



Canadian Journal of Physics

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Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2018-0586.R1
Manuscript Type:	Article
Date Submitted by the Author:	13-Oct-2018
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Keyword:	Powell-Eyring Liquid, Cattaneo-Christov Heat Flux Model, Two Line Current, Thermal Relaxation, Ferrofluid
Is the invited manuscript for consideration in a Special Issue? :	Not applicable (regular submission)

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Analysis of modified Fourier law in flow of ferromagnetic Powell-Eyring fluid considering two equal magnetic dipoles

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Abstract:- The target of the current study is to inspect theoretically 2D boundary layer flow of a Eyring-Powell ferromagnetic liquid over a flat plate. An external magnetic field due to two magnetic dipole is applied. Modified Fourier law of heat flux model is employed. Constitutive relations for Eyring-Powell fluid are considered in the boundary layer flow analysis. Series results to the nonlinear formulation are derived and scrutinized by homotopic scheme. Characteristics of various parameters like magneto-thermomechanical (ferrohydrodynamic) interaction parameter, Prandtl number, and dimensionless thermal relaxation on temperature profile is displayed via graphs. It is noted that temperature field decays via thermal relaxation factor.

Keywords : Powell-Eyring Liquid; Modified Fourier Heat Flux; Two Line Current; Thermal Relaxation; Ferrofluid.

Introduction

Ferrofluids have emerged remarkable physical properties and usages in many smart appliances and electrical fascinate engineers and scientists over the years. The Ferroliquid are used in various equipment like cooling agent, laser, avionics, nuclear power plants, filtration, semiconductor processing, robotics, computer peripherals and metal spinning etc. Thermophysical characteristics of ferrohydrodynamic inspected by Neuringer and Rosensweig [1]. The impact of heat transfer and magnetic dipole of ferrofluid flow due to stretchable surface is explored by Anderson and Valnes [2]. Natural convective flow of ferromagnetic in square cavity with four rigid walls is examined by Yamaguchi et al. [3]. Zeeshan et al. [4] analyzed simultaneous effects of radiation and magnetic dipole in flow of viscous ferroliquid. Vtulkina and Elfimova [5] examined thermodynamic and magnetic features of ferroliquids via impact of external magnetic field. Several studies discovering ferroliquid notion of heat and mass transfer (see refs.[6-14]).

It is familiar nowadays that all non-Newtonian materials in view of their various characteristics are not scrutinized by one constitutive relation. The non-Newtonian materials are classified for differential, rate and integral types [15-17]. The examples of non-Newtonian materials include drilling mud, printer ink, hydrogenated colloidal suspension, blood, butter, mayonnaise, cheese, soup, ketchup, jam etc. The notion of Eyring-Powell fluid model is known by Eyring and Powell in 1944. This model obtains an attention due to its constitutive relationship is given empirically and it shows Newtonian behavior for both high and low shear rates. Some studies appropriate to

Eyring-Powell liquid model (see refs.[18-20]). Investigation of heat transfer in boundary layer flow is phenomenon observed in the nature because of temperature difference between objects inside the same body. It has a remarkable practical application like energy fabrication, heat exchangers for the packed bed, drying technology, atomic reactor cooling, catalytic reactors, such as, medication and heat conduction in tissues has been focused by many scientists. The heat transfer characteristic was first proposed by Fourier [21] to forecast the heat transfer mechanism in several connected circumstances. However, one of the important drawbacks of this model is that it yields a parabolic energy transport for temperature profile and consequently it has weakness in the sense that any unsettling influence is lost. To overcome that paradox Cattaneo [22] remodel the Fourier law by containing thermal relaxation term to indicate thermal inertia. The expansion of thermal relaxation time converts the energy transport in form of thermal waves with finite speed. The material-invariant formula of Cattaneo's law through Oldroyd's upper-convected model has been established by Christov [23]. Ciarletta and Straughan [24] confirmed uniqueness and mechanical immovability of the solution by utilizing non-Fourier heat flux model. Han et al. [25] scrutinized heat transfer characteristics of viscoelastic liquid in view of non-Fourier heat flux model enclosed by stretching plate with velocity slip effect. Further recent researches covering improved Fourier expressions are given in Refs. [26-31].

