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ANALYSIS OF MIXED INTEGER PROGRAMMING FORMULATIONS FOR SINGLE MACHINE SCHEDULING PROBLEMS WITH SEQUENCE DEPENDENT SETUP TIMES AND RELEASE DATES

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ABSTRACT. The scheduling of jobs over a single machine with sequence dependent setups is a classical problem setting that appears in many practical applications in production planning and logistics. In this work, we analyze six mixed-integer formulation paradigms for this classical context considering release dates and two objective functions: the total weighted completion time and the total weighted tardiness. For each paradigm, we present and discuss a MIP formulation, introducing in some cases new constraints to improve performance. A dominance hierarchy in terms of strength of their linear relaxations bounds is developed. We report extensive computational experiments on a variety of instances to capture several aspects of practical situations, allowing a comparison regarding size, linear relaxation and overall performance. Based on the results, discussions and recommendations are made for the considered problems.

Keywords: Single machine scheduling, Sequence-dependent setup, Release dates.

1 INTRODUCTION

Scheduling research is concerned with the allocation of scarce resources to activities over time with the goal of optimizing one or more objectives. This vast family of problems is explicitly or implicitly present in countless applications, from production planning to bioinformatics related problems. Its study goes back to early the 1950s, were, from the perspective of Operations Research, the first problems on industrial applications began to be identified and formulated. This article deals with one of its simplest forms, a single machine environment, which is a challenging combinatorial optimization problem. Furthermore, we deal with mixed integer programming

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(MIP) formulations for the single machine scheduling problem with sequence-dependent setup times and release dates (SMSDRD).

For the considered problem we define a set $J = \{1, \ldots, n\}$ of jobs to be processed on a single machine. Preemptions are not allowed, i.e., once the job is allocated to the machine, the job holds the machine busy until the task is completed. The following data are associated with job j: the processing time p_j, i.e., the amount of time in which the job holds the machine; the release date r_j, i.e., the earliest time at which job j can start its processing; and the weight w_j, i.e., importance of job j relative to the other jobs in the system. Moreover, the non-symmetric sequence-dependent setup time s_{ij} is associated with jobs i and j. The setup times represent the clean-up time between two distinct jobs. We consider the SMSDRD with two variants for the objective function. The first objective to be considered is the total weighted completion time. The second one is the total weighted tardiness, where due date d_j is associated with job j. This date may represent the committed shipping or completion date (date promised to the customer).

Blazewicz et al. [1] were the first, to the best of our knowledge, to compile MIP formulations for machine scheduling problems. Queyranne et al. [2] analyzed MIP formulations for machine scheduling problems from a polyhedral theory point of view. Allahverdi et al. [3] provided a comprehensive review involving different setup considerations on several machines scheduling settings. The review was next expanded and updated by Allahverdi et al. [4] to cover several features such as static, dynamic, deterministic, and stochastic problems for all shop environments. Keha et al. [5] and Unlu & Mason [6] compare the computational performance of (MIP) formulations for machine scheduling. The first work address several single machine problems, while the latter focuses on parallel machines environment. No sequence-dependent setup is considered. Adamu & Adewumi [7] proposed a review focused on the weighted number of tardy jobs on a single machine.

As discussed above, several works are found in the literature on similar problems; however only a few of them proposed mathematical formulations considering sequence-dependent setups, even on a single machine environment. When considering the makespan (the maximum completion time) as objective function, the problem can be treated as the classical traveling salesman problem (TSP). In the survey of Öncan et al. [8] a comparison of mixed integer programming formulations (MIP) for the TSP problem is analyzed; however, the characteristics of the problem diverge from the single machine scheduling problem (SMSP) discussed in this work. Therefore, all formulations presented in this work reflect in a specific concept on how the variables and parameters are defined, requiring particular changes and definitions.

The MIP formulations for the SMSDRD we investigate can be grouped into four paradigms according to their decision variables: (i) completion time and precedence; (ii) assignment and positional date; (iii) time-indexed; and (iv) arc-time-indexed. A fifth paradigm formulation denominated “Linear Ordering” by Keha et al. [5] and Unlu & Mason [6] is not considered, as in a single machine scenario with sequence-dependent setup times, this formulation is equivalent to the “Completion Time and Precedence”. Our purpose is to analyze the dominance relationships concerning strength of their linear relaxations bounds, and then to compare the computational performance when trying to solve them with a standard optimization package.

A summary of the literature review with these MIP formulations approaches for scheduling problems is presented in Tables 1 and 2. Table 1 depicts research works in alphabetical order, with the same objective functions adopted in this article ($\sum_j w_j C_j$ and $\sum_j w_j T_j$) and Table 2 organizes other related works.

Table 1 – Previous specific research works for scheduling problems. The field "MIP Formulation" indicates the formulation paradigm. The field "Problem Parameters" is divided into "no parameters", "r_j and s_{ij}" and "with s_{ij}". The first presents works without parameters in the scheduling environment, the second presents works with both parameters, and the last presents works with the parameter s_{ij} in the formulation. The field "Performance Measures" defines the objective functions.

MIP Formulations	Problem Parameters	Performance Measures $\sum_j w_j C_j$	Performance Measures $\sum_j w_j T_j$
Completion Time and Precedence	no parameters	[5], [9]	[5], [10]
	r_j and no s_{ij}	[5]	[5]
	with s_{ij}	[11]	[11]
Linear Ordering	no parameters	[1], [12], [5]	[1], [5], [10]
	r_j and no s_{ij}	[13], [5], [2], [6]	[5]
	with s_{ij}		[14]
Assignment and Positional Date	no parameters	[5], [10], [15], [2]	[5]
	r_j and no s_{ij}	[5]	[5]
	Others		[16]
Time-Indexed	no parameters	[5], [10]	[17], [16], [5], [18], [19], [20], [21], [22], [23]
	r_j and no s_{ij}	[24], [5], [2]	[5], [2]
Arc-Time-Indexed	no parameters		[25]

Manne [28] initially proposes the completion time and precedence (CTP) formulation for the job shop problem, see also Balas et al. [33]. It is characterized by continuous variables defining the completion time of each job, and by binary variables describing the precedence relations between pairs of jobs. Formulations according to this paradigm have been proposed for a variety of scheduling problems. For instance, Maffioli & Sciomachen [40] used this formulation as an exact approach for solving the sequential ordering problem. In the assignment and positional date (APD) formulation, introduced by Wagner [48], a sequence to be processed on the machine is a permutation of the n jobs. Binary variables assign jobs to positions in the permutation.

Table 2 – Previous general research works for scheduling problems. The field "MIP Formulation" indicates the formulation paradigm. The field "Problem Parameters" is divided into "no parameters", "r_j and s_{ij}" and "with s_{ij}". The first presents works without parameters in the scheduling environment, the second presents works with both parameters, and the last presents works with the parameter s_{ij} in the formulation. The field "Performance Measures" defines the objective functions.

MIP Formulations	Problem Parameters	Performance Measures Other Objective Functions
Completion Time and Precedence	no parameters	[26], [7], [5], [27], [28]
	r_j and no s_{ij}	[7], [29], [13], [5], [30]
	with s_{ij}	[31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [11], [2], [43], [44], [45], [30]
Linear Ordering	no parameters	[5]
	r_j and no s_{ij}	[5], [46], [19]
Assignment and Positional Date	no parameters	[47], [5], [48]
	r_j and no s_{ij}	[49], [5], [50], [6]
	with s_{ij}	[51], [52], [44], [53]
	Others	[54], [55], [56], [57]
Time-Indexed	no parameters	[58], [59], [60], [61], [62], [5], [63], [64], [21], [22], [65], [23], [66]
	r_j and no s_{ij}	[67], [68], [69], [5], [70], [2], [6], [71]
	with s_{ij}	[72], [73], [74], [70], [63], [75], [76], [77], [78], [79]
Arc-Time-Indexed	with s_{ij}	[80], [81], [82], [83], [42]

Lee & Asllani [52] modeled a dual criteria problem – minimizing the number of tardy jobs and makespan – based on APD formulation. More recently, Dauzère-Pérès & Mönch [47] modeled a single batch processing problem.

The time index (TI) formulation is based on a time-discretization of the planning horizon. TI formulations have been investigated in the literature because they are likely to provide better LP-relaxation bounds than other formulations for scheduling problems. Sousa & Wolsey [22] proposed a variety of valid inequalities derived from the knapsack problem. Van den Akker et al. [84] developed column generation techniques to deal with the models of large dimensions yielded by such formulations, see also Bigras et al. [17]. Avella et al. [24] and Sourd [21] used TI formulations into Lagrangean relaxation schemes. Paula et al. [78] proposed a non-delayed relax-and-cut algorithm, based on a Lagrangean relaxation of a time-indexed formulation for scheduling problems on unrelated parallel machines. Tanaka et al. [23] proposed a TI formulation and successive sublimation dynamic programming method to minimize the total job completion cost, see also Tanaka & Araki [14]. Davari et al. [16] developed branch-and-bound techniques based on TI and APD formulations for single-machine scheduling with time windows and precedence constraints. Recently, Cota et al. [85] proposed a TI formulation for scheduling trucks on a crossdocking facility, modeling as a flow shop scheduling problem with precedence constraints. Pessoa et al. [25] proposed the arc-time-indexed (ATI) formulation where each variable is indexed by a pair of jobs and a completion time. The authors prove that ATI formulation dominates the TI formulation. Pessoa et al. [25] developed a powerful branch-and-price algorithm making use of a number of techniques to deal with highly degenerated problems yielded by formulations of pseudo-polynomial size. Keshavarz et al. [74] used ATI formulation into a Lagrangian-based branch-and-bound algorithm for a group scheduling problem. Nogueira et al. [42] proposed an ATI formulation with real applications for scheduling trains on a single track-line, modeling as a SMSP with sequence-dependent setup times and release dates.

This article is organized as follows. Section 2 presents the MIP formulations for the SMSDRD. In Section 3 the strengths of their linear relaxations are analyzed. In Section 4 we report computational experiments comparing their performances. Finally, Section 5 presents our conclusions remarks.

2 MATHEMATICAL FORMULATIONS

In all single machine scheduling problem environments, n jobs must be processed without preemption. We further assume that all parameters are known and given in integer values. The following notation summarizes the sets, parameters, and variables used in all mathematical formulations:

Sets

J – set of jobs, indexed $j \in \{1, \dots, n\}$.
H – set of time periods, indexed $t \in \{0, \dots, h\}$.

Parameters

h – time horizon length.
p_j – processing time of job j.
d_j – due date of job j.
r_j – release date of job j.
w_j – priority or weight of job j.
s_{ij} – sequence-dependent setup time between jobs i and j.

Decision variables

C_j – Completion time of job j; non-negative; used in minimizing total weighted completion time.

T_j – Tardiness of job j; $T_j = max\{0, C_j - d_j\}$; used in minimizing total weighted tardiness.

We assume setup times satisfying the triangle inequality, i.e., $s_{ij} \leq s_{il} + p_l + s_{lj}$, for any given triple $i, j, l \in J$, $i \neq j \neq l$. It is important to point out that, except for Assignment and Positional Date, and Arc-Time-Indexed formulations, all others require a setup time that satisfy the triangle inequality. The total weighted completion time and the total weighted tardiness are regular performance measures, which means they are non-decreasing functions of the completion time. A scheduling concept to be used in the sequel is that of active schedule (see, for instance, Pinedo [86]). A feasible non preemptive schedule is active if by changing the order of jobs, it is not possible to construct a schedule with at least one job finishing earlier without delaying another job. Given that the objective functions considered are regular, there exists an optimal schedule for the SMSDRD that is active.

In the next sections we present the constraint set of each formulation analyzed in this study. The continuous variables C_j are common to all models, and give the completion time of each job j. The objective of total weighted completion time is given by

$$\min \sum_{j \in J} w_j C_j \tag{1}$$

while the objective of total weighted tardiness is given by

$$\min \sum_{j \in J} w_j T_j \tag{2}$$

where $T_j = \max(C_j - d_j, 0)$ is the tardiness of job j.

2.1 Completion time and precedence formulation

The completion time and precedence (CTP) formulation is characterized by the binary variables γ_{ij} that describe precedence relations between each pair of jobs i and j. Given a pair i, j of jobs,

γ_{ij} assumes 1 if i is processed before j (not necessarily immediately before), and 0 otherwise. The constraint sets (3)-(6) composes the CTP formulation:

$$C_j \geq C_i + s_{ij} + p_j - M_{ij}(1-\gamma_{ij}) \qquad \forall\, i, j \in J, i \neq j, \tag{3}$$
$$\gamma_{ij} + \gamma_{ji} = 1 \qquad \forall\, i, j \in J, i < j, \tag{4}$$
$$C_j \geq r_j + p_j \qquad \forall\, j \in J, \tag{5}$$
$$\gamma_{ij} \in \{0, 1\} \qquad \forall\, i, j \in J, i \neq j. \tag{6}$$

Constraint set (3) makes use of a large positive constant M_{ij} defined for each pair $(i, j) \in J \times J$, as it can be asymmetric. These constraints ensure that if job j is to be processed after job i, then it finishes no earlier than the completion time of job i plus the sequence-dependent setup time and its processing time. Constraint set (4) imposes that either job i is processed before job j or vice versa. Constraint set (5) ensures that completion time of job j is greater than or equal to its release date plus its processing time. Constraints (6) impose the integrality of variables γ_{ij}.

We next give a proposition to compute a value for M_{ij} that preserves all active schedules.

Proposition 1. *All feasible active schedules for SMSDRD satisfy constraint (3) if for all $(i, j) \in J \times J$ the value of M_{ij} is computed as follows:*

$$M_{ij} = M_i - r_j + s_{ij},$$

with

$$M_i = \max\left\{r_i, \max_{l\in J, l\neq i,j} r_l + \sum_{l\in J, l\neq i,j} p_l + \sum_{l\in J, l\neq i,j} \max_{k\in J, k\neq l,j} s_{lk}\right\} + p_i.$$

Proof. For a given job i, M_i is an upper bound for its completion time C_i, as it considers the relation of its release time. If $\gamma_{ij} = 1$, constraints 3 generates the following constraint $C_j \geq C_i + s_{ij} + p_j$. On the contrary, if $\gamma_{ij} = 0$, constraints 3 generates the following constraint $C_j \geq C_i + s_{ij} + p_j - M_{ij}$. M_{ij} has to be big enough so that the completion time C_i does not generate a restriction in the completion time C_j. Besides the relation with a job i, C_j needs to satisfy its relation with j's release date, that is, $C_i + s_{ij} + p_j - M_{ij} \leq r_j + p_j$, thus $M_{ij} \geq M_i - r_j + s_{ij}$. □

Within the CTP paradigm, γ_{ij} can be used to indicate that job j follows immediately job i, when equal to one. Such a formulation, which we denote as arc-flow completion time and precedence (AFCTP), has been used to model the asymmetric traveling salesman problem, see for instance Ascheuer et al. [31]. In this case, completion time variables are redefined as C_{ij}. If $\gamma_{ij} = 1$, variable C_{ij} gives the completion time of job i, and job j starts at $\min(C_{ij}, r_j)$. Otherwise, $\gamma_{ij} = 0$ implies $C_{ij} = 0$. Completion time variables in CTP and AFCTP formulations are related by $C_j = \sum_{k\in J:k\neq j} C_{jk}$. The AFCTP formulation uses a fictitious job 0 indicating only the starting and ending point of the sequence, therefore its parameter values must be null for no impact in objective function values. For this reason, the new set J' is defined as $J \cup \{0\}$. The constraint sets (7)-(11) composes the AFCTP formulation:

$$\sum_{i\in J, i\neq j} C_{ij} + \sum_{i\in J', i\neq j} (p_j + s_{ij})\gamma_{ij} \leq \sum_{k\in J', k\neq j} C_{jk} \qquad \forall j \in J, \tag{7}$$

$$\sum_{j\in J', i\neq j} \gamma_{ij} = 1 \qquad \forall i \in J', \tag{8}$$

$$\sum_{i\in J', i\neq j} \gamma_{ij} = 1 \qquad \forall j \in J', \tag{9}$$

$$\gamma_{ij}(r_i + p_i) \leq C_{ij} \leq \gamma_{ij} M_i \qquad \forall i, j \in J', i \neq j, \tag{10}$$

$$\gamma_{ij} \in \{0, 1\} \qquad \forall i, j \in J', i \neq j. \tag{11}$$

Constraints (7) have the same meaning as (3). Constraints (8) and (9) establish that each job is succeeded and preceded by exactly one job. Constraint set (10) defines the C_{ij} domain, where M_i is a large positive constant as defined in Proposition 1. Constraints (11) impose the integrality of variables γ_{ij}.

2.2 Assignment and positional date formulation

The assignment and positional date (APD) formulation makes use of binary variables to represent the assignment of the n jobs to the n positions of the production sequence. A binary variable ν_{jk} assumes 1 if job j is assigned to the k^{th} position, and 0 otherwise. Variable C'_k defines the completion time of the job at position k. The constraint sets (12)-(17) composes the APD formulation:

$$\sum_{k=1}^{n} \nu_{jk} = 1 \qquad \forall j \in J, \tag{12}$$

$$\sum_{j\in J} \nu_{jk} = 1 \qquad k = 1, \ldots, n, \tag{13}$$

$$C'_k \geq \sum_{j\in J} (r_j + p_j)\nu_{jk} \qquad k = 1, \ldots, n, \tag{14}$$

$$C'_k \geq C'_{k-1} + (\nu_{i(k-1)} + \nu_{jk} - 1)(s_{ij} + p_j) \qquad \forall i, j \in J, i \neq j, k = 2, \ldots, n, \tag{15}$$

$$C_j \geq C'_k - M_k(1 - \nu_{jk}) \qquad \forall j \in J, k = 1, \ldots, n, \tag{16}$$

$$\nu_{jk} \in \{0, 1\} \qquad \forall j \in J, k = 1, \ldots, n. \tag{17}$$

Constraints (12) and (13) establish that a job is assigned to exactly one position in the production sequence and that each position is occupied by exactly job, respectively. Constraint set (14) ensures that the completion time of a job at position k is greater than or equal to its release date plus its processing time. Constraints (15) compute completion times for the jobs at positions $2, \ldots, n$. Constraints (16) relate the completion time of job j with its assigned position. Constraint set (17) imposes the integrality of variables ν_{jk}.

Completion times in successive positions can also be modeled by introducing auxiliary continuous variables. A variable β_{ij}^k assumes 1 if job i is assigned to the k^{th} position and job j to the $(k+1)^{\text{th}}$ position, an 0 otherwise. In all feasible solutions to APD formulation, β_{ij}^k assumes naturally a binary value when defined by the following constraints:

$$\beta_{ij}^k \leq \nu_{ik} \qquad \forall i, j \in J, i \neq j, k = 1, \ldots, n, \tag{18}$$

$$\beta_{ij}^k \leq \nu_{j(k+1)} \qquad \forall i, j \in J, i \neq j, k = 1, \ldots, n-1, \tag{19}$$

$$\beta_{ij}^k \geq 1 - (2 - \nu_{ik} - \nu_{j(k+1)}) \qquad \forall i, j \in J, i \neq j, k = 1, \ldots, n-1, \tag{20}$$

$$\beta_{ij}^k \geq 0 \qquad \forall i, j \in J, i \neq j, k = 1, \ldots, n. \tag{21}$$

Then, constraints (15) are replaced by

$$C'_k \geq C'_{k-1} + \sum_{j \in J} p_j \nu_{jk} + \sum_{i \in J} \sum_{\substack{j \in J, \\ j \neq i}} \beta_{ij}^{k-1} s_{ij} \qquad k = 2, \ldots, n. \tag{22}$$

Analogously to the CTP formulation, in the APD formulation it is necessary to give a value for the positive large constant M_k. So, the next proposition shows how to compute a value for M_k that preserves all active schedules.

Proposition 2. *All feasible active schedules for SMSDRD satisfy constraint (16) if for each position k, $k = 1, \ldots, n$, M_k is computed as follows:*

$$M_k = \mathscr{S}_{j \in J}^1 (p_j + r_j) + \mathscr{S}_{j \in J}^{k-1} (p_j + s_j^{max}), \tag{23}$$

where function $\mathscr{S}_{j \in J}^l (x_j)$ returns the sum of the l larger values of a parameter or variable x_j, for $j \in J$, and $\mathscr{S}^0 = 0$.

Proof. Let (k', k'') be two adjacent job positions in an active schedule and $(j_{k'}, j_{k''})$ its respective jobs. For each position k'' an upper bound $M_{k''}$ for the completion time at position k'' can be defined. The completion time $(C'_{k''})$ is at least $\max\{C'_{k'} + p_{j_{k''}} + s_{j_{k'} j_{k''}}, r_{j_{k''}} + p_{j_{k''}}\}$, as $\mathscr{S}_{j \in J}^1 (p_j + r_j) + \mathscr{S}_{j \in J}^{k''-1} (p_j + s_j^{max})$ is an upper bound for $\max\{C_{j'_k} + p_{j_{k''}} + s_{j_{k'} j_{k''}}, r_{j_{k''}} + p_{j_{k''}}\}$, $C_{k''}$ can be redefined. Thereby, generalizing for all n positions, $M_k = \mathscr{S}_{j \in J}^1 (p_j + r_j) + \mathscr{S}_{j \in J}^{k-1} (p_j + s_j^{max})$ is a valid upper bound. □

Keha et al. [5] and Unlu & Mason [6] showed that the APD formulation usually provides stronger linear relaxation lower bounds. However, when release dates and sequence-dependent setup times are introduced it is necessary to establish a positive large constant M in the constraint set (16). Keha et al. [5] mentions that the linear relaxation bound performance of the formulation decreases with the increase of M's value. Therefore, the weaker linear relaxation of an APD formulation in the problem treated in this work can be justified by Keha et al. [5] work.

2.3 Time-indexed formulation

Integer programming formulations making use of variables indexed by a job and a discrete time period have been proposed to a variety of scheduling problems, see Sousa & Wolsey [22]. In the time-indexed (TI) formulation, the planning horizon is divided into periods. Let H denote the set of periods. The duration of each period is Δ, and it is assumed that release and due dates and processing and setup times for all jobs are multiple of Δ. Let $h_j = M_j/\Delta$, $j \in J$, where M_j is given in Proposition 1. The set of time periods is defined as $H = \{0, \ldots, h\}$, where $h = \max_{j \in J}\{h_j\}$. A binary variable x_j^t assumes 1 if job j starts at time period t, and 0 otherwise. The constraint sets (24)-(27) composes the TI formulation:

$$\sum_{t=r_j}^{h_j-p_j+1} x_j^t = 1 \qquad \forall j \in J, \tag{24}$$

$$x_j^t + \sum_{s=\max\{r_i, t-p_i-s_{ij}+1\}}^{\min\{t+p_j+s_{ji}-1, h_i-p_i+1\}} x_i^s \leq 1 \quad \forall i, j \in J, i \neq j, t \in \{r_j, \ldots, h_j - p_j + 1\}, \tag{25}$$

$$C_j \geq \sum_{t=r_j}^{h_j-p_j+1} (t x_j^t) + p_j \qquad \forall j \in J, \tag{26}$$

$$x_j^t \in \{0, 1\} \qquad \forall j \in J, t \in \{r_j, \ldots, h_j - p_j + 1\}. \tag{27}$$

Constraints (24) ensure that each job j is assigned to a time period t. Constraint set (25) avoids overlaps, since given a job j assigned to a period t no other job i $(i \neq j)$ can be scheduled between periods $t - p_i - s_{ij} + 1$ and $t + p_j + s_{ji} - 1$. Constraints (26) computes the completion time of a job j as its starting time plus its processing time. Constraint set (27) imposes the integrality of variables x_j^t. It must be highlighted that for Time-indexed formulations the continuous variables C_j and T_j are unnecessary, once they can be indirectly defined as parameters (C_j^t and T_j^t, respectively). These parameters define the values that the continuous variables can assume for each job j in each period of time t. The C_j^t can be defined as $w_j t$ and T_j^t as $max\{t - d_j, 0\}t$. Therefore, the constraints associated for continuous variables C_j and T_j becomes unnecessary, and, the objective function changes from $\sum_j w_j C_j$ to $\sum_{jt} x_{jt} C_j^t$ and from $\sum_j w_j T_j$ to $\sum_{jt} x_{jt} T_j^t$.

Keha et al. [5] and Unlu & Mason [6] showed that the TI formulation usually provides stronger linear relaxation lower bounds compared to other formulations, but the linear programming problems associated are harder to solve. However, the computational experiments reported by Paula et al. [78] suggested that when sequence-dependent setup times are introduced the linear relaxation bounds provided by TI formulation are not as strong. Because the machine is available immediately after the completion of a job when no setup is involved, the non overlapping constraint in such cases can be verified for each period t taking the sum over all jobs, see Sousa & Wolsey [22]. In the presence of dependent sequence setup times, however, the constraint have to be verified for each period and each pair of jobs separately since the machine may take more or fewer periods to become available depending upon the pair of jobs being processed subsequently. This may explain a weaker linear relaxation of TI formulations in the presence of sequence-dependent setup times. We try to somehow overcome this difficult by introducing a valid inequality to improve the lower bounds provided by TI formulation for the SMSDRD.

