Analysis of MIMO Beamforming with Channel Response Variations over the Frame Interval

Eduardo Martos-Naya, José F. Paris, Unai Fernández-Plazaola and Andrea J. Goldsmith*

Dept. Ingeniería de Comunicaciones
Universidad de Málaga,
Campus de Teatinos, Málaga (Spain)
{eduardo,paris,unai}@ic.uma.es

* Dept. of Electrical Engineering
Stanford University
Stanford CA, 94305
andrea@ee.stanford.edu

***Abstract*— Important throughput improvements in multiple-input multiple-output (MIMO) fading channels can be obtained by merging beamforming at the transmitter and maximal ratio combining (MRC) at the receiver. However, to attain these performance gains it is important to obtain accurate channel state information (CSI). For this purpose a channel estimation technique is used at the receiver: channel prediction to feed back to the transmitter for beamforming, and channel interpolation for MRC at the receiver. In this paper, the impact of imperfect channel prediction on bit error probability (BEP) is analyzed in Rayleigh fading, taking into account the channel response variations over the frame interval. An exact closed-form expression for BEP is obtained, and we evaluate this expression assuming both time-variant and time-invariant channel models. These results indicate that the BEP performance degrades on the order of 1.5 dB due to channel variations.**

I. Introduction

One of the most promising and powerful techniques in wireless communications is multiple-input multiple-output (MIMO) systems [1]. When channel state information (CSI) is available to both the transmitter and the receiver, it can be used for beamforming at the transmitter and maximal ratio combining (MRC) at the receiver.

The quality of the CSI at the transmitter impacts beamforming performance [2]. In most systems CSI is predicted by the receiver and it is sent back to the transmitter by a feedback channel. However, the knowledge of CSI at the transmitter can never be perfect due to unavoidable prediction errors, for example using a pilot aided modulation (PSAM) in a noisy channel [3]. Adaptive modulation in MIMO systems has been studied in [2][4]-[7]. These works are based on approximated expressions for the BEP under imperfect CSI, usually starting from exponential type bounds [8, eq.17].

In [9] exact closed-form BEP expressions for MIMO beamforming with MRC under imperfect CSI due to prediction errors are derived, assuming a fixed modulation scheme and transmit power and a time-invariant channel response over the frame interval. This paper considers a model where the channel response varies over the frame interval. Under this assumption, an exact closed-form BEP expression is obtained for MIMO Beamforming with MRC employing BPSK or M-QAM over Rayleigh fading channels. The expression is evaluated for different modulation types and the degradation in BEP due to channel variation shown to be on the order of 1.5 dB.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III the BEP expressions are derived. Section IV presents numerical results which exploit the analytical expressions derived in previous sections. Conclusions are provided in Section V.

II. System Model

The system model for MIMO beamforming with MRC is depicted in Figure 1. The following channel model is assumed. We consider N_T transmit antennas and N_R receive antennas, and the channel gain is modelled by an $N_R \times N_T$ complex matrix $\mathbf{H}$, so that each entry $H_{i,j}$ is the channel gain between the j-th transmit and the i-th receive antennae. These channel gains exhibit frequency-flat slowly time-varying fading, according to Jake's model. The entries $H_{i,j}$ are assumed independent and identically-distributed (i.i.d) complex Gaussian circularly symmetric random variables (RVs), with zero-mean and unity-variance, i.e. $H_{i,j} \sim \mathcal{C}N(0,1)$, where the symbol $\sim$ means *statistically distributed as*. Noise is modelled by an additive N_R-dimensional vector $\boldsymbol{\varsigma}$, whose entries ς_i are i.i.d. complex Gaussian circularly symmetric RVs $\sim \mathcal{C}N(0,N_0)$.

Assuming a data stream parsed into frames of P symbols, the received signal corresponding to the symbol interval n $(0 \leqslant n \leqslant P-1)$ of the frame l is

$$\mathbf{y}[l,n] = \mathbf{H}[l,n]\mathbf{x}[l,n] + \boldsymbol{\varsigma}[l,n], \qquad (1)$$

where $\mathbf{y}[l,n]$ is the received N_R-dimensional complex vector and $\mathbf{x}[l,n]$ is the transmitted N_T-dimensional complex vector.

