. Such weapons included catapults, pulleys, and hooks strong enough to hoist a Roman ship out of the water and smash it against the rocks. During his dialogue with the Roman soldier, Archimedes seems to be deriving the formula for the area of a parabolic segment.

The philosophical dialogue between Lucius and Archimedes begins with the salutation "Greetings, Archimedes" (p. 57). Lucius goes on to cajole Archimedes to join the Roman side of the conflict. Greatly offended, Archimedes replies that he is proud of his Greek heritage as he states, "We Syracusans happen to be Greeks" (p. 58).

Through the dialogue, the author presents the geometric content of his short story. Lucius persists in trying to convince Archimedes to construct war machines for the glory of Rome. This work of mathematical fiction contains a philosophical dialogue about the application of mathematics during wartime.

[1979] *Convergent Series* Larry Niven **[P/G]**

The plot of this story involves the geometric construction of a pentagram that has magical properties (p. 162). The main character constructs such a pentagram on his basement floor using chalk. Suddenly, to the horror of the graduate student, a demon appears inside the pentagram.

Specifically, the geometry in this story concerns the convergence of a geometric sequence rather than a geometric series (as the title suggests). Having no trouble with a pentagram that measures two feet between alternate vertices, the demon easily "spread-eagles" himself within the five-pointed star. Now the graduate student diminishes the length of each diagonal of the pentagon and continues this sequence of decreasing lengths so that the pentagram becomes smaller and smaller.

The cultural theme of this story is connected to literature. For example, Goethe's *Faust* draws a pentagram on the entrance of his study but fails to connect all the vertices. This allows Mephistopheles to enter the study by means of the pentagram, but then becomes ensnared in the star's inner pentagon.

The mathematics of this story is tied in with the magical properties of the pentagram. The statement "Magic fascinates me" (p. 162) sets the mood of the short story. Intuitive explanations of a converging sequence that approaches zero is illustrated with the demon converging at the same rate as the pentagram.

[1986] *Tangents* Gregory Bear **[S/G]**

Bear wrote about the hypercube in connection with the computer in this story. A young Korean boy, Pal Tremont, is adopted by an American family that does not support his interest in mathematics. Pal befriends Peter, a computer expert and hacker who has similar characteristics as Alan Turing.

Geometry of higher dimensions is explained via computer technology and is contrasted with Abbott's *Flatland.* In fact, Pal tells Peter how a Flatlander can view a cube intersecting a plane. After he talks about the cube intersecting the plane, he deals with the intersection of a sphere and a plane from the perspective of a Flatlander so that when the plane is tangent to the sphere, the Flatlander sees an "invisible" point (p. 40).

One of the cultural themes in *Tangents* is society's attitude toward homosexuality. Pal does some research on the Enigma machine and finds a picture of Peter as part of the Bletchley

Park cryptanalysis team. Upon hearing this discovery from Pal, Peter quickly admits that he is a homosexual. Pal replies, "Oh, so what."

Higher dimensional geometry is presented through computer graphics. Peter and Pal have bonded due to their mutual interest in the fourth dimension. The computer is used to visualize the intersection of a cube with a plane at various angles where they intersect. In addition, the hypercube of this story provides an excellent application of computer graphics.

[1988] *Thomas Gray: Philosopher Cat* Philip Davis **[HF/G]**

The story line deals with a work of Theodorus of Cyrene "who was writing out for us something about square roots, such as the square roots of three or of five showing that they are incommensurable with the units" (p. 43). In this sense, Theodorus geometrically constructs the sequence $\sqrt{2}$, $\sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \ldots \sqrt{15}, \sqrt{16}, \sqrt{17}.$ Notice that he stops at $\sqrt{17}$. Why?

Theodorus was Plato's (427-347 B.C.) teacher and also the instructor of the skilled mathematician, Theaetetus (417-369 B.C.) The author briefly discusses that the $\sqrt{2}$ is irrational and can be represented geometrically as the diagonal of a unit square. Davis writes that according to the ancient Greek mathematicians, however, such a length is incommensurable.

The cultural theme is that Thomas Gray, the poet, connects with the title of this work of mathematical fiction. Davis explains the connection between cat and poet when both were at Cambridge. It seems that Thomas Gray, the "Pembroke College Cat," chose to remain at Pembroke College and was named by the students at the college (p. 13) after the famous alumnus, Thomas Gray, the poet.

The geometry of the "square root spiral" suggests the solution for why Theodorus stopped at 17 (see Appendix). Using an Arabic translation of Theodorus's work, the author notes that the "words for root 17 overlapped the words for root 18" (p. 47). The right triangle whose sides are 1 and $\sqrt{17}$ intersect the first right triangle of the spiral. After the square root of 17, the drawing of the spiral "becomes messy" and so $\sqrt{17}$ is the greatest irrational number to produce the sequence of right triangles, none of which overlap.

[1989] *The Eight* Katherine Neville **[HF/G]**

Catherine Vellis, the heroine of this story, is a computer expert with a flair for art, chess, mathematics, and music. She is asked by her employer to find a mythical chess set, once owned by Charlemagne, which was hidden in an abbey during the French Revolution. Blending the action between the 1970s and the late 18th century, the reader is introduced to Euler, one of the greatest mathematicians of all time; Bach, a composer whose name is synonymous with Baroque music; and Philidor, a master chess player.

On page 296, Neville mentions the mystical number, "one-half the square root of five minus one." She then goes on to say that the ancient Greeks called this number the "*aurio sectio*"—the golden section. The term "golden mean" is often referred to as the "golden ratio" or the "golden proportion."

The cultural theme of this novel is the connection between mathematics, music, and chess. Euler is the representative of mathematics, Bach the representative of music and Philidor the representative of chess. The common thread that connects this famous trio is, according to the author, pattern recognition that comprises symmetry and logic.

The mathematics is presented in this novel through a fictional conversation between Euler, Bach, and Philiodor which takes place at the court of Frederick the Great. A mathematical puzzle which is well known to chess players called the "Knights Tour"—where the tour of a knight must traverse the entire chessboard, never landing on the same square twice—was described by Euler and given to Bach as a challenge. Upon hearing the puzzle, Bach numbers each of the 64 squares on the chessboard and turns the knight's tour into octaves and chords. Euler is incredulous and exclaims "to turn mathematics into music—it is sheer magic" (p. 172). Bach replies, "And the reverse is also true."

[1989] *The Blind Geometer* Kim S. Robinson **[Pj]**

The reader encounters Carlos Oleg Nevsky who was born blind and is also a professor of mathematics; his specialty is geometry, with his research being mainly in topology. A jealous colleague, Jeremy Blasingame, constantly picks Carlos' brains to gain knowledge about multi-dimensional geometries. Jeremy has a mysterious woman, Mary, seduce Carlos in the hope of stealing his research.

Carlos discusses the proof of Desargues' theorem together with a diagram used to prove the theorem (p. 16). Carlos' appreciation of Desargues' Theorem is poetically described as "a beautiful theorem, with the purity and elegance characteristic of Renaissance mathematics" (p. 16). As the mystery progresses, Carlos educates the reader about the philosophy of geometry by citing Derrida's text (p. 23).

A cultural theme in *The Blind Geometer* is overcoming blindness to lead a productive life. Carlos reminds the reader that geometry is indeed possible for a blind person to comprehend since "my hands are my eyes." Throughout this work of mathematical fiction, Carlos never thinks of himself as disabled.

The mathematics is described in this story in conjunction with a diagram illustrating Desargues' theorem that actually is a coded warning to Carlos using the letters of the diagram. Mary reveals to Carlos that he is triangle ABC and she is triangle A´B´C´ in the projective plane. Observing that Carlos and Mary are, however, in two different planes that intersect but are "controlled" by the point of projection (perspective point), the reader can connect Desargues' theorem with the story line plot. Therefore, the geometric diagram is a metaphor in which Blasingame (evil) can try to control both Carlos and Mary (goodness) from the point of perspective.

[2004] *The Curious Incident of the Dog in the Night-Time* Mark Haddon **[P/G]**

Christopher, the narrator of this novel, has Asperger's Syndrome (high-functioning autism), and he wishes to take an advanced test in mathematics (A-level mathematics exam). Due to his autism, he experiences resistance from his school's administration to participate in the exam. His quirky behavior prevents him from having normal social relationships, but this does not impact his mathematical competence.

As part of the A-level mathematics exam, Christopher is asked to prove: "A triangle with sides that can be written in the form $2n, n^2+1,$ and $n^2-1,$ $n > 1$ is right angled. Show by means of a counterexample that the converse is false." He offers an excellent solution, which appears in the appendix of the story and a discussion of Pythagorean triples (pp. 223-226).

The cultural theme of this novel is that autism is an often misunderstood condition, which is not synonymous with mental retardation. Despite his autism, Christopher has no difficulty in explaining sophisticated mathematical concepts such as the Monty Hall problem (pp. 63-65), tessellations (pp. 202-204), and the solution of quadratic equations with three-digit coefficients. His knowledge of prime numbers is related to his explanation of the Sieve of Eratosthenes (pp. 11-12).

Mark Haddon, through Christopher's narration, explains all the mathematical themes in the story that the reader can understand and appreciate. It is clear that Christopher, although autistic, can function at a high mathematical level. Reading James Gleick's book *Chaos* (p. 96) implies that Christopher's reading comprehension is also at a high level. Perhaps the most remarkable geometry in the story is his counterexample on the A-level examination: namely, let $\overline{AC} = 25$, $\overline{BC} = 60$ then $\overline{AB} = 65$ and $(25)^2 + (60)^2 = (65)^2$ but the sides are not of the form $2n, n^2 - 1, n^2 + 1$.

Category III

In this category, the core is dominated by a cultural theme with geometry playing a subordinate role. The cultural theme drives the story line of the novels and short stories contained in this category. (Cultural themes such as art, music, chess, gender bias, religion, academia, physical disability, mysticism, and so forth play a major role, whereas geometry plays a supporting role.) As expected, Category III contains fewer sources than the two previous categories.

1885 *The Sirdar's Chess-Board* E.W.Latimer **[P/G]**

A sirdar, a high-ranking leader of an Afghani community, invites Sophia, the wife of a British colonel, to play a game of chess in his elaborate "audience-chamber." After the chess game, Sophia muses that the chessboard is an 8 by 8 square with an area equal to 64 (square units). She then claims that with three cuts on an 8 by 8 square, she can rearrange the four remaining geometric pieces so that the sum of their resulting area is 65 (square units).

As the title suggests, the geometric theme involves a chessboard. A geometric paradox is described using a chessboard where the result is 64 = 65. The reader is given a diagram showing that the rectangle formed by the four pieces has dimensions 13 by 5.

Attuned to the cultural climate of her time, Sophia believes that "no woman has ever been a great chess player" (p. 359). Gender bias and the low esteem of women in this story are recurring cultural themes. The women in the harem are uneducated and completely tyrannized by any male, even their own sons (p. 368).

The geometry is presented through a dialogue between Sophia and the sirdar. Sophia asserts to the astonished sirdar that “I could cut it so that there would be sixty-five” (p. 369). Laughing, the sirdar replies that it is impossible. Using a pair of scissors and a sheet of paper marked with 64 squares, Sophia uses “three-snips” and arranges the resulting four figures before the sirdar. Instead of an 8 by 8 square, the sirdar observes a 5 by 13 rectangle having an area of 65 square units rather than the original 64 square units.

[1928] *The Appendix and the Spectacles* Miles J. Breuer **[S/G]**

In this story, the reader encounters a cruel and greedy banker by the name of Cladgett who interrupts the studies of a medical student by demanding that he repay a student loan immediately. As a result, the student must abandon medical school; instead of becoming a medical doctor, the student whose name is Bookstrom pursues advanced mathematics. Some years later, Cladgett is in need of emergency surgery due to acute appendicitis, and by sheer coincidence, Cladgett is introduced to Dr. Bookstrom who at this time has mastered higher mathematics and specializes in the fourth dimension.

The geometric theme is the fourth dimension. Dr. Bookstrom claims that he can move the patient “along the fourth dimension” (p. 24) and then remove the appendix without making any incision. The fourth dimension is described at right angles to the x-y-z coordinate system.

Ironically, Dr. Bookstrom’s title of “doctor” was not of the M.D. variety. It seems that Bookstrom earned a Ph.D. in mathematics and hence the honorific title of doctor. There are many types of doctorates, and two such degrees are Doctor of Medicine (M.D.) and Doctor of Philosophy (Ph.D.). In today’s culture, most people assume the title “doctor” refers to an M.D.

The geometry described in this story deals with the "fantasy" of the fourth dimension. Dr. Bookstrom claims that in two dimensions, all points inside a circle can be visualized. However, if we were two-dimensional beings as in Flatland, then we would not be able to "see all points inside a circle" (p. 32).

[1962] *The Mathematician's Nightmare: The Vision of Professor Squarepunt*

Bertrand Russell **[P/G]**

This story is a satire of Pythagoras' mysticism. After a long day of research into Pythagorean numerology, Squarepunt falls asleep from sheer exhaustion. He proceeds to have a strange dream in which numbers assume human form with geometric properties.

Geometrical representations of numbers by points in a plane are called figurate numbers and had special appeal for the Pythagoreans. Russell mentions perfect numbers, perfect squares, and cubes, primes, and pyramidal numbers. For this large array of numbers, Pi is the master of ceremonies who has the numbers perform a ballet.

Odd numbers were male and were considered "good," while even numbers had the distinction of being both female and "evil." This type of ancient Greek dualism cast women on the side of evil. The Pythagoreans attributed mind/spirit to men and body/matter to women. Russell is opposed to number mysticism, which he considers not rational.

The geometry in this story is presented by Pi, the master of ceremonies. Pi introduces Squarepunt to each number starting with one. He then proceeds to describe properties of square numbers, triangular numbers, and perfect cubes. For example, eight proudly mentions that she is the first of the cubes—"except for poor old one."

[1998] *The Wild Numbers* Philibert Schogt **[Ax]**

In this novel, the plot concerns an obsessive desire to prove a mathematical theorem. A young mathematics professor attempts to prove such a theorem with the aid of his mentor who is the chairperson of the mathematics department. "Mathematics is the love of patterns," utters the young mathematics professor who becomes obsessed with his research (p. 19).

The geometric theme of this novel is the use of circular reasoning to establish the truth of a theorem. An egregious flaw is so obvious that Swift's colleague states, "Your proof is circular" (p. 130). Circular reasoning is when one assumes that the conclusion is true and then establishes the conclusion based on those assumptions.

Coping with a disruptive student in a mathematics class is an important theme in academia. Such a student appears in this work of mathematical fiction. He is a 53-year-old student who happens to be a retired mathematics teacher who opts to retake mathematics courses for a deeper understanding and incessantly asks irrelevant questions, establishing himself as a class nuisance.

The most blatant error in constructing a proof of a theorem is assuming the truth of what one is trying to prove. The chairperson who is the young professor's mentor also falls into the trap of circular reasoning. Tragically, the professor is devastated since his proof is bogus and his mentor, due to his embarrassment, announces his retirement (p. 156).

Summary

The three categories for the 26 sources of mathematical fiction were analyzed in the section in this chapter. For each source, the geometric theme and the cultural theme were analyzed. After reading each source, the reader may write about both the geometric and cultural themes. Since the geometry presented in each source is non-fiction, the reader is exposed to real mathematics. Consequently, reading and writing across the curriculum may become a reality. Literature and geometric concepts are connected through mathematical fiction as analyzed in this chapter.

Chapter V

THE AUTHORS' BACKGROUNDS AND LINKS TO MATHEMATICS

Overview

This chapter presents relevant biographical data about the authors of the mathematical fiction contained in this present study. These authors constitute a diverse group of people with different backgrounds ranging from theology to professional mathematicians with doctorates. The 26 authors may be divided into three groups:

Group 1: Writers with little mathematical or scientific training

Group 2: Writers with a background in a related field to mathematics; such as physics, engineering, biology, astronomy, and computer science

Group 3: Writers with a strong background in mathematics

Each group of authors is identified in chronological order as it connects to their respective works of mathematical fiction. Discovering the background of the author may explain why his or her mathematical fiction was sorted in the given category discussed in the previous chapter. While authors with little mathematical or scientific training (Group 1) seem to be the weakest in terms of geometry, within this group are gifted and famous writers who have written compelling stories with geometric themes and powerful cultural connections.

Group 1: Writers with Little Mathematical or Scientific Training

Born in England in 1819, Charles Kingsley's expertise was in ancient history and religion. He did research on the life of Hypathia and 5th-century Alexandria. Kingsley was appointed Professor of History at Cambridge in 1860 (Ufferman, 2007). Reverend Charles Kingsley applied his erudition in his 1852 novel *Hypathia: New Foes with an Old Face*. History of mathematics in Alexandria during the time of Hypathia provides a link between the title character and the religious/philosophical environment, which is a crucial element of Kingsley's novel.

Another author trained in theology and classics was Edwin A. Abbott (1838-1926). Over 120 years ago, Abbott introduced his readers to dimensional geometry using mathematical fiction as a conduit. His mathematical fiction is unique for its creativity in anticipating the properties of mathematical figures of higher dimensions. Abbott was neither a mathematician nor a physicist, but a renowned Shakespearean scholar and social critic of Victorian England. In his classic work *Flatland: A Romance of Many Dimensions* (Abbott, 1884), the geometric theme is linked with gender bias, which was prevalent during the time *Flatland* was published. Abbott demonstrated a sense of humor when he authored *Flatland* under the pseudonym A. Square; his middle initial A. stands for Abbott (same as his last name—Edwin Abbott Abbott), thus making his name EA^2 (Stewart, 2007).