The above-mentioned studies inspire the present work. The goal is to examine two-dimensional Powell-Eyring Liquid saturated with ferromagnetic nanoparticles flowing due to stretched surface with the appearance of two-line dipole current, which has not been discussed so far. We used modified Cattaneo's flux model to explore the heat transfer characteristics. Modelled equations have been solved by using Homotopic procedure. The behavior of numerous sundry parameters appearing in the problem statement is discussed in detail with the help of graphs.

Mathematical Formulation

Here we consider two dimensional an incompressible flow of Powell-Eyring ferromagnetic liquid due to a flat plate with two magnetic dipole as shown in Fig. 1. Wall temperature is $T_w = T_\theta \left(1 - \frac{x}{l}\right)$, in which l represents plate length ($l \ll d$). Heat transfer analysis is considered, because the outcome of applied force on the flow field is controlled to a tinny region which in very near to the wall where temperature T of the fluid is reasonably less from Curie temperature ($T < T_\theta$). Viscous dissipation is not considered. The governing equations describing the physical situation of the flow problem in term of Powell-Eyring ferromagnetic liquid are [12].

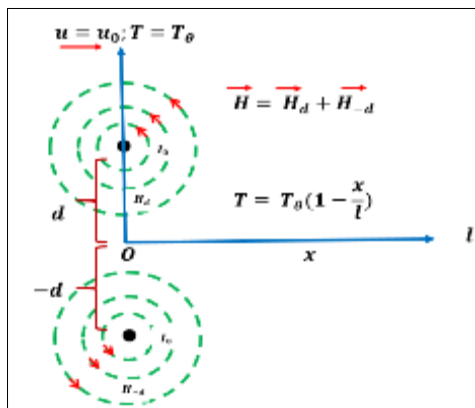


Fig. 1. Flow geometry

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho\beta c} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2\rho\beta c^3} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + \frac{\mu_0 M}{\rho} \frac{\partial H}{\partial x}, \quad (2)$$

$$\rho c_p (\mathbf{V} \cdot \nabla) T = -\nabla \cdot \mathbf{q}, \quad (3)$$

where component of velocity along x and y directions are u and v , respectively, μ_0 the magnetic permeability, ρ the density of fluid, T the temperature, c_p the specific heat, M the magnetization, H magnetic field, β and c are the fluid parameters. For the non-Fourier heat flux \mathbf{q} , we have

$$\mathbf{q} + \lambda [(\mathbf{V} \cdot \nabla) \mathbf{q} - (\mathbf{q} \cdot \nabla) \mathbf{V} + (\nabla \cdot \mathbf{V}) \mathbf{q}] = -\kappa \nabla T, \quad (4)$$

where λ signifies thermal relaxation time. For $\lambda = 0$, Eq.(4) is reduced to the classical Fourier's law.

$$\mathbf{q} + \lambda [(\mathbf{V} \cdot \nabla) \mathbf{q} - (\mathbf{q} \cdot \nabla) \mathbf{V}] = -\kappa \nabla T. \quad (5)$$

Comparing Eqs.(5) and (3), then excluding \mathbf{q} , we get the energy conservation law corresponding to Cattaneo-Christov heat flux model in the form. The boundary conditions applicable to the present flow are

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \lambda \left(u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + v \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} \right) = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2}. \quad (6)$$