In the linear relaxation of TI formulation the time index variables of two distinct jobs $i, j \in J, i \neq j$ with strictly positive values may be feasible for constraints (25), which means both i and j scheduled in the interval between $\max\{r_i, t - p_i - s_{ij} + 1\}$ and $\min\{t + p_j + s_{ji} - 1, h_i - p_i + 1\}$. The following constraints limit to 1 the sum between $\max\{t - p_i - SMin_i + 1, r_i\}$ and $\min\{t, h_i - p_i + 1\}$ of the time indexed variables for all $i \in J$, where $SMin_i$ is the minimum setup time from i for any other job:

$$\sum_{i \in J} \sum_{s=\max\{t-p_i-SMin_i+1, r_i\}}^{\min\{t, h_i-p_i+1\}} x_i^s \leq 1 \qquad \forall t \in H. \tag{28}$$

The motivation is to reduce the number of jobs sequenced simultaneously for a given time interval. Constraints (28) are valid for TI formulation, as $SMin_i$ does not depend on the sequence. We refer to TI formulation plus constraint (28) as time-indexed improvement (TII) formulation.

2.4 Arc-time-indexed formulation

The arc-time-indexed (ATI) formulation proposed by Pessoa et al. [25] consists in an extended network-flow based formulation assigning jobs to time periods while considering precedence relations. As in the TI formulation, the planning horizon is divided into a set of time periods $H = \{0, \ldots, h\}$. Given two jobs i and j, $i \neq j$, x_{ij}^t assumes 1 if, at time t, job i and the setup to job j has been completed and job j starts, and 0 otherwise. We remark that in the formulation proposed by Pessoa et al. [25] x_{jj}^t is not defined. Indeed, the authors showed by an example that such variables would weaken the formulation. A fictitious job 0 is created and variables x_{i0}^t and $x_{0j}^{t+\delta}$ take into account δ periods of idle time between jobs i and j. In the presence of sequence setup times, however, we cannot use this approach since we would lose the sequence information to carry setup times. Thus, given jobs i and j to be processed subsequently, a variable x_{ii}^t assumes 1 for each period the machine is idle, if any, before starts job j. The set J' is defined as $J \cup \{0\}$, and the fictitious job 0 with $p_0 = 0$ and $s_{0j} = s_{j0} = 0$, $j \in J$, starts and ends the sequence. Our formulation uses the parameter s'_{ij} which is $p_i + s_{ij}$ if $i \neq j$, and 1 if $i = j$. The constraint sets (29)-(32) composes the ATI formulation:

$$\sum_{\substack{i \in J' \\ i \neq j}} \sum_{t=\max\{r_i+s'_{ij}, r_j\}}^{h_j-p_j+1} x_{ij}^t = 1 \qquad \forall j \in J', \tag{29}$$

$$\sum_{\substack{j \in J' \\ t \geq r_j + s'_{ji}}} x_{ji}^t - \sum_{\substack{j \in J' \\ r_j \leq t+s'_{ij} \leq h_j-p_j+1}} x_{ij}^{t+s'_{ij}} = 0 \; \forall i \in J, t \in \{r_i, \ldots, h_i - p_i + 1\}, \tag{30}$$

$$C_j \geq \sum_{\substack{i \in J' \\ i \neq j}} \sum_{t=\max\{r_i+s'_{ij}, r_j\}}^{h_j-p_j+1} (t x_{ij}^t) + p_j \qquad \forall\, j \in J, \tag{31}$$

$$x_{ij}^t \in \{0, 1\} \qquad \forall\, i, j \in J' \text{ with } i \neq 0 \text{ or } j \neq 0, \quad t \in \{\max\{r_i + s'_{ij}, r_j\}, \ldots, h_j - p_j + 1\}. \tag{32}$$

Constraint set (29) ensures that every job is processed. Constraint set (30) is the flow conservation constraint establishing the sequence and avoiding overlaps. Idle times and setup times are taken into account in (30) with the use of parameters s'_{ij}. Constraints (31) compute the completion time of a job j as its starting time plus its processing time. Constraints (32) impose the integrality of variables x_{ij}^t.

We illustrate the use of variables x_{ii}^t with an example with two jobs, $J = \{1, 2\}$, and the following data:

$$p_j = \begin{bmatrix} 2 & 1 \end{bmatrix} \qquad s_{ij} = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \qquad s'_{ij} = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \qquad r_j = \begin{bmatrix} 0 & 6 \end{bmatrix}$$

In this example, independent of the objective function (1) or (2), an optimal solution is obtained by starting job 1 at time 0 and job 2 at time 6. From constraint (30), with $i = 1$ and $x_{1,2}^6 = 1$, $t + s'_{1,2} = 6$, and we have that $\sum_{\substack{j \in J' \\ 2 \geq r_j + s'_{j1}}} x_{j1}^2 = 1$. Since $x_{0,1}^2 = 1$ would delay the completion time of job 1, we have that $x_{1,1}^2 = 1$. Analogously, with $i = 1$ and $x_{1,1}^2 = 1$, $t + s'_{1,1} = 2$, and we have $x_{1,1}^1 = 1$. Finally, with $i = 1$ and $x_{1,1}^1 = 1$, $t + s'_{1,1} = 1$, and since job 0 is the one that satisfies $\sum_{\substack{j \in J' \\ 0 \geq r_j + s'_{j1}}} x_{j1}^0 = 1$, we have $x_{0,1}^0 = 1$. Note that variables $x_{1,1}^1 = 1$ and $x_{1,1}^2 = 1$ account for the two periods the machine is idle between processing jobs 1 and 2. But this does not mean that the machine is idle exactly in periods 1 and 2. In fact, variable $x_{0,1}^0 = 1$ indicates that job 1 starts at time 0, and its completion time after two periods is correctly computed due to constraint (31). Thus, a variable $x_{ii}^t = 1$ indicates the machine is idle during a period before processing the next job, but not necessarily during period t itself.

3 DOMINANCE HIERARCHY

In Öncan et al. [8] is defined that the efficiency of the enumeration depends on the linear relaxation of a given formulation. Furthermore, the author state that for minimization problems the larger relaxation values are better. The strengths of LP relaxations, or equivalently the strengths of two formulations, can also be compared by using polyhedral information. The authors define that one formulation is a better formulation than other since the lower bound obtained by solving its LP relaxation is at least equal to the one obtained by solving the LP relaxation of other. Briefly, dominance is defined in terms of the strength of their linear relaxations, therefore a given mathematical formulation dominates another if its solution space is contained within the other one.

We develop dominance relationships between some formulations presented in Section 2. To help in this analysis we define an instance with three jobs, $J = \{1, 2, 3\}$, where the objective is to minimize the total weighted completion time with the following data:

$$p_j = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix} \quad s_{ij} = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 4 \\ 3 & 5 & 0 \end{bmatrix} \quad r_j = \begin{bmatrix} 2 & 3 & 4 \end{bmatrix} \quad w_j = \begin{bmatrix} 10 & 30 & 50 \end{bmatrix}$$

An optimal sequence is given by processing first job 3, followed by job 1, and at last job 2. These proofs apply for the total weighted tardiness or any other regular objective as well. Furthermore, these propositions are based on methodology used by Öncan et al. [8] and Pessoa et al. [25], which converts one formulation for the space of the variables of other to aim compares them.

Proposition 3. *The ATI formulation dominates the TII formulation.*

Proof. Any solution $\overline{x_{ij}^t}$ of the linear relaxation of ATI formulation with cost z can be converted into a solution $\overline{x_j^t}$ of the linear relaxation of TII formulation with the same cost by setting

$$\overline{x_j^t} = \sum_{i \in J', i \neq j} \overline{x_{ij}^t}, \forall j \in J, t \in \{r_j, \ldots, h_j - p_j + 1\}.$$

As $\overline{x_{ij}^t}$ satisfies constraints (29), $\overline{x_j^t}$ satisfy (24). Likewise, the scheduling constraints (30) on $\overline{x_{ij}^t}$ imply constraints (25) and (28) on $\overline{x_j^t}$. Thus, all feasible solutions for the linear relaxation of ATI can be converted to a feasible solution of TII with the same objective function value.

On the other hand, the value of the linear relaxation bound provided by TII for the proposed instance is lower than the one provided by ATI. The solution of the linear relaxation of TII, with cost $1,210.9$, is a combination of several pseudo-schedules: x_1^2, x_1^9, x_1^{12}, x_1^{14} for job 1, x_2^3, x_2^5, x_2^9, x_2^{10}, x_2^{16} for job 2, and x_3^4, x_3^5, x_3^{11}, x_3^{13}, x_3^{18} for job 3. Therefore, the linear relaxation of TI allows the schedule of jobs and idle times when jobs may repeat. This occurs as TII allows for any pair of distinct jobs $i, j \in J, i \neq j$, that the sum of jobs scheduled simultaneously in the time interval between $\max\{r_i, t - p_i - s_{ij} + 1\}$ and $\min\{t + p_j + s_{ji} - 1, h_i - p_i + 1\}$ may be larger than 1. Constraints (28) reduce this time interval but does not eliminate the effect. For the ATI, however, the sum of jobs scheduled simultaneously in this time interval is at most 1 (see constraint (30)). The optimal solution of the ATI relaxation is integral for this instance. □

Proposition 4. *The TII formulation dominates the TI formulation.*

Proof. The constraint set (28) is valid and restrict the time interval in which the sum of the scheduled jobs may be larger than 1; therefore the TII formulation has a smaller solution space, and, consequently dominates TI. □

Proposition 5. *The TI formulation dominates the CTP formulation.*

Proof. Given a feasible solution $\overline{x_j^t}$ for the linear relaxation of TI with cost z, let $\overline{C_j} = \sum_{t \in r_j,\ldots,h_j-p_j+1} \overline{x_j^t}(t+p_j), \forall j \in J$. As $\overline{x_j^t}$ satisfy the assignment constraints (24), $\overline{C_j}$ satisfy the completion time constraints (5). Likewise, the scheduling constraints (25) and (28) on $\overline{x_j^t}$ imply constraints (3) and (4) on $\overline{C_j}$.

The linear relaxation bound provided by TI formulation is better than the one provided by CTP formulation for the proposed instance. The solution of the linear relaxation of CTP, with cost 750, is composed by the jobs starting their processing at the release dates, i.e., $C_1 = 4$, $C_2 = 7$ and $C_3 = 10$ ($C_j = r_j + p_j$). This fact occurs due to the relaxation of the precedence relation variables, γ_{ij} (see (6)), allowing that the constant M disable the schedule constraints (3) and (4). The solution of the linear relaxation of TI has an objective value of 1,060. □

Proposition 6. *The CTP formulation dominates the APD formulation.*

Proof. Any solution $\overline{C_j}$ of the linear relaxation of CTP with cost z can be converted into a solution of the linear relaxation of APD with the same cost. As $\overline{C_j}$ satisfy the completion time constraints (5), it also satisfies the assignment constraints (12) and (13), the completion time constraints (14) and (16). Likewise, the scheduling constraints (3) and (4) on $\overline{C_j}$ imply also constraints (20) and (15).

The solution of the linear relaxation of the APD for the proposed instance, with cost 0, is composed by the jobs finishing their processing at time 0, i.e., $C_1 = 0$, $C_2 = 0$ and $C_3 = 0$. This fact occurs due to the relaxation of the assignment position variables, v_{jk}, (see (17)). The relaxation allows the constant M to disable the completion time constraint set (16), removing any association between $\overline{C_j}$ and $r_j + p_j$. The relation is maintained only for $\overline{C'_k}$ (see 14). In CTP the constraint (5) takes into account this relationship. Therefore, in the linear relaxation of CTP, variables $\overline{C_j}$ respect the completion time conditions ($r_j + p_j$), obtaining a solution with cost 750. □

Proposition 7. *The AFCTP formulation dominates the CTP formulation.*

Proof. Any solution $\overline{C_{ji}}$ of the linear relaxation of AFCTP formulation with cost z can be converted into a $\overline{C_j}$ solution of the linear relaxation of CTP formulation with the same cost by setting $\overline{C_j} = \sum_{i \in J', i \neq j} \overline{C_{ji}}, \forall j \in J$. As $\overline{C_j}$ satisfy the completion time constraints (5), also satisfy constraints (10). Likewise, the scheduling constraints (3) and (4) on $\overline{C_j}$ also imply constraints (7), (8) and (9).

In the solution of the linear relaxation of the CTP formulation for the proposed instance, the jobs start their processing at their release dates, i.e., $C_1 = 4$, $C_2 = 7$ and $C_3 = 10$ ($C_j = r_j + p_j$), and the cost is 750. This solution is obtained due to the relaxation of the precedence relationships variables, γ_{ij} (see (6)), disabling the schedule constraints (3) and (4). However, in the AFCTP formulation the schedule constraint cuts-off such a solution, and an optimal solution is given by $C_1 = 9.74$, $C_2 = 7$ and $C_3 = 10$ with cost 807.4. □

Proposition 8. *The Time-Indexed and Arc-Flow Completion Time and Precedence formulations are incomparable.*

Proof. Consider, for example, the instances of class 4 with 5 jobs in Table A.4 in section Additional Tables (A). For these instances, the percentage gap from the optimum of the linear relaxation bounds corresponding to Arc-Flow Completion Time and Precedence, and Time-Indexed formulation is slightly lower for the first. Though for 7 jobs, the Time-Indexed formulation presents lower gap. For more details see the supplementary material in the section Additional Tables (A). □

Figure 1 summarizes the dominance relationships. In the Figure, we include an empirical result, the dominance of TI over AFCTP formulation. Although, we were not able to prove the propositional dominance, an extensive computational analysis supports the hypothesis.

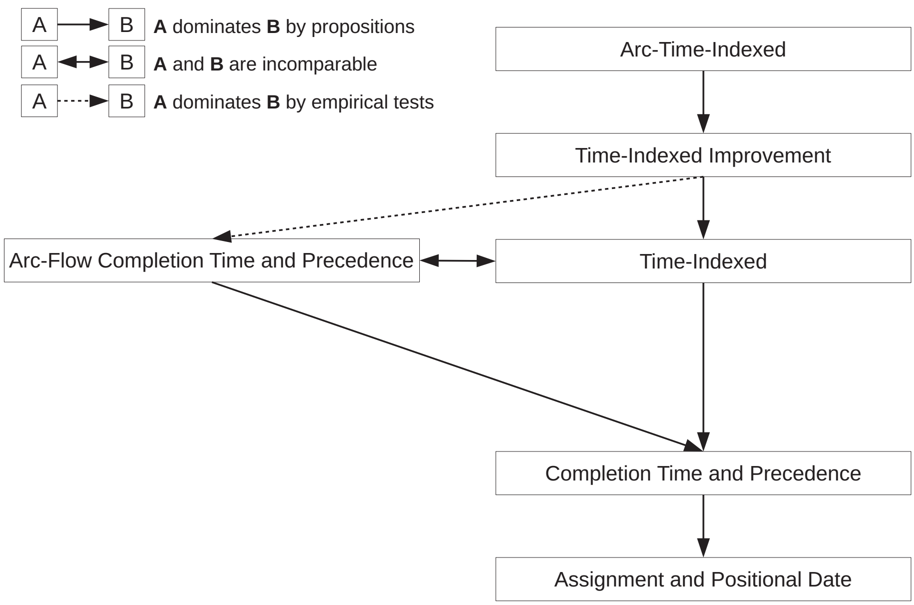

Figure 1 – Dominance relationships between SMSP formulations

4 SIZE OF FORMULATIONS

The sizes of the formulations of CTP, AFCTP and APD proposed in this article have a polynomial number of constraints and variables in the number of jobs. However, this is not the case for Time-indexed MIP based formulations, as they also are strongly dependent on h. Table 3 shows the number of constraints and binary variables associated with each paradigm. It is worth noting that as $h \ggg n$, $h \propto n$, TI, TII and ATI formulations will increase their size faster than other formulations.

Table 3 – Model Size for each Formulation Paradigm for Problems $1|r_j, s_{ij}|\sum_j w_j C_j$ and $1|r_j, s_{ij}|\sum_j w_j T_j$. For the formulations, "Variables" indicate the number of associated variables and "Constraints" the number of constraints with each formulation paradigm.

MIP Formulations	**Model Order Size for Both Problems**	
	Variables	**Constraints**
CTP	$O(n^2)$	$O(n^2)$
AFCTP	$O(n^2)$	$O(n^2)$
APD	$O(n^3)$	$O(n^3)$
TI	$O(nh)$	$O(n^2h)$
TII	$O(nh)$	$O(n^2h)$
ATI	$O(n^2h)$	$O(nh)$

5 COMPUTATIONAL EXPERIMENTS

We conduct computational experiments to validate the propositional dominance defined in Section 3 and, furthermore, to capture the strength and weaknesses of each formulation. For this purpose 660 instances divided into 6 classes are defined. The instances were randomly generated using uniform distribution as shown in Table 4.

5.1 Benchmark

Six different classes of instances are artificially created. All instances' parameters are randomly generated from a uniform distribution, and their minimal and maximal values are based on specific scale parameters. A similar methodology can be found in [6, 18, 44, 87–90] and [91]. The instance classes and its scale parameters are listed in Table 4.

Table 4 – Distribution values of the instances.

Input data	***Distribution value***
Processing Time (p_j)	$U(1, \alpha_1 50)$
Setup time (s_{ij})	$U(1, \alpha_2 10)$
Priority (w_j)	$U(1, n)$
Release date (r_j)	$U(0, \frac{\alpha_3 h'}{10})$
Due date (d_j)	$U(\max_j\{p_j\}, \frac{2h'}{\alpha_4})$

The h' was defined as the sum of processing times plus the sum of maximum setup times $(\sum_j p_j + \sum_i \max_j\{s_{ij}\})$. The scale parameters $\alpha_1, \alpha_2, \alpha_3$ and α_4 define the distribution scenario of "Processing Time", "Setup time", "Release date " and "Due date" respectively. $\alpha_1 \in \{1, 4\}$ modifies the process time extent, $\alpha_2 \in \{1, 5\}$ defines the setup time impact, $\alpha_3 \in \{1, 5\}$ the availability level and $\alpha_4 \in \{1, 4\}$ the congestion level.

In each class (1 to 6) there is a change in one scale parameter. The created classes are namely:

Class 1: all scale parameters have minimum values;

Class 2: α_1 has the maximum value (4) and other scale parameters have minimum values;

Class 3: α_2 has the maximum value (5) and other scale parameters have minimum values;

Class 4: α_3 has the maximum value (5) and other scale parameters have minimum values;

Class 5: α_4 has the maximum value (4) and other scale parameters have minimum values;

Class 6: all scale parameters have maximum values.

Each class presents special characteristics. The "Class 1" is our base scheduling system. "Class 2" considers a long planning horizon and the system is slightly affected by setup times. This class is closer to single machine scheduling problems without setup times ($p_j \ggg s_{ij}$) - $1|r_j|\sum_j w_jC_j$ and $1|r_j|\sum_j w_jT_j$. The "Class 3" considers a moderate planning horizon with setup times having a great impact in the scheduling system. This class is closest to the traveling salesman problem - $1|s_{ij}|\sum_j w_jC_j$ and $1|s_{ij}|\sum_j w_jT_j$. "Class 4" presents a moderate planning horizon with longer release dates. The "Class 5" defines a scheduling system with high congestion level, reducing its due date values. "Class 6" determines a scheduling system with emphasized conditions. The last defines a complex scheduling system, presenting long planning horizons, a moderate impact of setup times, an impact on the job's release dates and a considerable congestion level. For the problem $1|r_j, s_{ij}|\sum_j w_jC_j$ the classes **1** and **5** are redundant.

For each class, ten independent instances are considered with size $n \in \{5, 7, 9, 11, 13, 15, 20, 30, 50, 75, 100\}$. Thus, 660 instances are randomly and independently generated. All instances are slightly modified to satisfy the triangle inequality of the setup times ($s_{ij} \leq s_{ik} + p_k + s_{k,j}$, where i, j and $k \in J$ and $i \neq j \neq k$).

5.2 Results

The mathematical formulations are modeled and solved using AMPL and CPLEX 12.1 with default settings. Experiments are run on a Linux Maya with a single 2.4 GHz processor and 4GB memory. The runs are ended after one hour of CPU time.

To compare the performance of the different formulations, we compute the optimality gap after 3600 seconds, the linear programming relaxation GAP, CPU times and its dimensions. Linear programming relaxation gap is defined as the relative difference between the best integer solution found for each instance between all formulations analyzed and the LP (linear programming) relaxation value. The average results of the experiments are presented in Tables 6 and 7.

Table 6 depicts the average GAP results for the two problems considering both problems for each instance class, while Table 7 shows the average results for each size. Table 9 presents 95% confidence interval (CI) for all formulations in all sizes, while Table 8 presents 95% confidence interval (CI) for all formulations in all Classes. Finally, Table 5 presents the average GAP results for the two problems considering both problems for each instance class in small (until 15 jobs) and large sizes (larger than 15 jobs). The GAP is computed for each formulation and instance as the relative difference between the best integer solution found by all formulations and its LP relaxation value. It must be highlighted that in several occasions the Time-Indexed based formulations (TI, TII and ATI formulations) are unable to load the whole problem into the solver. In those cases, the GAP is defined as 100% and its computational time as 3600 seconds. Individual results for each class and each instance size are presented in Section Additional Tables (A).

5.2.1 Linear Programming Relaxation Problems

The analysis of the LP relaxation is presented in Tables 6 and 7. The ATI formulation presents a generally tighter linear relaxation GAP until the 30 jobs when the formulation is unable to solve the problem. Constraints (28) have significant impact strengthening TI formulation, improving GAP results around 40% to 5% (TII), but with the same disadvantages found in ATI.

Time-Indexed based formulations (TI, TII, and ATI) are not able to load the linear programming problems into the solver for most instances greater than 30 jobs. These formulations require column generation based methods to exploit their full potential to provide tight lower bounds in reasonable time, see Pessoa et al. [25] and Van den Akker et al. [84]. On the other hand, CTP, AFCTP, and APD linear programming relaxations are solved quickly for all instance sizes but leading to poor lower bounds (GAPs between 20% and 100% for F1 and between 1% and 100% for F2). Note that APD obtained zero as lower bound in almost all cases. We argue that stand-alone TI formulation is not the best choice in this scenario, as it gives lower bounds comparable to those obtained with CTP and AFCTP in much larger computational times.

When we analyze the effect of the instance classes, time-Indexed based formulations present better GAP results for instances with shortest and moderate planning horizon length (classes **1**, **4** and **5**) and worse results for long planning horizon (classes **2** and **6**), with its Gaps up to five times larger than others. These formulations present the worst results for relaxed problem **F2** in instances with high congestion level and emphasized conditions (classes **5** and **6**). Considering (class **3** (TSP scenario) the TI formulation worsens its GAPs in comparison to results in class **1**). For this class, ATI presents no significant variation.

The CTP, AFCTP and APD formulations have lower computational time values, but generally, producing weak lower bound results. Nevertheless, for the relaxed problem **F2**, the CTP and AFCTP formulations produce strong lower bounds for instances with shortest and moderate planning horizon length (classes **1** to **4**). The AFCTP formulation presents better GAP results in relaxed problem **F1** consuming more computational time than CTP. As the number of jobs increases, the GAP difference is irrelevant. There is no noticeable difference between the results for relaxed problem **F2**. However, the CTP formulation presents lower computational times. The APD formulation presents the worst GAP results in all classes.

Analyzing Tables 6 and 8 for the relaxed problem **F1**, the CTP and AFCTP formulations present better GAP results for instances with moderate and long planning horizon length (classes **4** and **6**) and worse results for base system (classes **1** and **5**), which have Gaps until three times larger than others. When considering the relaxed problem **F2**, they perform worse for high congestion level and emphasized conditions (classes **5** and **6**). In the TSP scenario, class **3**, for problem **F1** the formulations CTP, AFCTP and APD worsens their results.

Summarizing the results, we argue that when the planning horizon length is small or moderated, i.e., the number of jobs is low, or the processing time of the jobs are small, and the jobs are ready to start at the beginning of the planning horizon, we recommend time-indexed based formulations TII and ATI. Otherwise, the CTP and AFCTP formulations are a good alternative for generating lower bounds in small computational times. These formulations present their best results in scenarios with large planning horizon length or for difficult release dates. This analysis considers stand-alone formulations, but, CTP and AFCTP formulations can be incorporated in Relax-and-Fix framework (see [92]). In high congestion level scenario, all formulations have difficulties for solving the problems studied. Furthermore, with exception to ATI, all formulations present increase of its GAPs in the TSP scenario.

It is interesting to notice that for the cases where TII formulation solves the problem **F1**, the improvement over TI is less significant for class 2 in the $1|r_j|\sum_j w_jC_j$ scenario, and more significant for class 3 in the $1|s_{ij}|\sum_j w_jC_j$ scenario (see Table 8 for more details). Furthermore, for small instances in $1|r_j|\sum_j w_jC_j$ scenario (until 15 jobs) the linear relaxation GAP of TI and TII are similar, while for $1|s_{ij}|\sum_j w_jC_j$ scenario the TII GAP is half than the TI GAP. The low performance of TII in $1|r_j|\sum_j w_jC_j$ scenario occurs due to the improvement of its computational time by added constraints without important GAP improvements. In this scenario, the computational time of TII increases significantly more than TI. It is possible to state that constraints (28) are more effective for class **3**, that is with setup times varying in a wider range.

No formulations solve larger instances with emphasized conditions. In the problem **F2** is noticeable that all formulation present large GAPs in small-sized problems with high congestion level.