Only one year after *Flatland*, Elizabeth Wormeley Lattimer wrote a work of mathematical fiction dealing with the culture of Afghanistan. Born in 1822, she traveled extensively and became a writer of the history of Arabic societies. She was not trained in mathematics, but knew the fallacy of 64 = 65 in recreational mathematics. Her familiarity with chess and the customs of the people of Afghanistan together with the details of a geometric puzzle enabled her to weave a rich tapestry of geometry and Arabic culture in her short story, *The Sirdar's Chess-Board* (Lattimer, 1885).

One of the most influential Czech writers of the 20^{th} century, Karel Capek, born on January 9, 1890 in Bohemia, was a playwright, journalist, translator, and novelist. In 1936, he was nominated for the Nobel Prize. Recognized as an international writer, he introduced and popularized the work "robot" which appeared in his successful play *Rossum's Universal Robots: (R.U.R.).* Capek wrote about weapons, war machines, and conflicts between countries. Some of these themes appeared in his short story *The Death of Archimedes* (Capek, 1949). In addition, this short story deals with the philosophical implications of war. Capek was interested in philosophy and highlights Archimedes' mathematical focus on war machines. Being a straight A student in high school, Capek enrolled in Charles University of Prague as a philosophy major. After graduation with a bachelor's degree, he attended the Sorbonne of Paris and graduated with a master's of philosophy (Kussi, 1990).

Thus far, some of the writers in Group 1 have backgrounds in history, theology, philosophy, Arabic culture, and Shakespearean literature. The next writer in this group has a background in English literature. K.S. Robinson, born in Waukegan, Illinois in 1952, is best known as a writer of science and mathematical fiction, and in particular for his novel *The Blind Geometer* (Robinson, 1989). He instructs a writer's workshop and has written over 50 short stories. His B.A. is in literature from the University of California and his M.A. is in English from Boston University. In 1982, Robinson returned to the University of California to earn a Ph.D. in English. The major themes in his short stories include overcoming disabilities, achieving social justice, and scientist/mathematician as heroes by their research innovations. He has won over 11 major awards for his writings (Robinson, 2006). Recently, his writings have included ecological and sociological themes.

Another author of mathematical fiction with a degree in English is Mark Haddon. *The Curious Incident of the Dog in the Night-Time (CI)* (Haddon, 2004), which contains a mathematical proof, appeared on the coveted *New York Times* best-seller list in 2004 and won the Commonwealth Writer's Prize Overall Best First Book. Born in 1962, Haddon studied English at Oxford University and worked with autistic children. Describing his work as "suspenseful and harrowing as anything in Conan Doyle," CI became a highly rated novel in the mathematical fiction genre. Haddon's work with autistic children and his creative writing experience at Oxford, coupled with his obvious passion for mathematics, combined to make his first novel delightful and inspiring on two levels. First, the reader gains insight into autism and, second, the mathematics is genuinely interesting. In an interview (Weich, 2005), Haddon said that "if you enjoy math and you write novels, it's very rare that you'll get a chance to put your math in a novel. I leapt at the chance."

Rounding out Group 1 is Tina Chang, a published author and Brooklyn's poet laureate (*New York Times*, March 21, 2010). Tina Chang is neither a mathematician nor a scientist. She is an accomplished poet who holds an M.F.A. from Columbia University. Presently she teaches at Hunter College of the City University of New York. Combining her talent for poetry and her knowledge of the outline of Perelman's proof, Chang wrote an educational and entertaining short story that was published by the Mathematical Association of America (MAA). In fact, her work *Perelman's Song* (2008) is appropriate for a wide audience interested in the solution of one of the most famous problems in all of mathematics.

Group 2: Writers with a Background in a Related Field to Mathematics

The authors in this group are all accomplished writers of mathematical fiction. Whereas the writers in Group 1 have backgrounds in the humanities, the writers in Group 2 have training in the sciences other than mathematics.

The first work in this group is *Young Archimedes* (Huxley, 1924), whose geometric theme of proving the celebrated Pythagorean theorem by a pre-teen is connected to the cultural theme of musical talent. Connecting the link between excelling in both mathematics and music is the recognition of patterns, which Huxley explores in his short story. Aldous Leonard Huxley (1894-1963), the acclaimed novelist and literary icon, is best known for his classic novel *Brave New World*. While he was at Oxford, the noted circle of writers included Lytton Strachey, D.H. Lawrence, and Bertrand Russell. Of the three authors, Russell was famous both as a philosopher and a logician. Perhaps Huxley's interest in mathematics was due to Russell. Scientific and mathematical ideas were important to Huxley and he often used them in many of his books. During his long and distinguished career, Huxley wrote 47 books and received numerous awards for his writing. In 1959, Huxley was the recipient of the Award of Merit for the Novel. This prestigious award has been given every five years to such honorees as Ernest Hemmingway, Thomas Mann, and Theodore Dreiser (Atkins, 1956). Huxley's extensive research in such diverse areas as Greek, history, psychology, biology, and education are well documented.

The next work considered was written by a medical doctor. In *The Appendix and the Spectacles* (Breuer, 1928), the reader is introduced to the fourth dimension during the surgical removal of an appendix. Dr. Miles J. Breuer (1889-1947) was the author of many short stories dealing with both science and mathematical fiction. He earned an M.D. degree and was fascinated by the concept of higher dimensions. Many of his short stories appeared in popular magazines such as *Amazing Stories* and *Argosy*. Some reviewers have credited Breuer for his "new Ideas" (*The Encyclopedia of Science Fiction*, 1993).

In *A Subway Named Möbius* (Deutsch 1950), the author combines a popular song by the Kingston Trio with the properties of a Möbius strip. Armin Joseph Deutsch (1918-1969) is best known as an astronomer and author of many science fiction short stories. His degree in astronomy is from the University of Chicago, which he received in 1946. On the far side of the moon is a crater named after Deutsch and referred to as the "Deutsch crater." In 1951, he was nominated for the Hugo Award for his short story, which is included in the present study, *A Subway Named Möbius.* This quirky piece of mathematical fiction indicates that Deutsch was familiar with the geometric properties of the Möbius strip.

Group 2 thus far is comprised of a biologist/psychologist, a physician, and an astronomer. The last member of this group is a computer scientist. When Katherine Neville was 8 years old, she began writing stories and her goal was to become an author. After graduating from college, she entered the computer field as an analyst where she wrote programs for the New York Stock Exchange and the transportation industry. Leaving the computer field, she achieved her life-long ambition of a full-time professional writer and published her first novel, *The Eight,* which has been translated into 20 languages. Researching diverse areas such as chess, Russian history, music, art, the French Revolution, and mathematics, Neville's novel not only entertains but also educates her readers. The novel presents a wealth of mathematics, chess, and history (Neville, 2009).

Group 3: Writers with a Strong Background in Mathematics

This group consists entirely of professional mathematicians who have engaged in writing novels and short stories in the mathematical fiction genre. Because of their mathematical training, the geometric themes in their writings encompass the very foundations of mathematics relying on the philosophy and history of mathematics. Many authors in this group hold doctorates in mathematics and are professors of mathematics in colleges and universities in the United States and Europe. Writing on geometric themes in Euclidean and Non-Euclidean geometrics, these authors also blend their plots into a cultural context.

Robert Anson Heinlein (1907-1988) is representative of the mathematical fiction genre of Group 3. For many decades, the “Big Three” in science and mathematics fiction were Isaac Asimov, Arthur C. Clarke, and Robert Heinlein. Of the three, only Heinlein had the mathematical background which he used in his novels and short stories. His many accolades for his fiction include four Hugo awards and the Grand Master award given by the Science Fiction Writers of America for his lifetime achievements. After his discharge from the United States Navy, Heinlein took graduate courses in mathematics and physics. His writing output includes 32 novels and 59 short stories (Patterson, 1999). Perhaps Heinlein is best known for his 1940 work of mathematical fiction titled *And He Built a Crooked House*. In this work, he used his knowledge of physics and mathematics in describing the hypercube.

One of the leading figures of mathematical fiction is Martin Gardner whose two short stories are included in this present study, namely *The Island of Five Colors* (Gardner, 1952) and *No-Sided Professor* (Gardner, 1974). It is no exaggeration to claim that Gardner was the most influential person with regard to recreational mathematics. He single-handedly generated interest in mathematics through his “Mathematical Games” column, which appeared in *Scientific American* for 25 years (1956-1981). He introduced countless readers to such terms as flexagons, polyominoes, sona cubes, the board game hex (due to John Nash), tangrams, and Penrose tiles. Regarded as an authority on Lewis Carroll, Gardner’s *The Annotated Alice* is highly acclaimed. He has authored over 65 books and numerous articles.

Gardner’s background in recreational mathematics is evident in the two selections listed above. Moreover, he wrote in mathematical areas other than geometry such as number theory, algebra, probability, game theory, and many other topics in recreational mathematics. His writings on many of these topics are included in *The Colossal Book of Mathematics* (Gardner, 2001). He passed away on May 22, 2010 at age 95.

Another author in Group 3 is Arthur Porges who wrote *The Devil and Simon Flagg* in 1954. Born in Chicago on August 20, 1915, Porges is known today as a writer of short stories. His literary career spanned over a half-century, during which he published over 200 short stories, many in the science fiction genre. Earning a master’s degree in mathematics, Porges taught on the college level but abandoned his mathematics teaching profession to become a full-time writer. His interest in mathematics coupled with his life-long fascination with science fiction inspired him to write about Fermat’s Last Theorem (FLT). One of Porges’ best known stories is *The Devil and Simon Flagg*, which is representative of his mathematical fiction (Simms, 2002).

The next author is Bertrand Russell, who wrote a short story in 1962 with a title that surely suggests that mathematics is involved in this piece of fiction. *The Mathematician’s Nightmare: The Vision of Professor Squarepunt* deals with the blending of geometry and algebra through figurate numbers. Writing with Alfred North Whitehead (1861-1947), Russell (1872-1970) coauthored the classic *Principia Mathematica* (1903), which attempted to reduce all of mathematics to the laws of logic. A descendant of an aristocratic family, Russell was one of the greatest logicians of the 20th century. He was a graduate of Trinity College, Cambridge University, and wrote numerous books on logic, philosophy, and mathematics (Eves, 1990). In 1950, he won the Nobel Prize for Literature "in recognition of his varied and significant writing in which he champions humanitarian ideals and freedom of thought." Three decades before receiving the Nobel Prize, Russell was dismissed from his teaching position at Trinity College because of his defense of being a conscientious objector. From G.H. Hardy's (1970) account of Russell's prosecution and the public's view of pacifism, the reader can gain a complete account of Russell's trial.

Magic has fascinated people since the time of the Pythagoreans. Capturing the mystical spirit, Larry Niven (1979) has written a short story titled *Convergent Series*. Niven obtained an undergraduate degree in mathematics and completing graduate work in applied mathematics. In 1967, he won the prestigious Hugo Award for Best Short Story and repeated this honor in 1976. In addition, Niven is known for his script for *Star Trek: The Animated Series.* Combining mathematical knowledge with a whimsical writing style, *Convergent Series* amuses and educates his readers.

Forty-some years after Heinlein's story, Gregory Bear (b. 1951) wrote about the hypercube in connection with the computer in his 1986 story titled *Tangents*. Bear is best known for his science fiction in novels, short stories, television, and film. He has written many novels and short stories in this genre, but has also written mathematical fiction. To his many credits is his involvement with *Star Trek: The Original Series* on television. Awarded two Hugos and five Nebulas for his fiction, Bear has been called the best working writer of "hard" science fiction. As a teenager, he started writing short stories and, to date, Bear has published more than two dozen novels and a much larger number of short stories. His works have been translated into 19 languages including Chinese, Dutch, Greek, and Hebrew. One of his outstanding stories concerns the fourth dimension and applications of the computer.

The next author is an accomplished mathematician who has won many literary awards. Throughout his long career as a mathematician, distinguished professor at Brown University, and author of numerous mathematics texts and monographs, Philip J. Davis is also an accomplished writer of mathematical fiction. Born in 1923, Davis is known for his work in numerical analysis as well as his research in the history and philosophy of mathematics. Among his awards is the Chauvenet Prize for his work on the gamma function. *The Mathematical Experience,* co-authored with Reuben Hersh, won an American Book Award in 1984 (Davis & Hersh, 1981). Two years later, his book titled *Descartes' Dream*, also written with Hersh (Davis & Hersh, 2005), described the connections between mathematics and technology. In 1991, Davis delivered the Hendrick Lectures of the MAA, which led to the publication of his book *Spirals: From Theodorus to Chaos* (Davis, 1993), which in turn became one of the main themes in his mathematical fiction story titled *Thomas Gray: Philosopher Cat* (Davis, 1988). His view of mathematics is best expressed in his own words: "To me, mathematics has always been more than its form, or its content, its logic, its strategies, or its application. Mathematics is one of the greatest of human intellectual experiences, and as such merits and requires a rather liberal approach" (Davis, 1993).

The Wild Numbers (Schogt, 1998) deals with publishing a mathematical proof as a creative activity with the distractions of academia. The combination of a mathematical theme with a philosophical approach to an ethical question of plagiarism is handled effectively by the author. Born in 1960 in Amsterdam, Schogt earned an undergraduate degree with a minor in mathematics. His M.A. was in philosophy from the University of Amsterdam. *The Wild Numbers* was his first novel dealing with mathematical fiction. The *Math Horizons* magazine, published by the MAA, gave his novel a favorable review (Amazon, 2004).

Alex Kasman, a mathematics professor at the College of Charleston, is one of today's most prolific writers of mathematical fiction. Thanks to Professor Kasman's efforts, a database of over 700 novels and short stories dealing with mathematical fiction is available online. Kasman's own short stories reveal a love of fiction and an expertise in mathematics. Recently, the MAA published a collection of his short stories (Kasman, 2005) titled *Reality Conditions.* As a professional mathematician, Kasman never trivializes mathematical ideas. His writings have modernized the entire genre of mathematical fiction in the sense that the mathematics is richer in content and diversity. He has also added a new dimension to his stories by enhancing their pedagogical value by including "Author's Notes" at the end of his book to further explain the mathematics inherent in each story. His stories dealing with geometric themes are woven with cultural themes such as gender bias, aesthetics, and the culture of academia. Kasman's *Reality Conditions* received favorable reviews in the AMS (Swallow, 2006). His two works of mathematical fiction included in this present study, *The Object* and *Murder She Conjectured* (Kasman, 2005), reflect his skill as an expositor of mathematical fiction.

Tefcros Michaelides is a skilled author who is also a mathematician. Earning a Ph.D. in mathematics, he is presently a Professor of Mathematics at Athens College. Added to his expertise in the history of mathematics is his extensive knowledge of art and politics. In the genre of mathematical fiction, Michaelides has also written on the theory of numbers, analysis, and geometry. Chief among his translations are *The Parrot's Theorem* (Guendj, 2002) and

D'Alembert Principle (Crumey, 1998). His mathematical fiction, *Pythagorean Crimes* (Michaelides, 2006), combines axiomatics with the history of mathematics.

The last two authors in Group 3 are both Ph.D.s in mathematics and are interested in improving the quality of mathematics education. Arturo Sangalli has written a work of mathematical fiction that raises some hypothetical questions. He combines mystery with knowledge of the history of Pythagorean mathematics. Pythagorean mathematics is presented in his novel in a pedagogical manner since the author strives to educate his readers. Sangalli's literary and pedagogical skills are presented in his mathematical fiction titled *Pythagoras' Revenge: A Mathematical Mystery.* The reader may be reminded of Dan Brown's *The DaVinci Code* while reading Sangalli's novel.

The last work in this group is *Pythagoras' Darkest Hour*, a short story by Colin Adams (2009). Adams received his Ph.D. in 1983 and is an active mathematician. Presently, he is the Thomas T. Read Professor of Mathematics at Williams College. His expertise lies in knot theory and hyperbolic geometry. Among his many accolades are the Deborah and Franklin Tepper Halmo Distinguished Teaching Award from the MAA. In addition, he was a Polya lecturer for the MAA from 1998 to 2000. His first work of mathematical fiction was published by the American Mathematical Society (AMS) and is titled *Riot at the Calc Exam and Other Mathematically Bent Stories* (Adams, 2009). His short story included in this present study was chosen because of the author's humor, pedagogy, and geometric content. In Adams' own words, "this book gives a window into mathematics and the culture of mathematicians. Appropriate for mathematicians, math students, math people, lay people with an interest in mathematics, and indeed everyone else" (Adams, 2009).

Summary

All of the authors of the 26 novels and short stories were not known to the researcher at the start of this study. A presumption was held that, in general, authors of mathematical fiction would have a strong background in mathematics. The researcher wanted to see if this was the case for all of the mathematical fiction selected. While most of the authors have a strong background in mathematics, three distinct groups of authors were found. The authors range from having little mathematics or scientific background to having a strong background in mathematics. In between these two groups are the authors who have a background in science.

This chapter demonstrated that any author, regardless of mathematical background, can write a mathematical fiction novel or short story, including stories with a dominant geometric theme. Naturally, it was rare for the authors with a strong mathematical background to have a dominant cultural theme in their writing. The authors with strong mathematical backgrounds also covered the entire span of geometric categories; that is, all six geometric categories contained at least one author of Group 3.