At the receiver, two different channel estimation processes are required: prediction and interpolation. The predicted channel is employed to obtain the beam-steering vector which must be fed back to the transmitter. The interpolated channel is needed to carry out the MRC at the receiver. Both channel estimates are obtained by filtering a previous channel estimate obtained by adapting classical pilot symbol assisted modulation (PSAM) to a MIMO channel. This adaptation is done as follows. A known pilot symbol s_P is inserted within each frame. The pilot symbol spreads along the first

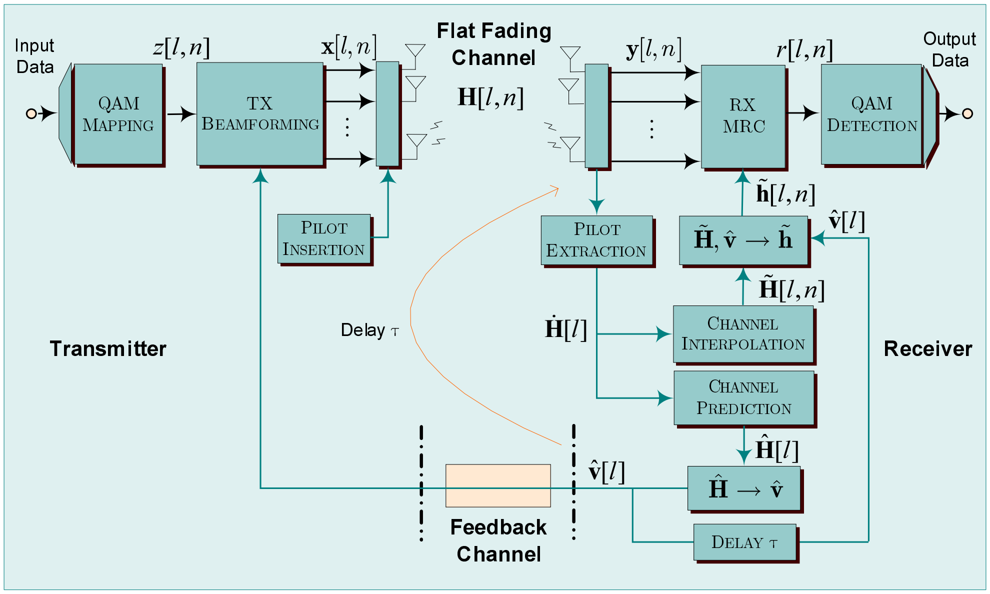

Fig. 1. System model for MIMO beamforming with MRC.

N_T symbol intervals, since orthogonal signatures are used to decouple the MIMO channel estimation problem into N_T single transmit-antenna problems at each receiver branch. Once the pilot symbols are extracted and decoupled, an initial channel estimation $\dot{\mathbf{H}}$ is obtained by dividing the pilots by s_P and it can be expressed as

$$\dot{\mathbf{H}}[l] = \mathbf{H}[l] + \dot{\mathbf{\Xi}}[l], \tag{2}$$

where $\mathbf{H}[l]$ is the channel matrix during the pilot interval of the l-th frame, assuming the channel response is invariant along the pilot interval, and $\dot{\mathbf{\Xi}}[l]$ is the estimation error matrix due to noise. As the entries of the noise vector $\varsigma_i[l,n]$ are i.i.d complex Gaussian circularly symmetric RVs and there is no correlation between time samples, the entries of estimation error matrix $\dot{\Xi}_{i,j}$ are i.i.d complex Gaussian circularly symmetric RVs $\sim \mathcal{C}N(0, N_0/E_P)$, where E_P is the energy of pilot symbol s_P.

To perform beamforming at the transmitter, the receiver needs to predict the channel matrix ahead of time. We further assume that the transmitter only adapts its beam-steering vector once per frame. Hence, the feedback delay is a multiple τ of the frame interval. The predicted channel matrix is obtained applying a digital filter, usually a Wiener prediction filter, to the initial channel estimates $\dot{\mathbf{H}}[l]$, i.e.

$$\hat{\mathbf{H}}[l] = \sum_{m=0}^{N-1} a_m \dot{\mathbf{H}}[l-m-\tau], \tag{3}$$

where N is the number of taps of the prediction filter, a_m is a real-value coefficient of the prediction filter, $\hat{\mathbf{H}}[l]$ is the predicted channel matrix at l-th frame. The prediction error matrix can be expressed as

$$\hat{\mathbf{\Xi}}[l,n] = \mathbf{H}[l,n] - \hat{\mathbf{H}}[l]. \tag{4}$$

The joint distribution of each entry of the predicted channel matrix $\hat{H}_{i,j}[l]$ and its corresponding entry of the prediction error matrix $\hat{\Xi}_{i,j}[l,n]$ is required to carry out the BEP analysis. Both RVs are jointly Gaussian with zero-mean, because they can be expressed as linear combinations of jointly Gaussian RVs with zero mean, but they are generally correlated. Under these considerations and assuming the well-known Jake's channel correlation model, we can obtain after some algebra their variances and covariance to set their joint probability density function (pdf). First, the variance of each entry of the predicted channel matrix $\hat{H}_{i,j}[l]$ can be obtained as