The subjected boundary relations to the cold flat plate are

$$u = v = 0, \quad T = T_0 \left(1 - \frac{x}{l} \right) \quad \text{at } y = 0 \quad (7)$$

$$u = u_0, \quad T = T_0 \quad \text{as } y \rightarrow \infty.$$

The similarity transformations are

$$\eta = \left(\frac{u_0}{\nu x}\right)^{\frac{1}{2}} y, u = u_0 f'(\eta), v = \left(\frac{\nu u_0}{x}\right)^{\frac{1}{2}} \left[\frac{\eta f'(\eta)}{2} - \frac{f(\eta)}{2} \right], T = T_\theta \left[1 - \frac{x}{l} \theta(\eta) \right]. \quad (8)$$

Substituting Eq. (8) into the Eqs. (2) to (7), we obtain

$$\left(\frac{1}{2} + K\right) f''' + K \lambda f'' f''' + \frac{1}{2} f f'' - \Lambda \theta = 0, \quad (9)$$

$$\theta'' + \frac{1}{2} \text{Pr} f \theta' - \text{Pr} f' \theta + \frac{\text{Pr} \gamma}{4} (2 f f'' + 2 f f' \theta' - f^2 \theta'') = 0. \quad (10)$$

The transformed boundary relations given in Eq. (7), take the form

$$f = f' = 0, \theta = 1 \text{ at } \eta = 0, f' \rightarrow 1, \theta \rightarrow 0, \text{ as } \eta \rightarrow \infty, \quad (11)$$

Here $K (= \frac{1}{\mu \beta c})$ and $\lambda (= \frac{u_0^3 x^2}{2c^2 \nu})$ are the fluid parameters, $\Lambda (= \frac{I_0 \mu_0 k T_0}{\pi \rho \mu_0^2})$ the ferrohydrodynamic interaction parameter, $\text{Pr} (= \frac{\mu c_p}{k})$ the Prandtl number, $\gamma (= \lambda u_0 x)$ the dimensionless thermal relaxation time. The friction factor is computed as

$$C_f = \frac{2\tau_w}{\rho u_0^2}, \quad (12)$$

where shear stress τ_w is

$$\tau_w = \left(\mu \left(\frac{\partial u}{\partial y} \right) + \frac{1}{\beta c} \left(\frac{\partial u}{\partial y} \right) - \frac{1}{6\beta} \left(\frac{1}{c} \frac{\partial u}{\partial y} \right)^3 \right) \Big|_{y=0}. \quad (13)$$

By Eq. (13) in Eq. (12) the skin friction coefficient in dimensionless forms is

$$C_f \text{Re}_x^{\frac{1}{2}} = 2(1+K) f''(0) - \frac{2K}{3} \lambda (f''(0))^3, \quad (14)$$

where

$$\text{Re}_x = \frac{\rho \mu_0 x}{\mu}, \quad (15)$$

is the local Reynolds number.

Magnetic Potential

Magnetic scalar potential φ ($H = -\nabla \varphi$) at any point (x, y) under the effect of dipole is

$$\varphi = -\frac{I_0}{2\pi} \left[\text{Tan}^{-1} \left(\frac{y+d}{x} \right) + \text{Tan}^{-1} \left(\frac{y-d}{x} \right) \right], \quad (16)$$

where I_0 is the strength of magnetic field and magnetic field H_x and H_y have the form:

$$H_x = -\frac{\partial \varphi}{\partial x} = -\frac{I_0}{2\pi} \left[\frac{y+d}{x^2 + (y+d)^2} + \frac{y-d}{x^2 + (y-d)^2} \right], \quad (17)$$

$$H_x = -\frac{\partial\varphi}{\partial x} = -\frac{I_0}{2\pi} \left[\frac{x}{x^2 + (y-d)^2} + \frac{x}{x^2 + (y+d)^2} \right]. \quad (18)$$

The resulting magnitude (H) of the force field strength is taken as

$$H = \left[\left(\frac{\partial\varphi}{\partial x} \right)^2 + \left(\frac{\partial\varphi}{\partial y} \right)^2 \right]^{\frac{1}{2}}. \quad (19)$$