Chapter VI

DISCUSSION

This chapter presents and discusses the data obtained in the study. The first part of the chapter is devoted to the study of textbooks, that is, of formal learning of geometry. This study was motivated by the idea of comparing and contrasting informal and formal learning. To this end, some textbooks were analyzed which seemed to some extent relevant to the analyzed fictions. The second part of the chapter presents some results of the coding of the fictions. Some basic information on the fictions in question is summarized below, which permits a later look at patterns and theoretical generalizations.

Mathematical Fiction and Formal Learning

As noted, a goal in this study is to explore mathematical fiction as an educational tool and a source of mathematical knowledge. Since the mathematics embedded in a mathematical fiction is factual, the geometric themes can be found in mathematics textbooks which can be used inside and outside the classroom to further these concepts.

While any reader of mathematical fiction can take part in informal learning and gain geometric knowledge without any association to a classroom, this section focuses on the connection between mathematical fiction and formal learning. Inside the college classroom, the main source of formal learning in geometry is the textbook. The next section of this chapter aims to unite and explore the geometric topics found in the mathematical fiction with the same topics in textbooks. Textbooks were chosen for all six geometric categories. As noted in Chapter II, while textbooks and mathematical fiction vary in terms of appearance, the mathematical topics found on the pages of these two different sources can be the same.

The researcher identified a few popular geometry textbooks of the same time period as the fictions analyzed. A selection of mathematical topics from the works of mathematical fiction was examined in comparison with these geometry textbooks. Three dimensions were explored: formal or informal presentation, reader's prerequisite knowledge, and presentation of the geometric theme.

The geometric themes presented in the works of mathematical fiction are connected with the geometry textbooks of that time period. It appears that the authors of mathematical fiction borrowed the geometric themes that were already published in geometry textbooks, and wove a story around these themes. Through the connection of the geometry found in a textbook coupled with the author's skill in mathematical fiction, the geometric theme took on relevance to the plot of the story. This connection of combining geometric themes found in geometry textbooks with mathematical fiction is present in all 26 novels and short stories examined in the present study. Some of the popular geometry textbooks in use during the same time period as a specific work of mathematical fiction are presented in the section titled Geometry Textbooks Aligned with Plane Geometry. College textbooks and high school textbooks were considered, as the audience at this level is the target for this study.

In the next section, geometry textbooks will be sorted into six categories that are matched with the six categories of mathematical fiction presented in this study. For each textbook, the following two questions are addressed:

1) Why was the textbook selected?

2) How was the content aligned with the geometric theme of the mathematical fiction?

Geometry Textbooks Aligned with Plane Geometry [P/G]

During Abbott's time, there were two popular English editions of the *Elements* (Heath, 1956, p. 111). Robert Simson's textbook *Euclid's Elements* went through 26 editions and its formal presentation of the material consisted of constructing proofs of theorems using the statement and reason format (Simson, 1844). This text was selected because it contained a faithful translation of Euclid's *Elements* and included material on regular polygons that appeared in Abbott's *Flatland* (1884). *A Sequel to Euclid* (Casey, 1881) is similar in style and content to Simson's text and was also used during the time of *Flatland*. Both textbooks were used in London, where Abbott worked and studied. In Casey's text, the theorems were demonstrated in a formal style. His text went through six editions, with the last edition published posthumously in 1891 (Johnson, 1929).

Properties of regular polygons appeared in both textbooks and, consequently, the geometric theme dealing with polygons in *Flatland* was present before Abbott's novel was published. It should be noted that Simson's textbook was "so well known as not to need any further description" (Heilbron, 1998). It is probable that Abbott's introduction to Euclidean geometry was from Simson's textbook.

Corresponding to the time period of *Young Archimedes* (Huxley, 1924), two geometry textbooks were widely used. First, *Plane Geometry* (Betz & Webb, 1912) offered more explanation of the proofs compared with its predecessors. Competing with Betz and Webb was the text *Plane and Solid Geometry* (Schultze & Sevenoak, 1913). This text had many more diagrams used for motivation and an expanded set of exercises. According to Schultze and

Sevenoak, the main objective of their text was "to introduce the student systematically to original geometric work" (p. vii). Both texts were selected because they contained proofs of the Pythagorean theorem that were mentioned by Huxley. In particular, Schultze and Sevenoak's proof of the Pythagorean theorem (p. 188) is connected with Huxley's sketch (p. 24) using similar triangles. Both this geometry textbook and Huxley's mathematical fiction sketch out the proof of the Pythagorean theorem. In the textbook, all statements are given, but some of the reasons are left to the reader to supply. Huxley, on the other hand, writes that the proof follows from similar triangles used in high school geometry. Schultze and Sevenoak's geometry text was selected because it provides "a more pedagogic sequence of propositions" (p. vii).

The Curious Incident of the Dog in the Night-Time (CI) dealt with the converse of the Pythagorean theorem (Haddon, 2004). The textbook aligned with this geometric theme is *College Geometry* (Denney & Minor, 1991). This text was selected because it devotes a section to the converse of the Pythagorean theorem and proves the converse of the Pythagorean theorem on page 165 (Theorem 6-9).

Geometry Textbooks Aligned with Solid Geometry [S/G]

At the writing of *And He Built a Crooked House* (Heinlein, 1940), several geometry texts dealt with solid geometry. A textbook popular during Heinlein's time that covered solid geometry was authored by Herbert E. Hawkes (Hawkes et al., 1922) and was called *Solid Geometry*. Solid geometry is the traditional name for the geometry of three-dimensional Euclidean space. This text was selected because it contained properties of planes and solids with proofs of important theorems. Another textbook on solid geometry titled *Systematic Study of Geometry: Part II* was chosen because it dealt with the analysis of drawing space figures (Gurvitz & Gangnus, 1933). The content of the text is aligned with Heinlein's short story because the authors describe properties of a cube intersected by a plane (pp. 27-28, 56). In Heinlein's mathematical fiction, the hypercube is described as a four-dimensional shape. The geometry of 4-space appeared in a textbook titled *Geometry of Four Dimensions* (Manning, 1914). It was chosen because it described the hypercube (tesseract). Heinlein's description of a hypercube corresponds to Manning's geometry text.

Inspired by computer graphics, *Tangents* (Bear, 1986) dealt with exploring higher dimensions using the computer. Among the many textbooks containing the geometry of higher dimensions, Ian Stewart's (1975) book *Concepts of Modern Mathematics* was selected because it is written for a general audience. For example, in chapter 14 (pp. 200-214), "Into Hyperspace," the reader travels from the Euclidean plane to three-dimensional space and then into four-dimensional space and beyond. This chapter's content aligns with Bear's mathematical fiction.

Geometry Textbooks Aligned with Historical Foundations of Geometry [HF/G]

Thomas Gray: Philosopher Cat (Davis, 1988) connects with a few textbooks such as *Spirals: From Theodorus to Chaos* (Davis, 1993). The reason that this text was selected is that it was written by the same author as the connecting mathematical fiction. Concepts such as the spiral of Theodorus are explained and illustrated, as are Spirals of Archimedes and Bernoulli. The content of Davis' mathematical fiction is expanded in his later text. Irrational numbers play an important part in the spiral of Theodorus and, consequently, in the main theme of Davis' mathematical fiction. A textbook that combines geometry and numbers is appropriately titled *Numbers and Geometry* (Stillwell, 1998). Inside is a section that deals with irrational numbers such as the square root of 2. Stillwell's text was selected because he writes in a non-rigorous manner. The writing style of Davis' textbook is more rigorous and demands a strong background in mathematics.

Davis' geometric theme in his mathematical fiction is concerned with the spiral of Theodorus and why he stopped at the square root of 17. Connecting this question to *A History of Mathematics* (Boyer, 1968), the researcher points out that the geometric theme of Davis' mathematical fiction is stated on page 95 of Boyer's book as "It is not known how he (Theodorus) did this, nor why he stopped with the square root of 17." The content of Davis' fiction and Boyer's historical analysis is similar.

Katherine Neville's (1989) novel *The Eight* mentions "one-half the square root of five minus one" on page 296. She then goes on to write that the ancient Greeks called this number the "aurio sectio" (the golden section). *History of Mathematics, Vol. I* was chosen as a textbook in this category since it describes the golden section (Smith, 1951)—in particular, $\phi = \frac{\sqrt{5}-1}{2}$. On pages 4 and 218, Smith describes the number phi. In Smith's (1953) second volume of the *History of Mathematics*, he writes that the construction of the pentagon depends on the Golden Section (the division of a line segment into extreme and mean ratio). He then goes on to say that the golden section is sometimes referred to as the divine proportion (p. 291).

A second textbook that relates to Neville's mathematical fiction is *An Introduction to the History of Mathematics* by Howard Eves (1990), a noted historian of mathematics. The text is now in its sixth edition and contains material on Euler's life that parallels Neville's novel. A part of *The Eight* takes place at the Prussian Academy and the St. Petersburg Academy where Euler did much of his mathematical research. In the novel, Euler is connected with the Konigsberg Bridge problem and the Knight's Tour problem. This content of the novel is connected to the textbook since Eves describes Euler's positions at both academies (pp. 432-433). In addition, the Konigsberg Bridge problem is discussed and illustrated in Eves' text on pages 459-460.

Alex Kasman's (2005) short story *The Object* has as its main mathematical theme the properties of regular polyhedra. Investigating such polyhedra, Kasman introduces Euler's formula. Using modern notation, his formula is $F - E + V = 2$, where F is the number of faces, E is the number of edges, and V is the number of vertices.

Many textbooks contain material about regular polyhedra and Euler's formula. Both geometry and liberal arts textbooks discuss this topic. Textbooks dealing with the history of mathematics present the material on polyhedra in an historical context (Katz, 1998). Katz's text *A History of Mathematics: An Introduction* (2nd ed.) covers the five regular polyhedra (pp. 94-95) as well as Euler's mathematical contributions (p. 553). *Geometry: An Investigative Approach* (O'Daffer & Clemens, 1977) takes the point of view that geometry should involve interesting classroom experiences. For example, the author of this text provides discovery lessons that highlight the creative side of geometry. Providing a rich environment for the study of solid shapes using models of the five regular polyhedra, the student can observe, touch, and make conjectures. Euler's formula may be verified by actually constructing the polyhedra and, for this reason, this text is aligned with *The Object.*

Another textbook selected to align with Kasman's mathematical fiction is *Mathematical Ideas* (11th ed.) (Miller et al., 2008). In the chapter on geometry, topics include plane and solid geometry, transformations, constructions, tessellations, non-Euclidean geometry, and polyhedra. Regular polyhedra are defined and the five Platonic solids are shown—namely, the tetrahedron, cube, octahedron, dodecahedron and icosahedron (p. 579). The authors discuss the number of faces, vertices, and edges for each polyhedron, which are part of the plot of Kasman's *The Object*.

Geometry Textbooks Aligned with Topology [Tp]

A Subway Named Möbius (Deutsch, 1950) deals with the mathematical properties of the Möbius strip. An elementary textbook on topology, *Mathematical Snapshots* (Steinhaus, 1941), describes "the science of knots" and contains photographs of various configurations. The Möbius strip (ribbon) is explained in this book by twisting a strip of paper through an angle of 180 degrees (a half-twist) and then pasting the edges together. One of the descriptions of this strip (p. 114) is that "a fly may crawl over the whole of it without the inconvenience of crossing its edge." It is for these reasons that Steinhaus' book is aligned with Deutsch's short story.

Martin Gardner's (1952) story *The Island of Five Colors* is concerned with the four-color problem. This problem was well known before his piece of mathematical fiction, although the four-color theorem was not proved until after this short story was published. A textbook that devotes space to this problem was published in 1955 by Meserve (pp. 288-306), titled *Fundamental Concepts of Geometry*. This book's main audience is prospective teachers of secondary mathematics and "those desiring a broad liberal education." Meserve's presentation is informal. The goal of the material is to illustrate the four-color problem (theorem) by drawing maps and having the reader experiment with different colors.

In her short story *Perelman's Song*, Tina Chang (2008) celebrates Perelman's proof of the Poincare conjecture. Textbooks related to Chang's piece of mathematical fiction are of a high level and involve many courses in mathematics beyond calculus. Nevertheless, some texts suitable for liberal arts students present material dealing with the topology in Chang's story. *Geometry* (Lang & Murrow, 1988) is one such text because it deals with some elegant ideas about finding "the volume of the ball (sphere)." Near the end of this text (pp. 281-290), Lang and

Murrow carefully develop an appropriate procedure for the ball of radius one (unit sphere) using the upper half (hemi-sphere) and taking n-cylindrical shells to calculate its volume.

Another textbook whose content is aligned with the Poincare conjecture, the theme of Chang's mathematical fiction, is *Euclidean and Non-Euclidean Geometries: Development and History* (Greenberg, 2008). In his fourth edition, Greenberg describes models due to Henri Poincare. Many diagrams are included in the treatment of the "disk model" due to Poincare (p. 302).

Geometry Textbooks Aligned with Axiomatics [Ax]

At the time of *Euclid Alone* (Orr, 1975), many textbooks covered the subject of axiomatics. Two texts on axiomatics relate to Orr's mathematical fiction: the first is *An Introduction to the Foundations and Fundamental Concepts of Mathematics* (Eves & Newsom, 1961) and the second is *An Introduction to the Foundations of Mathematics* (Wilder, 1952, rev. 1967). Both texts were selected with relation to Orr's short story because they explained the properties of a set of postulates. Both Wilder's and Eves' texts formally explain the properties of a set of postulates: consistency, independence, and completeness. In this regard, the texts are similar to each other. By contrast, Eves' text gives the reader more illustrative examples and the exercises are graded in difficulty and are more extensive.

Many textbooks explore the themes of *Pythagorean Crimes* (Michaelides, 2006) that deal with the foundations of geometry (axiomatics) and the early history of Greek geometry. One textbook that deals with the murder of Hippasus—part of the plot of *Pythagorean Crimes*—is *A History of Mathematics* (Suzuki, 2002). According to Suzuki, who is a professor of mathematics at Brooklyn College, Hippasus was mysteriously drowned at sea for revealing that the diagonal of a unit square is incommensurable (pp. 48-50). Other texts in the history of mathematics that deal with the Pythagoreans and their philosophy are *A History of Mathematics* (Boyer, 1968) and *A History of Mathematics: An Introduction* (Katz, 1998). In the study of axiomatics, a set of postulates must be consistent, independent, and complete. A textbook that deals with the study of axiomatics (besides Wilder's book, who was previously mentioned) is *An Introduction to the History of Mathematics* (6th ed.) (Eves, 1990).

Another text that deals with axiomatics is *From Here to Infinity: A Guide to Today's Mathematics* (Stewart, 1987). In this text, mention is made of the coordinate geometry model in the two-dimensional plane so that all of Euclid's axioms hold. Stewart concludes by stating that "if arithmetic is consistent, then so is Euclid" (p. 57). Likewise, non-Euclidean geometry is consistent provided that Euclidean geometry is consistent, which is part of Michaelides' geometric theme. The consistency of an axiomatic system is dealt with in *Euclidean and Non-Euclidean Geometries: Development and History* (Greenberg, 2008), which aligns with the geometric theme in *Pythagorean Crimes*.

Geometry Textbooks Aligned with Projective Geometry [Pj]

The *Blind Geometer* (K.S. Robinson, 1989) connects mathematical fiction with projective geometry through Desargues' (1591-1662) theorem. Among the many geometry texts that align with this novel, the following four texts illuminate this celebrated theorem. In *Geometry Revisited* (Coxeter & Greitzer, 1967), the reader comes upon theorem 3.61 (Desargues' theorem) along with a clear proof. Both authors of this textbook are well-respected scholars in the field of geometry. A second textbook that covers Desargues' theorem in a thorough manner is *College Geometry* (Kay, 1969). The pedagogy contained in Kay's text combines a great deal of historical material that gives the theorems a humanistic context. Kay's writing philosophy, as stated in the preface, is that "the many aspects of geometry should be as visual as possible…" Both texts were selected since Desargues' theorem is proven and presented in historical context.

The third textbook is *Geometry* (Pogorelov, 1984) and the main focus is on Chapter XVI, which deals with projective geometry. First, Pogorelov gives the axioms of incidence for projective geometry together with a brief history of the subject. In the preface, Pogorelov states that his book should be used as a "manual for students of universities and teachers' training colleges." A distinguishing characteristic of Pogorelov's text is the emphasis on the axiomatic method.

The fourth text is *Mathematics in Western Culture* (Kline, 1953), in which historical material is aligned with Desargues' theorem (pp. 145-147). This text was selected because the diagram used to prove Desargues' theorem is carefully explained in terms of two triangles being "in perspective from a point" (p. 148), which connects to Robinson's diagram in his novel.

The geometry textbooks discussed in the previous sections were grouped according to representative works of mathematical fiction included in this present study. These textbooks were used to gain a better understanding of the geometric theme presented in each source of mathematical fiction. In general, the texts evolved from a dense format with little motivation to a multi-colored format with many diagrams and instructional aids such as geo-boards, geometric models, and the geometry sketchpad. The advent of the computer also increased the sophistication of the geometric themes of mathematical fiction. This is evident in *Tangents* (Bear, 1986) and *The Object* (Kasman, 2005). Geometry textbooks used today have many educational ancillaries such as the use of the graphing calculator and visualizing multi-dimensional shapes via the computer.