5.2.2 Mixed Integer Programming Problems

Tables 6 and 7 show how in average all formultations have difficulties as the number of jobs increases. It is possible to notice that the TII and ATI formulations managed to optimality solve some instances, but as the number of variables and constraints increase, the MIP problems become rapidly unmanageable by the commercial solver. Section A presents a detailed description of the results for each instance size in each class. In Tables A.1 to A.6 we can see that the time-indexed based formulations (TI, TII and ATI formulations) can solve instances of up to 20 jobs for both MIP problems (**F1** and **F2**), depending on the class. These formulations present better GAP results for MIP problem **F1** for instance with shortest and moderate planning horizon length (classes **1**, **4** and **5**) and worse performance for long planning horizon (classes **2** and **6**. When considering the MIP problem **F2**, they perform worse for instances with high congestion level and emphasized conditions (classes **5** and **6**). In the TSP scenario (class **3**) for problem **F1** only TI worsens its GAPs significantly when compared with its best scenario. The linear relaxation

Table 5 – Average Relaxation GAP Results for Single Machine Scheduling Problems for Six MIP Formulations for All Classes in Small and Large Sizes. The GAP indicates the average value of the average linear relaxation gap for each classes in Small and Large sizes. The GAP is computed for each formulation and instance as the relative difference between the best integer solution and its LP relaxation value. The Small sizes are the instances with until 15 jobs, while Large sizes with more than 15 jobs. T(s) indicates the average value of the average CPU time. $F1$ and $F2$ denote the objective functions $\sum_j w_j C_j$ and $\sum_j w_j T_j$, respectively.

			Mixed Integer Program Formulations											
Objective Function	Instance Class	Size Problem	CTP		AFCTP		APD		TI		TII		ATI	
			GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)
F1	1	Small	61.3%	0.0	45.8%	1.3	100.0%	0.0	52.7%	140.8	4.7%	711.7	0.0%	25.8
		Large	77.0%	0.2	73.8%	13.7	100.0%	138.4	96.4%	3324.2	82.7%	3450.5	80.0%	2932.7
	2	Small	60.3%	1.1	44.2%	5.3	100.0%	5.4	64.7%	1844.1	66.9%	2742.2	0.0%	125.4
		Large	76.2%	6.8	72.7%	17.2	100.0%	176.3	100.0%	3600.0	100.0%	3600.0	96.0%	3472.7
	3	Small	60.1%	1.5	46.8%	0.0	100.0%	0.4	50.9%	756.9	21.7%	1079.7	0.3%	37.3
		Large	68.2%	6.3	65.9%	8.3	100.0%	292.3	90.8%	3203.4	100.0%	3600.0	84.9%	3179.1
	4	Small	27.0%	0.4	21.8%	0.8	100.0%	0.3	21.7%	240.2	3.7%	638.7	0.0%	11.1
		Large	26.1%	1.0	25.5%	30.7	100.0%	285.2	85.0%	3186.7	81.3%	3466.9	80.0%	2887.3
	5	Small	60.6%	5.8	45.6%	5.4	100.0%	0.5	51.6%	104.0	5.7%	585.2	0.0%	26.8
		Large	76.4%	6.8	73.1%	62.7	100.0%	153.2	96.9%	3330.5	86.4%	3449.0	84.0%	2921.5
	6	Small	25.6%	0.0	20.8%	2.7	100.0%	0.3	39.5%	1794.4	60.7%	2834.4	0.2%	151.8
		Large	23.8%	1.8	23.1%	130.0	100.0%	248.9	100.0%	3600.0	100.0%	3600.0	100.0%	3532.3
F2	1	Small	4.0%	0.0	4.0%	0.0	10.0%	0.0	7.3%	1040.1	7.1%	1312.6	3.7%	68.2
		Large	8.6%	0.1	8.6%	26.8	52.0%	120.6	52.0%	3600.0	52.0%	3600.0	48.2%	3467.6
	2	Small	0.0%	0.0	0.0%	0.2	3.3%	0.1	3.3%	1845.7	3.3%	2324.5	0.0%	621.5
		Large	18.2%	0.1	18.2%	35.7	68.0%	90.4	64.0%	3600.0	64.0%	3600.0	67.4%	3600.0
	3	Small	6.7%	0.0	6.7%	0.0	20.0%	0.1	13.3%	1867.1	20.0%	2003.6	2.1%	210.8
		Large	0.9%	0.1	0.9%	21.6	72.0%	105.6	72.0%	3457.1	72.0%	3600.0	68.0%	3229.5
	4	Small	3.7%	0.0	3.7%	1.2	56.7%	0.3	22.6%	1124.5	17.7%	1274.6	0.9%	142.9
		Large	5.6%	0.1	5.6%	35.9	100.0%	147.7	100.0%	3600.0	100.0%	3600.0	80.8%	3274.4
	5	Small	99.0%	0.0	99.2%	0.0	100.0%	0.2	98.9%	928.7	75.8%	1467.7	48.0%	178.6
		Large	97.6%	9.7	97.8%	129.2	100.0%	187.1	100.0%	3600.0	100.0%	3600.0	88.2%	3464.0
	6	Small	59.7%	0.0	59.5%	0.8	100.0%	0.9	86.2%	2340.3	82.4%	2789.7	18.3%	782.8
		Large	41.1%	12.0	41.1%	53.9	100.0%	194.8	100.0%	3600.0	100.0%	3600.0	92.6%	3600.0

of these formulations are not strong enough to avoid a significant number of branching, and to solve the linear relaxation in each node of the branch-and-bound tree is very time-consuming.

As mentioned before, even presenting weak lower bound values, the linear relaxation of CTP and AFCTP are solved much faster, around a few seconds. These formulations solve more instances, especially in large sizes, and gets better gaps than TI or TII even though generating a much larger number of nodes in the branch-and-bound tree (see Table 3), especially the CTP formulation. CTP and AFCTP formulations can solve instances of up to 100 jobs, depending on the MIP problem and the class. For the MIP problem **F1**, formulations based on completion time variables (CTP, AFCTP formulations) can solve instances of up to 50 jobs. For the MIP problem **F2** the number increases to 100 jobs for completion time-based formulations. Analyzing the results for the MIP problem, in **F1** the CTP and AFCTP formulations present better GAP results for instance classes with moderate and long planning horizon length (classes **4** and **6**) and worse results for base system instances (classes **1** and **5**). When the **F2** problem, they perform worse for classes with high congestion level and emphasized conditions (classes **5** and **6**), specially when we consider high congestion level scenario. It seems that for using a pure solver to minimize the completion time CTP is the best alternative when ATI generates large linear programs, as it obtains better gaps than the other formulations.

The size of the M constant impacts directly in the bounds quality. In the analysis of the Tables 6 and 7, it is noticeable that the mathematical formulations without constant M (time-indexed based formulations) present tighter bounds. However, the Time-Indexed based formulations can solve the smallest number of instances. In the AFCTP formulation, the value of the constant M is smaller than the CTP formulation value. AFCTP formulation presents lower bounds stronger or equal in all analyzed instances compared to CTP formulation. This difference is more apparent in the problem **F1** for small instances. For the problem **F2** these formulations present the same LP relaxation results. As the number of jobs increases, the GAP and the computational time increase faster for AFCTP. Therefore, the AFCTP formulation solves a smaller number of instances for LP and MIP than CTP.

Summarizing the MIP formulations results, we highlight that some formulations are affected by the differences between classes. When the linear relaxation problem can be solved efficiently by time-indexed based formulations, it is possible to solve the MIP problem. Therefore, such as in linear problem relaxation analysis, the larger instances with a long planning horizon are more difficult for these formulations. Furthermore, the TI formulation is the most influenced by setup times presence. The MIP problems generated with time-indexed based formulation are bigger for classes with a long planning horizon since processing times tend to be longer. In general, all formulations perform better in class **4** since release dates are spread over time. Considering the point of view of an optimization package user, we argue that the CTP formulation is the best choice to tackle total weighted tardiness, problem **F2**. In this problem, the CTP also obtained better results for classes with moderate planning horizon (classes **3** and **4**), solving all instances. Minimizing total weighted tardiness when we have early due dates, high congestion level, is a challenging problem.

Table 6 – Average GAP Results for Single Machine Scheduling Problems for Six MIP Formulations for All Classes in All Sizes. For the LP (linear programming) relaxation problem, the GAP indicates the average value of the average linear relaxation gap for all classes in all sizes, computed for each formulation and instance as the relative difference between the best integer solution and its LP relaxation value. For the MIP (mixed integer programming) problem, the GAP is the average value of the average optimality gap for all classes in all sizes. T(s) indicates the average value of the average CPU time for all classes in all sizes, and **SD** is the Standard Deviation for each metric. $F1$ and $F2$ denote the objective functions $\sum_j w_j C_j$ and $\sum_j w_j T_j$, respectively.

	Instance Classes	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD
LP RELAXATION PROBLEM	1	F1	68.5%	10.4%	0.1	0.2	58.5%	17.7%	6.9	15.3	100.0%	0.0%	62.9	149.5	72.5%	24.3%	1587.8	1713.5	40.1%	47.5%	1956.6	1565.3	36.4%	50.4%	1347.1	1787.7
	2	F1	67.5%	10.5%	3.7	8.1	57.2%	18.1%	10.7	15.2	100.0%	0.0%	83.1	193.2	80.8%	24.6%	2642.2	1471.0	81.9%	39.7%	3132.1	1038.5	43.7%	50.4%	1646.9	1761.6
	3	F1	63.8%	5.7%	3.7	6.7	55.5%	12.4%	3.8	8.0	100.0%	0.0%	133.1	326.2	69.0%	23.8%	1869.0	1414.3	57.3%	42.6%	2225.3	1503.1	38.7%	49.1%	1465.4	1745.8
	4	F1	26.6%	4.6%	0.6	0.9	23.5%	4.8%	14.4	30.0	100.0%	0.0%	129.8	353.2	50.4%	39.3%	1579.5	1658.8	39.0%	48.4%	1924.2	1580.5	36.4%	50.4%	1318.5	1759.9
	5	F1	67.8%	9.7%	6.3	9.4	58.1%	17.0%	31.4	65.1	100.0%	0.0%	69.9	163.1	72.2%	24.6%	1570.6	1728.9	42.4%	46.3%	1886.9	1618.6	38.2%	49.4%	1342.5	1754.3
	6	F1	24.8%	3.1%	0.8	1.9	21.9%	3.4%	60.6	87.0	100.0%	0.0%	113.3	279.9	67.0%	38.3%	2615.1	1349.1	78.5%	37.7%	3182.4	962.6	45.5%	52.1%	1688.4	1774.9
	F1 Average		**53.2%**	**7.4%**	**2.5**	**4.5**	**45.8%**	**12.2%**	**21.3**	**36.8**	**100.0%**	**0.0%**	**98.7**	**244.2**	**68.7%**	**29.1%**	**1977.4**	**1555.9**	**56.6%**	**43.7%**	**2384.6**	**1378.1**	**39.8%**	**50.3%**	**1468.1**	**1764.0**
	Standard Deviation		**21.3%**	**3.3%**	**2.4**	**4.0**	**17.9%**	**6.6%**	**21.5**	**32.0**	**0.0%**	**0.0%**	**30.7**	**87.2**	**10.1%**	**7.5%**	**516.9**	**164.5**	**19.5%**	**4.4%**	**610.5**	**295.8**	**3.9%**	**1.1%**	**163.3**	**15.0**
	1	F2	6.1%	8.6%	0.0	0.1	6.1%	8.6%	12.2	28.1	29.1%	32.7%	54.8	135.5	27.6%	33.5%	2203.7	1602.5	27.5%	33.6%	2352.4	1581.0	23.9%	34.3%	1613.3	1787.2
	2	F2	8.3%	12.2%	0.1	0.1	8.3%	12.2%	16.3	36.7	32.7%	39.3%	41.1	103.4	30.9%	38.3%	2643.1	1333.3	30.9%	38.3%	2904.3	1297.0	30.7%	40.2%	1975.4	1666.3
	3	F2	4.1%	7.9%	0.0	0.1	4.1%	7.9%	9.8	27.1	43.6%	35.6%	48.1	133.1	40.0%	38.0%	2589.9	1472.5	43.6%	35.6%	2729.2	1372.7	32.1%	42.4%	1583.0	1673.8
	4	F2	4.6%	4.4%	0.1	0.1	4.6%	4.4%	17.0	33.1	76.4%	32.0%	67.3	139.7	57.8%	45.4%	2249.7	1534.9	55.1%	46.7%	2331.6	1512.5	37.2%	49.8%	1566.3	1705.9
	5	F2	98.3%	1.2%	4.4	12.6	98.6%	1.3%	58.7	178.3	100.0%	0.0%	85.2	212.1	99.4%	1.4%	2142.9	1680.4	86.8%	15.6%	2436.9	1530.7	66.3%	29.5%	1672.0	1735.0
	6	F2	51.3%	12.9%	5.5	13.5	51.2%	12.6%	24.9	65.7	100.0%	0.0%	89.0	206.4	92.5%	12.5%	2912.9	1239.0	90.4%	21.6%	3158.0	997.4	52.1%	40.3%	2063.3	1647.4
	F2 Average		**28.8%**	**7.8%**	**1.7**	**4.4**	**28.8%**	**7.8%**	**23.2**	**61.5**	**63.6%**	**23.3%**	**64.3**	**155.1**	**58.0%**	**28.2%**	**2457.0**	**1477.1**	**55.7%**	**31.9%**	**2652.1**	**1381.9**	**40.4%**	**39.4%**	**1745.5**	**1702.6**
	Standard Deviation		**38.7%**	**4.5%**	**2.5**	**6.7**	**38.7%**	**4.4%**	**18.2**	**59.0**	**32.7%**	**18.2%**	**19.7**	**43.9**	**31.2%**	**17.3%**	**305.2**	**166.0**	**27.3%**	**11.4%**	**336.0**	**216.3**	**15.8%**	**7.0%**	**216.9**	**51.8**
	LP Relaxation Average		***41.0%***	***7.6%***	***2.1***	***4.5***	***37.3%***	***10.0%***	***22.2***	***49.1***	***81.8%***	***11.6%***	***81.5***	***199.6***	***63.3%***	***28.7%***	***2217.2***	***1516.5***	***56.1%***	***37.8%***	***2518.3***	***1380.0***	***40.1%***	***44.9%***	***1606.8***	***1733.3***
	Standard Deviation		***32.4%***	***3.7%***	***2.4***	***5.3***	***30.1%***	***5.8%***	***19.0***	***47.0***	***29.1%***	***17.3%***	***30.5***	***80.6***	***22.8%***	***12.7%***	***476.0***	***162.9***	***22.6%***	***10.3%***	***490.1***	***247.1***	***11.0%***	***7.4%***	***233.4***	***48.5***
MIP PROBLEM	1	F1	33.3%	35.6%	2070.6	1787.3	40.6%	34.2%	2532.0	1639.6	50.0%	44.2%	2357.4	1734.8	42.1%	47.9%	2281.0	1651.1	38.6%	48.9%	2470.9	1625.7	36.4%	50.5%	1474.6	1714.6
	2	F1	32.0%	35.0%	2049.5	1800.0	40.1%	34.4%	2588.5	1657.6	49.4%	46.6%	2337.5	1736.1	73.5%	37.4%	3396.8	673.8	50.2%	48.6%	2774.9	1266.0	45.5%	52.2%	1881.9	1693.4
	3	F1	28.0%	32.4%	1888.7	1800.4	35.1%	30.9%	2524.3	1644.7	48.9%	43.1%	2414.6	1682.4	53.7%	42.5%	2896.0	1317.6	48.4%	44.2%	2899.1	1331.7	38.3%	49.3%	1664.7	1745.9
	4	F1	8.8%	12.4%	1583.1	1813.2	12.2%	13.9%	2083.0	1764.9	41.4%	45.5%	2218.6	1747.0	38.6%	49.0%	2039.4	1653.7	36.9%	50.1%	2087.0	1652.0	36.4%	50.5%	1435.2	1748.1
	5	F1	32.0%	35.6%	1978.3	1796.8	38.8%	34.6%	2497.0	1641.0	50.1%	45.1%	2350.3	1741.9	46.3%	48.2%	2272.7	1574.4	39.0%	48.7%	2376.4	1579.1	38.2%	49.4%	1428.7	1745.0
	6	F1	8.00%	10.80%	1600.7	1808.3	11.16%	12.35%	2098.9	1770.3	42.96%	46.82%	2194.0	1723.9	62.78%	44.06%	3374.7	585.0	70.21%	43.61%	3261.1	960.9	45.56%	52.12%	1951.5	1662.5
	F1 Average		**23.7%**	**27.0%**	**1861.8**	**1801.0**	**29.7%**	**26.7%**	**2387.3**	**1686.4**	**47.1%**	**45.2%**	**2312.1**	**1727.7**	**52.8%**	**44.9%**	**2710.1**	**1242.6**	**47.2%**	**47.3%**	**2644.9**	**1402.6**	**40.0%**	**50.7%**	**1639.4**	**1718.2**
	Standard Deviation		**12.0%**	**12.0%**	**218.6**	**9.0**	**14.1%**	**10.6%**	**231.5**	**63.3**	**3.9%**	**1.4%**	**86.4**	**23.5**	**13.4%**	**4.5%**	**595.5**	**491.4**	**12.6%**	**2.7%**	**418.3**	**269.0**	**4.3%**	**1.3%**	**232.4**	**35.0**
	1	F2	0.4%	1.3%	188.3	350.9	24.7%	39.0%	1105.7	1547.4	29.2%	45.9%	1208.5	1612.8	43.6%	50.5%	1713.6	1709.3	49.1%	50.1%	1868.5	1755.3	49.1%	50.1%	1935.1	1761.7
	2	F2	3.1%	7.3%	648.1	1280.0	22.3%	38.4%	1077.9	1527.5	28.8%	42.8%	1160.7	1548.8	58.2%	47.7%	2434.2	1413.9	65.5%	45.7%	2541.3	1514.1	60.0%	49.0%	2513.5	1506.4
	3	F2	0.0%	0.0%	52.9	117.8	14.9%	31.3%	904.1	1399.1	25.5%	43.7%	1107.0	1592.1	47.3%	47.6%	1874.5	1644.5	51.6%	48.0%	2042.7	1657.2	54.5%	47.4%	2272.2	1645.0
	4	F2	0.0%	0.0%	172.1	378.5	16.0%	26.7%	1804.4	1701.7	34.4%	47.6%	1461.8	1756.1	42.1%	49.5%	1677.7	1748.5	40.0%	49.0%	1740.6	1708.7	49.1%	50.1%	1896.6	1735.7
	5	F2	42.1%	46.5%	1838.5	1758.3	49.8%	45.8%	2275.7	1723.7	50.7%	49.6%	2177.0	1745.3	62.9%	45.5%	2791.9	1410.9	65.5%	43.3%	2948.0	1253.6	55.4%	44.1%	2780.3	1300.9
	6	F2	13.53%	17.75%	1546.2	1795.6	26.09%	28.78%	2042.6	1709.5	43.89%	47.80%	2052.6	1735.8	84.35%	26.56%	3482.8	388.7	82.73%	32.13%	3371.8	572.2	59.05%	45.00%	2797.6	1292.5
	F2 Average		**9.9%**	**12.1%**	**741.0**	**946.8**	**25.6%**	**35.0%**	**1535.1**	**1601.5**	**35.4%**	**46.2%**	**1527.9**	**1665.2**	**56.4%**	**44.5%**	**2329.1**	**1386.0**	**59.1%**	**44.7%**	**2418.8**	**1410.2**	**54.5%**	**47.6%**	**2365.9**	**1540.4**
	Standard Deviation		**16.6%**	**18.1%**	**769.9**	**755.7**	**12.7%**	**7.3%**	**578.0**	**131.2**	**9.9%**	**2.6%**	**472.2**	**90.9**	**16.0%**	**9.0%**	**716.4**	**509.7**	**15.2%**	**6.6%**	**649.2**	**448.7**	**4.7%**	**2.6%**	**398.6**	**208.8**
	MIP Average		***16.8%***	***19.6%***	***1301.4***	***1373.9***	***27.6%***	***30.9%***	***1961.2***	***1643.9***	***41.3%***	***45.7%***	***1920.0***	***1696.4***	***54.6%***	***44.7%***	***2519.6***	***1314.3***	***53.1%***	***46.0%***	***2531.9***	***1406.4***	***47.3%***	***49.1%***	***2002.7***	***1629.3***
	Standard Deviation		***15.6%***	***16.6%***	***796.1***	***677.2***	***12.9%***	***9.7%***	***611.8***	***107.7***	***9.4%***	***2.1%***	***522.0***	***71.2***	***14.2%***	***6.8%***	***658.8***	***483.2***	***14.7%***	***5.0%***	***533.9***	***352.8***	***8.7%***	***2.5%***	***490.6***	***170.3***

Table 7 – Average GAP Results for Single Machine Scheduling Problems for Six MIP Formulations for All Sizes in All Classes. For the LP (linear programming) relaxation problem, the GAP indicates the average value of the average linear relaxation gap for all sizes in all classes, computed for each formulation and instance as the relative difference between the best integer solution and its LP relaxation value. For the MIP (mixed integer programming) problem, the GAP is the average value of the average optimality gap for all sizes in all classes. T(s) indicates the average value of the average CPU time for all sizes in all classes, and **SD** is the Standard Deviation for each metric. $F1$ and $F2$ denote the objective functions $\sum_j w_j C_j$ and $\sum_j w_j T_j$, respectively.

	Instance Sizes	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD
LP RELAXATION PROBLEM	5	F1	42.1%	13.9%	0.6	1.0	24.7%	6.6%	0.9	1.1	100.0%	0.0%	0.2	0.3	33.0%	9.7%	137.5	302.4	3.5%	3.1%	101.7	168.1	0.0%	0.1%	1.6	3.6
	7	F1	49.0%	14.2%	1.0	1.9	34.4%	8.6%	0.0	0.0	100.0%	0.0%	0.0	0.0	40.5%	13.5%	236.2	350.3	5.9%	3.9%	954.8	1410.1	0.0%	0.0%	2.3	2.8
	9	F1	49.6%	18.1%	0.2	0.5	37.0%	12.6%	0.0	0.0	100.0%	0.0%	0.0	0.0	43.9%	13.7%	873.1	1242.7	31.1%	38.8%	1280.4	1593.1	0.1%	0.1%	13.4	18.1
	11	F1	50.2%	18.3%	0.0	0.1	40.5%	13.8%	1.3	1.9	100.0%	0.0%	0.5	0.8	44.1%	17.4%	757.9	1095.4	36.9%	46.2%	1515.2	1359.2	0.1%	0.1%	35.4	32.9
	13	F1	51.0%	21.0%	1.9	2.1	42.8%	17.2%	2.0	1.7	100.0%	0.0%	4.0	8.9	52.3%	22.2%	1149.8	1282.6	40.1%	47.0%	2135.5	1185.8	0.1%	0.1%	96.8	85.6
	15	F1	52.9%	21.6%	4.9	10.9	45.7%	17.8%	11.4	13.2	100.0%	0.0%	2.1	2.7	67.2%	29.2%	1725.9	1539.0	46.0%	45.7%	2604.2	789.4	0.2%	0.3%	228.5	222.8
	20	F1	52.8%	22.8%	5.0	8.9	47.2%	19.8%	20.1	40.7	100.0%	0.0%	1.8	1.8	75.5%	28.6%	2575.4	839.5	58.7%	46.0%	3238.7	397.1	37.4%	42.4%	1449.4	1369.9
	30	F1	54.7%	25.3%	5.1	11.1	51.4%	23.4%	22.1	49.0	100.0%	0.0%	7.2	5.9	98.7%	3.1%	3495.3	256.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3522.0	122.4
	50	F1	55.9%	28.6%	4.6	8.9	54.5%	27.8%	6.5	4.5	100.0%	0.0%	40.6	16.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	61.7%	26.2%	2.2	2.5	61.0%	25.9%	79.4	91.7	100.0%	0.0%	223.9	43.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	64.7%	26.1%	2.1	1.8	64.3%	25.8%	90.8	78.4	100.0%	0.0%	804.9	297.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**53.2%**	**21.5%**	**2.5**	**4.5**	**45.8%**	**18.1%**	**21.3**	**25.7**	**100.0%**	**0.0%**	**98.7**	**34.3**	**68.7%**	**12.5%**	**1977.4**	**628.0**	**56.6%**	**21.0%**	**2384.6**	**627.5**	**39.8%**	**3.9%**	**1468.1**	**168.9**
	Standard Deviation		**6.2%**	**4.9%**	**2.0**	**4.4**	**11.7%**	**7.2%**	**32.6**	**34.0**	**0.0%**	**0.0%**	**243.5**	**88.1**	**27.3%**	**11.1%**	**1429.4**	**583.4**	**37.9%**	**22.9%**	**1264.1**	**652.5**	**49.0%**	**12.8%**	**1724.5**	**404.5**
	5	F2	30.1%	46.1%	0.0	0.0	30.1%	46.2%	0.4	0.5	40.0%	49.0%	0.2	0.2	29.3%	44.9%	26.4	55.8	21.3%	32.9%	143.9	320.0	16.2%	30.1%	0.9	1.0
	7	F2	29.5%	41.1%	0.0	0.0	29.5%	41.1%	0.1	0.1	36.7%	49.7%	0.1	0.0	30.5%	41.9%	297.4	388.0	21.5%	28.1%	444.8	610.9	16.2%	21.8%	14.4	22.4
	9	F2	26.9%	42.0%	0.0	0.0	26.9%	42.0%	0.3	0.6	46.7%	46.8%	0.2	0.4	37.3%	49.1%	1088.0	1326.7	30.5%	41.9%	1670.9	1343.0	10.5%	17.4%	44.8	45.0
	11	F2	28.5%	41.2%	0.0	0.0	28.5%	41.2%	0.3	0.7	56.7%	42.7%	0.1	0.1	40.5%	44.7%	1859.7	1064.7	36.8%	42.3%	2426.4	1069.9	12.2%	21.4%	173.3	163.3
	13	F2	29.6%	40.1%	0.0	0.0	29.6%	40.1%	0.2	0.4	56.7%	42.7%	0.3	0.4	47.3%	39.8%	2610.9	679.4	46.2%	36.8%	3220.2	470.4	9.6%	13.0%	738.8	710.7
	15	F2	28.6%	39.1%	0.0	0.0	28.6%	39.1%	1.0	1.5	53.3%	45.0%	0.8	1.4	46.8%	46.8%	3264.2	429.9	50.0%	43.4%	3266.5	367.6	8.2%	11.8%	1032.5	911.6
	20	F2	36.3%	34.8%	0.3	0.6	36.5%	35.2%	3.5	7.7	70.0%	35.2%	0.8	0.7	66.7%	37.2%	3480.9	291.6	66.7%	37.2%	3600.0	0.0	31.1%	26.9%	2796.2	788.2
	30	F2	26.6%	37.3%	0.4	0.6	26.6%	37.3%	0.4	0.2	70.0%	41.5%	2.5	1.4	70.0%	41.5%	3600.0	0.0	70.0%	41.5%	3600.0	0.0	70.0%	41.5%	3600.0	0.0
	50	F2	23.9%	37.5%	0.3	0.5	23.9%	37.5%	10.7	11.2	80.0%	25.3%	24.8	13.5	80.0%	25.3%	3600.0	0.0	80.0%	25.3%	3600.0	0.0	80.0%	25.3%	3600.0	0.0
	75	F2	28.5%	36.5%	14.6	22.4	28.5%	36.5%	35.8	20.5	100.0%	0.0%	180.8	96.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	28.2%	39.0%	2.8	4.8	28.2%	39.0%	202.2	198.8	90.0%	16.7%	496.4	146.4	90.0%	16.7%	3600.0	0.0	90.0%	16.7%	3600.0	0.0	90.0%	16.7%	3600.0	0.0
	F2 Average		**28.8%**	**39.5%**	**1.7**	**2.6**	**28.8%**	**39.6%**	**23.2**	**22.0**	**63.6%**	**35.9%**	**64.3**	**23.7**	**58.0%**	**35.3%**	**2457.0**	**385.1**	**55.7%**	**31.5%**	**2652.1**	**380.2**	**40.4%**	**20.5%**	**1745.5**	**240.2**
	Standard Deviation		**3.0%**	**3.1%**	**4.4**	**6.7**	**3.1%**	**3.1%**	**60.3**	**59.0**	**20.4%**	**15.7%**	**153.1**	**49.7**	**24.6%**	**15.2%**	**1405.4**	**463.4**	**27.4%**	**13.4%**	**1318.0**	**468.0**	**36.6%**	**10.8%**	**1666.9**	**367.7**
	LP Relaxation Average		**41.0%**	**30.5%**	**2.1**	**3.6**	**37.3%**	**28.8%**	**22.2**	**23.8**	**81.8%**	**17.9%**	**81.5**	**29.0**	**63.3%**	**23.9%**	**2217.2**	**506.6**	**56.1%**	**26.2%**	**2518.3**	**503.8**	**40.1%**	**12.2%**	**1606.8**	**204.6**
	Standard Deviation		**13.3%**	**10.1%**	**3.3**	**5.6**	**12.1%**	**12.2%**	**47.3**	**47.0**	**23.3%**	**21.3%**	**199.2**	**70.0**	**25.9%**	**17.4%**	**1404.9**	**529.0**	**32.3%**	**19.1%**	**1267.6**	**568.4**	**42.2%**	**14.3%**	**1661.2**	**379.0**

Table 7 – (Continuation).