The elements of ∇H in term of scalar potential are

$$(\nabla H)_x = \frac{\frac{\partial\varphi}{\partial x} \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial\varphi}{\partial y} \frac{\partial^2\varphi}{\partial x\partial y}}{\left[\left(\frac{\partial\varphi}{\partial x} \right)^2 + \left(\frac{\partial\varphi}{\partial y} \right)^2 \right]^{\frac{1}{2}}}. \quad (20)$$

$$(\nabla H)_y = \frac{\frac{\partial\varphi}{\partial y} \frac{\partial^2\varphi}{\partial y^2} + \frac{\partial\varphi}{\partial x} \frac{\partial^2\varphi}{\partial x\partial y}}{\left[\left(\frac{\partial\varphi}{\partial x} \right)^2 + \left(\frac{\partial\varphi}{\partial y} \right)^2 \right]^{\frac{1}{2}}}. \quad (21)$$

Utilizing Eq. (16) in Eqs. (20) and (21) and expanding up to order x^2 , we obtain

$$(\nabla H)_y = 0. \quad (22)$$

Since, at the surface of wall $\left(\frac{\partial\varphi}{\partial x} \right)_{y=0} = \left(\frac{\partial^2\varphi}{\partial y^2} \right)_{y=0} = 0$ and it is assumed that $x \gg d$ so that from Eq.

(20) we have

$$(\nabla H)_x = -\frac{I_0}{\pi} \frac{1}{x^2}. \quad (23)$$

Also, the change in magnetization M can be taken as a linear function of temperature T .

$$M = K^* (T_\theta - T), \quad (24)$$

where, K^* is a pyro magnetic constant and T_θ is Curie temperature.

Homotopic solutions

We implemented the HAM scheme for development of convergent solutions of the expressions in Eqs.(9) and (10) with the conditions given in Eq. (11). The initial guesses and operators are

$$f_0(\eta) = (1 - e^{-\eta}), \quad \theta_0(\eta) = e^{-\eta}, \quad (25)$$

$$\mathcal{L}_f = \frac{d^3 f}{d\eta^3} - \frac{df}{d\eta}, \quad \mathcal{L}_\theta = \frac{d^2 \theta}{d\eta} - \theta, \quad (26)$$

where

$$\mathcal{L}_f [C_1 + C_2 e^\eta + C_3 e^{-\eta}] = 0, \quad \mathcal{L}_\theta [C_4 e^\eta + C_5 e^{-\eta}] = 0, \quad (27)$$

in which C_i ($i = 1-5$) elucidate the arbitrary constants given as

$$C_2 = C_4 = 0, \quad C_3 = \left. \frac{\partial f_m^*(\eta)}{\partial \eta} \right|_{\eta=0}, \quad C_1 = -C_3 - f_m^*(0), \quad C_5 = -\theta_m^*(0). \quad (28)$$

Convergence analysis

No doubt the homotopic scheme deals notable choice to select the auxiliary parameters \hbar_f and \hbar_θ (both control the convergence). The \hbar -curves have been displayed to control the appropriate values of \hbar_f and \hbar_θ in Fig. 2. It is observed that permissible values are $-1.0 \leq \hbar_f \leq -0.5$ and $-1.5 \leq \hbar_\theta \leq -0.4$. Table 1 discloses the convergence of the series solutions for different orders of approximations. It is obvious that the 25th and 35th order of approximations are enough for the convergence of momentum and energy equations, respectively.