$$\sigma_{\hat{H}}^2 = \mathbf{a}\mathbf{W}\mathbf{a}^t, \tag{5}$$

where $\mathbf{a}$ is the N-dimensional coefficient row vector of the prediction filter used in expression (3) and the entries of the $N \times N$ dimensional matrix $\mathbf{W}$ results as

$$W_{m,m'} = J_0\left(2\pi\left(m-m'\right)T_D\right) + N_0/E_P\delta[m-m'], \tag{6}$$

where $T_D = PT_S f_D$ is the frame interval normalized to the Doppler frequency f_D and T_S is the symbol interval. The variance of each entry of the prediction error matrix $\hat{\Xi}_{i,j}[l,n]$ is given by

$$\sigma_{\hat{\Xi}}^2[n] = 1 + (\mathbf{a}\mathbf{W} - 2\mathbf{w}[n])\mathbf{a}^t, \tag{7}$$

where the entries of the N-dimensional row vector $\mathbf{w}$ results as

$$w_m[n] = J_0\left(2\pi\left[m+\tau+\frac{n}{P}\right]T_D\right). \tag{8}$$

Finally, the correlation between each entry of the prediction error matrix $\hat{\Xi}_{i,j}[l,n]$ and the corresponding entry of the channel predicted matrix $\hat{H}_{i,j}[l]$ can be obtained as

$$\sigma_{\hat{\Xi}\hat{H}}[n] = \tfrac{1}{2}(\sigma_H^2 - \sigma_{\hat{\Xi}}^2[n] - \sigma_{\hat{H}}^2) = \left(\mathbf{w}[n] - \mathbf{a}\mathbf{W}\right)\mathbf{a}^t. \tag{9}$$

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

The channel interpolated matrix $\tilde{\mathbf{H}}[l,n]$ is obtained by applying a different filter for each symbol n of the frame to the estimated channel matrix $\dot{\mathbf{H}}[l]$. In this paper we assume perfect interpolation, so the interpolated channel matrix can be expressed as

$$\tilde{\mathbf{H}}[l,n] = \mathbf{H}[l,n]. \tag{10}$$

From now on, frame index l will be omitted in all expressions without generality loss for simplicity reasons.

Using the predicted channel matrix $\hat{\mathbf{H}}$, the optimum beam-steering vector $\hat{\mathbf{v}}$ is the N_T-dimensional eigenvector corresponding to the largest eigenvalue $\hat{\lambda}$ of the matrix $\hat{\mathbf{H}}^{\mathcal{H}}\hat{\mathbf{H}}$, which is given by $\hat{\lambda} = \hat{\mathbf{v}}^{\mathcal{H}}\hat{\mathbf{H}}^{\mathcal{H}}\hat{\mathbf{H}}\hat{\mathbf{v}}$ [10]. Each frame the transmitter performs beamforming using the predicted beam-steering vector $\hat{\mathbf{v}}$, so that the transmitted vector becomes $\mathbf{x}[n] = \hat{\mathbf{v}}z[n]$, where $z[n]$ is the transmitted symbol. The effective channel gain is a N_R-dimensional vector defined as $\mathbf{h}[n] = \mathbf{H}[n]\hat{\mathbf{v}}$ and the predicted effective channel gain is the vector $\hat{\mathbf{h}} = \hat{\mathbf{H}}\hat{\mathbf{v}}$, whose square Euclidean norm is $||\hat{\mathbf{h}}||^2 = \hat{\lambda}$. The effective channel gain can also be expressed as $\mathbf{h}[n] = \mathbf{H}[n]\hat{\mathbf{v}} = (\hat{\mathbf{H}} + \hat{\boldsymbol{\Xi}}[n])\hat{\mathbf{v}} = \hat{\mathbf{h}} + \hat{\boldsymbol{\psi}}[n]$, where $\hat{\boldsymbol{\psi}}[n]$ is the prediction error of the effective channel gain vector.