MIP PROBLEM

Instance Sizes	Objective Function	Mixed Integer Program Formulations																							
		CTP				AFCTP				APD				TI				TII				ATI			
		GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD	GAP	SD	T(s)	SD
5	F1	0.0%	0.0%	0.4	0.6	0.0%	0.0%	0.3	0.5	0.0%	0.0%	0.1	0.1	0.0%	0.0%	546.3	769.5	0.0%	0.0%	81.2	158.7	0.0%	0.0%	4.2	6.9
7	F1	0.0%	0.0%	0.5	1.1	0.0%	0.0%	1.1	0.5	0.0%	0.0%	1.0	0.3	5.2%	9.3%	1223.0	1638.6	0.3%	0.8%	888.1	1193.2	0.0%	0.0%	19.4	29.4
9	F1	0.0%	0.0%	1.5	2.5	0.0%	0.0%	37.1	30.2	0.0%	0.0%	24.4	13.8	11.4%	14.0%	1741.5	1671.7	3.3%	6.3%	1416.0	1383.1	0.0%	0.0%	61.1	72.9
11	F1	0.0%	0.0%	15.1	12.1	3.1%	3.3%	1778.9	1369.1	0.0%	0.0%	625.0	394.0	17.0%	26.3%	2261.4	1472.0	11.2%	21.8%	2116.0	1329.3	0.0%	0.0%	143.7	152.5
13	F1	0.9%	1.4%	458.7	492.5	19.1%	14.7%	2857.4	1151.0	20.7%	15.5%	3182.3	617.3	27.3%	33.9%	2623.3	1205.6	25.1%	38.1%	3124.5	848.6	0.1%	0.3%	389.9	451.3
15	F1	9.0%	8.2%	2214.9	1651.0	29.8%	18.3%	3585.4	35.8	40.3%	18.4%	3600.0	0.0	51.8%	41.6%	3415.5	393.6	31.2%	37.3%	3468.2	322.9	0.1%	0.2%	777.4	705.3
20	F1	31.2%	20.2%	3388.8	327.9	40.6%	19.2%	3600.0	0.0	73.8%	8.7%	3600.0	0.0	68.5%	31.8%	3600.0	0.0	48.4%	43.3%	3600.0	0.0	40.2%	47.2%	2238.1	1275.1
30	F1	44.7%	23.8%	3600.0	0.0	50.0%	23.3%	3600.0	0.0	88.1%	10.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F1	51.3%	27.9%	3600.0	0.0	56.3%	26.8%	3600.0	0.0	95.5%	4.6%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F1	59.9%	25.9%	3600.0	0.0	62.2%	25.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F1	63.7%	26.0%	3600.0	0.0	65.2%	24.9%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F1 Average		**23.7%**	**12.1%**	**1861.8**	**226.2**	**29.7%**	**14.2%**	**2387.3**	**235.2**	**47.1%**	**5.2%**	**2312.1**	**93.2**	**52.8%**	**14.3%**	**2710.1**	**650.1**	**47.2%**	**13.4%**	**2644.9**	**476.0**	**40.0%**	**4.3%**	**1639.4**	**244.9**
Standard Deviation		**26.7%**	**12.4%**	**1741.0**	**501.4**	**26.5%**	**11.2%**	**1621.6**	**509.2**	**44.7%**	**6.9%**	**1716.9**	**210.0**	**42.3%**	**16.2%**	**1117.9**	**721.0**	**44.2%**	**18.0%**	**1300.7**	**588.3**	**49.0%**	**14.2%**	**1675.3**	**411.9**
5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.3	0.5	0.0%	0.0%	0.1	0.1	3.9%	9.6%	396.3	938.0	3.3%	8.2%	313.1	707.2	0.0%	0.0%	3.8	2.6
7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.9	1.5	0.0%	0.0%	0.6	0.8	7.9%	19.3%	744.2	1413.0	3.5%	8.6%	679.0	1161.8	1.8%	3.0%	343.8	480.2
9	F2	0.0%	0.0%	0.2	0.4	0.0%	0.0%	5.3	6.6	0.0%	0.0%	6.2	9.6	11.5%	28.3%	1071.6	1381.9	13.8%	27.6%	1202.5	1525.9	4.3%	7.6%	826.0	890.1
11	F2	0.0%	0.0%	1.5	3.2	0.0%	0.0%	175.8	219.9	0.0%	0.0%	78.1	133.2	22.6%	37.2%	1521.7	1680.5	31.2%	40.0%	1636.5	1691.1	7.5%	14.9%	1408.9	1312.6
13	F2	0.0%	0.0%	76.7	186.1	4.5%	9.1%	805.5	1132.2	0.4%	1.0%	531.8	902.0	37.3%	38.9%	1907.7	1572.9	44.6%	42.0%	2225.9	1582.0	26.4%	23.7%	2304.5	1464.9
15	F2	2.6%	6.4%	295.6	716.4	11.2%	20.1%	1426.9	1562.5	12.8%	23.3%	1160.9	1760.6	50.0%	48.6%	2269.5	1344.1	63.3%	42.7%	2639.0	1109.5	59.7%	26.6%	3137.8	601.5
20	F2	12.6%	25.8%	1031.5	1629.3	28.9%	29.7%	2166.6	1633.9	33.1%	41.1%	1841.3	1473.1	87.2%	15.4%	3309.4	321.0	90.0%	24.5%	3510.9	218.4	100.0%	0.0%	3600.0	0.0
30	F2	20.1%	37.5%	1201.4	1857.9	27.4%	37.3%	2085.8	1533.5	51.0%	46.5%	2571.8	1167.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F2	20.3%	38.1%	1433.8	1705.1	41.3%	32.7%	3018.7	885.2	92.1%	10.0%	3416.4	285.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F2	26.4%	37.2%	2155.5	1560.3	77.4%	21.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F2	26.4%	39.8%	1954.8	1578.8	91.1%	10.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F2 Average		**9.9%**	**16.8%**	**741.0**	**839.8**	**25.6%**	**14.6%**	**1535.1**	**634.2**	**35.4%**	**11.1%**	**1527.9**	**521.1**	**56.4%**	**17.9%**	**2329.1**	**786.5**	**59.1%**	**17.6%**	**2418.8**	**726.9**	**54.5%**	**6.9%**	**2365.9**	**432.0**
Standard Deviation		**11.4%**	**18.5%**	**841.7**	**820.5**	**32.4%**	**14.3%**	**1444.7**	**717.8**	**43.1%**	**17.7%**	**1534.7**	**675.1**	**41.5%**	**18.0%**	**1268.2**	**720.4**	**41.1%**	**18.1%**	**1281.1**	**707.6**	**46.5%**	**10.2%**	**1455.0**	**565.7**
MIP Average		**16.8%**	**14.5%**	**1301.4**	**533.0**	**27.6%**	**14.4%**	**1961.2**	**434.7**	**41.3%**	**8.1%**	**1920.0**	**307.2**	**54.6%**	**16.1%**	**2519.6**	**718.3**	**53.1%**	**15.5%**	**2531.9**	**601.4**	**47.3%**	**5.6%**	**2002.7**	**338.4**
Standard Deviation		**21.3%**	**15.6%**	**1452.5**	**734.1**	**29.0%**	**12.5%**	**1560.9**	**640.7**	**43.2%**	**13.5%**	**1639.0**	**534.8**	**40.9%**	**16.8%**	**1182.8**	**706.8**	**42.1%**	**17.8%**	**1265.1**	**647.9**	**47.2%**	**12.1%**	**1575.7**	**492.3**

Table 8 – Confidence Interval (CI) for Single Machine Scheduling Problems for Six MIP Formulations for All Classes. For the LP (linear programming) relaxation problem, the "**GAP**" indicates the 95% confidence interval value of the average linear relaxation gap, computed for each formulation and instance as the relative difference between the best integer solution and its LP relaxation value. For the MIP (mixed integer programming) problem, the "**GAP**" indicates the 95% confidence interval value of the average optimality gap. "**T(s)**" indicates 95% confidence interval value of the average CPU time. $F1$ and $F2$ denote the objective functions $\sum_j w_j C_j$ and $\sum_j w_j T_j$, respectively.

	Instance Classes	Objective Function	Mixed Integer Program Formulations											
			CTP		AFCTP		APD		TI		TII		ATI	
			GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)
LP RELAXATION PROBLEM	Class 1	F1	(62.1%,74.9%)	(0.0,0.2)	(47.6%,69.5%)	(0.0,16.4)	(100.0%,100.0%)	(0.0,155.6)	(57.5%,87.6%)	(525.8,2649.8)	(10.7%,69.6%)	(986.4,2926.7)	(5.1%,67.6%)	(239.1,2455.1)
	Class 2	F1	(61.0%,74.1%)	(0.0,8.7)	(46.0%,68.4%)	(1.3,20.2)	(100.0%,100.0%)	(0.0,202.8)	(65.5%,96.0%)	(1730.5,3554.0)	(57.3%,100.0%)	(2488.5,3775.8)	(12.4%,74.9%)	(555.1,2738.7)
	Class 3	F1	(60.2%,67.3%)	(0.0,7.8)	(47.8%,63.1%)	(0.0,8.8)	(100.0%,100.0%)	(0.0,335.3)	(54.3%,83.7%)	(992.4,2745.6)	(30.9%,83.7%)	(1293.7,3156.9)	(8.3%,69.1%)	(383.4,2547.4)
	Class 4	F1	(23.7%,29.5%)	(0.1,1.2)	(20.5%,26.5%)	(0.0,33.0)	(100.0%,100.0%)	(0.0,348.7)	(26.1%,74.8%)	(551.4,2607.6)	(9.0%,69.0%)	(944.6,2903.8)	(5.2%,67.7%)	(227.7,2409.3)
	Class 5	F1	(61.7%,73.8%)	(0.5,12.1)	(47.5%,68.7%)	(0.0,71.8)	(100.0%,100.0%)	(0.0,170.9)	(56.9%,87.4%)	(499.1,2642.2)	(13.7%,71.1%)	(883.7,2890.1)	(7.6%,68.8%)	(255.3,2429.8)
	Class 6	F1	(22.8%,26.7%)	(0.0,2.0)	(19.8%,24.0%)	(6.7,114.5)	(100.0%,100.0%)	(0.0,286.8)	(43.2%,90.7%)	(1779.0,3451.3)	(55.2%,100.0%)	(2585.8,3779.0)	(13.2%,77.9%)	(588.3,2788.5)
	F1 Average		**(48.6%,57.7%)**	**(0.0,5.3)**	**(38.2%,53.4%)**	**(0.0,44.1)**	**(100.0%,100.0%)**	**(0.0,250.0)**	**(50.6%,86.7%)**	**(1013.0,2941.7)**	**(29.5%,83.6%)**	**(1530.5,3238.7)**	**(8.6%,71.0%)**	**(374.8,2561.5)**
	Standard Deviation		**(19.3%,23.4%)**	**(0.0,4.9)**	**(13.8%,22.0%)**	**(1.7,41.4)**	**(0.0%,0.0%)**	**(0.0,84.8)**	**(5.4%,14.7%)**	**(414.9,618.9)**	**(16.8%,22.3%)**	**(427.1,793.8)**	**(3.2%,4.5%)**	**(154.0,172.6)**
	Class 1	F2	(0.8%,11.4%)	(0.0,0.1)	(0.8%,11.4%)	(0.0,29.6)	(8.8%,49.4%)	(0.0,138.8)	(6.9%,48.4%)	(1210.5,3196.9)	(6.7%,48.3%)	(1372.5,3332.2)	(2.7%,45.2%)	(505.6,2721.1)
	Class 2	F2	(0.7%,15.8%)	(0.0,0.1)	(0.7%,15.8%)	(0.0,39.1)	(8.4%,57.1%)	(0.0,105.2)	(7.2%,54.7%)	(1816.8,3469.5)	(7.2%,54.7%)	(2100.4,3708.1)	(5.8%,55.6%)	(942.6,3008.1)
	Class 3	F2	(0.0%,9.0%)	(0.0,0.1)	(0.0%,9.0%)	(0.0,26.7)	(21.6%,65.7%)	(0.0,130.6)	(16.5%,63.5%)	(1677.2,3502.5)	(21.6%,65.7%)	(1878.4,3580.0)	(5.8%,58.3%)	(545.5,2620.4)
	Class 4	F2	(1.9%,7.3%)	(0.0,0.1)	(1.9%,7.3%)	(0.0,37.5)	(56.5%,96.2%)	(0.0,153.9)	(29.6%,86.0%)	(1298.5,3201.0)	(26.2%,84.1%)	(1394.2,3269.0)	(6.4%,68.1%)	(509.0,2623.6)
	Class 5	F2	(97.6%,99.1%)	(0.0,12.2)	(97.7%,99.4%)	(0.0,169.2)	(100.0%,100.0%)	(0.0,216.6)	(98.5%,100.0%)	(1101.5,3184.4)	(77.1%,96.4%)	(1488.2,3385.7)	(48.0%,84.6%)	(596.6,2747.3)
	Class 6	F2	(43.3%,59.2%)	(0.0,13.8)	(43.3%,59.0%)	(0.0,65.6)	(100.0%,100.0%)	(0.0,216.9)	(84.8%,100.0%)	(2144.9,3680.8)	(77.0%,100.0%)	(2539.8,3776.2)	(27.1%,77.1%)	(1042.3,3084.4)
	F2 Average		**(23.9%,33.6%)**	**(0.0,4.4)**	**(23.9%,33.6%)**	**(0.0,61.3)**	**(49.2%,78.1%)**	**(0.0,160.4)**	**(40.6%,75.5%)**	**(1541.6,3372.5)**	**(36.0%,75.5%)**	**(1795.6,3508.5)**	**(16.0%,64.8%)**	**(690.3,2800.8)**
	Standard Deviation		**(35.9%,41.4%)**	**(0.0,6.7)**	**(36.0%,41.4%)**	**(0.0,54.7)**	**(21.4%,44.0%)**	**(0.0,47.0)**	**(20.6%,41.9%)**	**(202.4,408.1)**	**(20.2%,34.4%)**	**(201.9,470.0)**	**(11.5%,20.1%)**	**(184.8,249.0)**
	LP Relaxation Average		**(36.3%,45.7%)**	**(0.0,4.9)**	**(31.1%,43.5%)**	**(0.0,52.7)**	**(74.6%,89.0%)**	**(0.0,205.2)**	**(45.6%,81.1%)**	**(1277.3,3157.1)**	**(32.7%,79.6%)**	**(1663.0,3373.6)**	**(12.3%,67.9%)**	**(532.5,2681.1)**
	Standard Deviation		**(30.1%,34.7%)**	**(0.0,5.7)**	**(26.5%,33.7%)**	**(0.0,48.2)**	**(18.4%,39.8%)**	**(0.0,80.5)**	**(15.0%,30.7%)**	**(375.0,576.9)**	**(16.2%,29.0%)**	**(337.0,643.3)**	**(6.4%,15.6%)**	**(203.4,263.5)**
MIP PROBLEM	Class 1	F1	(11.2%,55.4%)	(962.9,3178.4)	(19.4%,61.8%)	(1515.7,3548.2)	(22.6%,77.4%)	(1282.2,3432.7)	(12.4%,71.8%)	(1257.7,3304.3)	(8.3%,68.9%)	(1463.2,3478.5)	(5.1%,67.6%)	(411.9,2537.3)
	Class 2	F1	(10.3%,53.7%)	(933.9,3165.1)	(18.8%,61.5%)	(1561.1,3615.9)	(20.6%,78.3%)	(1261.4,3413.5)	(50.4%,96.7%)	(2979.2,3814.5)	(20.1%,80.3%)	(1990.3,3559.6)	(13.1%,77.8%)	(832.4,2931.4)
	Class 3	F1	(7.9%,48.1%)	(772.9,3004.6)	(16.0%,54.3%)	(1504.9,3543.7)	(22.2%,75.6%)	(1371.9,3457.4)	(27.4%,80.1%)	(2079.4,3712.7)	(21.0%,75.8%)	(2073.7,3724.4)	(7.7%,68.9%)	(582.6,2746.8)
	Class 4	F1	(1.2%,16.5%)	(459.3,2706.9)	(3.6%,20.8%)	(989.1,3176.9)	(13.2%,69.6%)	(1135.8,3301.4)	(8.2%,68.9%)	(1014.4,3064.3)	(5.9%,67.9%)	(1063.1,3110.9)	(5.1%,67.6%)	(351.8,2518.7)
	Class 5	F1	(9.9%,54.1%)	(864.6,3091.9)	(17.3%,60.2%)	(1479.9,3514.1)	(22.1%,78.1%)	(1270.6,3429.9)	(16.4%,76.2%)	(1296.9,3248.5)	(8.8%,69.2%)	(1397.7,3355.2)	(7.6%,68.8%)	(347.2,2510.3)
	Class 6	F1	(1.3%,14.7%)	(479.9,2721.5)	(3.5%,18.8%)	(1001.7,3196.2)	(13.9%,72.0%)	(1125.5,3262.5)	(35.5%,90.1%)	(3012.1,3737.2)	(43.2%,97.2%)	(2665.5,3856.6)	(13.3%,77.9%)	(921.1,2981.9)
	F1 Average		**(7.0%,40.4%)**	**(745.6,2978.1)**	**(13.1%,46.2%)**	**(1342.1,3432.5)**	**(19.1%,75.1%)**	**(1241.2,3382.9)**	**(25.0%,80.6%)**	**(1940.0,3480.2)**	**(17.9%,76.6%)**	**(1775.6,3514.2)**	**(8.6%,71.4%)**	**(574.5,2704.4)**
	Standard Deviation		**(4.5%,19.4%)**	**(213.0,224.2)**	**(7.5%,20.6%)**	**(192.3,270.7)**	**(3.0%,4.8%)**	**(71.9,101.0)**	**(10.6%,16.1%)**	**(290.9,900.1)**	**(10.9%,14.2%)**	**(251.5,585.0)**	**(3.5%,5.1%)**	**(210.7,254.1)**
	Class 1	F2	(0.0%,1.2%)	(0.0,405.8)	(0.5%,48.8%)	(146.6,2064.8)	(0.8%,57.6%)	(208.9,2208.1)	(12.4%,74.9%)	(654.2,2773.1)	(18.0%,80.1%)	(780.6,2956.4)	(18.0%,80.1%)	(843.2,3027.0)
	Class 2	F2	(0.0%,7.6%)	(0.0,1441.4)	(0.0%,46.1%)	(131.2,2024.6)	(2.3%,55.3%)	(200.8,2120.7)	(28.6%,87.8%)	(1557.9,3310.6)	(37.1%,93.8%)	(1602.9,3479.7)	(29.6%,90.4%)	(1579.9,3447.2)
	Class 3	F2	(0.0%,0.0%)	(0.0,125.9)	(0.0%,34.3%)	(36.9,1771.2)	(0.0%,52.6%)	(120.2,2093.8)	(17.8%,76.7%)	(855.2,2893.8)	(21.8%,81.4%)	(1015.5,3069.8)	(25.2%,83.9%)	(1252.6,3291.8)
	Class 4	F2	(0.0%,0.0%)	(0.0,406.7)	(0.0%,32.5%)	(749.7,2859.1)	(4.9%,63.9%)	(373.4,2550.2)	(11.4%,72.8%)	(594.0,2761.5)	(9.6%,70.4%)	(681.6,2799.6)	(18.0%,80.1%)	(820.8,2972.4)
	Class 5	F2	(13.3%,70.9%)	(748.7,2928.3)	(21.4%,78.2%)	(1207.4,3344.0)	(20.0%,81.4%)	(1095.3,3258.7)	(34.6%,91.1%)	(1917.4,3666.3)	(38.6%,92.4%)	(2171.0,3725.0)	(28.0%,82.8%)	(1974.0,3586.6)
	Class 6	F2	(2.5%,24.5%)	(433.3,2659.0)	(8.3%,43.9%)	(983.1,3102.2)	(14.3%,73.5%)	(976.7,3128.5)	(67.9%,100.0%)	(3241.9,3723.7)	(62.8%,100.0%)	(3017.2,3726.5)	(31.2%,86.9%)	(1996.5,3598.7)
	F2 Average		**(2.3%,17.4%)**	**(154.2,1327.9)**	**(3.9%,47.3%)**	**(542.5,2527.7)**	**(6.8%,64.0%)**	**(495.9,2560.0)**	**(28.8%,84.0%)**	**(1470.1,3188.2)**	**(31.3%,86.8%)**	**(1544.8,3292.8)**	**(25.0%,84.0%)**	**(1411.2,3320.6)**
	Standard Deviation		**(5.4%,27.9%)**	**(301.6,1238.3)**	**(8.2%,17.2%)**	**(496.7,659.2)**	**(8.3%,11.5%)**	**(415.9,528.6)**	**(10.4%,21.5%)**	**(400.5,1032.3)**	**(11.1%,19.3%)**	**(371.1,927.4)**	**(3.1%,6.3%)**	**(269.2,528.1)**
	MIP Average		**(4.7%,28.9%)**	**(449.9,2153.0)**	**(8.5%,46.8%)**	**(942.3,2980.1)**	**(12.9%,69.6%)**	**(868.6,2971.4)**	**(26.9%,82.3%)**	**(1705.0,3334.2)**	**(24.6%,81.7%)**	**(1660.2,3403.5)**	**(16.8%,77.7%)**	**(992.8,3012.5)**
	Standard Deviation		**(5.3%,25.9%)**	**(376.4,1215.8)**	**(6.9%,19.0%)**	**(545.0,678.5)**	**(8.1%,10.7%)**	**(477.8,566.1)**	**(10.0%,18.3%)**	**(359.4,958.3)**	**(11.6%,17.8%)**	**(315.3,752.6)**	**(7.2%,10.3%)**	**(385.1,596.2)**

Table 9 – Confidence Interval (CI) for Single Machine Scheduling Problems for Six MIP Formulations for All Sizes. For the LP (linear programming) relaxation problem, the "**GAP**" indicates the 95% confidence interval value of the average linear relaxation gap, computed for each formulation and instance as the relative difference between the best integer solution and its LP relaxation value. For the MIP (mixed integer programming) problem, the "**GAP**" indicates the 95% confidence interval value of the average optimality gap. "**T(s)**" indicates 95% confidence interval value of the average CPU time. $F1$ and $F2$ denote the objective functions $\sum_j w_j C_j$ and $\sum_j w_j T_j$, respectively.