Table 1: Homotopy solutions convergence when $K = 0.2$, $\lambda = 0.3$, $\Lambda = 0.1$, $\gamma = 0.4$ and $Pr = 0.9$.

order of approximations	$-f''(0)$	$\theta'(0)$
1	0.6044	0.3910
5	0.5176	0.3230
10	0.4980	0.3370
15	0.4928	0.3458
20	0.4922	0.3466
25	0.4922	0.3472
30	0.4922	0.3474
35	0.4922	0.3474

Results and discussion

This portion addresses the HAM solutions for velocity and temperature distributions are inspected in this section for various values of K , λ , Λ , γ and Pr arising in the resulting problems. Fig. 3 shows the behavior of (K) on $f'(\eta)$. It is investigated that $f'(\eta)$ and its corresponding layer thickness increases. For higher (K) the viscosity decreases and so, velocity and momentum layer thickness are increased. Fig. 4 discloses the characteristics of (λ) on $f'(\eta)$. Here velocity and its related layer thickness shows decreasing behavior for larger (λ) . Fig. 5 is showed to focus influence of ferromagnetic interaction parameter $0 \leq \Lambda \leq 0.25$ on velocity field. It is just observed from the figure that in the existence of magnetic dipole velocity profile decreases for higher values of (Λ) , this occur due to the presence of Lorentz force, which opposes the flow and this force has tendency to change a resistance and increase the temperature field. Therefore, flattening the axial velocity $f'(\eta)$. It is more interesting to observe that the existence of applied magnetic field reduces velocity as related with the hydrodynamic case ($\Lambda = 0$) because there is interference between the fluid motion and the stroke of magnetic field. This kind of intervention reduces the velocity and raising the frictional heating within the fluid layers which are responsible for the increment in thermal boundary layer thickness as displayed

in Fig.6. Fig. 7 illustrates the effect of thermal relaxation parameter on temperature field $\theta(\eta)$. It is clearly evident that temperature of the fluid decreases by increasing thermal relaxation parameter(γ). The physical thinking behind it is that by increasing the value of(γ), material particles want additional time to exchange energy to their neighbouring particles. So, for larger value of relaxation parameter(γ) effects reduction in temperature profile and opposite trend is noted for velocity profile as shown in Fig. 8. Prandtl (Pr) number impact on temperature is revealed in Fig. 9. Higher estimation of (Pr) yields lower diffusivity, which reduces temperature and related thickness layer. Figs. 10 describe the impact of (K) and (λ) on skin friction. It is illustrated that skin friction is decreasing function of (λ).

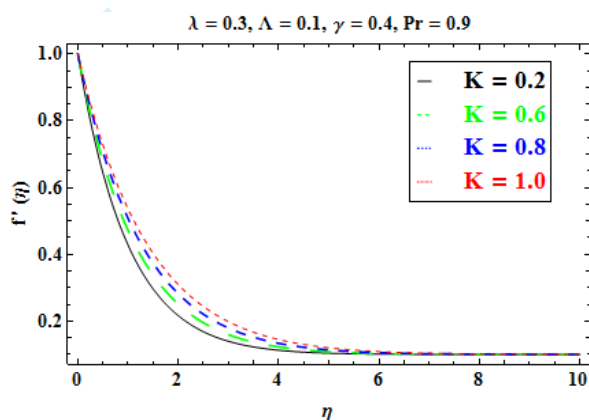


Fig.3. Outcome of K on $f'(\eta)$

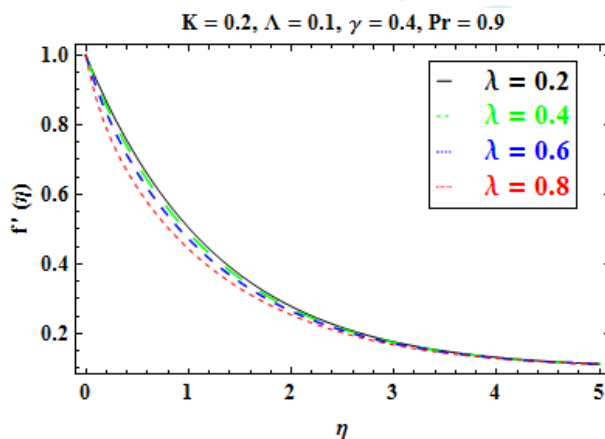


Fig.4. Outcome of λ on $f'(\eta)$.