Throughout this analysis of mathematical fiction with geometric themes, the geometric concept first appeared in the textbook and was later used by the writers of novels and short stories. It may be desirable to connect the geometric theme of a particular work of mathematical fiction with a more formal textbook presentation. Readers of mathematical fiction may broaden their understanding of the geometric theme by consulting a text that aligns with the novel or short story. Although most of the sources of mathematical fiction present geometric themes informally, connecting them with formal textbook presentations is desirable (as explained in Chapter II of this study).

In general, the geometric topic presented in the works of mathematical fiction was presented in an informal fashion, whereas the textbook's presentation was formal. Also, mathematical fiction assumes little prerequisite knowledge of geometry. The themes in mathematical fiction are explained in the context of the story's plot and cultural setting. Many geometric themes of mathematical fiction are presented using the history of mathematical ideas as a supporting component. The author often connects the history of a geometric theme with a work of mathematical fiction. Geometry textbooks, on the other hand, rarely integrate the geometric topic to its history. There are notable exceptions, namely *College Geometry: A Discovery Approach* (Kay, 2001), which contains numerous historical notes and biographical data that humanize the subject. For example, unusual details concerning the lives of the geometers and their contributions are presented alongside key theorems. Another textbook that contains the history of geometry is *Euclidean and Non-Euclidean Geometry: Development and History* (Greenberg, 2008). The foundations of geometry are presented and this text serves as a reference for many of the works of mathematical fiction considered in this current study.

Before the advent of the computer, the geometric themes of mathematical fiction were in stark contrast to later works. After computer technology flourished, the geometric themes became more sophisticated and less fanciful. In like fashion, geometry textbooks were aligned with computer technology. The Geometer's Sketchpad and Dynamic Geometry software are often referred to in geometry textbooks and applied to many geometric transformations. Higher dimensions as a geometric theme in mathematical fiction and geometry texts are connected because of computer technology.

The advent of the computer accounted for a shift in geometric themes in mathematical fiction. Some authors included in this study pivoted their geometric themes around the computer. Utilizing the computer as part of a geometric theme resulted in connecting the mathematical fiction to computer graphics. It is no accident that some geometric themes drastically changed because of computer technology. After the advent of the computer, there was less mathematical fantasy (cf. Abbott, Latimer, Heinlein, Breuer) because now higher dimensions could be visually explored. In *Tangents* (Bear, 1986) and *The Object (*Kasman, 2005), the reader experiences getting "lost in the fourth dimension" through the computer. Contrast Bear's story to Heinlein's where the hypercube is an object in 4-space that was described as fantasy.

After 1952, the computer played an important role in some of the sources included in this study. The following chart lists mathematical fiction from this study in which the computer was an integral part of the story.

Year	[ ][17]	Source	Role of the computer
1975	[Ax]	*Euclid Alone*	The computer is used to check the validity of a geometric proof. Data were postulates for Euclidean Geometry.
1986	[S/G]	*Tangents*	The main character is a computer 'hacker" who broke the Enigma code of the Nazis. The intersection of a plane and a cube is described using a computer.
2005	[S/G]	*The Object*	Various models of polyhedra are studied via the computer. Euler's formula is checked via computer graphics.
2006	[Ax]	*Pythagorean Crimes*	A mathematician tries to solve Hilbert's second problem using a computer. An algorithm is sought to check if a set of postulates are complete.

Organizing the Collected Data of the Study

On the Topics in the Fictions

Question 1 of this study was "What geometric topics are used in the mathematical fiction genre with regard to the selected novels and short stories?" To answer this question in Chapter IV, the 26 sources were analyzed with regard to the geometric themes connected with the plot of the novel or short story. Three categories were examined and presented chronologically (see the section titled The Three Categories of Mathematical Fiction with Geometric Themes). For each source, mathematical themes were identified. The following section presents the patterns of the geometric themes contained in the 26 sources. As recommended by the grounded theory methodology, it was possible to identify several groups of the themes which unite the collected data. In this way, six categories (groups) were found: Dimensionality, Famous Theorems and

[17] This represents which of the six the geometric themes are used (as documented throughout the study, Solid Geometry is coded as [S/G] and Axiomatics is [Ax]).

Problems, Fallacies and Mysticism, Pythagoras and the Pythagoreans, Famous Mathematicians, and Axiomatics.

Dimensionality Connected to Mathematical Fiction

Flatland: A Romance of Many Dimensions (Abbott, 1884)*, The Appendix and the Spectacles* (Breuer, 1928)*, And He Built a Crooked House* (Heinlein, 1940), and *Tangents* (Bear, 1986) are sources that contain dimensionality. E.A. Abbott (1838-1926) is the first author of the 26 examined here to introduce readers to dimensionality with his classic work *Flatland: A Romance of Many Dimensions.* The narrator of Flatland, A. Square, travels between Lineland and Spaceland and is exposed to both two- and three-dimensional space. *The Appendix and the Spectacles* deals with both three-dimensional and four-space. Dr. Bookstrom, who holds a Ph.D. in mathematics with a specialization in the fourth dimension, claims that he can move a patient "along the fourth dimension" (p. 24). In *And He Built a Crooked House,* an eccentric architect, Quintus Teals, has a vision of designing an extraordinary house in the shape of an unfolded tesseract or hypercube (p. 70). This four-dimensional object contains eight cubes and is probably the most familiar four-dimensional object (p. 74). *Tangents* also deals with the hypercube but this time in connection with a computer. In this story, a young Korean teenager, Pal, is fascinated with a hypercube, which he describes as "a four-dimensional analog of a cube" (p. 39). He also describes how a Flatlander can view a cube intersecting a plane. After he talks about the cube intersecting a plane, Pal deals with the intersection of a sphere and a plane from the perspective of a Flatlander (p. 40).

Famous Theorems and Problems Connected to Mathematical Fiction

The Island of Five Colors (Gardner, 1952), *The Devil and Simon Flagg* (Porges, 1954), *The Blind Geometer* (Robinson, 1989), *The Object* (Kasman, 2005), and *Perelman's Song* (Chang, 2008) all discuss famous theorems or problems. *The Island of Five Colors* deals with the four-color problem.[18] The mathematician in this fiction emphatically states that only four colors are necessary because "it is a theorem in topology" (p. 198). *The Devil and Simon Flagg* deals with Fermat's Last Theorem (FLT). Simon Flagg, a professor of mathematics, and the devil enter into a strange bargain (p. 63) in which the devil needs to either prove or disprove FLT. *The Blind Geometer* concerns Desargues' Theorem.[19] Carlos' appreciation of this celebrated theorem is described as "a beautiful theorem, with the purity and elegance characteristic of Renaissance mathematics" (p. 16). A brief discussion of Euclid's fifth postulate (parallel postulate) and the advent of Non-Euclidean geometry also add to the theme of the novel (pp. 40-41). Combining computer graphics with Euler's formula for regular polyhedra, *The Object* explains that regular polyhedra have "faces each of which is a copy of the same regular polygon" (p. 195). The strange polyhedron in this story has 32 faces that are shaped like triangles. Alice writes down Euler's formula V – E + F = 2, where V, E, and F are the number of vertices, edges, and faces of the polyhedron. *Perelman's Song* uses a dialogue between a god and a goddess to describe the Poincare Conjecture, which is the theme of her story.

[18]It is known today that four colors will suffice when coloring a map so that no two bordering regions share the same color. In the case where two regions touch at a single point, they have a common color.

[19]A diagram of Desargues' Theorem appears in the novel and Robinson uses the letters of the diagram as the heading of each section of the story.

Geometric Fallacies and Mysticism Connected to Mathematical Fiction

Geometric fallacies and mysticism connected to mathematical fiction were present in *The Sirdar's Chess-Board* (Latimer, 1885), *No-Sided Professor* (Gardner, 1974), *Convergent Series* (Niven, 1979), and *The Wild Numbers* (Schogt, 1998). *The Sirdar's Chess-Board* involves the 64 = 65 fallacy (see Appendix). Using a pair of scissors and a sheet of paper with 64 squares, Sophia uses "three-snips" and arranges the resulting four figures before the sirdar. Instead of an 8 by 8 square, the sirdar observes a 5 by 13 rectangle. The *No-Sided Professor* deals with a Möbius strip or unilateral surface as its geometric theme. Professor Slapernaski announces that he will demonstrate a surface with no sides called a "nonlateral surface." This topologist then pastes these ends together and declares to his audience of fellow topologists: "you are about to witness the first public demonstration of the Slapernarski surface" (p. 174).

Convergent Series deals with magical rituals with geometric overtones. The main character states "magic fascinates me" (p. 162). After he constructs a pentagram on the basement floor using chalk, a demon appears inside the pentagram.[20] The student diminishes the length of each diagonal of the pentagram and continues this sequence of decreasing lengths so that the dimensions of the pentagram become smaller and smaller. Possessing special magical powers, the demon shrinks his body so that he can fit within any size pentagram. The geometric theme centers on how the student outsmarts the demon to win the bargain. The geometric theme of *The Wild Numbers* deals with an obsessive desire to prove a theorem.[21] Swift is driven to prove the theorem and states that "Mathematics is the love of patterns" (p. 19). In this novel, Swift's mentor tells him (p. 92): "partial success can be more frustrating than no success at all." His

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[20]The demon's arms, legs, and head occupy all five points of the mystical five-pointed star. Both the demon and the student enter into a bargain. The demon claims that he can inscribe himself in any sized pentagram.

[21]Part ego, part obsessive desire to prove a theorem, and part the lure of invisible mathematical patterns drive the young mathematics professor, Isaac Swift, to uncover an unseen pattern.

colleague discovers a serious error and states that "your proof is circular" (p. 130). Swift's fallacious reasoning makes his proof worthless.

Pythagoras and the Pythagoreans Connected to Mathematical Fiction

Pythagoras and the Pythagoreans connected to mathematical fiction is a popular geometric theme. In *Young Archimedes* (Huxley, 1924), Guido at age 7 proves the Pythagorean theorem by using a burnt stick on smooth paving stones (p. 24). The proof described by Huxley is the classic proof of Euclid. Guido remarks that "it's so beautiful," and offers another proof of the celebrated theorem that falls under the heading of a dissection proof. Based on similar triangles, Guido yet offers a third proof of the Pythagorean theorem. *The Mathematician's Nightmare: The Vision of Professor Squarepunt* (Russell, 1962) is a satire of Pythagoras' mysticism, in which numbers assume human form. The Pythagorean view that "all is number" is satirized throughout this short story. *The Curious Incident of the Dog in the Night-Time* (CI) (Haddon, 2004) contains a proof of the converse of the Pythagorean theorem. Haddon, through Christopher's narration, explains Pythagorean triples (pp. 223-226) and tessellations (pp. 202-204) clearly so that the general public can both appreciate and understand the mathematics that is presented. As part of the A-level mathematics examination, Christopher is asked to prove: "A triangle with sides that can be written in the form $n^2 + 1$, $n^2 - 1$, and $2n$, $n > 1$ is right angled. Show by means of a counterexample that the converse is false." The counterexample that Christopher offers is the Pythagorean triple 25, 60, 65, but the sides are not of the form shown above.

As an ancient icon of numerology, philosophy, and mathematics, Pythagoras is a fascinating figure in the history of mathematics as well as mathematical fiction. Although Pythagoras' name is associated with the celebrated theorem concerning a right triangle, he left no mathematical writings. For this reason, people do not speak of Pythagoras' works but rather of "the contributions of the Pythagoreans" (Boyer, 1968). The absence of any of Pythagoras' manuscripts provides the geometric theme of Arturo Sangalli's novel titled *Pythagoras' Revenge: A Mathematical Mystery* (Sangalli, 2009). The information presented in the novel is historically accurate when compared to leading texts on the history of mathematics (Maor, 2007). The reaffirmation that the study of numbers and number theory is considered to be "the source and root of all things" (p. 55) is part of the theme. The geometric theme contained in *Pythagoras' Revenge* is woven into the history of Greek mathematics and its transmission to the Latin West through the translations of Arabic scholars. Sangalli presents various geometrical number shapes that are called figurate numbers (pp. 90-91). He also mentions the "perfection of the *Tetrakyts,"* a Pythagorean sacred symbol that is connected with the first four triangular numbers: 1, 3, 6, and 10. Furthermore, the *Tetrakyts* contains the ratios that are used in the harmony of the musical scale. For instance, 1:2, the octave 2:3, and the perfect fifth 3:4 are clearly contained in the *Tetrakyts*. Part of the Pythagorean philosophy was the belief that this equilateral triangle composed of 4 points on each side represents the "musical order of the cosmos" (p. 91).

The geometric theme in *Pythagoras's Darkest Hour* (Adams, 2009) is a fictional account of how Pythagoras discovered the theorem that bears his name. The initial problem was that Pythagoras thought that $x^2 + y^2 = z^2$ related to a circle. Testing some values, Pythagoras saw the error of this interpretation. Both Trianguluis (his servant) and Pythagoras tried other geometric objects. They both hit on the idea of a triangle (p. 9). Trying to relate x, y, and z to the measures of the three angles, making z a right angle, also resulted in a false conclusion. Instead of angles, they tried using lengths of sides of a right triangle. Playing with triples such as 3, 4, and 5, they noted that indeed $a^2 + b^2 = c^2$. They also discovered that multiples of the triple worked as well.

Famous Mathematicians Connected to Mathematical Fiction

As Pythagoras seems to be the most popular figure represented in the novels and short stories in this study, he was given an entire grouping. All the mathematical fiction found in the section titled Pythagoras and the Pythagoreans Connected to Mathematical Fiction can therefore be sorted into this category as well. *Hypatia: New Foes with an Old Face* (Kingsley, 1852) deals with the life and times of the first female mathematician, Hypatia. The geometric theme associated with this novel is the teachings of Hypatia. Attracting a large and loyal following, Hypatia lectured on the geometry of Euclid and Apollonius at the famous Alexandrian Museum (University of Alexandria), the center of Greek learning for many centuries (Deakin, 1994).

On page 172 of Kingsley's novel, Hypatia responds to a student's question on conic sections by stating, "In what does my commentary differ from the original text of Apollonius, on which I have so faithfully based it." Throughout this novel, Hypatia's views of mathematics in general and geometry in particular are filtered through the lens of her philosophy. Being a Neoplatonist, she lectures on abstract structures and forms that are well suited to the geometry of Apollonius.

The Death of Archimedes (Capek, 1949) is a fictional account of the last day of Archimedes' (287-212 B.C.) life. Written as a dialogue between the Roman soldier Lucius and Archimedes, the greatest mathematician of antiquity (Boyer, 1968), it is evident that the soldier has great admiration for the old geometer. The dialogue presented in this short story is a philosophical discussion about the application of mathematics during a time of war and the geometric theme deals with the war machines that Archimedes invented.

The geometric theme of *A Subway Named Möbius* (Deutsch, 1950) deals with the Möbius strip. This quirky story deals with the disappearance of a subway in Boston from the Metropolitan Transit Authority (MTA). A mathematics professor explains this disappearance due to the complex topology of the subway system (p. 223). Properties of a Möbius strip are part of the geometric theme.

Deutsch explains how to construct a Möbius strip and describes its properties. Möbius' name is often linked with his discovery of a surface with one side, called the Möbius strip. Such a strip or band is easily constructed by taking a strip of paper, giving it a half-twist, and taping the ends together. Most mathematics texts describe a Möbius strip as having one face, one side, and one surface. The geometric theme of the novel *Thomas Gray: Philosopher Cat* (Davis, 1988) deals with a 9^{th}-century Arabic manuscript due to the mathematician and translator Thâbit ibn Qurra (826-901). In this manuscript, Thâbit translated a work of Theodorus of Cyrene. "Theodorus was writing out for us something about square roots, such as the square roots of three or of five showing that they are incommensurable with the units" (p. 43). In this sense, Theodorus geometrically constructed the sequence $\sqrt{2},\sqrt{3},\sqrt{4},\sqrt{5},...,\sqrt{15},\sqrt{16},\sqrt{17}$. Why did he stop at the square root of 17? This very question concerns Davis who reminds the reader that Theodorus was Plato's (427-347 B.C.) teacher and also the instructor of the skilled mathematician, Theaetetus (417-369 B.C.). Davis raises the question: does "17" has any special significance? Davis, through his main character Dr. Fysst, offers an interesting solution to this question.

The Eight (Neville, 1989) deals with the interaction of Euler, Bach, and Philidor. Blending the action between the 1970s and the late 18^{th} century, the reader is introduced to Euler, one of the greatest mathematicians of all time; Bach, a composer whose name is synonymous with Baroque music; and Philidor, a master chess player whose name is associated with a classic chess defense. Leonhard Euler (1707-1783) made ground-breaking contributions to all fields of mathematics during the 18th century. Two powerful European royal courts coveted Euler's remarkable mathematical talent. Both royal academies at St. Petersburg and Berlin are included in *The Eight* and it was at the Prussian Academy where Euler and Bach discussed music and mathematics.

The first mathematical reference in *The Eight* appears on pages 170-174. It recounts a fictional meeting between Euler, Bach, and Philidor at the court of Frederick the Great. Bach is referred to as the "Kapellmeister of Leipzig" who is fond of puzzles. Euler describes a mathematical puzzle which is well known to chess players called the "Knight's Tour," where the tour of a knight must traverse the entire chessboard, never landing on the same square twice; this puzzle was given to Bach as a challenge. Upon hearing the puzzle, Bach numbered each of the 64 squares on the chessboard and turned the knight's tour into octaves and chords. Euler was incredulous and exclaims "to turn mathematics into music—it is sheer magic" (p. 172). Bach replied, "And the reverse is also true."