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations											
			CTP		AFCTP		APD		TI		TII		ATI	
			GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)	GAP	T(s)
LP RELAXATION PROBLEM	5	F1	(33.4%,50.7%)	(0.0,1.2)	(20.6%,28.8%)	(0.2,1.6)	(100.0%,100.0%)	(0.1,0.4)	(27.0%,39.0%)	(0.0,324.9)	(1.6%,5.4%)	(0.0,205.9)	(0.0%,0.1%)	(0.0,3.8)
	7	F1	(40.2%,57.8%)	(0.0,2.2)	(29.0%,39.7%)	(0.0,0.0)	(100.0%,100.0%)	(0.0,0.1)	(32.2%,48.8%)	(19.0,453.3)	(3.5%,8.3%)	(80.9,1828.8)	(0.0%,0.0%)	(0.6,4.0)
	9	F1	(38.4%,60.8%)	(0.0,0.5)	(29.1%,44.8%)	(0.0,0.0)	(100.0%,100.0%)	(0.0,0.1)	(35.3%,52.4%)	(102.9,1643.4)	(7.0%,55.2%)	(293.0,2267.9)	(0.0%,0.1%)	(2.2,24.6)
	11	F1	(38.9%,61.5%)	(0.0,0.1)	(32.0%,49.0%)	(0.1,2.5)	(100.0%,100.0%)	(0.0,1.0)	(33.4%,54.9%)	(79.0,1436.8)	(8.3%,65.6%)	(672.7,2357.6)	(0.0%,0.2%)	(15.0,55.8)
	13	F1	(38.0%,64.0%)	(0.6,3.3)	(32.2%,53.5%)	(0.9,3.0)	(100.0%,100.0%)	(0.0,9.5)	(38.5%,66.0%)	(354.9,1944.8)	(10.9%,69.2%)	(1400.6,2870.5)	(0.0%,0.2%)	(43.8,149.8)
	15	F1	(39.6%,66.3%)	(0.0,11.7)	(34.7%,56.7%)	(3.3,19.6)	(100.0%,100.0%)	(0.5,3.8)	(49.1%,85.3%)	(772.0,2679.7)	(17.6%,74.3%)	(2115.0,3093.5)	(0.0%,0.4%)	(90.5,366.6)
	20	F1	(38.7%,66.9%)	(0.0,10.5)	(35.0%,59.5%)	(0.0,45.3)	(100.0%,100.0%)	(0.7,2.9)	(57.8%,93.2%)	(2055.1,3095.8)	(30.2%,87.2%)	(2992.6,3484.8)	(11.2%,63.7%)	(600.3,2298.4)
	30	F1	(39.0%,70.4%)	(0.0,12.0)	(37.0%,65.9%)	(0.0,52.5)	(100.0%,100.0%)	(3.5,10.9)	(96.8%,100.0%)	(3336.3,3654.3)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3446.2,3597.9)
	50	F1	(38.1%,73.6%)	(0.0,10.2)	(37.3%,71.7%)	(3.7,9.2)	(100.0%,100.0%)	(30.7,50.6)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	75	F1	(45.4%,78.0%)	(0.7,3.8)	(45.0%,77.1%)	(22.6,136.3)	(100.0%,100.0%)	(196.7,251.1)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	100	F1	(48.5%,80.9%)	(1.0,3.3)	(48.3%,80.3%)	(42.2,139.4)	(100.0%,100.0%)	(620.7,989.2)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	F1 Average		**(39.9%,66.5%)**	**(0.0,5.3)**	**(34.6%,57.0%)**	**(5.4,37.2)**	**(100.0%,100.0%)**	**(77.4,119.9)**	**(60.9%,76.4%)**	**(1588.1,2366.6)**	**(43.6%,69.6%)**	**(1995.7,2773.5)**	**(37.4%,42.3%)**	**(1363.4,1572.8)**
	Standard Deviation		**(3.1%,9.2%)**	**(0.0,4.8)**	**(7.3%,16.2%)**	**(11.5,53.7)**	**(0.0%,0.0%)**	**(188.8,298.1)**	**(20.4%,34.1%)**	**(1067.8,1791.0)**	**(23.7%,52.1%)**	**(859.7,1668.5)**	**(41.0%,56.9%)**	**(1473.8,1975.2)**
	5	F2	(1.5%,58.6%)	(0.0,0.0)	(1.4%,58.7%)	(0.1,0.7)	(9.6%,70.4%)	(0.0,0.3)	(1.4%,57.1%)	(0.0,61.0)	(0.9%,41.7%)	(0.0,342.2)	(0.0%,34.8%)	(0.3,1.6)
	7	F2	(4.1%,55.0%)	(0.0,0.0)	(4.1%,55.0%)	(0.0,0.1)	(5.9%,67.4%)	(0.0,0.1)	(4.5%,56.5%)	(56.9,537.9)	(4.0%,38.9%)	(66.2,823.5)	(2.8%,29.7%)	(0.6,28.3)
	9	F2	(0.9%,52.9%)	(0.0,0.0)	(0.9%,52.9%)	(0.0,0.7)	(17.7%,75.6%)	(0.0,0.4)	(6.9%,67.8%)	(265.7,1910.3)	(4.5%,56.5%)	(838.5,2503.2)	(0.0%,21.2%)	(16.9,72.7)
	11	F2	(3.0%,54.0%)	(0.0,0.0)	(3.0%,54.0%)	(0.0,0.7)	(30.2%,83.2%)	(0.0,0.2)	(12.7%,68.2%)	(1199.8,2519.6)	(10.6%,63.0%)	(1763.3,3089.6)	(0.0%,25.5%)	(72.0,274.5)
	13	F2	(4.8%,54.5%)	(0.0,0.0)	(4.8%,54.5%)	(0.0,0.4)	(30.2%,83.2%)	(0.1,0.6)	(22.7%,72.0%)	(2189.8,3032.0)	(23.4%,69.0%)	(2928.7,3511.8)	(1.5%,17.7%)	(298.3,1179.3)
	15	F2	(4.3%,52.8%)	(0.0,0.0)	(4.3%,52.8%)	(0.1,1.9)	(25.4%,81.2%)	(0.0,1.6)	(17.8%,75.8%)	(2997.7,3530.6)	(23.2%,76.9%)	(3038.7,3494.4)	(0.9%,15.6%)	(467.5,1597.5)
	20	F2	(14.7%,57.9%)	(0.0,0.7)	(14.7%,58.3%)	(0.0,8.3)	(48.2%,91.8%)	(0.4,1.2)	(43.6%,89.7%)	(3300.2,3661.7)	(43.6%,89.7%)	(3600.0,3600.0)	(14.5%,47.8%)	(2307.7,3284.8)
	30	F2	(3.5%,49.7%)	(0.0,0.7)	(3.5%,49.7%)	(0.3,0.5)	(44.3%,95.7%)	(1.6,3.3)	(44.3%,95.7%)	(3600.0,3600.0)	(44.3%,95.7%)	(3600.0,3600.0)	(44.3%,95.7%)	(3600.0,3600.0)
	50	F2	(0.6%,47.1%)	(0.0,0.6)	(0.6%,47.1%)	(3.8,17.6)	(64.3%,95.7%)	(16.4,33.1)	(64.3%,95.7%)	(3600.0,3600.0)	(64.3%,95.7%)	(3600.0,3600.0)	(64.3%,95.7%)	(3600.0,3600.0)
	75	F2	(5.8%,51.1%)	(0.7,28.5)	(5.8%,51.1%)	(23.1,48.5)	(100.0%,100.0%)	(121.2,240.4)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	100	F2	(4.0%,52.4%)	(0.0,5.8)	(4.0%,52.4%)	(79.0,325.4)	(79.6%,100.0%)	(405.7,587.2)	(79.6%,100.0%)	(3600.0,3600.0)	(79.6%,100.0%)	(3600.0,3600.0)	(79.6%,100.0%)	(3600.0,3600.0)
	F2 Average		**(4.3%,53.3%)**	**(0.1,3.3)**	**(4.3%,53.3%)**	**(9.5,36.8)**	**(41.4%,85.9%)**	**(49.6,78.9)**	**(36.2%,79.9%)**	**(2218.4,2695.7)**	**(36.2%,75.2%)**	**(2416.4,2887.7)**	**(27.7%,53.1%)**	**(1596.7,1894.4)**
	Standard Deviation		**(1.1%,5.0%)**	**(0.2,8.5)**	**(1.2%,5.0%)**	**(23.8,96.9)**	**(10.6%,30.1%)**	**(122.3,183.9)**	**(15.2%,34.0%)**	**(1118.2,1692.6)**	**(19.1%,35.7%)**	**(1027.9,1608.0)**	**(29.9%,43.3%)**	**(1439.0,1894.8)**
	LP Relaxation Average		**(22.1%,59.9%)**	**(0.0,4.3)**	**(19.4%,55.2%)**	**(7.5,37.0)**	**(70.7%,92.9%)**	**(63.5,99.4)**	**(48.5%,78.1%)**	**(1903.2,2531.2)**	**(39.9%,72.4%)**	**(2206.0,2830.6)**	**(32.5%,47.7%)**	**(1480.1,1733.6)**
	Standard Deviation		**(7.1%,19.6%)**	**(0.0,6.8)**	**(4.5%,19.6%)**	**(18.2,76.5)**	**(10.1%,36.5%)**	**(155.8,242.6)**	**(15.1%,36.7%)**	**(1077.1,1732.8)**	**(20.5%,44.1%)**	**(915.3,1619.9)**	**(33.3%,51.1%)**	**(1426.3,1896.0)**

Table 9 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations: CTP GAP	CTP T(s)	AFCTP GAP	AFCTP T(s)	APD GAP	APD T(s)	TI GAP	TI T(s)	TII GAP	TII T(s)	ATI GAP	ATI T(s)
MIP PROBLEM	5	F1	(0.0%,0.0%)	(0.0,0.8)	(0.0%,0.0%)	(0.0,0.6)	(0.0%,0.0%)	(0.1,0.2)	(0.0%,0.0%)	(69.4,1023.3)	(0.0%,0.0%)	(0.0,179.5)	(0.0%,0.0%)	(0.0,8.5)
	7	F1	(0.0%,0.0%)	(0.0,1.1)	(0.0%,0.0%)	(0.7,1.4)	(0.0%,0.0%)	(0.8,1.1)	(0.0%,10.9%)	(207.3,2238.6)	(0.0%,0.8%)	(148.5,1627.6)	(0.0%,0.0%)	(1.1,37.6)
	9	F1	(0.0%,0.0%)	(0.0,3.1)	(0.0%,0.0%)	(18.4,55.8)	(0.0%,0.0%)	(15.9,33.0)	(2.7%,20.0%)	(705.4,2777.6)	(0.0%,7.2%)	(558.7,2273.2)	(0.0%,0.0%)	(15.9,106.3)
	11	F1	(0.0%,0.0%)	(7.6,22.6)	(1.1%,5.2%)	(930.4,2627.5)	(0.0%,0.0%)	(380.8,869.2)	(0.7%,33.2%)	(1349.1,3173.8)	(0.0%,24.7%)	(1292.1,2939.8)	(0.0%,0.0%)	(49.2,238.2)
	13	F1	(0.0%,1.7%)	(153.4,763.9)	(9.9%,28.2%)	(2144.0,3570.8)	(11.1%,30.3%)	(2799.7,3564.9)	(6.3%,48.3%)	(1876.1,3370.5)	(1.4%,48.7%)	(2598.6,3650.5)	(0.0%,0.3%)	(110.2,669.6)
	15	F1	(3.9%,14.1%)	(1191.6,3238.2)	(18.4%,41.1%)	(3563.2,3607.6)	(28.9%,51.7%)	(3600.0,3600.0)	(26.0%,77.6%)	(3171.6,3659.5)	(8.1%,54.3%)	(3268.0,3668.3)	(0.0%,0.2%)	(340.3,1214.6)
	20	F1	(18.7%,43.7%)	(3185.6,3592.0)	(28.7%,52.4%)	(3600.0,3600.0)	(68.5%,79.2%)	(3600.0,3600.0)	(48.8%,88.2%)	(3600.0,3600.0)	(21.5%,75.2%)	(3600.0,3600.0)	(11.0%,69.5%)	(1447.8,3028.4)
	30	F1	(30.0%,59.5%)	(3600.0,3600.0)	(35.5%,64.4%)	(3600.0,3600.0)	(81.8%,94.3%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	50	F1	(34.0%,68.6%)	(3600.0,3600.0)	(39.6%,72.9%)	(3600.0,3600.0)	(92.6%,98.3%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	75	F1	(43.8%,75.9%)	(3600.0,3600.0)	(46.5%,77.9%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	100	F1	(47.6%,79.8%)	(3600.0,3600.0)	(49.8%,80.6%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	F1 Average		**(16.2%,31.2%)**	**(1721.6,2002.0)**	**(20.9%,38.4%)**	**(2241.5,2533.1)**	**(43.9%,50.4%)**	**(2254.3,2369.9)**	**(44.0%,61.7%)**	**(2307.2,3113.0)**	**(38.9%,55.5%)**	**(2349.9,2939.9)**	**(37.4%,42.7%)**	**(1487.7,1791.2)**
	Standard Deviation		**(19.0%,34.5%)**	**(1430.3,2051.7)**	**(19.6%,33.4%)**	**(1306.0,1937.2)**	**(40.4%,48.9%)**	**(1586.7,1847.0)**	**(32.2%,52.3%)**	**(671.1,1564.8)**	**(33.0%,55.4%)**	**(936.1,1665.3)**	**(40.2%,57.8%)**	**(1419.9,1930.6)**
	5	F2	(0.0%,0.0%)	(0.0,0.0)	(0.0%,0.0%)	(0.0,0.6)	(0.0%,0.0%)	(0.1,0.1)	(0.0%,9.8%)	(0.0,977.6)	(0.0%,8.4%)	(0.0,751.4)	(0.0%,0.0%)	(2.2,5.5)
	7	F2	(0.0%,0.0%)	(0.0,0.1)	(0.0%,0.0%)	(0.0,1.8)	(0.0%,0.0%)	(0.1,1.1)	(0.0%,19.8%)	(0.0,1619.9)	(0.0%,8.8%)	(0.0,1399.1)	(0.0%,3.7%)	(46.2,641.4)
	9	F2	(0.0%,0.0%)	(0.0,0.5)	(0.0%,0.0%)	(1.2,9.4)	(0.0%,0.0%)	(0.3,12.2)	(0.0%,29.0%)	(215.2,1928.1)	(0.0%,30.9%)	(256.8,2148.3)	(0.0%,9.0%)	(274.3,1377.7)
	11	F2	(0.0%,0.0%)	(0.0,3.5)	(0.0%,0.0%)	(39.5,312.1)	(0.0%,0.0%)	(0.0,160.6)	(0.0%,45.6%)	(480.2,2563.3)	(6.4%,55.9%)	(588.4,2684.6)	(0.0%,16.8%)	(595.4,2222.5)
	13	F2	(0.0%,0.0%)	(0.0,192.0)	(0.0%,10.2%)	(103.7,1507.2)	(0.0%,1.0%)	(0.0,1090.9)	(13.2%,61.5%)	(932.8,2882.6)	(18.5%,70.6%)	(1245.4,3206.5)	(11.7%,41.1%)	(1396.5,3212.4)
	15	F2	(0.0%,6.5%)	(0.0,739.6)	(0.0%,23.7%)	(458.5,2395.4)	(0.0%,27.3%)	(69.7,2252.1)	(19.9%,80.1%)	(1436.4,3102.6)	(36.8%,89.8%)	(1951.4,3326.7)	(43.3%,76.2%)	(2765.0,3510.6)
	20	F2	(0.0%,28.6%)	(21.7,2041.3)	(10.5%,47.3%)	(1153.9,3179.2)	(7.6%,58.6%)	(928.3,2754.3)	(77.6%,96.7%)	(3110.5,3508.4)	(74.8%,100.0%)	(3375.5,3646.2)	(100.0%,100.0%)	(3600.0,3600.0)
	30	F2	(0.0%,43.3%)	(49.9,2353.0)	(4.3%,50.5%)	(1135.4,3036.3)	(22.1%,79.8%)	(1848.4,3295.3)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	50	F2	(0.0%,43.9%)	(377.0,2490.7)	(21.0%,61.6%)	(2470.0,3567.3)	(85.9%,98.3%)	(3239.4,3593.4)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	75	F2	(3.4%,49.5%)	(1188.4,3122.6)	(64.1%,90.7%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	100	F2	(1.7%,51.0%)	(976.3,2933.4)	(84.9%,97.3%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)	(100.0%,100.0%)	(3600.0,3600.0)
	F2 Average		**(0.0%,20.3%)**	**(220.5,1261.5)**	**(16.6%,34.6%)**	**(1142.0,1928.1)**	**(28.5%,42.3%)**	**(1205.0,1850.9)**	**(45.3%,67.5%)**	**(1841.7,2816.6)**	**(48.2%,70.0%)**	**(1968.3,2869.3)**	**(50.3%,58.8%)**	**(2098.1,2633.6)**
	Standard Deviation		**(0.0%,22.9%)**	**(333.1,1350.2)**	**(23.5%,41.3%)**	**(999.8,1889.6)**	**(32.1%,54.1%)**	**(1116.3,1953.1)**	**(30.4%,52.6%)**	**(821.7,1714.7)**	**(29.9%,52.4%)**	**(842.5,1719.6)**	**(40.2%,52.8%)**	**(1104.3,1805.6)**
	MIP Average		**(7.8%,25.7%)**	**(971.1,1631.7)**	**(18.7%,36.5%)**	**(1691.8,2230.6)**	**(36.2%,46.3%)**	**(1729.6,2110.4)**	**(44.6%,64.6%)**	**(2074.4,2964.8)**	**(43.5%,62.8%)**	**(2159.1,2904.6)**	**(43.8%,50.8%)**	**(1792.9,2212.4)**
	Standard Deviation		**(11.6%,30.9%)**	**(997.5,1907.5)**	**(21.2%,36.7%)**	**(1163.7,1958.0)**	**(34.9%,51.6%)**	**(1307.5,1970.4)**	**(30.5%,51.3%)**	**(744.8,1620.9)**	**(31.1%,53.1%)**	**(863.6,1666.7)**	**(39.7%,54.7%)**	**(1270.5,1880.8)**

6 CONCLUDING REMARKS

In this article, we proposed and compared six different MIP formulations for two single machine scheduling problems with sequence-dependent setup times and release dates. Not only extensive computational experiments were performed but also their dominance relations regarding the strength of their linear relaxations bounds were analyzed. Aforementioned allowed illustrating the wealth of the formulations. We provided a comparative literature review of several works about SMSP formulations. Besides, we presented a new Arc-Time-Indexed formulation for the single machine scheduling scenario treated, proving its dominance. The analyzed MIP formulations could be easily adapted to other objective functions and machine environments (i.e., parallel machines, flow-shop, and job-shop). The performances of MIP formulations depend on the problem, the number of jobs, the characteristic of the instances (class) and the length of the planning horizon.

The formulations “Completion Time and Precedence” and “Time-Indexed” seems the most widely used formulations in the Scheduling literature. “Completion Time and Precedence” and “Assignment and Positional Date” formulations are the oldest and “Arc-Time-Indexed” proposed formulation is the newest one.

Time-Indexed based formulations (TI, TII, and ATI) present better bounds in general. However, these formulations cannot be directly applied to many instances due to their large number of variables, preventing the use of commercial solvers within a reasonable computational time. Therefore, they recommend using when the length of the time horizon is small or when integrated into a methodology that could deal with their size, as column-generation, Lagrangean relaxation algorithms, and heuristic combinations.

Even though providing weaker lower bounds, CTP and AFCTP formulations managed to solve a significant number of instances. Methods that could take advantage of their capacity in generating feasible solutions in a reduced computational time will best fit with these paradigms. Future directions of research include their integration with heuristic approaches, for instance, in a relax-and-fix framework.

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A ADDITIONAL TABLES

Table A.1 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 1.

LP RELAXATION PROBLEM

Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
		CTP				AFCTP				APD				TI				TII				ATI			
		GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
5	F1	49.0%	4.5%	0.0	0.0	28.0%	4.4%	0.0	0.0	99.9%	0.1%	0.0	0.0	37.3%	6.8%	0.6	1.0	1.6%	1.3%	2.0	2.4	0.0%	0.0%	0.0	0.0
7	F1	57.7%	2.0%	0.0	0.0	37.2%	0.9%	0.0	0.0	100.0%	0.0%	0.0	0.0	47.3%	2.4%	8.5	13.8	3.2%	2.3%	34.0	36.8	0.0%	0.0%	0.2	0.1
9	F1	61.4%	4.3%	0.0	0.0	44.8%	3.9%	0.0	0.1	100.0%	0.0%	0.0	0.0	52.1%	5.4%	26.8	26.6	5.7%	2.5%	220.6	195.8	0.0%	0.0%	1.2	0.3
11	F1	63.0%	1.9%	0.0	0.0	49.6%	2.6%	4.4	9.6	100.0%	0.0%	0.0	0.0	55.4%	2.4%	84.9	147.5	4.9%	2.1%	508.4	445.0	0.0%	0.0%	21.8	27.9
13	F1	68.4%	3.7%	0.0	0.0	56.9%	3.3%	1.8	0.9	100.0%	0.0%	0.0	0.0	61.2%	5.0%	203.8	138.0	6.4%	2.4%	1449.2	965.7	0.2%	0.2%	60.9	51.2
15	F1	68.5%	3.1%	0.0	0.0	58.2%	2.8%	1.4	0.7	100.0%	0.0%	0.1	0.0	62.5%	3.1%	520.2	874.7	6.3%	1.7%	2055.9	1478.5	0.1%	0.1%	70.6	65.4
20	F1	70.0%	1.9%	0.0	0.0	62.0%	1.6%	0.2	0.2	100.0%	0.0%	0.3	0.1	82.0%	14.7%	2221.0	1888.3	13.5%	7.9%	2852.3	1175.1	0.1%	0.1%	263.3	52.6
30	F1	73.5%	1.1%	0.0	0.0	68.9%	1.6%	0.5	0.1	100.0%	0.0%	2.1	0.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F1	76.4%	1.8%	0.1	0.0	74.5%	1.8%	3.6	1.8	100.0%	0.0%	30.0	8.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F1	81.6%	1.0%	0.2	0.0	80.7%	1.0%	12.3	3.8	100.0%	0.0%	173.6	94.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F1	83.7%	1.4%	0.7	0.5	83.1%	1.4%	51.8	44.0	100.0%	0.0%	485.8	191.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F1 Average		**68.5%**	**2.4%**	**0.1**	**0.1**	**58.5%**	**2.3%**	**6.9**	**5.6**	**100.0%**	**0.0%**	**62.9**	**26.8**	**72.5%**	**3.6%**	**1587.8**	**280.9**	**40.1%**	**1.8%**	**1956.6**	**390.9**	**36.4%**	**0.0%**	**1347.1**	**18.0**
Standard Deviation		**10.4%**	**1.3%**	**0.2**	**0.1**	**17.7%**	**1.2%**	**15.3**	**13.1**	**0.0%**	**0.0%**	**149.5**	**61.3**	**24.3%**	**4.4%**	**1713.5**	**592.0**	**47.5%**	**2.3%**	**1565.3**	**552.9**	**50.4%**	**0.1%**	**1787.7**	**26.3**
5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.6	0.7	0.0%	0.0%	3.4	4.4	0.0%	0.0%	0.1	0.0
7	F2	20.0%	44.7%	0.0	0.0	20.0%	44.7%	0.0	0.0	20.0%	44.7%	0.0	0.0	20.0%	44.7%	33.8	31.2	20.0%	44.7%	89.5	43.0	20.0%	44.7%	1.1	0.7
9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	202.5	194.3	0.0%	0.0%	193.2	226.0	0.0%	0.0%	3.6	1.2
11	F2	4.2%	9.4%	0.0	0.0	4.2%	9.4%	0.0	0.0	20.0%	44.7%	0.0	0.0	3.9%	8.6%	826.6	571.2	2.7%	6.0%	1432.3	425.7	2.3%	5.1%	40.2	46.1
13	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	2295.3	1400.8	0.0%	0.0%	3149.5	673.7	0.0%	0.0%	50.3	34.9
15	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	20.0%	44.7%	0.0	0.0	20.0%	44.7%	2882.0	1605.5	20.0%	44.7%	3008.1	1323.6	0.0%	0.0%	313.7	113.3
20	F2	24.6%	36.9%	0.0	0.0	24.6%	36.9%	0.1	0.1	40.0%	54.8%	0.2	0.0	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0	21.1%	30.2%	2937.8	1111.2
30	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.3	0.2	0.0%	0.0%	0.7	0.2	0.0%	0.0%	3600.0	0.0	0.0%	0.0%	3600.0	0.0	0.0%	0.0%	3600.0	0.0
50	F2	4.8%	10.8%	0.0	0.0	4.8%	10.8%	3.9	1.6	60.0%	54.8%	12.8	5.2	60.0%	54.8%	3600.0	0.0	60.0%	54.8%	3600.0	0.0	60.0%	54.8%	3600.0	0.0
75	F2	8.6%	10.0%	0.1	0.0	8.6%	10.0%	41.5	21.8	100.0%	0.0%	148.4	152.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F2	5.2%	10.3%	0.3	0.1	5.2%	10.3%	88.3	53.3	60.0%	54.8%	441.1	521.3	60.0%	54.8%	3600.0	0.0	60.0%	54.8%	3600.0	0.0	60.0%	54.8%	3600.0	0.0
F2 Average		**6.1%**	**11.1%**	**0.0**	**0.0**	**6.1%**	**11.1%**	**12.2**	**7.0**	**29.1%**	**27.1%**	**54.8**	**61.8**	**27.6%**	**23.9%**	**2203.7**	**345.8**	**27.5%**	**23.6%**	**2352.4**	**245.1**	**23.9%**	**17.2%**	**1613.3**	**118.9**
Standard Deviation		**8.6%**	**15.5%**	**0.1**	**0.0**	**8.6%**	**15.5%**	**28.1**	**16.7**	**32.7%**	**26.3%**	**135.5**	**159.1**	**33.5%**	**26.1%**	**1602.5**	**599.1**	**33.6%**	**26.3%**	**1581.0**	**421.8**	**34.3%**	**23.8%**	**1787.2**	**331.0**
LP Relaxation Average		***37.3%***	***6.8%***	***0.1***	***0.0***	***32.3%***	***6.7%***	***9.6***	***6.3***	***64.5%***	***13.6%***	***58.9***	***44.3***	***50.1%***	***13.7%***	***1895.8***	***313.3***	***33.8%***	***12.7%***	***2154.5***	***318.0***	***30.2%***	***8.6%***	***1480.2***	***68.4***
Standard Deviation		***33.2%***	***11.6%***	***0.2***	***0.1***	***30.1%***	***11.7%***	***22.3***	***14.6***	***42.7%***	***22.8%***	***139.3***	***119.0***	***36.7%***	***21.0%***	***1649.3***	***582.2***	***40.7%***	***21.3%***	***1548.5***	***485.6***	***42.6%***	***18.6%***	***1749.7***	***234.9***