At the receiver, the effective channel gain vector $\mathbf{h}[n]$ is estimated to perform MRC. Using the interpolated channel $\tilde{\mathbf{H}}[n]$ and the beam-steering vector $\hat{\mathbf{v}}$ sent to the transmitter, according to our system model, the interpolated effective channel gain vector $\tilde{\mathbf{h}}[n]$ results as

$$\tilde{\mathbf{h}}[n] = \tilde{\mathbf{H}}[n]\hat{\mathbf{v}} = \mathbf{H}[n]\hat{\mathbf{v}} = \mathbf{h}[n], \tag{11}$$

and the symbol $r[n]$ which results from applying MRC to the received vector $\mathbf{y}[n]$ is given by

$$r[n] = \frac{\tilde{\mathbf{h}}^{\mathcal{H}}[n]\mathbf{y}[n]}{\left\|\tilde{\mathbf{h}}[n]\right\|^2} = \frac{\mathbf{h}^{\mathcal{H}}[n]\mathbf{y}[n]}{\|\mathbf{h}[n]\|^2} = z[n] + \varsigma'[n] \tag{12}$$

where

$$\varsigma'[n] \triangleq \frac{\left(\hat{\mathbf{h}} + \hat{\boldsymbol{\psi}}[n]\right)^{\mathcal{H}} \boldsymbol{\varsigma}[n]}{\left\|\hat{\mathbf{h}} + \hat{\boldsymbol{\psi}}[n]\right\|^2} \tag{13}$$

is the resultant noise after MRC. As is shown in [9], the pdf of the resultant noise $\varsigma'[n]$ has two important properties: the real and imaginary parts have the same pdf, which is an even function. Under these conditions and using a BPSK (L=2) or a square M-QAM ($L^2 = M$) modulation with independent mapping for the in-phase an quadrature components, the BEP analysis can be significantly simplified, as is shown in [11]. The BEP for the n-th symbol of the frame can be expressed as

$$BEP(n) = \sum_{k=1}^{L-1} \omega_k \mathcal{I}(k,n), \tag{14}$$

where $\mathcal{I}(k,n) \triangleq \Pr\left\{\Re\left\{\varsigma'[n]\right\} > (2k-1)d\right\}$ is named the component of error probability (CEP), the coefficients ω_k can be explicitly computed using the expressions proposed in [9] and d is the minimum symbol to decision boundary distance which can be expressed as $d = \sqrt{\kappa E_S}$, where $\kappa = 1$ for BPSK modulation or $\kappa = \frac{3}{2(M-1)}$ for M-QAM modulation and E_S is the average data symbol energy.

III. BEP Analysis

In this section, a exact closed-form BEP analysis is carried out. First, the CEP conditioned on the predicted channel $\hat{\mathbf{H}}$, which is called the conditioned component of error probability (CCEP), is computed using Proakis' analysis of complex Gaussian quadratic forms [12]. Then, the CCEP is averaged over the predicted channel and over all the data symbols of the frame to obtain the BEP.

As in [9][11], the CCEP can be computed as

$$\begin{aligned}&\mathcal{I}(k,n|\hat{\mathbf{H}}) = \Pr\left\{\Re\left\{\varsigma'[n]\right\} > (2k-1)d \,\middle|\, \hat{\mathbf{H}}\right\} = \\ &\Pr\left\{D[k,n] < 0 \,\middle|\, \hat{\mathbf{H}}\right\},\end{aligned} \tag{15}$$

where

$$D[k,n] \triangleq \sum\nolimits_{i=1}^{N_R} \mathbf{x}_i^H[n]\mathbf{Q}_k\mathbf{x}_i[n], \tag{16}$$

and

$$\mathbf{x}_i[n] \triangleq \begin{bmatrix} y_i[n] \\ \tilde{h}_i[n] \end{bmatrix}, \mathbf{Q}_k \triangleq \begin{bmatrix} 0 & -1/2 \\ -1/2 & {\scriptstyle (2(u+k-1)-L)d} \end{bmatrix}. \tag{17}$$

According to our system model, each entry of the received signal $y_i[n]$ and each entry of the interpolated effective channel gain $\tilde{h}_i[n]$ are

$$\begin{cases} y_i[n] = h_i[n]s_{u,v} + \varsigma_i[n] = \left(\hat{h}_i + \hat{\psi}_i[n]\right)z + \varsigma_i[n], \\ \tilde{h}_i[n] = h_i[n] = \hat{h}_i + \hat{\psi}_i[n], \end{cases} \tag{18}$$

when the symbol $z = s_{u,v} = {\scriptstyle (2u-L-1)}d + j{\scriptstyle (2v-L-1)}d$ is transmitted ($1 \leq \{u,v\} \leq L-1$).

Using Proakis' analysis of complex Gaussian quadratic forms [12] the CCEP can be calculated, but it is necessary to show that the vectors $x_i[n]$ are independent vectors of Gaussian RVs conditioned on $\hat{\mathbf{H}}$ and have the same covariance matrix. As the entries of $\boldsymbol{\varsigma}[n]$ are independent complex Gaussian variables with the same variance N_0, we only have to show that the entries of $\hat{\boldsymbol{\psi}}[n]$ conditioned on $\hat{\mathbf{H}}$ are also independent complex Gaussian RVs and have the same variance, which is shown as follows.