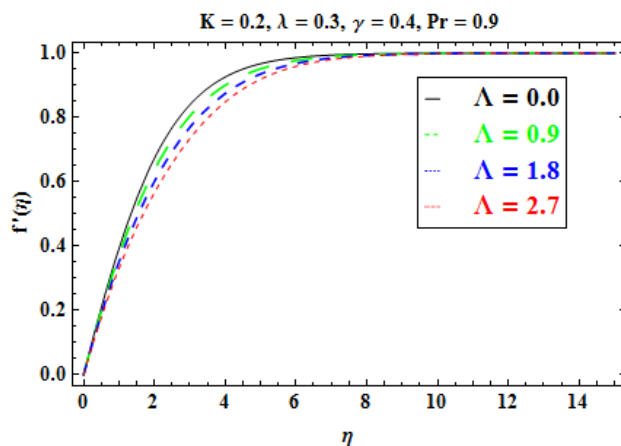


Fig.5. outcomes of Λ on $f'(\eta)$.

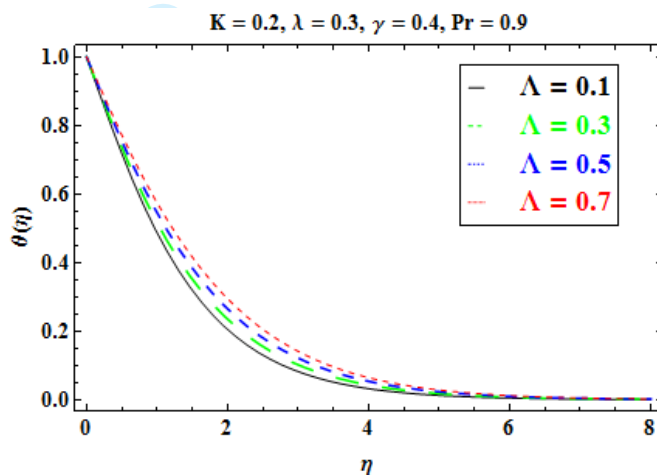


Fig.6. Outcome of Λ on $\theta(\eta)$.

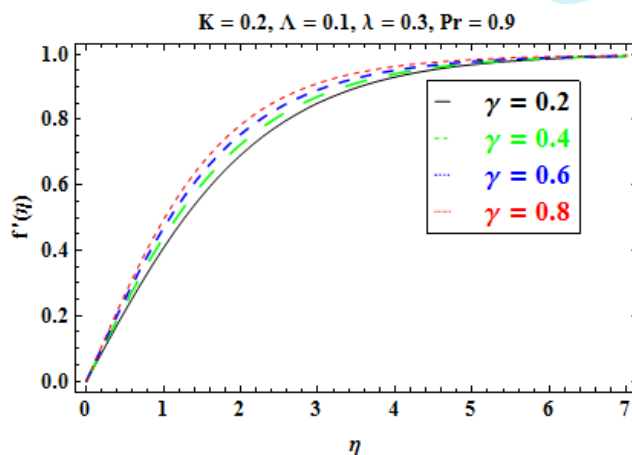


Fig.7. Outcome of γ on $f'(\eta)$.