Table A.1 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	5.0	1.4	0.0%	0.0%	3.9	2.2	0.0%	0.0%	0.4	0.1
	7	F1	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1.2	0.3	0.0%	0.0%	0.6	0.1	0.0%	0.0%	43.3	16.4	0.0%	0.0%	49.7	21.1	0.0%	0.0%	1.2	0.3
	9	F1	0.0%	0.0%	0.8	0.9	0.0%	0.0%	65.8	54.9	0.0%	0.0%	18.4	10.0	0.0%	0.0%	382.5	675.3	0.0%	0.0%	252.8	244.2	0.0%	0.0%	3.6	1.3
	11	F1	0.0%	0.0%	15.7	15.8	7.4%	7.9%	2584.4	1426.2	0.0%	0.0%	713.0	521.6	0.0%	0.0%	554.9	273.1	0.4%	0.9%	1673.0	1445.9	0.0%	0.0%	23.3	9.9
	13	F1	2.0%	4.5%	1160.3	1401.5	33.8%	7.0%	3600.0	0.0	36.4%	6.7%	3600.0	0.0	2.4%	5.4%	2505.0	957.5	3.8%	2.4%	3600.0	0.0	0.0%	0.0%	294.0	178.5
	15	F1	20.2%	10.6%	3600.0	0.0	43.2%	5.4%	3600.0	0.0	52.2%	7.4%	3600.0	0.0	12.8%	16.1%	3600.0	0.0	7.0%	2.3%	3600.0	0.0	0.0%	0.0%	382.6	204.9
	20	F1	47.4%	1.5%	3600.0	0.0	54.2%	3.3%	3600.0	0.0	78.2%	11.5%	3600.0	0.0	47.4%	11.4%	3600.0	0.0	13.2%	4.4%	3600.0	0.0	0.0%	0.0%	1115.3	275.7
	30	F1	62.8%	1.6%	3600.0	0.0	68.0%	1.6%	3600.0	0.0	86.8%	10.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	71.6%	1.8%	3600.0	0.0	75.8%	0.8%	3600.0	0.0	96.6%	7.6%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	79.6%	1.1%	3600.0	0.0	80.6%	1.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	82.6%	2.1%	3600.0	0.0	83.2%	1.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**33.3%**	**2.1%**	**2070.6**	**128.9**	**40.6%**	**2.6%**	**2532.0**	**134.7**	**50.0%**	**4.0%**	**2357.4**	**48.3**	**42.1%**	**3.0%**	**2281.0**	**174.9**	**38.6%**	**0.9%**	**2470.9**	**155.8**	**36.4%**	**0.0%**	**1474.6**	**61.0**
	Standard Deviation		**35.6%**	**3.1%**	**1787.3**	**422.1**	**34.2%**	**2.9%**	**1639.6**	**428.7**	**44.2%**	**4.8%**	**1734.8**	**157.0**	**47.9%**	**5.7%**	**1651.1**	**333.4**	**48.9%**	**1.5%**	**1625.7**	**434.0**	**50.5%**	**0.0%**	**1714.6**	**104.4**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	3.0	1.9	0.0%	0.0%	3.4	2.1	0.0%	0.0%	0.7	0.3
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.2	0.3	0.0%	0.0%	0.0	0.0	0.0%	0.0%	19.6	7.2	0.0%	0.0%	23.5	12.4	0.0%	0.0%	17.1	25.0
	9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.2	0.0%	0.0%	0.2	0.1	0.0%	0.0%	95.2	30.3	0.0%	0.0%	109.6	91.7	0.0%	0.0%	25.3	13.6
	11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	129.8	287.3	0.0%	0.0%	1.1	1.6	0.0%	0.0%	98.5	70.8	0.0%	0.0%	126.7	37.3	0.0%	0.0%	157.9	67.4
	13	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.5	0.5	0.0%	0.0%	0.7	0.6	0.0%	0.0%	361.7	254.2	0.0%	0.0%	206.5	111.2	0.0%	0.0%	393.2	130.3
	15	F2	0.0%	0.0%	0.1	0.0	0.0%	0.0%	4.6	5.5	0.0%	0.0%	14.9	25.0	0.0%	0.0%	927.3	1273.4	40.0%	54.8%	2084.0	1454.4	40.0%	54.8%	2692.2	1193.6
	20	F2	0.0%	0.0%	0.3	0.2	24.6%	37.0%	1444.8	1967.4	21.6%	36.0%	1492.4	1926.0	80.0%	44.7%	2944.4	1465.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F2	0.0%	0.0%	2.9	1.8	0.0%	0.0%	341.1	445.4	0.0%	0.0%	984.2	838.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	0.0%	0.0%	605.7	915.5	57.2%	52.3%	3041.9	1116.6	99.8%	0.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	4.4%	9.8%	1035.8	1456.9	91.8%	4.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	0.0%	0.0%	426.3	587.6	97.8%	3.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**0.4%**	**0.9%**	**188.3**	**269.3**	**24.7%**	**8.8%**	**1105.7**	**347.6**	**29.2%**	**3.3%**	**1208.5**	**253.8**	**43.6%**	**4.1%**	**1713.6**	**282.1**	**49.1%**	**5.0%**	**1868.5**	**155.4**	**49.1%**	**5.0%**	**1935.1**	**130.0**
	Standard Deviation		**1.3%**	**3.0%**	**350.9**	**500.8**	**39.0%**	**18.1%**	**1547.4**	**637.9**	**45.9%**	**10.8%**	**1612.8**	**608.7**	**50.5%**	**13.5%**	**1709.3**	**544.6**	**50.1%**	**16.5%**	**1755.3**	**432.7**	**50.1%**	**16.5%**	**1761.7**	**355.1**
	MIP Average		***16.8%***	***1.5%***	***1129.5***	***199.1***	***32.6%***	***5.7%***	***1818.8***	***241.1***	***39.6%***	***3.7%***	***1783.0***	***151.1***	***42.8%***	***3.5%***	***1997.3***	***228.5***	***43.8%***	***2.9%***	***2169.7***	***155.6***	***42.7%***	***2.5%***	***1704.9***	***95.5***
	Standard Deviation		***29.8%***	***3.0%***	***1583.6***	***457.7***	***36.7%***	***13.0%***	***1718.5***	***541.4***	***45.2%***	***8.2%***	***1737.1***	***446.3***	***48.0%***	***10.1%***	***1665.5***	***444.0***	***48.6%***	***11.6%***	***1679.5***	***422.9***	***49.5%***	***11.7%***	***1712.7***	***257.9***

Table A.2 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 2.

LP RELAXATION PROBLEM

Instance (# jobs)	Objective Function	CTP GAP	CTP SD	CTP T(s)	CTP SD(T(s))	AFCTP GAP	AFCTP SD	AFCTP T(s)	AFCTP SD(T(s))	APD GAP	APD SD	APD T(s)	APD SD(T(s))	TI GAP	TI SD	TI T(s)	TI SD(T(s))	TII GAP	TII SD	TII T(s)	TII SD(T(s))	ATI GAP	ATI SD	ATI T(s)	ATI SD(T(s))
5	F1	47.3%	3.7%	0.1	0.2	23.3%	4.5%	0.1	0.1	100.0%	0.0%	0.7	1.6	40.3%	4.8%	15.5	18.6	0.4%	0.4%	180.3	183.3	0.0%	0.0%	8.9	1.5
7	F1	56.2%	1.8%	0.0	0.0	38.4%	2.0%	0.0	0.0	100.0%	0.0%	0.0	0.0	50.3%	2.5%	405.9	572.8	2.8%	1.8%	2793.2	1379.9	0.0%	0.0%	7.3	8.4
9	F1	60.7%	1.9%	0.0	0.0	42.9%	1.3%	0.0	0.0	100.0%	0.0%	0.0	0.0	52.8%	1.0%	2622.1	1341.2	98.2%	4.1%	3600.0	0.0	0.2%	0.4%	47.4	68.3
11	F1	63.5%	1.8%	0.0	0.0	49.9%	2.2%	0.3	0.2	100.0%	0.0%	2.0	2.5	59.2%	1.8%	821.0	911.2	100.0%	0.0%	2680.0	1261.7	0.0%	0.0%	60.6	30.1
13	F1	65.6%	2.6%	4.0	5.3	53.1%	2.6%	1.0	1.5	100.0%	0.0%	22.1	44.0	85.6%	32.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	0.0%	0.0%	172.6	92.2
15	F1	68.5%	2.9%	2.4	4.2	58.0%	2.3%	30.7	67.0	100.0%	0.0%	7.5	7.8	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	0.0%	0.1%	455.4	240.0
20	F1	70.0%	2.3%	0.4	0.5	62.1%	2.9%	0.7	0.7	100.0%	0.0%	0.9	0.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	80.0%	44.7%	2963.6	871.5
30	F1	73.2%	3.4%	27.8	59.1	67.8%	3.3%	2.9	5.4	100.0%	0.0%	2.6	1.3	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F1	73.7%	1.5%	3.0	3.1	71.5%	1.6%	12.2	21.9	100.0%	0.0%	27.7	19.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F1	79.9%	1.7%	0.4	0.4	78.9%	1.8%	33.7	37.4	100.0%	0.0%	217.3	139.8	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F1	84.2%	1.0%	2.4	2.6	83.5%	1.0%	36.5	18.0	100.0%	0.0%	632.8	239.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F1 Average		**67.5%**	**2.2%**	**3.7**	**6.9**	**57.2%**	**2.3%**	**10.7**	**13.8**	**100.0%**	**0.0%**	**83.1**	**41.5**	**80.8%**	**3.8%**	**2642.2**	**258.5**	**81.9%**	**0.6%**	**3132.1**	**256.8**	**43.7%**	**4.1%**	**1646.9**	**119.3**
Standard Deviation		**10.5%**	**0.9%**	**8.1**	**17.4**	**18.1%**	**1.0%**	**15.2**	**21.5**	**0.0%**	**0.0%**	**193.2**	**77.7**	**24.6%**	**9.5%**	**1471.0**	**471.4**	**39.7%**	**1.3%**	**1038.5**	**529.5**	**50.4%**	**13.5%**	**1761.6**	**259.8**
5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.6	1.3	0.0%	0.0%	0.1	0.1	0.0%	0.0%	7.0	4.7	0.0%	0.0%	46.4	54.7	0.0%	0.0%	2.4	2.8
7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	767.8	1514.2	0.0%	0.0%	803.8	1564.5	0.0%	0.0%	19.6	11.6
9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1326.3	1412.5	0.0%	0.0%	2296.7	1785.9	0.0%	0.0%	83.3	74.5
11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	2489.1	1534.1	0.0%	0.0%	3600.0	0.0	0.0%	0.0%	214.0	128.8
13	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	20.0%	44.7%	0.1	0.0	20.0%	44.7%	2884.1	1600.7	20.0%	44.7%	3600.0	0.0	0.0%	0.0%	1805.3	1513.6
15	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.3	0.6	0.0%	0.0%	0.1	0.0	0.0%	0.0%	3600.0	0.0	0.0%	0.0%	3600.0	0.0	0.0%	0.0%	1604.7	1221.8
20	F2	36.3%	50.1%	0.0	0.0	36.3%	50.1%	19.3	43.0	60.0%	54.8%	0.2	0.1	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0	57.2%	52.5%	3600.0	0.0
30	F2	10.1%	22.7%	0.0	0.0	10.1%	22.7%	0.3	0.5	40.0%	54.8%	1.3	0.9	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0
50	F2	6.9%	15.3%	0.1	0.0	6.9%	15.3%	4.3	4.8	40.0%	54.8%	13.2	1.4	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0	40.0%	54.8%	3600.0	0.0
75	F2	24.1%	43.2%	0.1	0.0	24.1%	43.2%	32.5	17.9	100.0%	0.0%	96.8	54.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F2	13.4%	15.6%	0.3	0.1	13.4%	15.6%	122.4	89.4	100.0%	0.0%	340.6	97.3	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F2 Average		**8.3%**	**13.4%**	**0.1**	**0.0**	**8.3%**	**13.4%**	**16.3**	**14.3**	**32.7%**	**19.0%**	**41.1**	**14.0**	**30.9%**	**19.0%**	**2643.1**	**551.5**	**30.9%**	**19.0%**	**2904.3**	**309.6**	**30.7%**	**14.7%**	**1975.4**	**268.5**
Standard Deviation		**12.2%**	**18.5%**	**0.1**	**0.0**	**12.2%**	**18.5%**	**36.7**	**28.2**	**39.3%**	**26.5%**	**103.4**	**32.0**	**38.3%**	**26.5%**	**1333.3**	**765.4**	**38.3%**	**26.5%**	**1297.0**	**677.2**	**40.2%**	**25.2%**	**1666.3**	**548.9**
LP Relaxation Average		***37.9%***	***7.8%***	***1.9***	***3.4***	***32.7%***	***7.8%***	***13.5***	***14.1***	***66.4%***	***9.5%***	***62.1***	***27.7***	***55.8%***	***11.4%***	***2642.7***	***405.0***	***56.4%***	***9.8%***	***3018.2***	***283.2***	***37.2%***	***9.4%***	***1811.1***	***193.9***
Standard Deviation		***32.3%***	***14.0%***	***5.9***	***12.5***	***29.2%***	***14.0%***	***27.6***	***24.4***	***43.8%***	***20.7%***	***152.7***	***59.7***	***40.5%***	***20.9%***	***1370.0***	***638.2***	***46.2%***	***20.6%***	***1152.4***	***593.8***	***45.0%***	***20.5%***	***1681.7***	***426.0***

Table A.2 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	0.1	0.1	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1365.1	989.4	0.0%	0.0%	29.7	9.0	0.0%	0.0%	18.1	9.1
	7	F1	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1.9	1.0	0.0%	0.0%	0.7	0.2	22.8%	3.3%	3600.0	0.0	0.0%	0.0%	1436.5	1281.8	0.0%	0.0%	75.7	97.4
	9	F1	0.0%	0.0%	1.4	1.5	0.0%	0.0%	37.3	26.2	0.0%	0.0%	19.4	5.9	33.0%	2.7%	3600.0	0.0	0.0%	0.0%	1427.0	553.4	0.0%	0.0%	187.4	94.6
	11	F1	0.0%	0.0%	25.1	16.1	6.8%	4.4%	3233.8	818.9	0.0%	0.0%	612.5	393.5	68.0%	30.0%	3600.0	0.0	0.4%	0.5%	2431.2	1072.9	0.0%	0.0%	330.2	185.4
	13	F1	3.2%	7.2%	917.7	1514.3	28.8%	7.2%	3600.0	0.0	17.2%	11.3%	3479.3	269.9	85.2%	22.0%	3600.0	0.0	25.6%	42.4%	3600.0	0.0	0.0%	0.0%	545.5	277.2
	15	F1	12.8%	8.0%	3600.0	0.0	43.0%	3.5%	3600.0	0.0	49.0%	6.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	26.2%	41.5%	3600.0	0.0	0.0%	0.0%	1544.1	843.3
	20	F1	47.2%	5.4%	3600.0	0.0	56.0%	3.6%	3600.0	0.0	82.0%	8.2%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F1	61.8%	5.0%	3600.0	0.0	66.6%	3.4%	3600.0	0.0	99.4%	1.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	68.2%	1.8%	3600.0	0.0	73.8%	0.8%	3600.0	0.0	96.2%	8.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	76.8%	1.8%	3600.0	0.0	81.6%	0.9%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	82.4%	0.9%	3600.0	0.0	84.6%	0.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**32.0%**	**2.7%**	**2049.5**	**139.3**	**40.1%**	**2.2%**	**2588.5**	**76.9**	**49.4%**	**3.2%**	**2337.5**	**60.9**	**73.5%**	**5.3%**	**3396.8**	**89.9**	**50.2%**	**7.7%**	**2774.9**	**265.2**	**45.5%**	**0.0%**	**1881.9**	**137.0**
	Standard Deviation		**35.0%**	**3.1%**	**1800.0**	**456.1**	**34.4%**	**2.4%**	**1657.6**	**246.2**	**46.6%**	**4.4%**	**1736.1**	**136.7**	**37.4%**	**10.5%**	**673.8**	**298.3**	**48.6%**	**16.9%**	**1266.0**	**482.4**	**52.2%**	**0.0%**	**1693.4**	**251.9**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.2	0.5	0.0%	0.0%	0.1	0.0	0.0%	0.0%	31.1	33.4	0.0%	0.0%	18.9	7.2	0.0%	0.0%	7.4	4.2
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.1	0.0%	0.0%	0.2	0.2	0.0%	0.0%	287.6	169.2	0.0%	0.0%	129.2	82.5	0.0%	0.0%	91.8	69.2
	9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.3	0.4	0.0%	0.0%	0.3	0.2	0.0%	0.0%	1208.3	893.1	0.0%	0.0%	766.8	924.2	0.0%	0.0%	998.5	596.7
	11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.1	0.0%	0.0%	0.4	0.3	0.0%	0.0%	1522.8	1409.1	40.0%	54.8%	2085.2	1501.3	0.0%	0.0%	1712.0	651.7
	13	F2	0.0%	0.0%	0.1	0.1	0.0%	0.0%	2.8	2.2	0.0%	0.0%	3.5	3.8	60.0%	54.8%	2626.6	1333.8	80.0%	44.7%	3354.2	549.7	60.0%	54.8%	3239.1	711.5
	15	F2	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1.8	1.7	0.0%	0.0%	1.7	1.8	80.0%	44.7%	3100.1	1117.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	20	F2	0.0%	0.0%	0.4	0.2	16.2%	36.2%	737.4	1600.5	6.0%	13.4%	739.0	1599.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F2	0.0%	0.0%	3.7	1.6	10.8%	22.5%	960.1	1491.6	30.2%	44.8%	1814.5	1690.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	0.0%	0.0%	765.0	1411.3	20.8%	41.1%	2954.2	1255.8	80.2%	44.3%	3008.2	1323.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	22.4%	43.7%	3492.3	137.6	98.8%	1.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	11.6%	13.4%	2867.4	1591.1	98.2%	1.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**3.1%**	**5.2%**	**648.1**	**285.6**	**22.3%**	**9.3%**	**1077.9**	**395.7**	**28.8%**	**9.3%**	**1160.7**	**420.0**	**58.2%**	**9.0%**	**2434.2**	**450.6**	**65.5%**	**9.0%**	**2541.3**	**278.6**	**60.0%**	**5.0%**	**2513.5**	**184.8**
	Standard Deviation		**7.3%**	**13.4%**	**1280.0**	**603.7**	**38.4%**	**16.0%**	**1527.5**	**681.2**	**42.8%**	**17.9%**	**1548.8**	**723.0**	**47.7%**	**20.2%**	**1413.9**	**600.7**	**45.7%**	**20.2%**	**1514.1**	**506.3**	**49.0%**	**16.5%**	**1506.4**	**302.6**
	MIP Average		*17.6%*	*4.0%*	*1348.8*	*212.5*	*31.2%*	*5.8%*	*1833.2*	*236.3*	*39.1%*	*6.3%*	*1749.1*	*240.4*	*65.9%*	*7.2%*	*2915.5*	*270.3*	*57.8%*	*8.4%*	*2658.1*	*271.9*	*52.7%*	*2.5%*	*2197.7*	*160.9*
	Standard Deviation		*28.8%*	*9.6%*	*1684.4*	*527.5*	*36.8%*	*11.7%*	*1737.0*	*525.8*	*44.9%*	*13.1%*	*1714.7*	*540.0*	*42.6%*	*15.8%*	*1187.8*	*498.2*	*46.7%*	*18.2%*	*1367.2*	*482.6*	*50.0%*	*11.7%*	*1597.0*	*272.8*

Table A.3 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 3.

LP RELAXATION PROBLEM

Instance	Objective	Mixed Integer Program Formulations																							
(# jobs)	Function	CTP				AFCTP				APD				TI				TII				ATI			
		GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
5	F1	54.8%	2.5%	0.0	0.1	33.5%	1.2%	0.0	0.0	100.0%	0.1%	0.3	0.5	40.6%	4.7%	753.5	1592.0	8.6%	3.8%	4.3	6.6	0.3%	0.6%	0.2	0.0
7	F1	58.0%	4.4%	4.9	10.9	42.1%	4.8%	0.0	0.0	100.0%	0.0%	0.0	0.0	48.3%	4.9%	68.5	88.7	12.7%	3.8%	76.9	112.3	0.0%	0.0%	1.3	0.3
9	F1	61.6%	4.3%	0.0	0.0	46.5%	3.5%	0.0	0.0	100.0%	0.0%	0.0	0.0	53.2%	4.2%	160.9	342.1	14.3%	4.0%	496.3	784.8	0.1%	0.3%	6.8	2.9
11	F1	61.4%	2.5%	0.0	0.0	50.4%	3.2%	0.0	0.0	100.0%	0.0%	0.6	1.0	54.4%	2.7%	469.8	502.0	16.2%	3.4%	1497.7	1784.4	0.4%	0.5%	27.2	3.8
13	F1	60.2%	5.3%	3.7	7.0	51.5%	4.8%	0.0	0.0	100.0%	0.0%	0.2	0.1	50.4%	6.1%	1339.3	1313.4	24.0%	13.1%	2005.3	1487.7	0.0%	0.0%	54.4	16.7
15	F1	64.4%	1.5%	0.2	0.1	56.7%	1.0%	0.0	0.0	100.0%	0.0%	1.4	0.8	58.1%	1.3%	1749.5	1174.3	54.4%	18.8%	2397.8	1652.4	0.8%	1.2%	133.8	27.8
20	F1	61.8%	3.7%	6.3	10.5	55.9%	3.4%	0.1	0.0	100.0%	0.0%	2.3	2.5	61.5%	10.5%	2245.6	1857.0	100.0%	0.0%	3600.0	0.0	24.4%	43.2%	1495.7	1227.5
30	F1	65.0%	2.3%	1.3	1.4	61.7%	2.2%	0.9	1.1	100.0%	0.0%	17.1	16.8	92.3%	17.2%	2971.6	1405.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F1	69.3%	4.4%	22.7	47.8	68.1%	3.9%	2.7	0.8	100.0%	0.0%	69.3	117.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F1	71.4%	1.0%	0.3	0.1	70.8%	1.0%	13.2	11.4	100.0%	0.0%	291.9	162.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F1	73.4%	1.7%	1.0	1.0	73.1%	1.6%	24.9	7.0	100.0%	0.0%	1081.0	520.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F1 Average		**63.8%**	**3.0%**	**3.7**	**7.2**	**55.5%**	**2.8%**	**3.8**	**1.9**	**100.0%**	**0.0%**	**133.1**	**74.7**	**69.0%**	**4.7%**	**1869.0**	**752.2**	**57.3%**	**4.3%**	**2225.3**	**529.8**	**38.7%**	**4.2%**	**1465.4**	**116.3**
Standard Deviation		**5.7%**	**1.4%**	**6.7**	**14.1**	**12.4%**	**1.5%**	**8.0**	**3.8**	**0.0%**	**0.0%**	**326.2**	**158.1**	**23.8%**	**5.2%**	**1414.3**	**722.1**	**42.6%**	**6.2%**	**1503.1**	**752.9**	**49.1%**	**12.9%**	**1745.8**	**368.7**
5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.5	1.1	0.0%	0.0%	2.6	5.3	0.0%	0.0%	5.3	10.4	0.0%	0.0%	0.5	0.4
7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	85.8	177.9	0.0%	0.0%	125.2	239.7	0.0%	0.0%	3.5	1.8
9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	20.0%	44.7%	0.0	0.0	20.0%	44.7%	1033.4	1553.8	20.0%	44.7%	2519.6	1166.5	0.0%	0.0%	23.0	15.1
11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	40.0%	54.8%	0.0	0.0	40.0%	54.8%	2881.0	1607.7	40.0%	54.8%	2881.9	1605.6	0.0%	0.0%	96.1	72.5
13	F2	19.9%	27.4%	0.0	0.0	19.9%	27.4%	0.0	0.1	40.0%	54.8%	0.2	0.3	19.9%	27.4%	3600.0	0.0	40.0%	54.8%	3600.0	0.0	7.9%	10.8%	681.7	293.5
15	F2	20.0%	44.7%	0.0	0.0	20.0%	44.7%	0.0	0.0	20.0%	44.7%	0.0	0.0	0.0%	0.0%	3600.0	0.0	20.0%	44.7%	2889.4	1588.9	4.7%	10.5%	460.2	386.8
20	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.3	0.5	20.0%	44.7%	1.2	2.6	20.0%	44.7%	2885.6	1597.4	20.0%	44.7%	3600.0	0.0	0.0%	0.0%	1747.4	1192.6
30	F2	1.1%	2.5%	0.0	0.0	1.1%	2.5%	0.2	0.1	80.0%	44.7%	4.4	6.2	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0
50	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	4.1	3.1	80.0%	44.7%	14.6	18.6	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0
75	F2	2.1%	2.2%	0.1	0.0	2.1%	2.2%	12.8	9.2	100.0%	0.0%	62.4	72.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F2	1.5%	2.1%	0.3	0.0	1.5%	2.1%	90.8	52.0	80.0%	44.7%	445.5	298.5	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0	80.0%	44.7%	3600.0	0.0
F2 Average		**4.1%**	**7.2%**	**0.0**	**0.0**	**4.1%**	**7.2%**	**9.8**	**5.9**	**43.6%**	**34.4%**	**48.1**	**36.3**	**40.0%**	**27.8%**	**2589.9**	**449.3**	**43.6%**	**34.4%**	**2729.2**	**419.2**	**32.1%**	**14.1%**	**1583.0**	**178.4**
Standard Deviation		**7.9%**	**14.8%**	**0.1**	**0.0**	**7.9%**	**14.8%**	**27.1**	**15.5**	**35.6%**	**22.4%**	**133.1**	**89.5**	**38.0%**	**22.9%**	**1472.5**	**732.2**	**35.6%**	**22.4%**	**1372.7**	**677.3**	**42.4%**	**20.1%**	**1673.8**	**362.4**
LP Relaxation Average		***33.9%***	***5.1%***	***1.9***	***3.6***	***29.8%***	***5.0%***	***6.8***	***3.9***	***71.8%***	***17.2%***	***90.6***	***55.5***	***54.5%***	***16.2%***	***2229.4***	***600.7***	***50.5%***	***19.3%***	***2477.3***	***474.5***	***35.4%***	***9.1%***	***1524.2***	***147.4***
Standard Deviation		***31.3%***	***10.5%***	***5.0***	***10.4***	***28.2%***	***10.5%***	***19.8***	***11.2***	***37.9%***	***23.4%***	***247.0***	***126.9***	***34.3%***	***20.1%***	***1456.4***	***726.4***	***38.9%***	***22.2%***	***1428.2***	***701.1***	***44.9%***	***17.3%***	***1670.0***	***358.1***