Taking into account that the joint pdf of $\hat{H}_{i,j}$ and $\hat{\Xi}_{i,j}[n]$ is Gaussian, the pdf of $\hat{\Xi}_{i,j}[n]$ conditioned on $\hat{H}_{i,j}$ is also Gaussian. Thus, the $N_R \times N_T$ complex matrix $\mathbf{M}_{\hat{\Xi}|\hat{H}}[n]$, whose (i,j)-th entry is the mean of $\hat{\Xi}_{i,j}[n]$ conditioned on $\hat{H}_{i,j}$, can be expressed as

$$\mathbf{M}_{\hat{\Xi}|\hat{H}}[n] = \frac{\sigma_{\hat{\Xi}\hat{H}}[n]}{\sigma^2_{\hat{H}}[n]}\hat{\mathbf{H}}, \tag{19}$$

and the variance of $\hat{\Xi}_{i,j}[n]$ conditioned on $\hat{H}_{i,j}$ can be obtained as follows

$$\sigma^2_{\hat{\Xi}|\hat{H}}[n] = \sigma^2_{\hat{\Xi}}[n] - \frac{\sigma^2_{\hat{\Xi}\hat{H}}[n]}{\sigma^2_{\hat{H}}[n]}. \tag{20}$$

Recall that the predicted effective channel gain $\hat{\mathbf{h}} = \hat{\mathbf{H}}\hat{\mathbf{v}}$ and the prediction error of the effective channel gain

$\hat{\boldsymbol{\psi}}[n] = \hat{\boldsymbol{\Xi}}[n]\hat{\mathbf{v}}$. The entries $\hat{\Xi}_{i,j}[n]$ conditioned on the entries $\hat{H}_{i,j}$ are independent complex Gaussian variables with the same variance $\sigma^2_{\hat{\Xi}|\hat{H}}[n]$. Since the vector $\hat{\mathbf{v}}$ only depends on $\hat{\mathbf{H}}$ and is a unitary vector, the entries $\hat{\psi}_i[n]$ conditioned on $\hat{\mathbf{H}}$ are independent complex Gaussian RVs with the same variance as the entries $\hat{\Xi}_{i,j}[n]$ conditioned on $\hat{H}_{i,j}$, i.e.

$$\sigma^2_{\hat{\psi}|\hat{\mathbf{H}}}[n] = \sigma^2_{\hat{\Xi}|\hat{H}}[n], \tag{21}$$

and the mean of $\hat{\boldsymbol{\psi}}[n]$ conditioned on $\hat{\mathbf{H}}$, using (19), becomes

$$\boldsymbol{\mu}_{\hat{\psi}|\hat{\mathbf{H}}}[n] = \mathbf{M}_{\hat{\Xi}|\hat{H}}[n]\hat{\mathbf{v}} = \frac{\sigma_{\hat{\Xi}\hat{H}}[n]}{\sigma^2_{\hat{H}}[n]}\hat{\mathbf{h}}. \tag{22}$$

Now we can calculate the CCEP using Proakis' analysis of complex Gaussian quadratic forms [12], which results

$$\mathcal{I}(k,n|\hat{\mathbf{H}}) = \Pr\left\{D[k,n] < 0 \,\middle|\, \hat{\mathbf{H}}\right\} = Q_1(a,b) + \sum_{p=0}^{N_R-1} C_p(a,b,\eta) I_p(ab) \exp\left[-\frac{a^2+b^2}{2}\right], \tag{23}$$

where $Q_1(\cdot)$ is the Marcum Q function, $I_p(\cdot)$ is the modified Bessel function, and the parameters a, b, η and C_p can be obtained using the expressions that appear in [12, pag. 944-947] or the following alternative expressions proposed in [13]

$$\eta = \left|\frac{\delta_1}{\delta_2}\right|, \tag{24}$$

$$a = \sqrt{\frac{2\delta_2\left(\sum_{i=1}^{N_R}\boldsymbol{\mu}_i^H[n]\left[\mathbf{Q}_k - \delta_1\mathbf{R}^{-1}[n]\right]\boldsymbol{\mu}_i[n]\right)}{(\delta_1-\delta_2)^2}}, \tag{25}$$

$$b = \sqrt{\frac{2\delta_1\left(\sum_{i=1}^{N_R}\boldsymbol{\mu}_i^H[n]\left[\mathbf{Q}_k - \delta_2\mathbf{R}^{-1}[n]\right]\boldsymbol{\mu}_i[n]\right)}{(\delta_1-\delta_2)^2}}, \tag{26}$$