Murder She Conjectured (Kasman, 2005) deals with an unsolved mystery of the authorship of a theorem in geometry. It appears that the real author was murdered and the name associated with the theorem is fraudulent. As the story unfolds, Kasman hints that the real author may have been a woman. Lending credibility to the theme, Kasman mentions two prominent female mathematicians of the 19th century: Charlotte Angas Scott (1858-1930) and Emmy Noether (1882-1935), who are part of the theme of this story. Although both were gifted mathematicians, they experienced a great deal of gender bias; on page 13, Beth notes, "it must have been pretty hard to be a woman mathematician in those days."

Axiomatics Connected to Mathematical Fiction

Euclid Alone (Orr, 1975) deals with the consistency of Euclidean geometry. Orr focuses on the construction of a geometric proof by a non-tenured mathematics professor named David. After spending countless, agonizing hours reviewing David's proof, Dr. Lucas is convinced that the author's reasoning contains a flaw. In fact, if the proof is correct, then Euclidean geometry is inconsistent. This is clearly impossible since Euclid's geometry has been the *sine qua non* of deductive reasoning for over two millennia. Euclid's *Elements* is perhaps the most influential mathematics book ever written. It starts with basic definitions and then states only five postulates that are included in Orr's story.

In *Pythagorean Crimes* (Michaelides, 2006), the geometric theme deals with Hilbert's second problem. The author flashes back to August 8, 1900, the exact date when the two friends first met. This date is particularly significant in the history of mathematics since it was the first day of the Second International Congress of Mathematics held in Paris. David Hilbert, perhaps the most influential mathematician of his day, was the keynote speaker and the title of his lecture was "On the Future Problems in Mathematics." During his lecture, Hilbert presented a global overview of the seminal mathematics problems of the 20^{th} century.

At this historic lecture, Hilbert presented a list of 23 problems that served as a blueprint for mathematical research. The second problem is the focus of the geometric theme in this novel. Essentially, in the study of axiomatics, four properties are highly desirable: the axiom set should be consistent, independent, complete, and categorical. Each property is carefully explained in this novel. The property of completeness is of major importance in Hilbert's second problem. Fundamentally, an axiom system is complete if it is impossible to add a new independent axiom to the original system of axioms. Hilbert challenged his audience to find an algorithm that decides whether an axiom system is complete.

Concluding the first research question, the geometric themes given above for the 26 sources contained in this study exhibit some general patterns. Pythagoras as a central character never lost his popularity regarding his celebrated theorem and his mysticism (see section titled Pythagoras and the Pythagoreans Connected to Mathematical Fiction). Euclid's axioms became prevalent in this genre after Orr's *Euclid Alone* (see section titled Axiomatics Connected to Mathematical Fiction). Historical and philosophical themes became more sophisticated as mathematical fiction evolved. The most popular category in mathematical fiction for this study is the historical foundations of geometry, with emphasis on the contributions of famous mathematicians to geometry (see section titled Famous Mathematicians Connected to Mathematical Fiction). The evolution of dimensionality evolved from fantasy in *Flatland* to reality in *Tangents* (see section titled Dimensionality Connected to Mathematical Fiction). This, of course, is due to the advent of the computer.

The classic work of mathematical fiction is *Flatland* because its theme is found in many other works such as *An Episode of Flatland* (Hinton, 1907), *Sphereland: A Fantasy About Curved Spaces in an Expanding Universe* (Burger & Asimov, 1965), *The Annotated Flatland* (Stewart, 2001), *Flaterland: Like Flatland, Only More So* (Stewart, 2002), *Spaceland: A Novel of the Fourth Dimension* (Rucker, 2002), *Flatland: The Movie—A Journey of Many Dimensions* (Flat World Productions, 2007), and many other works too numerous to mention.

In the section titled Famous Theorems and Problems Connected to Mathematical Fiction, famous theorems and problems were represented accurately by the mathematical fiction sources in this category. Paradoxes and mysticism are popular with the general public, as are geometric puzzles (see section titled Geometric Fallacies Connected to Mathematical Fiction). The most prolific writer in mathematical fiction before 2000 was Martin Gardner. After 2000, Alex Kasman has published the most short stories in this genre. Mathematical Notes in short stories is contained in *Reality Conditions* (Kasman, 2005), which is unique in the genre of mathematical fiction.

On Pedagogical Approaches

The second question of the study was "What are the author's pedagogical approaches to delivering these topics? What are the author's mathematical backgrounds?" To address the first part of this for each fiction, the following questions will be answered:

a) What is the reader's prerequisite knowledge?

b) How is the geometric theme presented?

To answer these questions, the researcher divided the 26 sources of mathematical fiction into two groups: those utilizing formal pedagogical approach and those utilizing informal pedagogical approach.

Informal Presentations of the Geometric Themes

Flatland: A Romance of Many Dimensions (Abbott, 1884)

a) The reader should be familiar with identification of regular polygons and have some knowledge of cubes and spheres to gain more from the novel.

b) The geometric theme of dimensionality was presented by the narrator, A. Square, of the mathematical fiction to the readers.

The Sirdar's Chess-Board (Latimer, 1885)

a) The prerequisite knowledge should include finding the area of a square and a rectangle. Also required are elementary properties of a straight line.

b) The theme is presented through a familiar paradox; 64 = 65. The main character provides the details of cutting a square using "3 snips" so that the square becomes a rectangle to the sirdar.

Young Archimedes (Huxley, 1924)

a) The reader should be familiar with a proof of the Pythagorean Theorem. It would be helpful to know some elementary theorems of similar triangles.

b) The geometric theme is presented by Guido, a mathematical prodigy who is 7 years old. He outlines several proofs of this theorem for the reader.

The Appendix and the Spectacles (Breuer, 1928)

a) Knowledge of a three-dimensional coordinate system is helpful.

b) The geometric theme is presented by the main character. The setting for the theme involves removing an appendix in an operating room.

The Death of Archimedes (Capek, 1949)

a) Some elementary knowledge of the Second Punic War would be helpful.

b) The geometric theme is delivered through a dialogue between a Roman soldier and Archimedes.

The Island of Five Colors (Gardner, 1958)

a) No previous knowledge required.

b) The four-color problem (theorem) is presented using a fictional island of five regions and a conversation between two characters.

The Mathematician's Nightmare: The Vision of Professor Squarepunt (Russell, 1962)

a) Elementary knowledge of perfect squares and cubes is needed. Some background knowledge of the Pythagoreans might be helpful.

b) The geometric theme is presented through a dream where numbers take on human traits. The numbers talk about themselves to the narrator in this dream.

Tangents (Bear, 1986)

a) It would be helpful to have read *Flatland* and to have some prerequisite knowledge of a hypercube.

b) The geometric theme is described via computer graphics through conversations between two characters.

The Eight (Neville, 1989)

a) Knowledge of the "Knight's Tour" and some of the contributions of Euler are not necessary but would be helpful.

b) The geometric theme is presented by a fictional conversation between Bach, Philidor, and Euler at the court of Frederick the Great.

The Wild Numbers (Schogt, 1998)

a) No previous knowledge is required.

b) The geometric theme is presented through a conversation between two mathematics professors with regard to a fallacy in a proof.

The Object (Kasman, 2005)

a) The reader should be familiar with polyhedra and definitions of edges, vertices, and faces.

b) The geometric theme is described through different conversations about computer graphics of a regular polyhedron that contradict Euler's formula.

Murder She Conjectured (Kasman, 2005)

a) No previous knowledge is required.

b) The author provides the relevant historical information about the female mathematicians who experienced gender bias in the 19^{th} century.

Pythagoras' Revenge: A Mathematical Mystery (Sangalli, 2009)

a) The reader should have some previous knowledge of Pythagoras and the Pythagoreans.

b) The geometric theme is described by revealing the contents of a hypothetical scroll ascribed to Pythagoras by the author to the reader.

Pythagoras' Darkest Hour (Adams, 2009)

a) Knowledge of the Pythagorean Theorem and Pythagorean triples is helpful.

b) The geometric theme is presented via a conversation between Pythagoras and his servant, Triangulus. The problem solving method of trial and error is illustrated.

Formal Presentations of the Geometric Themes

Hypatia: New Foes with an Old Face (Kingsley, 1852)

a) A knowledge of the religious and philosophical turmoil of 5^{th} century Alexandria is crucial to understand the theme.

b) The geometric theme is presented through the teaching of Hypatia at the Alexandrian Museum.

And He Built a Crooked House (Heinlein, 1940)

a) Knowledge of the properties of a hypercube is important as well as three-dimensional visualization of solids.

b) The geometric theme is presented by the architect who is designing a four-dimensional house.

A Subway Named Möbius (Deutsch, 1950)

a) Knowledge of the properties of a Möbius strip is important.

b) The geometric theme is described by a mathematics professor.

The Devil and Simon Flagg (Porges, 1954)

a) The reader should have knowledge of the statement of Fermat's Last Theorem (FLT). Some familiarity with the history of the problem is desirable.

b) The geometric theme is presented through the collaboration of a mathematics professor and a devil.

No-Sided Professor (Gardner, 1974)

a) Knowledge of the properties of a Möbius strip and its construction is important.

b) The geometric theme is presented at a conference of topologists where a mathematics professor demonstrates a surface with "no sides."

Euclid Alone (Orr, 1975)

a) The reader should be familiar with the postulates of Euclid and the properties of a set of postulates.

b) The author carries a discussion to the reader of the fifth postulate (parallel postulate), which is essential for the presentation of the geometric theme.

Convergent Series (Niven, 1979)

a) The reader should have knowledge of an infinite series and an infinite sequence. The definition of convergence is also important.

b) The geometric theme is presented by describing a pentagram whose sides are decreasing geometrically.

Thomas Gray: Philosopher Cat (Davis, 1988)

a) The reader should be familiar with irrational numbers and geometric constructions.

b) The geometric theme is presented using a translation of an Arabic manuscript dealing with the spiral of Theodorus.

The Blind Geometer (Robinson, 1989)

a) The reader should be familiar with Desargues' Theorem.

b) Proof of the theorem is a metaphor for a mystery. The main character, a blind geometer, sketches the proof.

Curious Incident of the Dog in the Night-Time (Haddon, 2004)

a) The prerequisite knowledge includes the converse of the Pythagorean Theorem. The reader should also be familiar with tessellations.

b) The geometric theme is presented by Christopher, an autistic teenager who presents the proof of the converse of the Pythagorean Theorem.

Pythagorean Crimes (Michaelides, 2006)

a) The reader should be familiar with Euclid's five postulates and the properties of a postulate set.

b) The author carefully explains the importance of the properties of a postulate set: consistent, independent, complete, and categorical.

Perelman's Song (Chang, 2009)

a) The reader should be familiar with Poincare's conjecture.

b) The geometric theme is presented via a conversation between a god and a goddess.

When reading any sort of novel or short story, it helps to have some prerequisite knowledge about the topic because it will add to the reading experience and allow the reader to derive more from the story.

From the findings above, it is clear that, overall, novels and short stories in mathematical fiction use the narrator or main character to introduce the geometric theme. While there might be a preconceived notion that all mathematical fiction is informal, it has been shown that this is not the case.

What Are the Authors' Mathematical Backgrounds?

To address the second part of the second research question, Chapter V presented relevant biographical data about the authors of mathematical fiction contained in the present study. The authors of these sources constitute a diverse group of people with different backgrounds ranging from theology to professional mathematicians with doctorates. The 26 authors may be divided into three groups:

Group 1: Writers with little mathematical or scientific training

Group 2: Writers with a background in a related field to mathematics such as physics, engineering, biology, astronomy, and computer science

Group 3: Writers with a strong background in mathematics

The authors in Group 1 were (presented in chronological order of their works of mathematical fiction): Kingsley (historian and theologian), Abbott (schoolmaster), Lattimore (magazine writer), Capek (Master's in philosophy), Haddon (B.A. at Oxford in English), and Chang (poet laureate).

Of these authors, Abbott made the most impact on the genre of mathematical fiction with *Flatland: A Romance of Many Dimensions.* Haddon's novel *Curious Incident of the Dog in Night-Time* made *The New York Times* best-seller list in 2004. Chang's work is noteworthy because it presents Perelman's solution of the Poincare conjecture in a non-threatening manner. Although these authors had little mathematical training, their mathematical fiction received high ratings for both literary style and mathematical content. Surprisingly, most of the authors in this group did not place more emphasis on the cultural theme in their mathematical fiction. Overall, for this group of authors, the cultural theme was not the sole dominant theme. The geometric theme was employed in their mathematical fiction to support their cultural theme.

In addition, about half the authors in Group 1 presented their geometric themes informally. For this group, the prerequisite knowledge involved an understanding of elementary geometric concepts. The findings of this group were not surprising, given that authors tend to focus writing on what they know well.

The authors in Group 2 were (presented in chronological order of their works of mathematical fiction): Huxley (biologist), Breuer (physician), Deutsch (astronomer), and Neville (computer analyst).

It might be expected that the authors in Group 2 placed equal emphasis on both the cultural theme and the geometric theme. However, this was only the case for one of the four authors in this group. (Two had a dominant geometric theme and one had a dominant cultural theme.)

For this group of writers, the prerequisite knowledge involved an understanding of elementary to intermediate geometric concepts. In addition, the authors in Group 2 presented their geometric themes informally, with the exception of A.J. Deutsch's *A Subway Named*

Möbius. The findings for this group are not out of the ordinary, as someone might expect at least one or two formal deliveries of the geometric theme for these writers.

The authors in Group 3 were: Heinlein (graduate courses in mathematics and physics), Gardner (author of the "mathematical games" column and numerous books on recreational mathematics), Porges (M.A. in mathematics), Russell (author of *Principia Mathematica*), Niven (graduate work in mathematics), Bear (science fiction writer), Davis (distinguished professor of mathematics at Brown), Robinson (science fiction wrier), Schogt (graduate work in mathematics), Kasman (Ph.D. in mathematics and founder of the mathematical fiction website), Michaelides (Ph.D. in mathematics and translator of mathematical fiction novels), Sangalli (Ph.D. in mathematics), and Adams (distinguished professor of mathematics at Williams).

Cultural Themes

Research Question 3 was "What cultural themes are integrated with geometric topics in these mathematical fiction short stories and novels?"

The importance of cultural themes contained in works of mathematical fiction cannot be ignored. Linking mathematical fiction to geometric themes together with its cultural context is highly desirable because it educates while motivating the reader (Storey, 1996). For example, the classic *Flatland: A Romance of Many Dimensions* (Abbott, 1884) is concerned with both dimensionality and gender bias in the rigid class structure of Victorian England (Lindgren & Banchoff, 2010). This societal concern enriches the mathematical theme of geometric dimensionality in *Flatland* that was an integral part of geometry in mathematics education during Abbott's life. Another case in point is Kingsley's *Hypatia: New Foes with an Old Face*, where the geometric theme is blended with the religious and philosophical conflicts of 5th century

Alexandria. The religious confluence of Christianity, Paganism, and Judaism was examined with regard to its impact on Hypatia's mathematical lectures at the Library of Alexandria. It is interesting to note that in Alejandro Amenabar's 2010 film *Agora* presented the life of Hypatia interlacing her teaching of geometry with her philosophy that was similar to Kingsley's mathematical fiction written about 150 years earlier.

In the section titled The Three Categories of Mathematical Fiction with Geometrical Themes, the researcher connected the cultural theme with the geometric theme for each work of mathematical fiction. The coding of the sources permitted identification of some patterns and general categories (again, following the grounded theory methodology). Some cultural themes are shared by more than one work of mathematical fiction. These themes have been organized so that the theme, the work(s) of mathematical fiction, and the cultural context are all identified. After the cultural themes were organized into distinct categories, the researcher presented further analysis based on the 26 sources of mathematical fiction contained in this study.

Gender Bias Connected to Mathematical Fiction

At the very start of *Hypatia: New Foes with Old Faces* (Kingsley, 1852), the reader is informed that women were "the authors of all evil" (p. 6). Hypatia is portrayed as a heathen-sorceress who uses magical rites to corrupt "the souls of men" (p. 82). A mob of frenzied religious fanatics, who do not hide their misogyny, are against her teaching mathematics because of her gender and her philosophy. Hypatia is characterized as "subtler than the serpent, skilled in the tricks of logic" (p. 92). In this climate of vengeance, Hypatia prepared her last lecture. For example, a paroxysm of fury infused the mob of monks with shouts of "vengeance on all blaspheming harlots!" (p. 293). Hypatia rationalized that since she is a woman, the mob "will not dare to harm me" (p. 312). Tragically, Hypatia was murdered by a mob of religious fanatics (Chapter XXIX, pp. 360-371).

The cultural theme of gender bias continues in mathematical fiction with *Flatland: A Romance of Many Dimensions* (Abbott, 1884). Victorian culture during Abbott's time did not hold women in high regard. Quite the contrary, women and the poor were two highly disadvantaged groups in Victorian England. Snobbery was rife in this rigid socially stratified society where everyone knew their place and behaved accordingly. Consequently, Flatland was a mirror image of this hypocrisy.