Table A.3 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.2	0.2	0.0%	0.0%	207.6	134.7	0.0%	0.0%	17.8	10.7	0.0%	0.0%	1.8	0.7
	7	F1	0.0%	0.0%	0.0	0.0	0.0%	0.0%	1.2	0.5	0.0%	0.0%	1.0	0.6	0.0%	0.0%	442.7	412.6	0.0%	0.0%	591.3	700.3	0.0%	0.0%	6.0	1.2
	9	F1	0.0%	0.0%	0.3	0.2	0.0%	0.0%	31.7	15.1	0.0%	0.0%	49.0	35.2	13.0%	12.2%	2446.0	1581.5	4.4%	4.5%	2480.7	1534.2	0.0%	0.0%	38.5	30.3
	11	F1	0.0%	0.0%	17.6	14.2	1.8%	3.5%	2534.7	1007.1	0.0%	0.0%	1310.8	208.2	17.4%	13.2%	3560.3	88.8	11.4%	3.5%	3600.0	0.0	0.0%	0.0%	127.6	72.8
	13	F1	0.0%	0.0%	144.0	115.8	23.0%	8.9%	3600.0	0.0	34.4%	8.4%	3600.0	0.0	31.8%	11.8%	3600.0	0.0	19.2%	13.2%	3600.0	0.0	0.0%	0.0%	133.8	41.9
	15	F1	6.4%	8.8%	2614.2	990.7	41.6%	4.2%	3600.0	0.0	56.2%	3.0%	3600.0	0.0	58.8%	3.6%	3600.0	0.0	44.4%	17.2%	3600.0	0.0	0.0%	0.0%	663.3	484.0
	20	F1	37.6%	6.3%	3600.0	0.0	49.2%	4.5%	3600.0	0.0	79.4%	12.2%	3600.0	0.0	70.2%	3.8%	3600.0	0.0	53.4%	13.1%	3600.0	0.0	21.4%	44.0%	2940.7	1065.7
	30	F1	54.4%	3.0%	3600.0	0.0	59.2%	2.5%	3600.0	0.0	76.6%	11.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	64.6%	4.5%	3600.0	0.0	67.8%	3.3%	3600.0	0.0	91.2%	12.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	71.2%	2.2%	3600.0	0.0	70.6%	0.9%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	74.0%	1.9%	3600.0	0.0	73.0%	1.6%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**28.0%**	**2.4%**	**1888.7**	**101.9**	**35.1%**	**2.7%**	**2524.3**	**93.0**	**48.9%**	**4.3%**	**2414.6**	**22.2**	**53.7%**	**4.1%**	**2896.0**	**201.6**	**48.4%**	**4.7%**	**2899.1**	**204.1**	**38.3%**	**4.0%**	**1664.7**	**154.2**
	Standard Deviation		**32.4%**	**3.0%**	**1800.4**	**296.8**	**30.9%**	**2.7%**	**1644.7**	**303.2**	**43.1%**	**5.5%**	**1682.4**	**62.6**	**42.5%**	**5.6%**	**1317.6**	**474.4**	**44.2%**	**6.6%**	**1331.7**	**488.5**	**49.3%**	**13.3%**	**1745.9**	**334.1**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	4.8	2.1	0.0%	0.0%	6.4	3.7	0.0%	0.0%	3.7	2.0
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	28.4	4.3	0.0%	0.0%	28.6	8.2	0.0%	0.0%	48.1	37.6
	9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	73.2	14.4	0.0%	0.0%	118.5	87.0	0.0%	0.0%	218.0	124.6
	11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.2	0.2	0.0%	0.0%	0.3	0.1	0.0%	0.0%	151.3	96.8	0.0%	0.0%	203.0	137.5	0.0%	0.0%	630.0	249.4
	13	F2	0.0%	0.0%	0.1	0.0	0.0%	0.0%	108.2	200.9	0.0%	0.0%	5.7	7.5	20.0%	44.7%	1110.9	1410.6	27.6%	43.7%	2209.4	1744.4	40.0%	54.8%	2914.3	739.1
	15	F2	0.0%	0.0%	0.1	0.0	0.0%	0.0%	489.6	1093.9	0.0%	0.0%	12.3	25.3	20.0%	44.7%	1782.5	1698.0	40.0%	54.8%	1903.5	1591.2	60.0%	54.8%	3180.4	891.8
	20	F2	0.0%	0.0%	0.1	0.0	0.0%	0.0%	17.2	29.4	0.0%	0.0%	36.1	45.6	80.0%	44.7%	3068.4	1188.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F2	0.0%	0.0%	1.0	0.5	1.2%	2.7%	813.4	1561.5	1.2%	2.7%	1832.3	1173.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	0.0%	0.0%	10.4	11.0	12.2%	27.3%	1316.0	1281.0	79.0%	44.2%	3090.4	1139.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	0.0%	0.0%	342.9	332.8	55.8%	29.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	0.0%	0.0%	227.7	301.3	94.8%	6.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**0.0%**	**0.0%**	**52.9**	**58.7**	**14.9%**	**5.9%**	**904.1**	**378.8**	**25.5%**	**4.3%**	**1107.0**	**217.4**	**47.3%**	**12.2%**	**1874.5**	**401.3**	**51.6%**	**9.0%**	**2042.7**	**324.7**	**54.5%**	**10.0%**	**2272.2**	**185.9**
	Standard Deviation		**0.0%**	**0.0%**	**117.8**	**128.0**	**31.3%**	**11.2%**	**1399.1**	**611.4**	**43.7%**	**13.3%**	**1592.1**	**464.5**	**47.6%**	**20.9%**	**1644.5**	**672.6**	**48.0%**	**20.1%**	**1657.2**	**666.4**	**47.4%**	**22.2%**	**1645.0**	**322.6**
	MIP Average		***14.0%***	***1.2%***	***970.8***	***80.3***	***25.0%***	***4.3%***	***1714.2***	***235.9***	***37.2%***	***4.3%***	***1760.8***	***119.8***	***50.5%***	***8.1%***	***2385.3***	***301.5***	***50.0%***	***6.8%***	***2470.9***	***264.4***	***46.4%***	***7.0%***	***1968.5***	***170.1***
	Standard Deviation		***26.6%***	***2.4%***	***1559.7***	***224.1***	***32.1%***	***8.1%***	***1705.2***	***493.2***	***44.0%***	***9.9%***	***1732.9***	***338.5***	***44.2%***	***15.5%***	***1545.2***	***577.1***	***45.1%***	***14.7%***	***1531.1***	***573.5***	***47.9%***	***18.1%***	***1684.3***	***320.9***

Table A.4 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 4.

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
LP RELAXATION PROBLEM	5	F1	23.5%	8.0%	2.2	5.0	16.8%	6.4%	2.4	3.3	100.0%	0.0%	0.1	0.2	20.3%	8.0%	0.5	0.5	1.2%	2.0%	0.9	0.6	0.0%	0.0%	0.1	0.0
	7	F1	34.4%	5.5%	0.0	0.0	25.9%	4.7%	0.0	0.0	100.0%	0.0%	0.0	0.0	22.6%	2.6%	23.6	26.5	5.0%	1.9%	34.1	39.1	0.0%	0.0%	0.7	0.4
	9	F1	26.8%	10.7%	0.0	0.0	20.5%	8.6%	0.0	0.0	100.0%	0.0%	0.0	0.0	23.9%	12.7%	45.3	36.6	3.4%	1.9%	187.4	116.4	0.0%	0.0%	2.9	1.5
	11	F1	27.8%	3.4%	0.0	0.0	23.5%	2.9%	0.1	0.3	100.0%	0.0%	0.0	0.0	21.3%	4.8%	156.3	248.6	3.8%	0.8%	550.6	440.3	0.0%	0.1%	6.7	1.7
	13	F1	23.0%	4.2%	0.0	0.0	20.5%	4.2%	1.5	2.5	100.0%	0.0%	0.4	0.6	18.7%	4.5%	583.9	552.6	4.5%	1.3%	1183.4	1119.8	0.3%	0.5%	13.5	4.4
	15	F1	26.6%	3.0%	0.0	0.0	23.8%	2.3%	0.9	1.2	100.0%	0.0%	1.1	0.9	23.0%	3.2%	631.5	1028.6	4.5%	1.2%	1875.8	1228.8	0.0%	0.0%	42.6	16.3
	20	F1	23.5%	2.8%	0.1	0.1	22.0%	2.9%	3.1	4.5	100.0%	0.0%	5.0	9.3	24.9%	10.9%	1533.4	1291.4	6.7%	3.9%	2934.7	1487.7	0.2%	0.3%	301.6	46.5
	30	F1	22.9%	3.9%	0.4	0.4	22.1%	3.7%	2.5	2.7	100.0%	0.0%	8.6	15.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3335.1	592.3
	50	F1	19.9%	2.6%	0.4	0.2	19.7%	2.6%	2.3	0.8	100.0%	0.0%	38.8	36.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	31.9%	4.7%	2.0	1.0	31.6%	4.7%	90.7	122.5	100.0%	0.0%	193.2	89.8	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	32.4%	1.2%	2.0	0.6	32.3%	1.2%	55.2	47.3	100.0%	0.0%	1180.7	419.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**26.6%**	**4.6%**	**0.6**	**0.7**	**23.5%**	**4.0%**	**14.4**	**16.8**	**100.0%**	**0.0%**	**129.8**	**52.0**	**50.4%**	**4.2%**	**1579.5**	**289.5**	**39.0%**	**1.2%**	**1924.2**	**403.0**	**36.4%**	**0.1%**	**1318.5**	**60.3**
	Standard Deviation		**4.6%**	**2.7%**	**0.9**	**1.5**	**4.8%**	**2.1%**	**30.0**	**37.7**	**0.0%**	**0.0%**	**353.2**	**125.0**	**39.3%**	**4.5%**	**1658.8**	**466.1**	**48.4%**	**1.2%**	**1580.5**	**583.1**	**50.4%**	**0.2%**	**1759.9**	**177.0**
	5	F2	2.4%	5.3%	0.0	0.0	2.4%	5.3%	1.1	2.4	40.0%	54.8%	0.1	0.2	2.2%	5.0%	1.9	2.7	1.6%	3.6%	3.3	4.1	1.6%	3.6%	0.1	0.1
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	36.0	39.7	0.0%	0.0%	55.3	53.5	0.0%	0.0%	0.9	1.1
	9	F2	5.5%	12.3%	0.0	0.0	5.5%	12.3%	1.6	3.2	60.0%	54.8%	0.0	0.0	4.0%	9.0%	247.2	163.7	0.0%	0.0%	437.7	378.1	0.0%	0.0%	11.7	7.3
	11	F2	9.8%	21.9%	0.0	0.0	9.8%	21.9%	1.6	2.7	80.0%	44.7%	0.1	0.2	8.7%	19.5%	1656.8	1421.3	4.5%	10.1%	1785.0	1208.3	3.7%	8.3%	75.5	64.4
	13	F2	2.8%	4.5%	0.0	0.0	2.8%	4.5%	0.9	1.2	80.0%	44.7%	1.2	0.6	60.0%	54.8%	2203.4	1913.4	40.0%	54.8%	2464.9	1203.8	0.0%	0.0%	214.7	115.3
	15	F2	2.0%	4.2%	0.0	0.0	2.0%	4.2%	2.1	4.6	80.0%	44.7%	0.4	0.4	60.7%	53.8%	2602.0	1569.2	60.2%	54.5%	2901.7	1561.4	0.1%	0.2%	554.5	300.5
	20	F2	10.9%	7.0%	0.0	0.0	10.9%	7.0%	0.7	0.5	100.0%	0.0%	0.5	0.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	4.1%	9.0%	1972.2	1541.0
	30	F2	12.3%	14.5%	0.0	0.0	12.3%	14.5%	0.6	0.6	100.0%	0.0%	3.5	5.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	2.9%	2.1%	0.1	0.0	2.9%	2.1%	13.9	20.6	100.0%	0.0%	44.5	60.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	1.4%	1.5%	0.2	0.2	1.4%	1.5%	68.2	93.3	100.0%	0.0%	294.9	200.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	0.7%	0.6%	0.3	0.1	0.7%	0.6%	96.3	65.2	100.0%	0.0%	395.2	326.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**4.6%**	**6.7%**	**0.1**	**0.0**	**4.6%**	**6.7%**	**17.0**	**17.7**	**76.4%**	**22.2%**	**67.3**	**54.0**	**57.8%**	**12.9%**	**2249.7**	**464.6**	**55.1%**	**11.2%**	**2331.6**	**400.8**	**37.2%**	**1.9%**	**1566.3**	**184.5**
	Standard Deviation		**4.4%**	**6.8%**	**0.1**	**0.1**	**4.4%**	**6.8%**	**33.1**	**31.6**	**32.0%**	**25.7%**	**139.7**	**108.8**	**45.4%**	**21.3%**	**1534.9**	**761.5**	**46.7%**	**21.7%**	**1512.5**	**610.4**	**49.8%**	**3.5%**	**1705.9**	**459.1**
	LP Relaxation Average		***15.6%***	***5.6%***	***0.4***	***0.3***	***14.1%***	***5.4%***	***15.7***	***17.2***	***88.2%***	***11.1%***	***98.6***	***53.0***	***54.1%***	***8.6%***	***1914.6***	***377.0***	***47.1%***	***6.2%***	***2127.9***	***401.9***	***36.8%***	***1.0%***	***1442.4***	***122.4***
	Standard Deviation		***12.1%***	***5.2%***	***0.7***	***1.1***	***10.7%***	***5.1%***	***30.9***	***33.9***	***25.2%***	***21.0%***	***264.1***	***114.3***	***41.6%***	***15.7%***	***1596.8***	***622.6***	***47.1%***	***15.9%***	***1523.9***	***582.5***	***48.9%***	***2.6%***	***1696.1***	***345.5***

Table A.4 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	0.5	0.6	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.2	0.4	0.0%	0.0%	5.2	2.5	0.0%	0.0%	2.2	0.6	0.0%	0.0%	1.1	0.2
	7	F1	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.6	0.3	0.0%	0.0%	0.9	1.0	0.0%	0.0%	44.0	32.1	0.0%	0.0%	34.0	15.8	0.0%	0.0%	3.4	1.8
	9	F1	0.0%	0.0%	0.1	0.2	0.0%	0.0%	3.6	3.2	0.0%	0.0%	9.8	3.5	0.0%	0.0%	83.8	35.0	0.0%	0.0%	133.4	116.2	0.0%	0.0%	8.9	3.7
	11	F1	0.0%	0.0%	1.5	1.3	0.0%	0.0%	79.9	53.2	0.0%	0.0%	171.3	72.2	0.6%	0.0%	932.5	133.0	0.0%	0.0%	471.5	426.3	0.0%	0.0%	25.7	11.2
	13	F1	0.0%	0.0%	4.1	4.6	0.0%	0.0%	1316.5	1290.0	2.8%	4.8%	2622.1	1238.4	0.0%	1.3%	749.2	1407.1	0.2%	0.4%	1507.2	1329.8	0.0%	0.0%	82.0	74.9
	15	F1	0.0%	0.0%	75.1	63.3	6.6%	3.8%	3512.3	196.1	18.0%	1.9%	3600.0	0.0	6.0%	5.8%	2618.7	1193.4	1.2%	1.1%	2809.1	1085.6	0.0%	0.0%	99.7	39.5
	20	F1	5.2%	4.9%	2932.1	1493.5	15.4%	3.6%	3600.0	0.0	58.6%	26.3%	3600.0	0.0	17.8%	8.6%	3600.0	1118.8	4.4%	3.8%	3600.0	0.0	0.0%	0.0%	1166.7	872.9
	30	F1	14.0%	3.7%	3600.0	0.0	21.0%	3.5%	3600.0	0.0	76.0%	32.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	16.2%	2.6%	3600.0	0.0	24.0%	3.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	30.4%	4.9%	3600.0	0.0	33.6%	3.6%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	31.4%	1.7%	3600.0	0.0	33.8%	0.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**8.8%**	**1.6%**	**1583.1**	**142.1**	**12.2%**	**1.7%**	**2083.0**	**140.3**	**41.4%**	**5.9%**	**2218.6**	**119.6**	**38.6%**	**1.4%**	**2039.4**	**356.5**	**36.9%**	**0.5%**	**2087.0**	**270.4**	**36.4%**	**0.0%**	**1435.2**	**91.3**
	Standard Deviation		**12.4%**	**2.1%**	**1813.2**	**448.6**	**13.9%**	**1.8%**	**1764.9**	**385.9**	**45.5%**	**11.7%**	**1747.0**	**371.7**	**49.0%**	**2.9%**	**1653.7**	**572.5**	**50.1%**	**1.2%**	**1652.0**	**483.6**	**50.5%**	**0.0%**	**1748.1**	**260.3**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.1	0.0%	0.0%	1.9	1.1	0.0%	0.0%	2.3	1.6	0.0%	0.0%	1.0	0.3
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.1	0.0%	0.0%	18.5	9.7	0.0%	0.0%	19.1	10.5	0.0%	0.0%	8.6	4.4
	9	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	4.9	5.8	0.0%	0.0%	0.4	0.4	0.0%	0.0%	38.2	18.7	0.0%	0.0%	23.5	10.3	0.0%	0.0%	75.6	58.4
	11	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	19.6	41.0	0.0%	0.0%	8.6	14.8	0.0%	0.0%	199.2	271.4	0.0%	0.0%	204.1	271.3	0.0%	0.0%	146.8	88.8
	13	F2	0.0%	0.0%	0.0	0.0	0.6%	1.3%	782.3	1580.5	0.0%	0.0%	12.1	14.1	0.0%	0.0%	146.7	78.5	0.0%	0.0%	385.5	280.0	0.0%	0.0%	476.1	391.5
	15	F2	0.0%	0.0%	0.1	0.0	2.2%	4.4%	1441.1	1970.8	0.0%	0.0%	78.2	100.5	0.0%	0.0%	607.0	514.4	0.0%	0.0%	1046.7	1010.5	40.0%	54.8%	2154.1	1329.1
	20	F2	0.0%	0.0%	0.2	0.1	10.0%	7.3%	3600.0	0.0	2.8%	6.3%	1580.1	1697.7	63.0%	50.9%	3043.8	1243.6	40.0%	54.8%	3065.1	735.0	100.0%	0.0%	3600.0	0.0
	30	F2	0.0%	0.0%	1.0	0.8	14.0%	14.6%	3200.4	893.6	75.2%	33.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	0.0%	0.0%	21.9	30.5	15.2%	16.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	0.0%	0.0%	861.9	1393.5	52.2%	15.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	0.0%	0.0%	1007.6	1529.7	81.6%	8.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**0.0%**	**0.0%**	**172.1**	**268.6**	**16.0%**	**6.2%**	**1804.4**	**408.3**	**34.4%**	**3.6%**	**1461.8**	**166.2**	**42.1%**	**4.6%**	**1677.7**	**194.3**	**40.0%**	**5.0%**	**1740.6**	**210.8**	**49.1%**	**5.0%**	**1896.6**	**170.2**
	Standard Deviation		**0.0%**	**0.0%**	**378.5**	**590.7**	**26.7%**	**6.6%**	**1701.7**	**731.3**	**47.6%**	**10.2%**	**1756.1**	**508.8**	**49.5%**	**15.4%**	**1748.5**	**384.2**	**49.0%**	**16.5%**	**1708.7**	**349.9**	**50.1%**	**16.5%**	**1735.7**	**401.6**
	MIP Average		***4.4%***	***0.8%***	***877.6***	***205.4***	***14.1%***	***3.9%***	***1943.7***	***274.3***	***37.9%***	***4.8%***	***1840.2***	***142.9***	***40.3%***	***3.0%***	***1858.6***	***275.4***	***38.4%***	***2.7%***	***1913.8***	***240.6***	***42.7%***	***2.5%***	***1665.9***	***130.8***
	Standard Deviation		***9.7%***	***1.6%***	***1468.1***	***515.9***	***20.8%***	***5.3%***	***1697.8***	***586.9***	***45.6%***	***10.7%***	***1752.7***	***435.5***	***48.1%***	***10.9%***	***1671.0***	***483.0***	***48.4%***	***11.7%***	***1649.6***	***413.0***	***49.5%***	***11.7%***	***1716.3***	***332.7***

Table A.5 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 5.