$$C_p(a,b,\eta) = \begin{cases} -1 + \dfrac{1}{(1+\eta)^{2N_R-1}} \displaystyle\sum_{k=0}^{N_R-1} \begin{pmatrix} 2N_R-1 \\ k \end{pmatrix} \eta^k, & p = 0, \\ \dfrac{1}{(1+\eta)^{2N_R-1}} \displaystyle\sum_{k=0}^{N_R-1-p} \begin{pmatrix} 2N_R-1 \\ k \end{pmatrix} \times \left[\left(\dfrac{b}{a}\right)^l \eta^k - \left(\dfrac{a}{b}\right)^p \eta^{2N_R-1-k}\right], & p \neq 0, \end{cases} \tag{27}$$

where $\boldsymbol{\mu}_i[n]$ and $\mathbf{R}[n]$ are the mean and the covariance matrix of $\mathbf{x}_i[n]$, respectively (note that the covariance matrix has no dependence on i), δ_1 and δ_2 are the eigenvalues of the matrix $\mathbf{R}[n]\mathbf{Q}_k$ and $\delta_1 > \delta_2$ by definition.

At this point we use the statistical characterization of the RVs which appear in (18) given by (21) and (22). The mean $\boldsymbol{\mu}_i[n]$ and the covariance matrix $\mathbf{R}[n]$ of the vector $\mathbf{x}_i[n]$ can be expressed as follows

$$\boldsymbol{\mu}_i[n] = \hat{h}_i\left(1 + \frac{\sigma_{\hat{\Xi}\hat{H}}[n]}{\sigma^2_{\hat{H}}[n]}\right)\begin{bmatrix} s_{u,v} \\ 1 \end{bmatrix} = \hat{h}_i\mathbf{g}[n], \tag{28}$$

$$\mathbf{R}[n] = \begin{bmatrix} \sigma^2_{\hat{\psi}|\hat{\mathbf{H}}}[n]\left|s_{u,v}\right|^2 + N_0 & \sigma^2_{\hat{\psi}|\hat{\mathbf{H}}}[n]s_{u,v} \\ \sigma^2_{\hat{\psi}|\hat{\mathbf{H}}}[n]s^*_{u,v} & \sigma^2_{\hat{\psi}|\hat{\mathbf{H}}}[n] \end{bmatrix}. \tag{29}$$

Now we show that the CCEP dependence on $\hat{\mathbf{H}}$ can be expressed as a CCEP dependence on $\hat{\lambda}$, i.e.

$$\mathcal{I}(k,n|\hat{\mathbf{H}}) = \mathcal{I}(k,n|\hat{\lambda}). \tag{30}$$

The CCEP dependence on $\hat{\mathbf{H}}$ in expression (23) is contained in the parameters a and b, specifically, in the term $\hat{h}_i$ that appears in the expression of $\boldsymbol{\mu}_i[n]$ (28). Substituting expression (28) in the expression (25), the parameter a results

$$a = \sqrt{\frac{2\delta_2\left(\sum_{m=1}^{N_R}\left|\hat{h}_m\right|^2\right)\left(\mathbf{g}^H[n]\left[\mathbf{Q}_k - \delta_1\mathbf{R}^{-1}[n]\right]\mathbf{g}[n]\right)}{(\delta_1-\delta_2)^2}} = \sqrt{\frac{2\delta_2\hat{\lambda}\left(\mathbf{g}^H[n]\left[\mathbf{Q}_k - \delta_1\mathbf{R}^{-1}[n]\right]\mathbf{g}[n]\right)}{(\delta_1-\delta_2)^2}} = a_k[n]\sqrt{\hat{\lambda}}. \tag{31}$$

In the same way, the parameter b can be expressed as

$$b = b_k[n]\sqrt{\hat{\lambda}}, \tag{32}$$

and, therefore, it is shown the CCEP dependence on $\hat{\lambda}$.

After some algebra it can be shown that, for the complex Gaussian quadratic form defined in this analysis, the parameter η can be expressed as

$$\eta = \eta_k[n] = \frac{b_k[n]}{a_k[n]}. \tag{33}$$

We can obtain the CEPs averaging the CCEPs over all the data symbols of the frame ($N_T \le n \le P-1$) and over $\hat{\lambda}$. Then, using the expression (14), we can obtain the BEP from the CEPs as

$$BEP = \sum_{k=1}^{L-1}\omega_k \sum_{n=N_T}^{P-1}\tfrac{1}{(P-N_T)}\int_0^\infty \mathcal{I}(k,n|\hat{\lambda})p(\hat{\lambda})d\hat{\lambda}, \tag{34}$$

where $p(\hat{\lambda})$ is the pdf of the largest eigenvalue of complex a Wishart matrix. This pdf can be expressed as a weighted sum of elementary Gamma pdfs [14] as