Gender bias in Flatland is blatant. On page 12, "if a soldier is a wedge, a woman is a needle (line); being so to speak, *all* point, at least at the two extremities." Depending on the position of the "needle," the viewer may see it as a point—hence, practically invisible. The old adage "A woman's place is in the home" is both normal and expected in Flatland. Only during religious festivals are women permitted to go outside their homes. Women are perceived as witless and will always remain in their wretched state (p. 17, "once a woman, always a woman"). Abbott's intention is to satirize the social, moral, and religious values of Victorian England. Some of the statements are very disturbing with regard to women. For instance, on page 49, "since women are deficient in Reason but abundant in Emotion, they ought no longer to be treated as rational, nor receive any mental education." Many talented women in mathematics were denied access to study in universities during this period (Wertheim, 1995). The priests and women are colorless and remain pure "from the pollution of paint" (p. 33).

Gender bias in mathematical fiction is included in *The Sirdar's Chess-Board* (Latimer, 1885). Sophia believes that "no woman has ever been a great chess player" (p. 359) because of the cultural climate of her time. Gender bias is depicted by the women in the sirdar's harem who are uneducated and abused by any male. Even members of a woman's own family tyrannize her and the harem is depicted as life without moral principles (p. 368).

Gender bias during 19th century England is the main cultural theme of *Murder She Conjectured* (Kasman, 2005). Two non-fictional women mathematicians, Scott and Noether, who were both mathematically talented women, were denied positions because of their gender. Noether held a position similar to a guest lecturer (p. 15) for which she received no salary. Attributable to Kasman's knowledge of the history of mathematics, the narrative brims with historical material concerning the gender bias against Scott and Noether. In 1921, with the help of David Hilbert, Dr. Noether was given a university position, but again with no salary. Like Scott, Dr. Noether immigrated to America and was hired by Bryn Mawr in 1933, two years before her death. At Bryn Mawr she did receive a salary. Today, her name is associated with a special class of algebraic rings that play a vital role in abstract algebra.

Art and Music Connected to Mathematical Fiction

The cultural theme of *Young Archimedes* (Huxley, 1924) is the strong relationship between music and geometry as the study of patterns. Woven into this story is how Guido's insatiable appetite for mathematics is satisfied when he receives a gift of Euclid's *Elements* from Robin's father. The reader also encounters Guido's passion for music. His favorite pieces are Beethoven's Fifth and Seventh Symphonies (p. 21). His appreciation for music is surpassed, however, by his love of geometry. Guido's mentor, Robin's father, now realizes that Guido is "not a Mozart but a little Archimedes" (p. 25). According to Huxley, child prodigies usually are musical and mathematical.

In *And He Built a Crooked House* (Heinlein, 1940), the hypercube is used in architecture. Applications of the hypercube are described in architecture and art. Taking geometry to a new level inspires the modern design of a "dream house" in the fourth dimension. In art, Salvador Dalí used the unfolded hypercube to represent a crucifix in his famous painting titled "Christus Hypercubus" or "Corpus Hypercubus" (see Appendix). The design of the house is an inverted cross. The architect mentions that his design is "a hypercube unfolded in three dimensions" (p. 71).

A Subway Named Möbius (Deutsch, 1950) has a cultural theme related to music. In this short story, a subway is lost because of the complex topology of the pattern of tracks that resembles a Möbius strip. After a month, "the missing persons did not return" (p. 233) and the lyrics of the Kingston Trio's pop song comes to mind. The Kingston Trio[22] was formed in 1957 and their musical genre was folk songs. One of their most popular songs was the "MTA" that dealt with the story of a man who "never returned" from the Boston subway system. Connecting Deutsch's short story published in 1950 with the "MTA" song written in 1948 by Jacqueline Steiner and Bess Lomax Hawes, one readily notes the song's influence on Deutsch's mathematical fiction. The hit song was recorded by the Kingston Trio and known as "Charlie on the MTA." Comparing the chorus of the song to Deutsch's *A Subway Named Möbius* is astonishing for its similarities. The following are lyrics from this song:

> Did he ever return,
> No he never returned
> And his fate is still unlearn'd
> He may ride forever
> 'neath the streets of Boston
> He's the man who never returned.

[22]The Kingston Trio was popular at college concerts and in 1960 won the Grammy Award for the Best Ethnic or Traditional Folk Recording.

Here is another excellent example of how popular culture influenced mathematical fiction. In 1996, the Argentinean movie *Möbius* presented a similar theme as Deutsch's short story.

The cultural theme in the *No-Sided Professor* (Gardner, 1974) is the connection of paper folding (origami) to constructing a Möbius strip. Paper folding is often considered part of recreational mathematics. Nevertheless, paper folding may be both recreational and educational with regard to studying properties of geometric figures. Paper folding is very popular in different cultures, but the Japanese particularly popularized it into an art called origami.

The cultural theme of *Euclid Alone* (Orr, 1975) connects geometry to sculpture. On page 195, a diagram of the wild sphere appears. Connecting mathematics with art, the wild sphere is a sculpture by the world famous artist Helamen Ferguson.[23] Geometrically, the wild sphere is topologically a sphere and Dr. Lucas, the main character in the short story, muses that it is a perfect object for Dalí to paint. Obviously, Ferguson draws much of his inspiration from topology, as evidenced in the following works which have gained world-wide recognition: A Periodic Penrose, Ariadne's Torus, Papa Möbius, Fibonacci Box I, II, Hyperbolic Quilt, Esker Trefoil Torus, and the Wild Ball. Connecting mathematics to art is no accident and many believe that mathematics is in fact abstract art. Helaman Ferguson holds a Ph.D. in mathematics from the University of Washington and one of his sculptures may be seen at the headquarters of the National Council of Teachers of Mathematics in Reston, Virginia (Ferguson, 1994).

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[23]In the June 2008 issue of *Mathematics Magazine* (volume 81, number 3), an article appeared describing Ferguson's newest sculpture that was specifically designed for the outside of the Science and Engineering Center at the University of St. Thomas in St. Paul, Minnesota. Titled "Four Canoes," Ferguson's sculpture is comprised of "two linked donuts" made out of granite. Each donut is six feet in diameter and weighs about three tons. The sculpture rests on two granite pedestals that are each two feet high. On the ground are 30 hexagonal tiles arranged in a tessellation. His sculpture uses the mathematics of the Möbius band and the Klein bottle. A Klein bottle may be constructed by sewing two Möbius strips together. Ferguson's sculpture represents mathematical objects and the creation of his sculpture involves topology.

There are two cultural themes in *The Eight* (Neville, 1989). The first is the connection of geometry and music, and the second the connection of chess and mathematics. Euler, one of the greatest mathematicians of all time; Bach, a composer whose name is synonymous with Baroque music; and Philidor, a master chess player (best chess player of his century) whose name is associated with a classic chess defense, all appear in this novel. Leonhard Euler (1707-1783) made groundbreaking contributions to all fields of mathematics during the 18th century. Two powerful European royal courts coveted Euler's remarkable mathematical talent. Both royal academies at St. Petersburg and Berlin are included in *The Eight*, and it was at the Prussian Academy where Euler and Bach discussed music and mathematics. Johann Sebastian Bach (1685-1750) was also invited by Frederick the Great during Euler's tenure and was challenged to improvise on a musical theme that Frederick himself composed. Bach brilliantly improvised on Frederick's composition and the result was the classic *Musical Offering*. Bach, like Euler, was innovative and his cantatas, sacred choral music, sonatas, and much beloved Brandenburg Concertos are regarded as the most famous of his century. Joining Euler and Bach was Francois-Andre Danican Philidor (1726-1795), who was at that time the unofficial chess champion from 1747 to 1795 and a musician and composer of operas. As a chess master, Philidor is the first player to fully appreciate the powerful role of the pawn in a chess game with his famous dictum, "the pawns are the soul of chess."

The famous trio of Euler, Bach, and Philidor in *The Eight* is similar to the Pulitzer Prize-winning book by Hofstadter (1979), *Gödel, Escher, Bach: An Eternal Golden Braid* (GEB). The GEB discusses the logician Kurt Gödel, the artist M.C. Escher, and the composer J.S. Bach. According to Hofstadter, their creativity is a combination of factors dealing with thinking processes. Chief among these factors is pattern recognition. Euler, Bach, and Philidor are drawn to mathematics, music, and chess. Writing about the logical patterns and strategies of chess, Neville describes the connections between mathematics and the game.

Pythagorean Crimes (Michaelides, 2006) contains many cultural themes. This novel connects geometry to art, literature, and architecture. Halfway through the book, the author shifts his focus to *la bande a Picasso* (Picasso's gang) and discusses the art scene of Montmartre. Artists such as Van Gogh, Picasso, Toulouse-Lautrec, and Leon David (Max Jacob) contributed to the bon vivant atmosphere of Paris. Cubism is discussed from the artistic perspective of Picasso and the mathematical principles of the mathematician Maurice Princet, who was a close friend of his. Widely considered as "the mathematician of cubism," Princet had profound influence on Picasso's work. Blending geometry and art, the author constructs realistic dialogue between mathematicians and artists that takes place at the bohemian Zut, a famous "hangout" for artists and intellectuals during Picasso's time.

One of the problems that interested both artist and mathematician was which regular polygons can be used to cover a plane surface (p. 74). A helpful discussion of regular polygons is given and the conclusion is that only equilateral triangles, squares, and regular hexagons can fill a given space. When discussing geometric theorems, Michaelides connects them to art, architecture, and axiomatics. For instance, art is linked to the cubism of Picasso, and the Eiffel Tower is connected with a model of geometric architecture with its "curves and symmetrical lines." As a geometer, Michael felt an aesthetic link to the "sight of such an extravaganza of triangles dominating its skeleton" (p. 83). An excellent quotation about geometry appears on page 124: "geometry is to the visual arts what grammar is to writing."

Mysticism and Magic Connected to Mathematical Fiction

A reason Porges' short story *The Devil and Simon Flagg* (Porges, 1954) is compelling is the mathematical culture of proof. Proving a theorem is both challenging and frustrating. Often, an obsessive interest in solving the problem emerges. But once the theorem is proven, the overwhelming joy is expressed (usually with an AHA!). This sense of satisfaction is worth the time spent on solving the problem.

Fermat wrote in his copy of Diophantus' *Arithmeitca* (Book II, p. 241, problem 8) the most tantalizing marginal note in the history of mathematics. For $n \geq 3$ where n is a natural number, Fermat thought that the triple, $a^n + b^n = c^n$, did not exist. Teasingly, Fermat wrote in the margin that he discovered "a truly wonderful proof" of this theorem. Unfortunately, he went on to write that the margin "was too small to contain it" (Eves, 1990).

The collaboration of a mathematics professor and a devil at the end of the story adds to the mystical atmosphere of the story. At the conclusion, Porges adds a quirky twist by having the devil wish that he could "only prove one simple little lemma" (p. 69), implying that if one lemma can be established, then the devil may be able to prove FLT.

In *The Mathematician's Nightmare: The Vision of Professor Squarepunt*, the author presents a satire of Pythagorean numerology (Russell, 1962). Aligned with the Pythagorean belief, odd numbers were masculine while even numbers were feminine. Different types of numbers wore distinct uniforms of various colors. Russell mentions perfect numbers, perfect squares and cubes, primes, and pyramidal numbers. Introducing himself, 1 states that he is "the father of infinite progeny." Six states that she is the first perfect number and that the next two "upstart rivals" are 28 and 496 (p. 156).

The cultural theme of *Convergent Series* (Niven, 1979) deals with the pentagram. In the beginning of the story, the main character states that he is intrigued by magic. He constructs by geometric means a pentagram on the basement floor using chalk. The Pythagoreans knew the geometric properties of the regular pentagon. By drawing the five diagonals of the regular pentagon, the figure of a five-pointed star is formed and is called a pentagram (Boyer, 1968). This special symbol of the Pythagorean School represented health. In Renaissance magic, the pentagram was called the “pentalpha” because five capitalized A’s are embedded within the pentagram (Gardner, 1975).

The cultural theme in *Pythagoras’ Revenge: A Mathematical Mystery* (Sangalli, 2009) deals with numerology and how numbers are connected to the musical scale. The novel mentions that a manuscript actually written by Pythagoras was indeed translated into Arabic. This manuscript was in the form of a scroll. Despite the authenticity of this translation, the Pythagorean subject matter dealt with cosmology, mathematics, and reincarnation. Not at all surprising is the reaffirmation that the study of numbers, number theory, is considered to be “the source and root of all things” (p. 55). As a matter of fact, Pythagorean geometry was based on natural numbers. The diagonal of the unit square $\sqrt{2}$ is a non-commensurable number, that is, irrational, and this discovery by the Pythagorean Hippasus may have cost him his life. (p. 57)

Sangalli presents various geometrical number shapes that are called figurate numbers (pp. 90-91). He mentions a Pythagorean sacred symbol, the *Tetrakyts*, connected to the first four triangular numbers and the ratios that are used in the harmony of the musical scale. For instance, 1:2, the octave 2:3, and the perfect fifth 3:4 are clearly contained in the *Tetrakyts*. Part of the Pythagorean philosophy was the belief that this equilateral triangle composed of four points on each side represents the “musical order of the cosmos” (p. 91).

Academia Connected to Mathematical Fiction

In *The Appendix and the Spectacles* (Breuer, 1928), there is confusion with the title doctor. One of the characters assumes that Dr. Bookstrom is a medical doctor and holds the M.D. degree. Bookstrom who has mastered higher mathematics and specializes in the fourth dimension holds a Ph.D. Dr. Bookstrom claims that he can move the patient "along the fourth dimension" (p. 24) and then remove his appendix without making any incision. The fourth dimension is described as being at right angles to the x-y-z coordinate system.

The cultural theme in *The Island of Five Colors* deals with the collaboration of a mathematician and a cultural anthropologist (Gardner, 1958), who wish to draw a map of an island that is divided into five regions (p. 197). A tribe with unique cultural characteristics that distinguish it from the other tribes inhabits each region, and these tribes share common borders. Describing each tribe's unique cultural attributes, the reader is treated to a great deal of Gardner's humor.

The setting of *Thomas Gray: Philosopher Cat* takes place at Pembroke College at Cambridge. At Pembroke, don Lucas Fysst's pet cat, Thomas Gray, is no ordinary feline. It is indeed curious that a female feline is named after a male English poet and philosopher. Thomas Gray, the "Pembroke College Cat," chose to reside at Cambridge to be near the spirit of the logician-philosopher Wittgenstein. Pembroke College, established in 1347, has a long and distinguished number of graduates in arts and science (Attwater, 1936). It should be noted that Thomas Gray, the poet (1716-1771), was a fellow at Pembroke and a cultural icon who is best known for his masterwork, "Elegy Written in a Country Churchyard," which is still a popular and frequently quoted poem. Statements such as "ignorance is bliss" and "'tis folly to be wise" are due to Thomas Gray. The Pembroke students named the cat Thomas Gray in honor of the poet's "Ode on the Death of a Favorite Cat, Drowned in a Tub of Gold Fishes."

The cultural theme in *The Wild Numbers* is how to deal with a disruptive student in a college classroom (Schogt, 1998), Leonard Vale, a 53-year-old student who happens to be a retired mathematics teacher, opts to retake mathematics courses for a deeper understanding. Mr. Vale appears in Professor Isaac Swift's abstract algebra class, with his tape recorder taking down every word of the lecture. Incessantly, Vale asks irrelevant questions and establishes himself as a class nuisance. Other faculty also find Vale's behavior equally obnoxious, undermining the class lectures. One member of the mathematics faculty offers the following advice: "if his [Vale] words made too little sense to qualify as a question, ergo, they formed a distraction…and the instructor had the right to bar disruptive elements from the lecture" (p. 22).

The cultural theme of *The Object* deals with Alice Wu who is a computer geek and a dropout of MIT (Kasman, 2005). While examining a strange object on a computer screen (a polyhedron), she notices that it has 37 vertices. Clearly, this is mathematically impossible. Regular polyhedra have "faces each of which is a copy of the same regular polygon" (p. 195). Alice then writes down Euler's formula; $V - E + F = 2$, where V, E, and F represent the number of vertices, edges, and faces of the polyhedron. Alice is now frantic and meets with a former mathematics professor at MIT to discuss the remote possibility of having a regular polyhedron with 37 vertices and 32 triangular faces.

Social Issues Connected to Mathematical Fiction

Social issues connected to mathematical fiction are found in *Tangents, The Blind Geometer*, and *The Curious Incident of the Dog in the Night-Time.* The social issue in *Tangents* concerns a freelance computer analyst, Peter, who deals with codes, encryptions, and computer security (Bear, 1986). This main character is involved in breaking the Enigma code. Peter is no ordinary computer analyst. He and his teenage assistant, Pal, have bonded over their mutual interest in the fourth dimension via computer graphics. In the meantime, Pal does some research on the Enigma machine and finds a picture of Peter as part of the Bletchley Park cryptanalysis team. Upon hearing this discovery from Pal, Peter quickly admits that he is a homosexual. Pal replies, “Oh, so what” (p. 47). This description hints that Peter’s character is based on the life of Alan Turing, who is regarded as the “father of the modern computer” (Leavitt, 2006). In 1952, Turing was, in fact, arrested on “acts of gross indecency with another male” (Leavitt, pp. 268-269) and charged under Section 11 of the Criminal Amendment Act of 1885. Oscar Wilde was convicted of the same “crime” under the same ordinance a half-century before Turing’s conviction.

It is difficult to imagine a blind person excelling in geometry, which is both abstract and highly visual. In *The Blind Geometer*, the reader encounters Carlos Oleg Nevsky who was born blind and is also a professor of mathematics (Robinson, 1989). Without vision, Carlos relies on a special Xerox machine that produces geometric copies with raised ridges so that the three-dimensional copy enables him “to see” (touch) the various shapes. The blind geometer can visualize Desargues’ Theorem that he describes as “a beautiful theorem, with the purity and elegance characteristic of Renaissance mathematics” (p. 16).