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
LP RELAXATION PROBLEM	5	F1	52.6%	1.4%	1.4	2.9	28.9%	2.4%	0.4	0.6	100.0%	0.0%	0.2	0.2	38.7%	3.4%	3.7	3.7	4.2%	2.3%	9.3	14.0	0.0%	0.0%	0.1	0.0
	7	F1	60.3%	4.0%	1.2	1.7	41.2%	2.8%	0.0	0.0	100.0%	0.0%	0.1	0.0	50.6%	4.8%	28.7	41.9	3.5%	1.4%	33.8	30.9	0.0%	0.0%	0.4	0.2
	9	F1	61.4%	2.9%	1.2	2.3	46.2%	2.2%	0.0	0.0	100.0%	0.0%	0.1	0.0	52.5%	4.3%	62.5	58.6	6.6%	3.0%	142.5	141.0	0.0%	0.0%	1.8	0.5
	11	F1	60.0%	2.5%	0.2	0.4	47.5%	3.2%	3.0	6.4	100.0%	0.0%	0.2	0.2	51.9%	3.6%	98.4	115.3	4.1%	0.9%	254.3	254.9	0.0%	0.0%	6.9	3.5
	13	F1	63.7%	3.2%	4.0	5.7	54.0%	3.1%	5.1	5.4	100.0%	0.0%	0.7	0.6	56.3%	3.8%	176.8	309.5	5.3%	0.7%	975.3	1453.5	0.0%	0.0%	47.0	40.2
	15	F1	65.7%	3.7%	27.1	58.5	55.6%	3.6%	23.9	41.3	100.0%	0.0%	1.6	1.2	59.5%	4.6%	254.2	408.9	10.5%	8.9%	2095.8	1391.5	0.1%	0.1%	104.5	29.4
	20	F1	67.8%	3.4%	22.5	45.8	59.7%	3.8%	13.9	15.1	100.0%	0.0%	0.4	0.1	84.5%	18.6%	2252.7	1845.9	32.1%	14.1%	2845.1	1038.2	20.0%	44.7%	410.3	149.9
	30	F1	71.9%	1.9%	1.0	1.1	67.4%	1.7%	4.0	4.8	100.0%	0.0%	2.6	0.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3397.0	453.9
	50	F1	77.5%	1.3%	0.9	0.4	75.4%	1.3%	6.4	4.7	100.0%	0.0%	29.8	11.8	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	80.7%	1.7%	4.1	3.2	79.6%	1.6%	71.5	78.2	100.0%	0.0%	208.4	144.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	84.0%	0.5%	5.7	5.8	83.4%	0.5%	217.4	120.4	100.0%	0.0%	524.7	379.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**67.8%**	**2.4%**	**6.3**	**11.6**	**58.1%**	**2.4%**	**31.4**	**25.2**	**100.0%**	**0.0%**	**69.9**	**49.0**	**72.2%**	**3.9%**	**1570.6**	**253.1**	**42.4%**	**2.9%**	**1886.9**	**393.1**	**38.2%**	**4.1%**	**1342.5**	**61.6**
	Standard Deviation		**9.7%**	**1.1%**	**9.4**	**20.3**	**17.0%**	**1.0%**	**65.1**	**39.6**	**0.0%**	**0.0%**	**163.1**	**117.6**	**24.6%**	**5.3%**	**1728.9**	**546.3**	**46.3%**	**4.5%**	**1618.6**	**592.7**	**49.4%**	**13.5%**	**1754.3**	**137.6**
	5	F2	98.6%	3.1%	0.0	0.0	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.1	0.0	96.9%	6.9%	5.8	6.1	72.3%	32.5%	8.6	8.6	75.5%	19.7%	0.4	0.2
	7	F2	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.1	0.0	100.0%	0.0%	34.7	44.4	68.5%	31.3%	53.1	71.4	55.3%	38.9%	3.7	1.3
	9	F2	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.1	0.0	100.0%	0.0%	118.7	106.8	63.0%	11.6%	978.1	1108.2	41.2%	12.8%	30.6	7.5
	11	F2	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.0	0.0	100.0%	0.0%	0.1	0.1	100.0%	0.0%	420.6	478.5	73.7%	23.8%	1259.3	1584.2	54.8%	20.1%	129.8	47.8
	13	F2	98.4%	2.9%	0.1	0.1	98.4%	2.9%	0.0	0.0	100.0%	0.0%	0.4	0.5	96.3%	5.3%	1691.7	1742.6	77.2%	32.1%	2907.0	951.5	32.0%	11.7%	285.9	64.3
	15	F2	97.0%	5.9%	0.1	0.1	97.0%	5.9%	0.0	0.0	100.0%	0.0%	0.4	0.5	99.9%	0.2%	3301.0	668.7	100.0%	0.0%	3600.0	0.0	29.0%	9.7%	621.2	272.2
	20	F2	98.2%	1.7%	1.4	0.9	99.2%	1.2%	0.1	0.1	100.0%	0.0%	0.7	0.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	41.1%	16.6%	2919.9	619.3
	30	F2	97.1%	3.5%	0.6	0.7	97.1%	3.5%	0.6	0.4	100.0%	0.0%	1.9	0.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	96.8%	2.1%	0.5	0.6	96.8%	2.1%	5.9	2.4	100.0%	0.0%	36.2	25.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	97.4%	1.7%	42.2	85.0	97.4%	1.7%	44.3	23.4	100.0%	0.0%	197.3	104.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	98.3%	0.6%	3.7	6.0	98.3%	0.6%	594.9	1102.6	100.0%	0.0%	699.6	667.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**98.3%**	**2.0%**	**4.4**	**8.5**	**98.6%**	**1.6%**	**58.7**	**102.6**	**100.0%**	**0.0%**	**85.2**	**72.6**	**99.4%**	**1.1%**	**2142.9**	**277.0**	**86.8%**	**11.9%**	**2436.9**	**338.5**	**66.3%**	**11.8%**	**1672.0**	**92.1**
	Standard Deviation		**1.2%**	**1.8%**	**12.6**	**25.4**	**1.3%**	**1.9%**	**178.3**	**331.7**	**0.0%**	**0.0%**	**212.1**	**199.6**	**1.4%**	**2.5%**	**1680.4**	**536.8**	**15.6%**	**14.8%**	**1530.7**	**582.0**	**29.5%**	**12.1%**	**1735.0**	**192.6**
	LP Relaxation Average		***83.1%***	***2.2%***	***5.4***	***10.1***	***78.3%***	***2.0%***	***45.1***	***63.9***	***100.0%***	***0.0%***	***77.5***	***60.8***	***85.8%***	***2.5%***	***1856.8***	***265.0***	***64.6%***	***7.4%***	***2161.9***	***365.8***	***52.2%***	***7.9%***	***1507.2***	***76.8***
	Standard Deviation		***17.1%***	***1.5%***	***10.9***	***22.5***	***23.8%***	***1.5%***	***131.7***	***233.9***	***0.0%***	***0.0%***	***184.8***	***160.3***	***22.0%***	***4.3%***	***1689.3***	***528.7***	***40.7%***	***11.7%***	***1562.9***	***573.9***	***42.2%***	***13.1%***	***1710.9***	***164.1***

Table A.5 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	1.6	3.4	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	8.5	6.7	0.0%	0.0%	29.5	30.5	0.0%	0.0%	0.5	0.4
	7	F1	0.0%	0.0%	2.6	3.3	0.0%	0.0%	1.1	0.6	0.0%	0.0%	1.2	0.3	0.0%	0.0%	173.0	146.2	0.0%	0.0%	149.1	104.1	0.0%	0.0%	1.7	0.4
	9	F1	0.0%	0.0%	6.6	4.5	0.0%	0.0%	77.6	68.8	0.0%	0.0%	31.2	13.7	0.0%	0.0%	336.7	151.0	0.0%	0.0%	601.9	824.4	0.0%	0.0%	19.2	29.5
	11	F1	0.0%	0.0%	30.2	19.9	2.8%	3.8%	2188.3	1695.9	0.0%	0.0%	620.6	543.6	0.0%	0.0%	1320.9	633.7	0.0%	0.0%	920.1	503.7	0.0%	0.0%	18.8	7.3
	13	F1	0.0%	0.0%	515.6	444.2	27.6%	9.3%	3600.0	0.0	31.0%	15.2%	3600.0	0.0	0.4%	0.9%	1685.8	1211.9	1.6%	1.5%	2840.0	1227.1	0.0%	0.0%	51.5	17.8
	15	F1	14.6%	13.9%	3204.2	542.0	38.2%	7.4%	3600.0	0.0	51.0%	6.9%	3600.0	0.0	33.0%	42.5%	3474.5	280.6	8.2%	4.2%	3600.0	0.0	0.0%	0.0%	218.4	40.1
	20	F1	44.0%	4.8%	3600.0	0.0	52.0%	5.9%	3600.0	0.0	75.8%	12.5%	3600.0	0.0	75.6%	23.0%	3600.0	0.0	19.2%	15.2%	3600.0	0.0	20.0%	44.7%	1006.1	373.5
	30	F1	60.8%	1.6%	3600.0	0.0	65.8%	1.9%	3600.0	0.0	93.0%	11.2%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	72.6%	1.8%	3600.0	0.0	76.4%	1.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	78.0%	1.4%	3600.0	0.0	80.4%	1.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	82.4%	0.5%	3600.0	0.0	83.4%	0.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**32.0%**	**2.2%**	**1978.3**	**92.5**	**38.8%**	**2.9%**	**2497.0**	**160.5**	**50.1%**	**4.2%**	**2350.3**	**50.7**	**46.3%**	**6.0%**	**2272.7**	**220.9**	**39.0%**	**1.9%**	**2376.4**	**244.5**	**38.2%**	**4.1%**	**1428.7**	**42.6**
	Standard Deviation		**35.6%**	**4.1%**	**1796.8**	**199.4**	**34.6%**	**3.3%**	**1641.0**	**509.6**	**45.1%**	**6.1%**	**1741.9**	**163.5**	**48.2%**	**13.9%**	**1574.4**	**381.6**	**48.7%**	**4.6%**	**1579.1**	**423.5**	**49.4%**	**13.5%**	**1745.0**	**110.6**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	0.1	0.0	0.0%	0.0%	26.2	12.5	0.0%	0.0%	92.6	65.5	0.0%	0.0%	4.3	4.3
	7	F2	0.0%	0.0%	0.1	0.1	0.0%	0.0%	1.2	0.7	0.0%	0.0%	1.9	1.0	0.0%	0.0%	510.9	747.9	0.0%	0.0%	938.4	1404.0	7.0%	15.7%	1108.9	1571.2
	9	F2	0.0%	0.0%	1.0	1.2	0.0%	0.0%	15.7	8.1	0.0%	0.0%	22.0	9.1	0.0%	0.0%	1414.9	838.6	13.6%	19.6%	2596.8	1419.6	7.4%	16.5%	1380.7	1410.5
	11	F2	0.0%	0.0%	8.0	6.8	0.0%	0.0%	466.2	397.6	0.0%	0.0%	128.4	109.0	47.4%	38.4%	3558.5	92.8	47.0%	21.8%	3600.0	0.0	37.2%	28.8%	3284.7	705.0
	13	F2	0.0%	0.0%	456.5	979.6	22.8%	19.4%	2949.5	1454.7	2.4%	5.4%	2194.4	1113.6	44.0%	34.8%	3600.0	0.0	59.8%	39.2%	3600.0	0.0	22.4%	21.6%	3204.2	850.9
	15	F2	15.6%	21.4%	1757.8	1713.6	50.6%	14.3%	3600.0	0.0	57.8%	23.2%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	35.4%	19.3%	3600.0	0.0
	20	F2	64.4%	17.4%	3600.0	0.0	82.8%	9.4%	3600.0	0.0	98.2%	2.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F2	93.4%	7.3%	3600.0	0.0	97.2%	3.4%	3600.0	0.0	99.2%	1.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	94.8%	2.8%	3600.0	0.0	97.4%	1.7%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	96.8%	1.8%	3600.0	0.0	98.2%	1.1%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	98.0%	0.7%	3600.0	0.0	98.6%	0.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**42.1%**	**4.7%**	**1838.5**	**245.6**	**49.8%**	**4.5%**	**2275.7**	**169.2**	**50.7%**	**3.0%**	**2177.0**	**112.1**	**62.9%**	**6.6%**	**2791.9**	**153.8**	**65.5%**	**7.3%**	**2948.0**	**262.6**	**55.4%**	**9.3%**	**2780.3**	**412.9**
	Standard Deviation		**46.5%**	**7.7%**	**1758.3**	**568.6**	**45.8%**	**6.8%**	**1723.7**	**442.6**	**49.6%**	**6.9%**	**1745.3**	**333.7**	**45.5%**	**14.8%**	**1410.9**	**318.0**	**43.3%**	**13.4%**	**1253.6**	**568.5**	**44.1%**	**11.1%**	**1300.9**	**616.8**
	MIP Average		***37.1%***	***3.4%***	***1908.4***	***169.0***	***44.3%***	***3.7%***	***2386.3***	***164.8***	***50.4%***	***3.6%***	***2263.6***	***81.4***	***54.6%***	***6.3%***	***2532.3***	***187.4***	***52.2%***	***4.6%***	***2662.2***	***253.6***	***46.8%***	***6.7%***	***2104.5***	***227.8***
	Standard Deviation		***40.7%***	***6.1%***	***1736.3***	***423.1***	***40.0%***	***5.2%***	***1646.2***	***465.8***	***46.3%***	***6.4%***	***1703.9***	***258.4***	***46.5%***	***14.0%***	***1482.9***	***344.5***	***47.0%***	***10.2%***	***1421.8***	***489.3***	***46.5%***	***12.4%***	***1653.6***	***472.1***

Table A.6 – Average GAP Results for each Instance Size for Single Machine Scheduling Problems for Six MIP Formulations for Class 6.

LP RELAXATION PROBLEM

Instance (# jobs)	Objective Function	Mixed Integer Program Formulations: CTP GAP	CTP SD	CTP T(s)	CTP SD(T(s))	AFCTP GAP	AFCTP SD	AFCTP T(s)	AFCTP SD(T(s))	APD GAP	APD SD	APD T(s)	APD SD(T(s))	TI GAP	TI SD	TI T(s)	TI SD(T(s))	TII GAP	TII SD	TII T(s)	TII SD(T(s))	ATI GAP	ATI SD	ATI T(s)	ATI SD(T(s))
5	F1	25.3%	5.4%	0.0	0.0	17.7%	4.7%	2.2	3.0	100.0%	0.0%	0.1	0.0	20.8%	6.0%	51.1	88.4	5.1%	2.5%	413.2	551.5	0.0%	0.0%	0.4	0.1
7	F1	27.6%	1.6%	0.0	0.0	21.3%	1.8%	0.0	0.0	100.0%	0.0%	0.1	0.0	23.8%	2.2%	881.7	939.3	8.1%	3.9%	2757.1	983.0	0.0%	0.0%	4.0	2.7
9	F1	25.9%	7.8%	0.0	0.0	21.0%	7.0%	0.0	0.0	100.0%	0.0%	0.1	0.0	28.5%	6.7%	2321.2	1340.5	58.5%	8.7%	3036.0	1261.1	0.1%	0.1%	20.4	6.4
11	F1	25.6%	4.7%	0.0	0.0	22.1%	4.0%	0.0	0.0	100.0%	0.0%	0.1	0.0	22.5%	3.2%	2917.2	1526.8	92.4%	6.5%	3600.0	0.0	0.2%	0.4%	89.1	16.1
13	F1	25.2%	4.7%	0.0	0.0	21.0%	5.3%	2.4	3.5	100.0%	0.0%	0.5	0.4	41.2%	21.4%	995.2	1456.3	100.0%	0.0%	3600.0	0.0	0.3%	0.7%	232.3	107.3
15	F1	23.8%	4.5%	0.0	0.1	21.8%	4.4%	11.8	11.9	100.0%	0.0%	1.2	0.6	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	0.3%	0.5%	564.5	299.5
20	F1	23.9%	7.0%	0.5	0.4	21.9%	5.9%	102.4	211.5	100.0%	0.0%	2.1	1.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3261.6	756.6
30	F1	21.6%	9.6%	0.2	0.1	20.8%	9.0%	122.1	263.7	100.0%	0.0%	10.2	9.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F1	18.2%	4.4%	0.8	0.5	18.0%	4.3%	11.7	16.1	100.0%	0.0%	48.3	27.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F1	24.7%	3.3%	6.3	12.7	24.6%	3.3%	255.1	549.7	100.0%	0.0%	259.2	144.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F1	30.5%	5.0%	1.1	0.5	30.4%	4.9%	158.9	171.6	100.0%	0.0%	924.6	201.2	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F1 Average		**24.8%**	**5.3%**	**0.8**	**1.3**	**21.9%**	**5.0%**	**60.6**	**111.9**	**100.0%**	**0.0%**	**113.3**	**35.0**	**67.0%**	**3.6%**	**2615.1**	**486.5**	**78.5%**	**2.0%**	**3182.4**	**254.1**	**45.5%**	**0.2%**	**1688.4**	**108.1**
Standard Deviation		**3.1%**	**2.2%**	**1.9**	**3.8**	**3.4%**	**1.9%**	**87.0**	**175.7**	**0.0%**	**0.0%**	**279.9**	**69.7**	**38.3%**	**6.4%**	**1349.1**	**673.5**	**37.7%**	**3.1%**	**962.6**	**463.7**	**52.1%**	**0.3%**	**1774.9**	**233.6**
5	F2	79.4%	12.1%	0.1	0.1	78.2%	12.3%	0.8	0.5	100.0%	0.0%	0.1	0.1	76.4%	13.9%	140.3	258.4	54.1%	27.0%	796.2	1567.7	20.1%	9.8%	2.2	0.9
7	F2	57.3%	7.0%	0.0	0.0	57.3%	7.0%	0.3	0.4	100.0%	0.0%	0.1	0.1	63.0%	21.9%	826.5	1551.5	40.3%	36.2%	1542.0	1334.9	22.1%	16.3%	57.8	62.0
9	F2	55.7%	22.3%	0.0	0.0	55.7%	22.3%	0.0	0.0	100.0%	0.0%	0.9	1.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	21.6%	15.3%	116.8	86.4
11	F2	57.0%	10.1%	0.0	0.0	57.0%	10.1%	0.0	0.0	100.0%	0.0%	0.3	0.3	90.2%	22.0%	2884.0	1601.0	100.0%	0.0%	3600.0	0.0	12.7%	7.7%	484.1	292.2
13	F2	56.6%	9.5%	0.0	0.0	56.6%	9.5%	0.1	0.2	100.0%	0.0%	0.1	0.1	87.9%	27.1%	2990.9	1362.0	100.0%	0.0%	3600.0	0.0	17.8%	7.3%	1395.2	1311.6
15	F2	52.4%	13.7%	0.0	0.0	52.4%	13.7%	3.5	5.6	100.0%	0.0%	3.6	6.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	15.7%	13.5%	2640.6	1089.6
20	F2	47.8%	22.9%	0.4	0.5	47.8%	22.9%	0.8	0.9	100.0%	0.0%	1.8	1.1	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	63.2%	43.7%	3600.0	0.0
30	F2	38.8%	13.0%	1.5	1.0	38.8%	13.0%	0.4	0.2	100.0%	0.0%	2.9	1.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
50	F2	31.8%	5.8%	1.2	0.9	31.8%	5.8%	32.2	37.9	100.0%	0.0%	27.5	15.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
75	F2	37.2%	7.9%	44.7	93.5	37.2%	7.9%	15.5	10.1	100.0%	0.0%	285.1	232.7	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
100	F2	50.0%	5.0%	12.2	13.9	50.0%	5.0%	220.6	150.2	100.0%	0.0%	656.6	338.9	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
F2 Average		**51.3%**	**11.8%**	**5.5**	**10.0**	**51.2%**	**11.8%**	**24.9**	**18.7**	**100.0%**	**0.0%**	**89.0**	**54.4**	**92.5%**	**7.7%**	**2912.9**	**433.9**	**90.4%**	**5.8%**	**3158.0**	**263.9**	**52.1%**	**10.3%**	**2063.3**	**258.4**
Standard Deviation		**12.9%**	**6.1%**	**13.5**	**28.0**	**12.6%**	**6.1%**	**65.7**	**45.0**	**0.0%**	**0.0%**	**206.4**	**117.0**	**12.5%**	**11.1%**	**1239.0**	**694.3**	**21.6%**	**13.0%**	**997.4**	**589.4**	**40.3%**	**12.8%**	**1647.4**	**476.4**
LP Relaxation Average		***38.0%***	***8.5%***	***3.1***	***5.6***	***36.5%***	***8.4%***	***42.8***	***65.3***	***100.0%***	***0.0%***	***101.2***	***44.7***	***79.7%***	***5.6%***	***2764.0***	***460.2***	***84.5%***	***3.9%***	***3170.2***	***259.0***	***48.8%***	***5.2%***	***1875.9***	***183.2***
Standard Deviation		***16.4%***	***5.6%***	***9.7***	***20.0***	***17.5%***	***5.6%***	***77.4***	***133.9***	***0.0%***	***0.0%***	***240.3***	***94.5***	***30.7%***	***9.1%***	***1273.2***	***668.1***	***30.6%***	***9.4%***	***956.6***	***517.5***	***45.6%***	***10.2%***	***1682.1***	***374.1***

Table A.6 – (Continuation).

	Instance (# jobs)	Objective Function	Mixed Integer Program Formulations																							
			CTP				AFCTP				APD				TI				TII				ATI			
			GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))	GAP	SD	T(s)	SD(T(s))
MIP PROBLEM	5	F1	0.0%	0.0%	0.0	0.0	0.0%	0.0%	1.3	2.7	0.0%	0.0%	0.1	0.0	0.0%	0.0%	1686.7	1543.0	0.0%	0.0%	404.2	635.7	0.0%	0.0%	3.6	1.1
	7	F1	0.0%	0.0%	0.0	0.0	0.0%	0.0%	0.4	0.2	0.0%	0.0%	1.4	1.1	8.4%	7.8%	3034.7	860.6	2.0%	1.6%	3067.9	1189.9	0.0%	0.0%	28.1	17.8
	9	F1	0.0%	0.0%	0.1	0.1	0.0%	0.0%	6.4	9.2	0.0%	0.0%	18.8	9.7	22.2%	14.2%	3600.0	0.0	15.6%	18.8%	3600.0	0.0	0.0%	0.0%	109.0	50.6
	11	F1	0.0%	0.0%	0.4	0.5	0.0%	0.0%	52.3	27.3	0.0%	0.0%	321.5	290.5	15.8%	1.6%	3600.0	0.0	54.8%	6.2%	3600.0	0.0	0.0%	0.0%	336.6	178.2
	13	F1	0.0%	0.0%	10.2	15.1	1.2%	2.7%	1427.7	1694.6	2.6%	5.8%	2192.3	1279.6	44.2%	9.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	0.8%	1.8%	1232.5	1359.8
	15	F1	0.0%	0.0%	196.0	399.5	6.0%	3.9%	3600.0	0.0	15.6%	6.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	0.4%	0.9%	1756.6	1189.3
	20	F1	6.0%	5.1%	3000.9	1339.7	16.6%	5.5%	3600.0	0.0	69.0%	21.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F1	14.6%	8.3%	3600.0	0.0	19.2%	9.1%	3600.0	0.0	96.6%	4.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F1	14.8%	4.7%	3600.0	0.0	19.8%	4.7%	3600.0	0.0	88.8%	25.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F1	23.2%	3.6%	3600.0	0.0	26.6%	2.6%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F1	29.4%	5.3%	3600.0	0.0	33.4%	6.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F1 Average		**8.0%**	**2.5%**	**1600.7**	**159.5**	**11.2%**	**3.2%**	**2098.9**	**157.6**	**43.0%**	**5.8%**	**2194.0**	**143.7**	**62.8%**	**3.0%**	**3374.7**	**218.5**	**70.2%**	**2.4%**	**3261.1**	**166.0**	**45.6%**	**0.2%**	**1951.5**	**254.2**
	Standard Deviation		**10.8%**	**3.0%**	**1808.3**	**409.2**	**12.4%**	**3.1%**	**1770.3**	**509.8**	**46.8%**	**9.1%**	**1723.9**	**386.6**	**44.1%**	**5.1%**	**585.0**	**509.5**	**43.6%**	**5.7%**	**960.9**	**389.5**	**52.1%**	**0.6%**	**1662.5**	**508.6**
	5	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	1.2	0.7	0.0%	0.0%	0.2	0.1	23.4%	32.6%	2310.7	1767.0	20.0%	44.7%	1754.9	1697.9	0.0%	0.0%	5.9	2.8
	7	F2	0.0%	0.0%	0.0	0.0	0.0%	0.0%	3.7	2.8	0.0%	0.0%	1.3	0.5	47.2%	13.3%	3600.0	0.0	21.0%	17.5%	2935.4	1486.1	4.0%	8.9%	788.1	1573.5
	9	F2	0.0%	0.0%	0.1	0.1	0.0%	0.0%	10.7	14.3	0.0%	0.0%	14.6	2.3	69.2%	42.6%	3600.0	0.0	69.0%	42.5%	3600.0	0.0	18.6%	19.5%	2257.8	1840.9
	11	F2	0.0%	0.0%	0.9	1.3	0.0%	0.0%	438.9	530.6	0.0%	0.0%	329.7	188.5	88.0%	26.8%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	8.0%	12.0%	2522.2	1263.5
	13	F2	0.0%	0.0%	3.6	4.6	3.8%	8.5%	989.7	1553.9	0.0%	0.0%	974.4	655.4	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	36.0%	37.0%	3600.0	0.0
	15	F2	0.0%	0.0%	15.3	17.7	14.4%	10.4%	3024.6	1286.7	19.2%	13.6%	3258.6	763.5	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	83.0%	38.0%	3600.0	0.0
	20	F2	11.4%	19.9%	2587.9	1464.9	39.8%	16.3%	3600.0	0.0	69.8%	19.5%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	30	F2	27.0%	13.5%	3600.0	0.0	41.2%	14.3%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	50	F2	26.8%	6.9%	3600.0	0.0	44.8%	7.5%	3600.0	0.0	93.8%	13.9%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	75	F2	35.0%	8.2%	3600.0	0.0	67.4%	7.4%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	100	F2	48.6%	5.2%	3600.0	0.0	75.6%	3.9%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0	100.0%	0.0%	3600.0	0.0
	F2 Average		**13.5%**	**4.9%**	**1546.2**	**135.3**	**26.1%**	**6.2%**	**2042.6**	**308.1**	**43.9%**	**4.3%**	**2052.6**	**146.4**	**84.3%**	**10.5%**	**3482.8**	**160.6**	**82.7%**	**9.5%**	**3371.8**	**289.4**	**59.1%**	**10.5%**	**2797.6**	**425.5**
	Standard Deviation		**17.8%**	**6.8%**	**1795.6**	**441.0**	**28.8%**	**6.0%**	**1709.5**	**575.2**	**47.8%**	**7.5%**	**1735.8**	**285.0**	**26.6%**	**16.0%**	**388.7**	**532.8**	**32.1%**	**17.7%**	**572.2**	**645.7**	**45.0%**	**14.9%**	**1292.5**	**739.6**
	MIP Average		***10.8%***	***3.7%***	***1573.4***	***147.4***	***18.6%***	***4.7%***	***2070.8***	***232.9***	***43.4%***	***5.0%***	***2123.3***	***145.1***	***73.6%***	***6.7%***	***3428.7***	***189.6***	***76.5%***	***6.0%***	***3316.5***	***227.7***	***52.3%***	***5.4%***	***2374.6***	***339.9***
	Standard Deviation		***14.6%***	***5.3%***	***1758.7***	***415.3***	***22.9%***	***4.9%***	***1698.5***	***535.9***	***46.2%***	***8.2%***	***1689.8***	***331.4***	***37.2%***	***12.2%***	***487.8***	***509.6***	***37.9%***	***13.3%***	***773.8***	***524.2***	***48.0%***	***11.5%***	***1516.3***	***625.6***