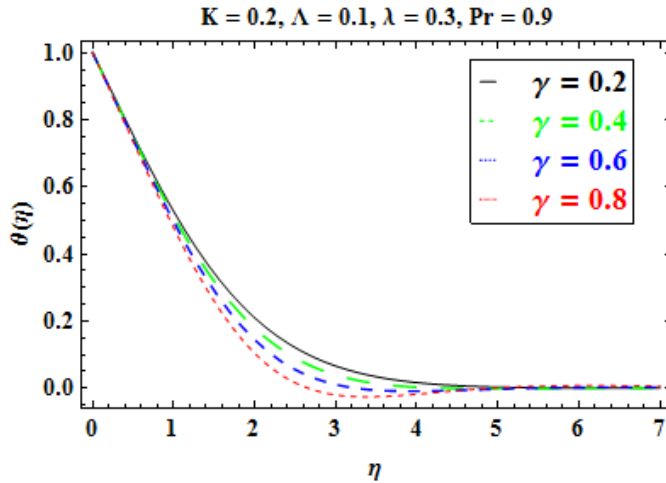


Fig.8. Outcomes of γ on $\theta(\eta)$.

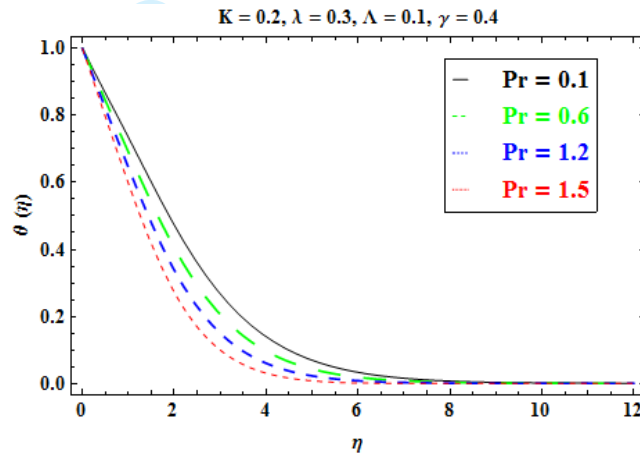


Fig.9. Outcomes of Pr on $\theta(\eta)$.

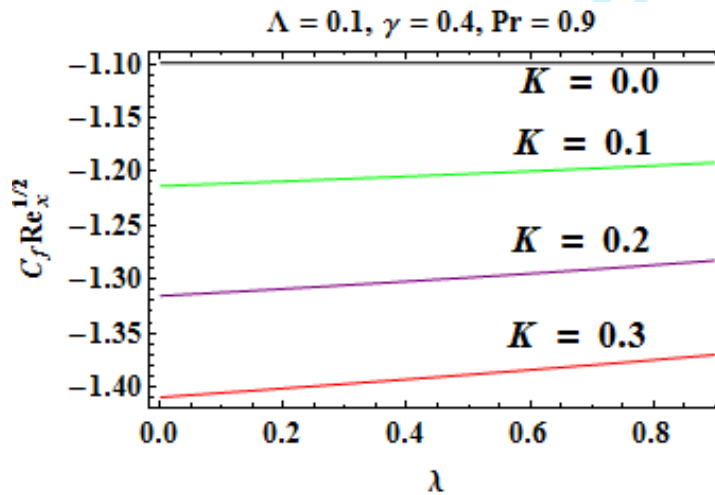


Fig.10. Outcomes of K and λ on $C_f \text{Re}_x^{\frac{1}{2}}$.

Conclusions

Here we investigated two-dimensional incompressible parallel flow of Powell-Eyring liquid saturated with ferromagnetic liquid due to a flat surface under the effect of two point line dipole.

Some of the important judgments of the present problem is as follows :

- Effects of (K) on velocity $f'(\eta)$ are opposite to that of (λ).
- The fluid Velocity decreases, whereas temperature rises by increasing the values of ferromagnetic interaction parameter (Λ).
- Temperature profile specifies falling behavior of non-Fourier model as compared with Fourier's law.
- The present analysis can be extended for the case of nanofluid, homogeneous-heterogeneous reactions and temperature dependent thermal conductivity. Besides, it can also be extended for rate type and differential type fluid models.

Acknowledgements

The authors are thankful to Dr. Muhammad Ayub, Professor of Mathematics in Quaid-i-Azam University Islamabad, Pakistan, for his insightful comments and helpful suggestions.

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