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Analysis of Multicriteria Choice Problems by Methods of the Theory of Criteria Importance, Based on Computer Systems of Decision-Making Support

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Abstract—A set of interrelated methods is presented for analyzing multicriteria decision-making problems on the basis of an information on the criteria importance and change of the preferences along their scales. Computer systems of decision-making support, implementing these methods within the methodology of progressive adequate modeling of preferences are briefly described. The paper is based on two presentations at the International Conference on Operations Research in Moscow (ORM-2007) [1, 2].

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INTRODUCTION

The methods for analyzing multicriteria decision-making problems can be divided into two large classes. The first class includes methods without a modeling of preferences of the decision-makern (DM) (one merely specifies a set of criteria to assess the solution variants). The modern methods of this class that employ the capabilities of computer graphics are described in [3]. The second class includes methods based on a modeling (with one or another completeness level) of DM preferences. For example, these methods include those constructing multicriteria value or utility functions [4].

One of the prospective approaches to constructing a model of multicriteria preferences is *the iterative–fragmentary approach*, which is a synthesis of the iterative and fragmentary approaches [5]. The iterative approach, which is based on the successive collection and use of information on preferences, was proposed and validated in [6] and then improved in a number of other works (for example, [5, 7–13]). This approach is based on the following two main ideas. The first idea is that one should first use simpler and more reliable information on preferences and accept weak assumptions on the structure of preferences and, then, if needed (when such information and/or assumptions are insufficient for the problem solution), use more complicated (and, consequently, less reliable) information with stronger assumptions. The second idea is that one needs to check whether the formulated assumptions, as well as the additional information obtained earlier, are consistent and make corrections if inconsistencies are revealed; in this case, a less reliable message should be checked with the help of more reliable ones or, at least, using several messages of the same reliability. In the iterative approach, the process of problem analysis is represented as a sequence of steps each yielding additional information on preferences and, then, these data are checked for consistency, corrected (if needed), and, only after this, the model of preferences is extended. Actually, the process of problem analysis in the iterative approach is an interactive procedure (as a man–computer dialog) alternating at each of its steps the stages of collecting information on DM and/or expert preferences with the stages of their computer processing and representation the results to DM. Here, the dialog with a human should be performed in a language conventional to him/her, and the analysis results should be represented in a form convenient and obvious to a human.

Often, the implementation of the interactive procedure is accompanied with heterogeneous and incomplete information on preferences, which can be represented as a set of information fragments. A general approach to constructing a model of preferences allowing one to use such information was proposed in the theory of criteria importance [14–16] and improved further in [5] (called here a *fragmentary* approach), as well as in [11, 12].

The name “iterative” for the approach under consideration does not seem to be quite adequate—it also encompasses a great number of methods that presume that, at each step of the procedure, homogeneous information on preferences are received. Examples of such methods are the method ZAPROS [17] and the method of successive concessions by E. S. Venttsel’ [18, 19]. In view of this, we propose to call the iterative–fragmentary approach and the set of all its supporting methods the *methodology of progressive adequate modeling of preferences* (PAMP).

For practical implementation of the PAMP methodology, we need a rather ample arsenal of suitable methods allowing one to correctly use different (both qualitative and quantitative) information on preferences. With the development of the theory of decision making, such methods appear in increasing numbers. A relatively full set of methods is that drawing information on the criteria importance (see the bibliography in [20] and our latest works [21–23]). This paper is devoted to methods of the theory of criteria importance within the framework of the PAMP methodology and briefly describes analytical computer systems for support of multicriteria decision-making (ACSSMDM) implementing these methods.

1. MATHEMATICAL MODEL OF MULTICRITERIA DECISION-MAKING SITUATION AND ITS DEVELOPMENT ON THE BASIS OF THE PAMP METHODOLGY

The further discussion is based on a mathematical model of (individual) decision-making under multiple criteria (under certainty), which is the basis of all ACSSMDMs considered hereafter:

$$\langle X, f_1, \dots, f_m, R \rangle, \tag{1.1}$$

where X is the set of variants (strategies, designs, alternatives, etc.); $f_1, \dots, f_m$ are the criteria (value functions, quality or efficiency indices, etc.); and R is a relation of the nonstrict DM preference. Let us analyze the meaning of elements of model (1.1) in more detail.

The set of variants X is assumed to be finite: $X = \{x^1, \dots, x^N\}$. Each variant $x \in X$ is characterized by the values of $m \geqq 2$ criteria f_i. By a *criterion* f_i, we mean a function defined on X and taking values of the set Z_0, which is usually called the scale of criterion[1]. Thus, all criteria have a common scale (or are reduced to such a scale), which is taken to be finite: $Z_0 = \{1, \dots, q\}$. This scale is primarily supposed to be an order scale: it is known merely that the preferences (the values of grading utility) grow with the grading number.