$$p(\hat{\lambda}) = \sum_{l=1}^{N_1}\sum_{r=N_2-N_1}^{(N_2+N_1-2l)l} B_{l,r}\frac{\hat{\lambda}^r}{(1-\chi)^{r+1}}\exp\left(\frac{-l\hat{\lambda}}{1-\chi}\right), \tag{35}$$

where $N_1 \triangleq \min\{N_T, N_R\}$, $N_2 \triangleq \max\{N_T, N_R\}$ and the constants $B_{l,r}$ are

$$B_{l,r}(N_1,N_2) \triangleq \frac{A_{l,r}}{\prod_{l=1}^{N_1}(N_1-l)!\prod_{l=1}^{N_1}(N_2-k)!}, \tag{36}$$

and coefficients $A_{l,r}$ can be exactly computed by the algorithm proposed in [14].

Finally, the BEP is obtained by substituting the expressions (23), (31)-(33) and (35) in (34), using the expression [15, eq.8.772-3] and the integrals given in [16] and [15, eq.8.914-1].

1930-529X/07/$25.00 © 2007 IEEE
This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE GLOBECOM 2007 proceedings.

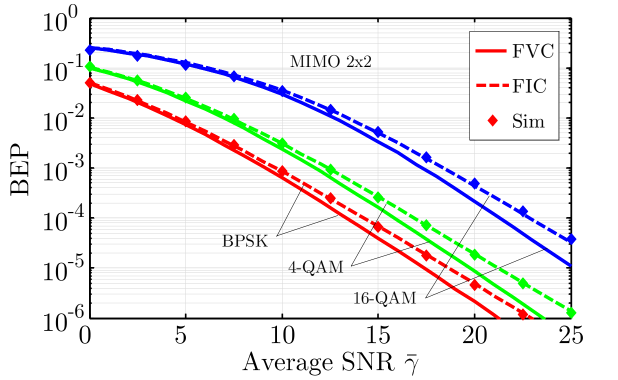

Fig. 2. BEP for different constellation sizes under imperfect prediction using a FVC and FIC models and simulation results (Sim)

The exact closed-form BEP expression is given by

$$\begin{aligned}
BEP = {} & \tfrac{1}{(P-N_R)} \sum_{n=N_R}^{P-1} \sum_{k=1}^{L-1} \sum_{l=1}^{N_1} \sum_{r=N_2-N_1}^{(N_2+N_1-2k)k} \frac{\omega_k B_{l,r}}{l^{r+1}} \times \\
& \Bigg\{ \frac{r!}{l^{1+r}} \Bigg\{ 1 + \frac{b_k^2[n]}{s_k[n]} \sum_{l=0}^{r} (l+1) \left(\frac{2l}{s}\right)^l \times \\
& \Bigg[\frac{a_k^2[n]}{s_k[n]} {}_2F_1\left(\frac{l+2}{2}, \frac{l+2}{2} + \frac{1}{2}; 2; \frac{4a_k^2[n]b_k^2[n]}{s_k^2[n]}\right) - \\
& \frac{1}{1+l} {}_2F_1\left(\frac{l+1}{2}, \frac{l+1}{2} + \frac{1}{2}; 1; \frac{4a_k^2[n]b_k^2[n]}{s_k^2[n]}\right)\Bigg]\Bigg\} + \\
& \frac{(r+l)!}{(a_k[n]b_k[n])^{r+1}} \frac{1}{l!} \left(\frac{w_k[n]-1}{w_k[n]+1}\right)^{l/2} \left(\frac{w_k[n]+1}{2}\right)^r \times \\
& {}_2F_1\left(-r, -r+l; l+1; \frac{w_k[n]-1}{w_k[n]+1}\right) \sqrt{(w_k^2[n]-1)^{r+1}} \Bigg\},
\end{aligned} \tag{37}$$

where ${}_2F_1$ is the Gauss hypergeometric function, $s_k[n] = a_k^2[n] + b_k^2[n] + 2k$ and $w_k[n] = \left(1 - \frac{4a_k^2[n]b_k^2[n]}{s_k^2[n]}\right)^{-1/2}$.

IV. Numerical results

Figure 2 shows the BEP for a 2x2 MIMO system for BPSK, 4-QAM and 16-QAM modulations as a function of the average SNR $\overline{\gamma} = E_S/N_0$. The remaining parameters correspond to a realistic scenario with equal power for pilot and data symbols ($E_P = E_S$), carrier frequency f_c=3 GHz, frame interval PT_S=2.56 ms, which corresponds to a system with P=256 and $1/T_S$=100 KHz, mobile speed v=36 Km/h, and feedback delay $\tau = 1$ frame. We consider imperfect prediction using a 16-taps Wiener prediction filter and two channel models: the usual frame-invariant channel model (FIC) [2][9], where the channel response remains invariant over the frame and the BEP is calculated using the expressions show in [9]; and a frame-variant channel model (FVC), in which the channel response is variable over the frame and the BEP is calculated using (37). Moreover, it is shown simulation results considering the FVC model. The figure indicates significant differences between the BEP calculated with both channel models, especially for high $\overline{\gamma}$ because the effects of prediction error due to the channel variation over the frame dominates over the noise effects.