Autism is the social issue presented in *The Curious Incident of the Dog in the Night-Time* (CI) (Haddon, 2004). The narrator of CI is a 15-year-old boy whose name is Christopher John Francis Boone. Christopher’s quirky behavior is due to Asperger’s Syndrome or high-functioning autism. Knowing all the prime numbers from 2 to 7507, Christopher is unable to develop normal social relationships or relationships with his environment. Chapters are numbered using primes 2, 3, 5, 7, 11, 13, so that Chapter 4 using cardinal numbers is actually Chapter 7 in Christopher's world. Nevertheless, he clearly explains how to generate prime numbers using the sieve of Eratosthenes, which is illustrated visually on pages 11-12.

Christopher is mathematically gifted but, on the other hand, he has no comprehension of human emotion. In fact, Christopher lists his "behavioral problems" (p. 46) from A to R, consecutively. For instance, (B) Not eating or drinking for a long time, (C) Not like being touched, (H) Not liking yellow or brown things, (L) Not smiling, and (O) Hitting other people. These are common characteristics of autism, as are reliance on routine, order, and a predictable daily schedule.

Warfare Connected to Mathematical Fiction

Warfare connected to mathematical fiction is present in *The Death of Archimedes.* The cultural theme in this story is the applications of geometry and physics to war machines used in the Second Punic War (Capek, 1949). Due to Archimedes' defense machines, the Roman general Marcellus has respect for his ingenious adversary. Some of the ingenious defense weapons invented by Archimedes were catapults, pulleys, and hooks strong enough to hoist a Roman ship out of the water and smash it against the rocks, as well as parabolic mirrors used to set an entire Roman fleet of ships aflame (Boyer, 1968).

Poetry Connected to Mathematical Fiction

Perelman's Song uses a dialogue between a god and a goddess to describe the solution of the Poincare Conjecture (Chang, 2008). Tina Chang is knowledgeable in mathematics and is also an accomplished poet. The goddess says, "Any simply connected closed three manifold is a sphere." She then clarifies her statement by morphing the blob using the Ricci flow method developed by Columbia University mathematics professor, Richard Hamilton. In fact, Perelman uses the Ricci flow method as developed by Hamilton in his proof of Poincare's Conjecture. Essentially, the Ricci flow equation changes the curvature of the blob so that it looks like a shrinking cylinder with an infinite number of "thin tubes." At this point, Perelman adds a song that is actually a poetic metaphor to connect the changing shape with the changing pitch of notes in the song.

Philosophy Connected to Mathematical Fiction

Another connection present is philosophy and mathematical fiction. The cultural theme in *Pythagoras' Darkest Hour* deals with the discovery of a number whose square is 2 (Adams, 2009). Examining an isosceles right triangle, each of whose sides is 1, Pythagoras noticed that the hypotenuse would have length whose square is 2. Puzzled by this strange-looking number, Pythagoras wondered what fraction, when squared, yields the number 2. Pythagoras mentions that he will challenge the Assembly to find this special fraction. Clearly, to the Pythagoreans and Pythagoras himself, such a number would upset their entire cosmology that "all is number." In fact, to the Pythagoreans, number meant natural number or their ratios.

It follows that the diagonal of a unit square by Pythagoras' own theorem yields the square root of 2. All lengths of triangles could be measured by natural numbers or some ratio of natural numbers, but they could not deny the existence of $\sqrt{2}$. A "legend" states that a member of the Pythagorean Brotherhood, Hippasus, who divulged the incommensurability of the square root of 2, was mysteriously drowned in a shipwreck (Calinger, 1995).

Cultural themes connected to mathematical fiction reveal some important results. Gender bias is used in this genre to act as a mirror of real bias during the time that the mathematical fiction was published. The cultural themes are very often connected to art, music, and chess, where pattern recognition is the common attribute. Magic and mysticism are popular cultural themes used to arouse the reader's interest and highlight the geometric content.

Academic life is often depicted as stressful for young faculty of a mathematics department because of the "publish or perish" syndrome. Autism (CI) is viewed as an acceptable social issue, not as an illness or a disease, and an autistic teenager may excel in mathematics as well as other areas.

Popular culture has an impact on mathematical fiction. The theme of Neville's work is similar to Brown's *Da Vince Code* that may have played a role in the mathematical fiction of Sangalli. Classical languages like Hebrew, Greek, Latin, Arabic, and Aramaic are important in the cultural transmission and translations of mathematical manuscripts. This is shown in the works of Davis, Michaelides, and Sangalli.

Chapter VII

SUMMARY, FINDINGS, AND RECOMMENDATIONS

This study was guided by the following research questions:

Question 1. What geometric topics are used in the mathematical fiction genre with regard to the selected novels and short stories?

Question 2. What are the authors’ pedagogical approaches to delivering these topics? What are the authors’ mathematical backgrounds?

Question 3. What cultural themes are integrated with geometric topics in these mathematical fiction stories and novels?

To answer Question 1, six major themes (topics) were identified. Dimensionality, Fallacies and Mysticism, Pythagoras and the Pythagoreans, Famous Mathematicians, Famous Theorems and Problems, and Axiomatics were the geometric topics presented in the 26 novels and short stories.

Figure 2 below depicts how prevalent the geometric topic was in this study. It is clear that stories involving famous mathematicians (including Pythagoras and the Pythagoreans) were by far the most popular. Axiomatics was the least popular topic.

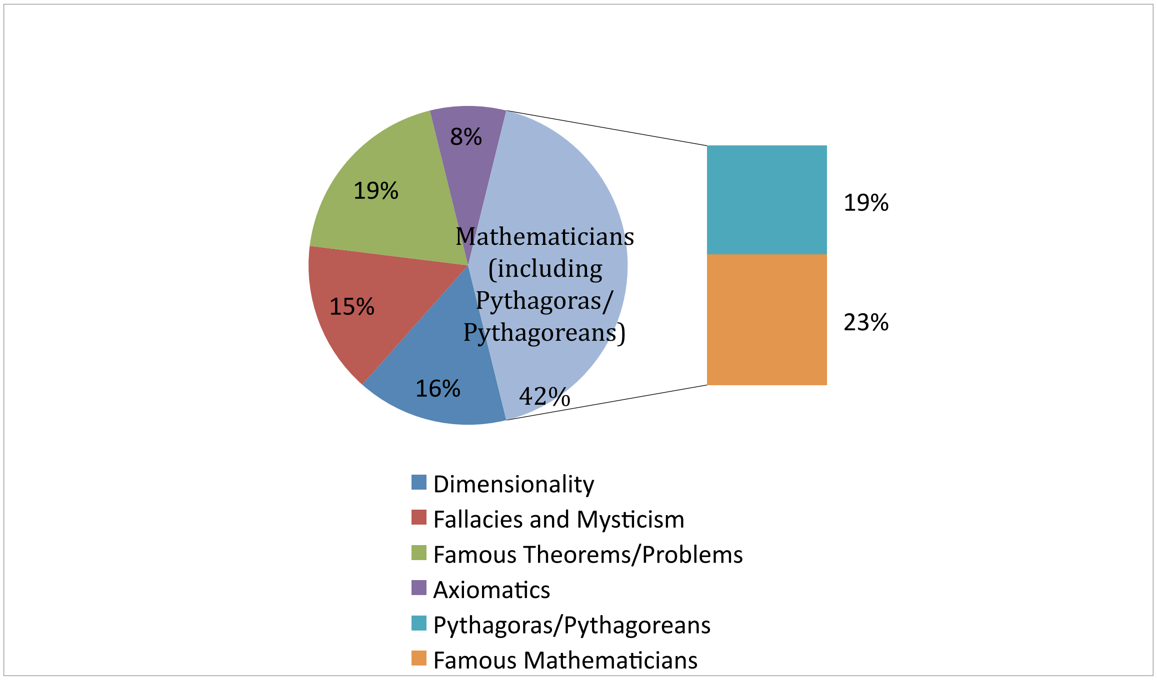

Figure 2. Prevalence of geometric topics

Flatland, The Appendix and the Spectacles, And He Built A Crooked House, and *Tangents* were the four sources (15.4%) that involved dimensionality. *The Sirdar's Chess-board, No-Sided Professor, Convergent Series,* and *The Wild Numbers* were the four sources (15.4%) that involved fallacies and mysticism. Pythagoras was the most popular mathematician, with five of the sources (19.2%) specifically about him (or the Pythagoreans); these five novels and short stories were *Young Archimedes, The Vision of Professor Squarepunt, C.I., Pythagoras' Revenge,* and *Pythagoras' Darkest Hour.* Other famous mathematicians were present in six of the sources (23.1%), namely *Hypatia, The Death of Archimedes, A Subway Named Möbius, The Eight, The Object,* and *Murder She Conjectured.* Famous Theorems and Problems were covered in 5 of the 26 sources (19.2%): *The Island of 5 Colors, The Devil and Simon Flagg, The Blind Geometer, The Object,* and *Perelman's Song.* Lastly, the novels *Euclid Alone* and *Pythagorean Crimes* made up the two sources (7.7%) involving Axiomatics. It is interesting to see many famous mathematicians not only inside the story, but also in these titles of mathematical fiction.

To answer *Question 2* in Chapter VI, all 26 sources were analyzed. As was demonstrated, there were both informal and formal deliveries. Informal deliveries involved casual conversations between characters and the narrator or author presenting information to the reader. Formal deliveries also included the narrator or author presenting information to the reader, but the tone was noticeably different. Other formal deliveries took place when a character lectured another character or when there was high-level mathematical discussion between characters. There seem to be two predominant ways to introduce geometric topics in the novels and short stories: author to reader (including using a narrator or main character) or dialogue between characters (including lectures, casual conversation or serious mathematical discussion).

Authors' backgrounds were broken down into three groups. Group 1 (Little to No Mathematics/Science Background) contained seven authors with backgrounds in history, theology, journalism, philosophy, English, and poetry. Group 2 (Science Background) had four authors who studied biology, psychology, medicine, astronomy, and computer science. Group 3 (Strong Mathematics Background) had 14 authors. Most authors of these mathematical fictions had a strong mathematical background, which seems likely. Similarly, authors who publish poetry are not limited to poets, although it would be expected that they comprise the most dominant group.

The pattern that emerged was that the more sophisticated geometric themes were due to the authors who have Ph.D.s in mathematics. These authors will next be connected with their geometric themes to illustrate their higher level of sophistication.

P.J. Davis is a distinguished professor at Brown who related the transmission of Greek geometry, the spiral of Theodorus in particular, to Western Europe through the translations of Arabic mathematicians. His novel dealt with an important event in the history of mathematics.

Alex Kasman is the founder of the mathematics fiction website that lists over 800 sources. His theme in both of his works, *The Object* and *Murder She Conjectured*, related historical material dealing with Euler, Noether, Scott, and Kovalevsky.

Tefcros Michaelides, professor of mathematics at Athens College, related Hilbert's second problem concerning the completeness of an axiom system. Moreover, his geometric theme dealt with the foundations of geometry as it relates to a set of postulates.

Colin Adams is a distinguished professor of mathematics at Williams College whose mathematical fiction dealt with geometric conjectures and mathematical discovery.

Four of the last five authors, in the analysis of mathematical fiction included in this study, hold Ph.D.s in mathematics and their mathematical fiction is more sophisticated than earlier authors of this genre. This latter group of authors injects a great deal of mathematical content and is more concerned with the geometric theme than the story itself. Consequently, the geometric theme is dominant, whereas cultural considerations serve as a contextual backdrop for the story. In contrast, earlier authors in this genre highlight the cultural theme. Furthermore, some of the professional mathematicians in this genre use axiomatics as it applies to geometry, both Euclidean and Non-Euclidean, as a dominant theme. Included in their geometric thematic expositions is historical and philosophical information usually found in history of mathematics textbooks. Connecting historical information to geometric themes may better educate the reader about the origin of geometric ideas.

The authors in Group 3 placed more emphasis on the historical foundations of geometry. Moreover, the majority of authors in Group 3 presented their geometric themes formally. For this group, the prerequisite knowledge involves a deeper understanding of the geometric concepts. While these findings were expected, something noteworthy emerged from this group. Another finding is that the professional mathematician usually has more than one mathematical theme in his or her story, but the dominant theme is still geometry.

It is interesting to note that authors from both Group 1 and Group 3 were present in five of the six geometric themes. All three groups wrote about Famous Mathematicians, Pythagoras and the Pythagoreans, and Dimensionality. The only geometric topic without author variety was Axiomatics.[24]

In Figure 3, authors' backgrounds are connected to the dominant theme of the novel or short story. Figure 4 shows the connection between authors' backgrounds and geometric theme.

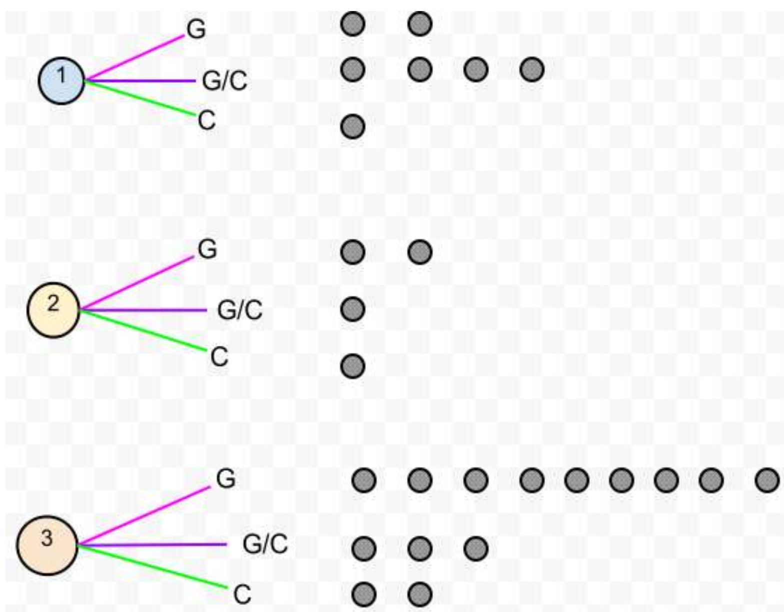

Figure 3. Connection between authors' background amd dominant theme

[24]Axiomatics as a geometric topic was present only in two pieces of mathematical fiction. While one of the authors is in Group 3, the background of the other author is unknown.

In this figure, 1 denotes the authors' background for Group 1, 2 for Group 2, and 3 for Group 3. The G represents geometry as the dominant theme, C represents cultural theme as dominant, and G/C demonstrates that they are of equal value. The grey circles represent the novels and short stories used in this study. There are only 25 circles because one of the authors was not sorted into a group given lack of biographical information. It seems logical that the geometric theme is dominant for Group 3 authors because they have strong mathematical backgrounds. It is interesting that most Group 1 authors chose to have the geometric theme so important to the plot of their mathematical fiction.

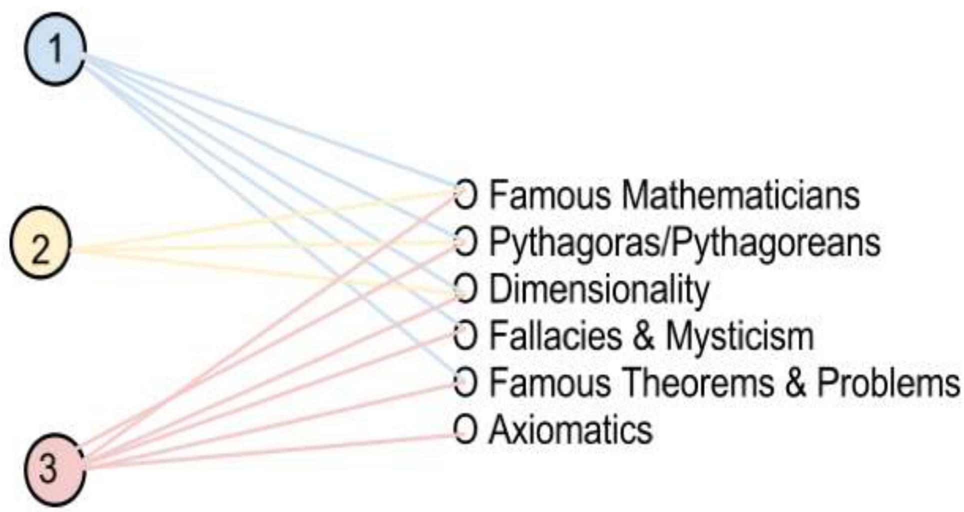

Figure 4. Connection between authors' background and geometric theme

In this figure, 1 denotes the authors' background for Group 1, 2 for Group 2, and 3 for Group 3. This figure illustrates that authors with strong mathematical background were represented in every geometric topic (all red lines connect to all the topics). It was surprising to the researcher that Group 1 authors wrote about five of the six topics. All groups wrote about the geometric topics of famous mathematicians (including Pythagoras and the Pythagoreans) and dimensionality. Axiomatics seemed left to writers with a strong mathematical background.

To answer *Question 3* in Chapter VI, the themes were united in several categories. Figure 5 below illustrates the categories and their dominance.