The criteria f_i (called *partial*) constitute a *vector criterion* $f = (f_1, \dots, f_m)$. Thus, each variant x is described by the values of $f_i(x)$ of all criteria constituting a *vector estimate* for this variant $f(x) = (f_1(x), \dots, f_m(x))$. Let us introduce the contracted term "vectestimate". The set of all vectestimates is $Z = Z_0^m$. The vectestimates $f(x)$ of variants called *attainable* or *realizable* form a *set of attainable vectestimates* $Y = f(X) = \{y \in Z_0^m \mid y = f(x), x \in X\}$. It is supposed that a variant is fully characterized by its vectestimate, so that the comparison between variants in terms of preference is achieved by a comparison between vectestimates. Thus, the problem of the selection of the best variant in the set X is reduced to the search for the most preferable vectestimate in the set $Y = f(X)$.

The DM preferences are modeled by the *nonstrict preference relation* R on Z: yRz means that the vectestimate y is no less preferable than z. It is assumed that the relation R is a (partial) quasiorder; i.e., the relation is reflexive (for any vectestimate y, yRy is true) and transitive (for any vectestimates y, z, and u, it follows from yRz and zRu that yRu). R generates relations of (strict) preference P and indifference I: xIy is true if xRy and yRx, and xPy is true if xRy is true but yRx is not true. The relation I is equivalence, and P is a (strict partial) order.

The elements $X, f_1, \dots, f_m$ of model (1.1) are given (formed) at the start of the problem formalization. The relation R is constructed (reproduced) during the problem analysis on the basis of DM and/or expert information on preferences. Then, one considers the case when these information include data about the relative importance of criteria and change in the preferences along their common scale, while the relation R is constructed in line with the PAMP methodology.

Let us assume that, at the kth step of the interactive procedure, one has some information on preferences $\Pi(k)$ and, based on this, constructs a nonstrict preference relation on Z (a quasiorder $R(k)$). Let us further assume that, at the next $(k + 1)$th step, one receives additional information and, based on all the data information $\Pi(k + 1)$, constructs a quasiorder $R(k + 1)$.The following *condition of consistency* between successively constructed quasiorders must be satisfied:

$$R(k) \subseteq R(k+1), \quad I(k) \subseteq I(k+1), \quad P(k) \subseteq P(k+1) \tag{1.2}$$

where the second inclusion follows from the first one. The relation $R(k + 1)$ is said to *consistently extend* or *consistently continue* the relation $R(k)$ (also, that the relation $R(k)$ is embedded in $R(k + 1)$ or is its subrelation); in this case, the following notation is used: $R(k) \preccurlyeq R(k + 1)$. The requirement that the first inclusion in (1.2) be strict is called the *condition of richness* of the additional data on preferences.

Let $Y(k)$ be the set of vectestimates that are nondominated or maximal with respect to $P(k)$. Because the finiteness of the set Y ensures that $Y(k)$ is externally stable, only the variants with vectestimates of $Y(k)$ can pretend to be optimal. If condition (1.2) is satisfied, the inclusion $Y(k) \supseteq Y(k + 1)$ is true. Thus, during the problem analysis, the set of nondominated variants is, generally speaking, contracted, but not a single "pretender" to the optimal variant is lost. The possibility of some weakening of the requirements of (1.2) is considered in [13].

At the start of the first step of the procedure, when there have been no additional information on preferences, the relation $R(0)$ can be represented by the Pareto relation R^0:

$$yR^0z \Leftrightarrow y_i \geqq z_i, \quad i = 1, \dots, m.$$

[1] More appropriately, the *set of scale estimates*.

Combinations of information types on criteria importance and their scale, as well as the corresponding nonstrict preference relations

Quantitative	Θ: point estimates for importance	R^{Θ}	$R^{\Theta D}$	$R^{\Theta[V]}$	$R^{\Theta V}$
	[Θ]: interval estimates for importance	$R^{[\Theta]}$	$R^{[\Theta]D}$	$R^{[\Theta][V]}$	$R^{[\Theta]V}$
Qualitative	Ω: criteria ordering by importance	R^{Ω}	$R^{\Omega D}$	$R^{\Omega[V]}$	$R^{\Omega V}$
∅: none		R^0			
Additional information on preferences ↑ on criteria importance		∅: none (the value grows with grading number)	D: the growth of value slows down	[V]: interval estimates for the growth of grading value	V: point estimates for the grading value
on scale grading ⟶			Qualitative	Quantitative	