V. Conclusions

Exact closed-form BEP expressions for MIMO beamforming with MRC systems under channel prediction errors have been derived. These results allow us to analyze the performance of BPSK and square M-QAM modulations using practical estimation methods under a channel model that includes channel response variation over the frame interval. The system performance has been analyzed for a realistic scenario with different constellations and our results show performance degradation of up to 1.5 dB due to imperfect channel estimates from prediction errors.

Acknowledgement

This work was partially supported by the Spanish Government and the European Union under project TEC2007-67289/TCM and by AT4 wireless. The work of A. Goldsmith was supported in part by LG Electronics.

References

[1] A. J. Goldsmith, *Wireless Communications*, 1st ed., Cambridge University Press, New York, 2005.

[2] S. Zhou, G.B. Giannakis,"How Accurate Channel Prediction Needs to be for Transmit-Beanforming With Adaptive Modulation Over Rayleigh Mimo Channel?"*IEEE Trans. Wireless Commun.*, vol. 3, pp. 1285-1294, July 2004.

[3] J. K. Cavers, "An analysis of pilot symbol assisted modulation for Rayleigh fading channels,"*IEEE Trans. Veh. Tech.*, pp. 686-693, Nov. 1991.

[4] J. F. Paris and A. J. Goldsmith, "Adaptive Modulation for MIMO Multiplexing under Average BER Constraints and Imperfect CSI," *Proc. of Int. Conf. Comm.* ICC 2006, pp.1318-1325, June 2006.

[5] Z. Zhou and B. Vucetic, "Design of adaptive modulation using imperfect CSI in MIMO systems," *IEE Electron. Lett.*, pp. 10731075, Aug. 2004.

[6] Z. Zhou, B. Vucetic, M. Dohler, Y. Li, "MIMO Systems with Adaptive Modulation," *IEEE Trans. Veh. Tech.*, vol. 54, pp. 1828-1842, Sep. 2005.

[7] J. F. Paris and A. J. Goldsmith, "Adaptive Modulation for MIMO Beamforming under Average BER Constraints and Imperfect CSI," *Proc. of Int. Conf. Comm.* ICC 2006, pp.1312-1317, June 2006.

[8] A. J. Goldsmith, S.-G. Chua, "Variable-Rate Variable-Power M-QAM for Fading Channels," *IEEE Trans. Commun.*, vol 45, pp. 1218-1230, Oct. 1997.

[9] E. Martos-Naya, J.F. Paris, U. Fernández-Plazaola, A. J. Goldsmith, "Exact Closed-Form Performance Analysis for MIMO Beamforming with MRC under Prediction Errors," submitted to *IEEE Trans. Wireless Commun.*

[10] S. Zhou and G. B. Giannakis, "Optimal Transmitter Eigen-Beamforming and Space-Time Block Coding Based on Channel Mean Feedback," *IEEE Trans. Signal Processing*, vol. 50, pp. 1673-1690 , Oct. 2002.

[11] L. Cao and C. Beaulieu, "Closed-Form BER Results for MRC Diversity with Channel Estimation Errors in Ricean Fading Channels," *IEEE Trans. Wireless Commun.*, vol. 4, pp. 1440-1447, July 2005.

[12] J.G. Proakis, *Digital Communications*, 4th ed., McGraw-Hill, New York, 2001.

[13] U. Fernández-Plazaola, E. Martos-Naya, J.F. Paris, "Another Look at the Analysis of the Characteristic Function of Complex Gaussian Quadratic Forms," submitted to *IEEE Trans. Commun.*

[14] A. Maaref and S. Aissa, "Closed-Form Expressions for the Outage and Ergodic Shannon Capacity of MIMO MRC Systems," *IEEE Trans. Commun.*, vol. 53, pp. 1092-1095, July 2005.

[15] I. S. Gradshteyn and I. M. Ryzhik,*Table of Integrals, Series, and Products*, 5th ed., Academic Press, San Diego, 1994.

[16] M. K. Simon and M.-S. Alouini, "Some New Results for Integrals Involving the Generalized Marcum Q Function and their Application to Performance Evaluation over Fading Channels," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 611-615, July 2003.