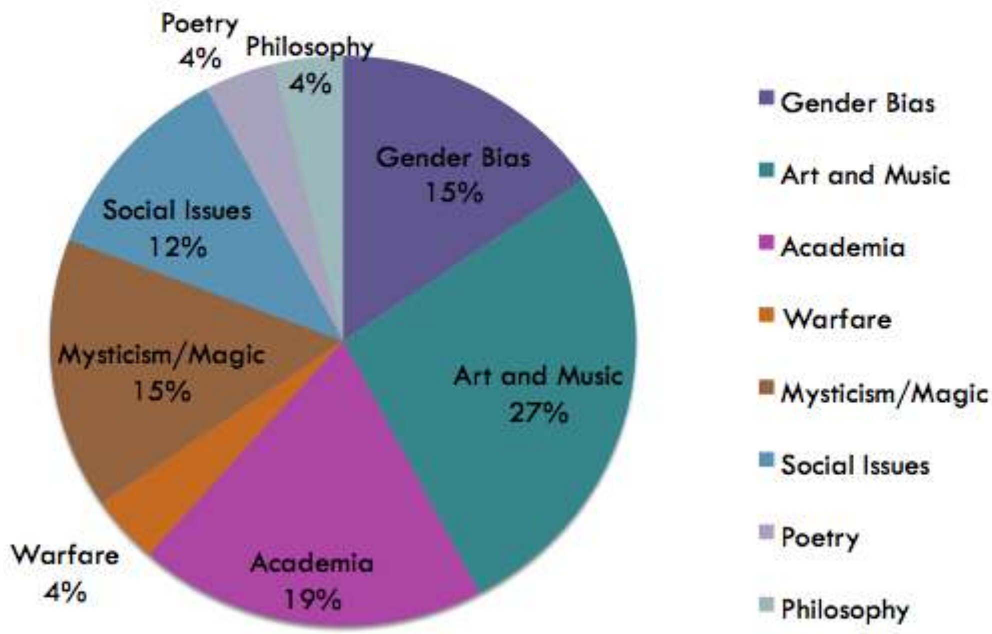

Figure 5. Categories and their dominance

Limitations of the Study

This study has some limitations. The first limitation is connected with the use of the existing database. Although the database of mathematical fiction is recognized for valuable research in mathematical fiction (Swallow, 2006), the researcher encountered a few issues. Readers, unknown to the researcher, scored the novel and short stories based on unidentified criteria. These ratings are subjective since one reader can believe a novel warranted a 4 in literary quality while another may have given the same story a 2. There is also no way to prove that those individuals submitting ratings for the novels and short stories read that piece of mathematical fiction. The researcher was cognizant that the rating scheme provided by the database has its downfalls. No specific guidelines explained how to quantify the mathematics present in the story or the literary content. For example, there was no indication (at least at the time the study was conducted) about what exactly being a 3 in a certain category meant. Again, this is subjective to the voter. It would be interesting to have a qualified panel judge the novels and short stories presented in the database, although this might be impossible given the sheer volume of mathematical fiction entries.

The second group of limitations is connected with the subjectivity of some decisions made in a qualitative study, despite the best efforts of the researcher. To identify categories, the researcher needs to establish some kind of a border between them which, to some extent, may be questioned (although the researcher believes that in general, the established categorization is sound).

The third group of limitations is connected with the relatively small number of sources analyzed. Although it was possible to make some generalizations (say, see the connection between the authors' mathematical education and the sophistication of mathematics in the fiction), in other cases it was not possible to see some general patterns. This may be explained by a simple lack of these patterns, but it is also possible that these patterns did not manifest themselves because the sample was not sufficiently large. Further study using more sources can be recommended.

Recommendations for Future Studies

One direction of studies has been mentioned already—exploring more fictions using geometrical themes. The number of "mathematical fictions" is increasing and relates to all mathematical topics. The following chart gives the number of works of mathematical fiction, both novels and short stories, that were published between 2000 and 2009. Using Kasman's database, there are eight possible categories: geometry, algebra, analysis, cryptology, mathematical physics, probability and statistics, logics and sets, and mathematical finance.

Table 1

Number of Works of Mathematical Fiction Published between 2000 and 2009

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	Totals
Geometry	5	5	6	4	2	7	9	7	3	4	52
Algebra	4	2	3	5	5	9	7	7	7	3	52
Analysis	1	1	4	3	4	1	1	1	2	1	19
Cryptology	1	0	1	4	6	7	3	5	1	3	31
Mathematical Physics	5	2	1	3	1	7	2	3	0	2	26
Probability and Statistics	3	0	2	0	2	2	0	0	0	1	10
Logics and Sets	3	0	2	2	3	2	1	0	2	1	16
Mathematical Finance	1	2	0	1	0	0	0	3	1	0	8
Totals	23	12	19	22	23	35	23	26	16	15	214

From these data, geometry comprises about 24% of all mathematical fiction published during this decade. Cryptography has become a popular core subject, accounting for approximately 15% of the 214 works of mathematical fiction. Due in part to Dan Brown's best-selling novel *The Da Vinci Code*, the public has generated an interest in cryptography. This suggests that pop culture has an impact on the content of mathematical fiction.

Although this research specifically dealt with geometric themes contained in mathematical fiction, the methodology used here may be applied to other core subjects. Indeed, research utilizing the methodology of this study may be devoted to other mathematical subjects. Specifically, Analysis may be a good theme for such research. Some categories that may emerge are differentiation, integration, historical foundations of analysis, applications to physics, among others

Additional directions from this study concerning mathematical fiction may include sources other than novels and short stories. For example, mathematical fiction themes may appear in plays, film, television, poetry, and other media. These resources provide fertile ground for further study.

One more direction is connected with the exploration of the impact of mathematical fictions and their pedagogical use. While the genre is growing, so is the attention given to mathematical fiction. The number of entries in the mathematical database has grown tremendously from the start of the study (over 300 additional entries and counting), and journals have published articles about the potential use of mathematical fiction inside the classroom. It would be interesting to see how the public interacts with and educators utilize mathematical fiction in the years to come. Moreover, it would be exciting to explore ways to market mathematical fiction to involve the general public in informal learning. Also, an issue of particular interest is the perception of mathematics. It would be worth exploring studies on whether or not views on mathematics and mathematicians are altered after reading mathematical fiction.

Exploring changes in mathematics education through the use of mathematical fiction to teach at the college level is a major avenue for future research. Studies can compare the use and exclusion of mathematical fiction in the same course. Other studies can utilize the informal knowledge gained when reading mathematical fiction outside the classroom to teach and develop topics inside the classroom. Another study of interest might be to research students identified with mathematics anxiety and how they respond to learning with mathematical fiction.

Recommendations for Educators

Many educators, even in the field of mathematics, are unaware of the mathematical fiction genre and its uses. Each mathematical fiction with geometric themes has pedagogical value. Consequently, it is recommended that teachers use these themes as a teaching tool to enhance a particular mathematical theme. A guide involving pedagogical considerations will now be provided. In this study, each geometric theme may be sharpened and further scrutinized within its particular category. Mathematics educators at both the high school and college level may use each of the 26 sources to extend the geometric theme embedded in the mathematical fiction. By careful analysis of geometric themes inherent in mathematical fiction, future researchers may extend the scope of this study to include other related geometric topics.

Pedagogical considerations may be connected to each mathematical fiction. For instance, the study of polygons is a natural extension of *Flatland* (Abbott, 1884). Properties of interior angles of regular polygons may be investigated. The paradox resulting from an 8 by 8 square into four plane figures so that when reassembled a 5 by 13 rectangle results is of pedagogical value in Latimer's short story *The Sirdar's Chess-Board*. A discussion of slopes and the colinearity of three points may emerge from this paradox. Huxley's *Young Archimedes* may be extended to include lessons on dissection proofs of the Pythagorean theorem. For example, Bhaskara's proof (Eves, 1990) and President Garfield's proof (Maor, 2007) provide examples of dissection proofs. Other pedagogical applications include the analysis of the hypercube (Gardner, 1975), which is the theme of Heinlein's short story. Map coloring is a pedagogical consideration after reading *The Island of Five Colors* by Gardner. Experimenting with four or five colors is a valuable discovery lesson with regard to the four-color theorem.

It is generally acknowledged that figurate numbers originated with the Pythagoreans. A pedagogical component to Russell's short story *The Mathematician's Nightmare: The Vision of Professor Squarepunt* (1962) is describing the properties of triangular, square, and pentagonal numbers. An enrichment exercise is discovering formulas for each type of figurate number. Construction of a Möbius strip and a double Möbius strip are pedagogical extensions of the *No-Sided Professor*. A class discussion of both the Möbius strip and the Klein bottle also has pedagogical value as it connects to the above short story.

In *Convergent Series* (Niven, 1979), the geometric theme deals with a pentagram. A lesson can involve having students construct a pentagon and connect it to the golden section. A discussion dealing with the distinction of an infinite sequence and an infinite series naturally follows from Niven's mathematical fiction. In *Tangents* (Bear, 1986), a pedagogical component is a discussion of Alan Turing and the Enigma code. Artificial intelligence (AI) also may be discussed with regard to computers. Analysis of the geometric theme of *Thomas Gray: Philosopher Cat* (Davis, 1988) leads to an application of geometric constructions of irrational numbers using straight edge and compass. The student would then discover "the square root spiral."

A pedagogical component of *The Blind Geometer* (Robinson, 1989) is connecting projective geometry with Renaissance art. An instructor may lead the discussion of relating Desargues' Theorem with Da Vinci's "The Last Supper." Another pedagogical application of this study is connecting the Knight's Tour with music and chess. This pattern appears in *The Eight* through a fictional conversation between Euler, Bach, and Philidor.

An analysis of *The Curious Incident of the Dog in the Night-Time* (Haddon, 2004) is focused on the Pythagorean theorem and its converse. Mathematical patterns in this novel are explained by an autistic teenager. Pedagogical considerations involve the blending of mathematical talent and autism.

Two short stories by Kasman analyzed in this study; namely *The Object* and *Murder She Conjectured,* have pedagogical applications as end notes given in the appendix to Kasman's novel *Reality Conditions*. A pedagogical component in the first story is an explanation of Euler's formula, $V - E + F = 2$. Historical events in the second story involve the women mathematicians Noether and Scott.

A pedagogical component of *Pythagorean Crimes* (Michaelides, 2006) is describing the characteristics of a set of postulates. The author formally defines the properties of an axiom set: consistent, independent, complete, and categorical. Inherent in *Pythagoras' Darkest Hour* (Adams, 2009) is the problem solving method of trial and error. This strategy is worthwhile to explain in mathematics education. The story shows the importance of this strategy when Pythagoras discovers the relationship of the two sides of a right triangle and its hypotenuse.

The pedagogical considerations are not limited to the ones mentioned above. Many lessons can develop from these novels and short stories. Also, extra-credit assignments or extensions can be given instead of or in addition to lessons.

A final implication from this study is that mathematical fiction may be used as an outreach vehicle. The general public may learn to appreciate mathematics through mathematical fiction. Popular culture has produced a new audience for mathematical fiction and the 26 sources in this present study may serve as a gateway for exploring ideas with geometric themes that blend mathematics and literature.

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APPENDIX

Images

Image of Salvador Dalí's painting titled, Crucifixion (Corpus Hypercubus)

Source: http://www.artchive.com/artchive/D/dali/crucifix.jpg.html

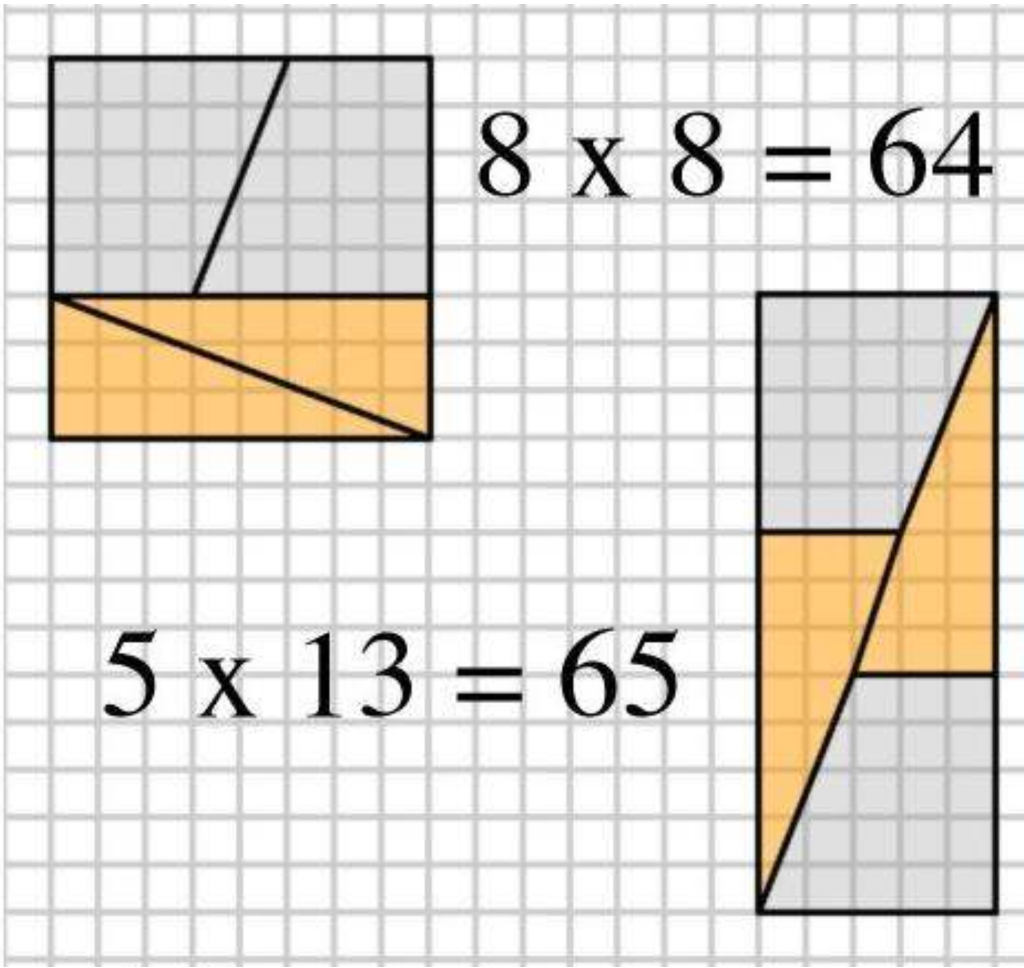

Image of the “64 equals 65” geometric paradox

Source: http://puzzles4you.blogspot.com/2009/08/picture-puzzles.html

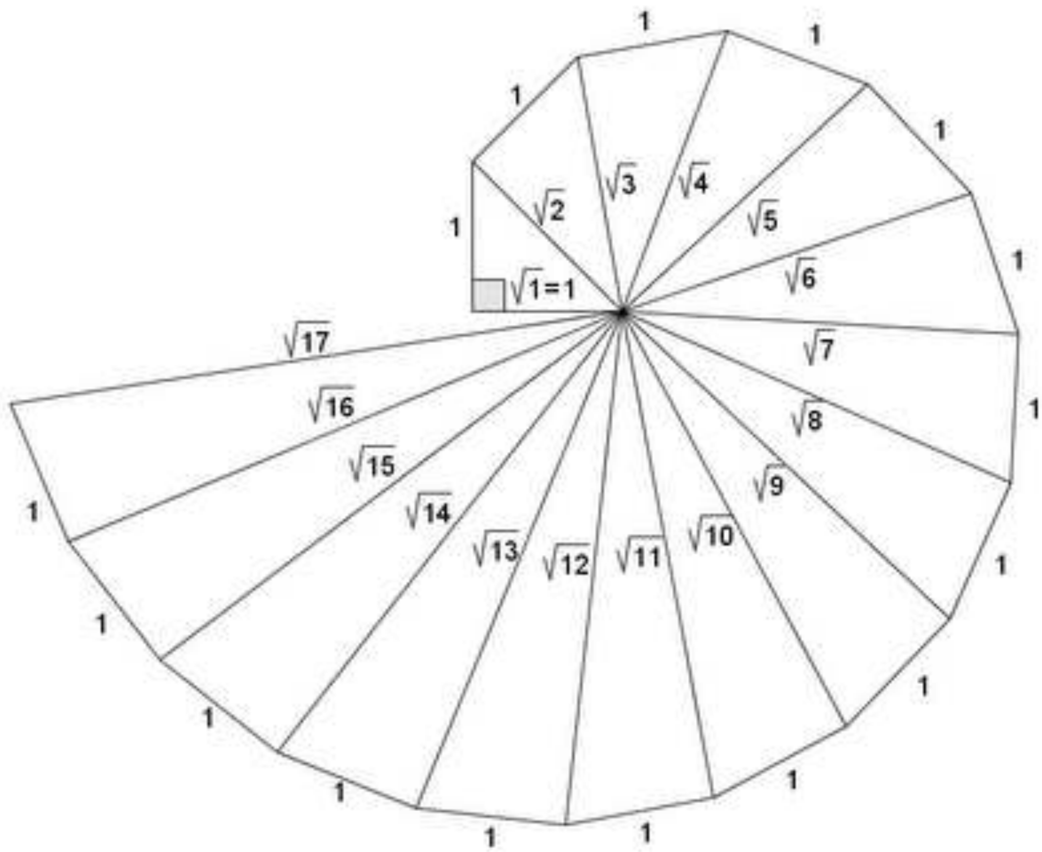

Image of the Spiral of Theodorus

Source: http://offthehypotenuse.blogspot.com/2010/02/7th-grade-theodorus-wheel-and-art.html