Let us suppose that, at some step of the interactive procedure, the information on preferences Π accumulated can be represented as a set of fragments (possibly, overlapping) $\Pi^1, \ldots, \Pi^g$, for which one has succeeded to construct suitable preferences $R^1, \ldots, R^g$ (in a special case, Π^j is R^j). Because of the assumption that the relation R to be reproduced is a quasiorder, it is appropriate to take the quasi-transitive closure of the union of all relations $R^1, \ldots, R^g$ as the reproduced (on the basis of information Π) part R^{Π}. If one of these relations is reflexive (in particular, if it contains the Pareto relation), we have

$$R^{\Pi} = \mathrm{TrCl} \cup_{j=1}^{g} R^j, \tag{1.3}$$

where TrCl is the operation of transitive closure of a binary relation. The consistency of data Π can be checked by the conditions

$$R_j \preceq R^{\Pi}, j = 1, \ldots, g.$$

The definition given by (1.3) is nonconstructive. However, the theory of criteria importance includes efficient methods for constructing relations defined by (1.3), as well as methods for checking the information consistency.

Hereafter, we will assume that, during the problem analysis, one can obtain additional information on preferences in the following forms.

Information on the criteria importance:

Ω: ranking (nonstrict ordering) of criteria by importance,

[Θ]: interval estimates for degrees of superiority in the importance of each criterion over the next criterion according to the Ω ranking,

Θ: exact estimates for degrees of superiority in the importance of each criterion over the next criterion according to the Ω ranking.

Information on the criteria scale:

D: the growth of the grading value (preferences along the scale) slows down (here, the scale is of the first-order metric [24]),

[V]: interval estimates for the growth of the grading value along the scale (here, the scale is of the first-order bounded interval metric [24]),

V: point estimates for the scale-grading value (here, the scale turns out to be quantitative: no less perfect than the scale of intervals).

Based on the information types listed above, we have constructed a table indicating the nonstrict preference relations that correspond to combinations of those types. These relations are constructed on the basis of suitable methods described in the above-mentioned works on the theory of criteria importance. The data Ω&V and [Θ]&V are based on the methods described in [24, 25]. The data on preference are corrected when moving left-to-right or bottom-to-top along the cells. Let us stress that, when no information on the criteria importance are available, the improvement of their scale fails to extend the relation R^0. The problem analysis is terminated as soon as a unique (accurate to equivalence) variant is obtained or it turns out that additional information on preferences cannot be obtained.

2. SYSTEMS IMPLEMENTING THE METHODS OF THE CRITERIA IMPORTANCE THEORY

The first ACSSMDM implementing the methods of the qualitative theory of criteria importance was the system based on information on the criteria importance for analysis of alternatives (SIVKA) [26]. Created in the late 1980s, this system is operating in the DOS environment. The system allows one to use only qualitative information on the criteria importance with an order scale (i.e., it "encompasses" only two cells in Table 1, ensuring the construction of the relations R^0 and R^{Ω} and separating out variants that are nondominated in P^0 and P^{Ω}). Further, the system can reduce heterogeneous criteria to a common scale and allows one to use only partial information on importance (partial criteria ordering), as well as constructs explaining chains indicating why one variant is preferable (or indifferent) to another.

During the last two years, we have developed two new systems implementing the methods of the criteria importance theory within the framework of the PAMP methodology. The first system is the training ACSSMDM "Burka" [2] (this name is a reminiscence of its "ancestor" SIVKA). The system requirements are the following. The operating systems supported are Windows 2000 Service Pack 3, Windows 98, Windows 98 Second Edition, Windows ME, Windows Server 2003, Windows Vista (all versions), and Windows XP Service Pack 2. The amount of memory required by.Net Framework 2.0 is 280 Mb (x86) and 610 Mb (x64). The disk space for "Burka" is some 10 Mb. The system encompasses three columns of the table (it cannot deal only with the [*V*] information). The system allows heterogeneous criteria to be reduced to a common scale and indicates a set of variants that are nondominated with respect to a corresponding preference relation; in this case, for each dominated variant, the system shows the variants that dominate over that variant. When additional information on preferences are introduced, the system checks the consistency between these and earlier accumulated information, and, if inconsistencies have been found, output a warning message. "Burka" is used as an educational system in the State University–Higher School of Economics for studies including the theory of decision-making.

Another ACSSMDM is the Decision Analysis Support System (DASS) [27], which is designed for running under Windows NT/2000/XP/2003 operating systems both in the "window" mode (module Dass.exe) and from the command-line prompt (module DassCon.exe). The latest version of the system covers the table completely and implements all the possibilities accessible in "Burka".

CONCLUSIONS

The PAMP methodology equipped with a set of methods of the theory of criteria importance seems to be highly promising for an analysis of complex multicriteria decision-making problems of a unique character. Such decisions can be supported by the DASS used as an analytical core in constructing problem-oriented ACSSMDMs.

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