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Analysis of Nonlinear Missile Guidance Systems Through Linear Adjoint Method

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In this paper, a linear simulation algorithm, the adjoint method, is modified and employed as an efficient tool for analyzing the contributions of system parameters to the miss - distance of a nonlinear time-varying missile guidance system model. As an example for the application of the linear adjoint method, the effect of missile flight time on the miss - distance is studied. Since the missile model is highly nonlinear and a time-varying linearized model is required to apply the adjoint method, a new technique that utilizes the time-reversed linearized coefficients of the missile as a replacement for the time-varying describing functions is applied and proven to be successful. It is found that, when compared with Monte Carlo generated results, simulation results of this linear adjoint technique provide acceptable accuracy and can be produced with much less effort.

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NOMENCLATURE

C = Aerodynamic coefficients with subscripts.
f_f = Fuel flow rate.
H.E. = Heading error angle or the initial deviation of the missile from the collision triangle.
I_Y = Missile mass moment of inertia.
l = Characteristic length.
L_a = Lead angle or the theoretical correct angle for the missile to be on a collision triangle.
m = Missile mass.
N′ = Effective navigation ratio. Usually taken to be 3-5 in value.
N_c = Guidance acceleration command perpendicular to the line-of-sight.
N_T = Target lateral acceleration or maneuverability.
q = Pitch rate.
S = Area.
t_f = Time at the end of the missile's flight.
T = Missile thrust.
V = Relative wind speed.
V_c = Closing velocity or the negative rate of change of the distance between missile and target.
V_T = Target relative wind speed.
w = Missile normal speed.
$\dot{w}$ = Missile normal acceleration.
X_{eM}, X_{eT} = Missile and target down ranges.
Y_{eM}, Y_{eT} = Missile and target cross ranges.
Y_{md} = Relative separation between the missile and target perpendicular to the fixed reference.
α = Angle of attack.
β = Target incidence angle.
δ = Missile elevator deflection.
γ = Flight path angle.
λ = Line-of-sight angle.
$\dot{\lambda}$ = Rate of change of line-of-sight angle.
θ = Pitch angle.
ρ = Atmospheric density.

Subscripts:

D_0 = Zero lift drag coefficient.
L = Aerodynamic lift coefficient.
L_0 = Zero lift coefficient.
$L\alpha$ = Lift curve slope $dC_L/d\alpha$.
$L\alpha^2$ = Lift curve second derivative $d^2C_L/d\alpha^2$.
$L\delta$ = Lift curve slope $dC_L/d\delta$.
M = Pitch moment.
Mq = Moment curve slope dC_M/dq.
$M\alpha$ = Moment curve slope $dC_M/d\alpha$.
$M\delta$ = Moment curve slope $dC_M/d\delta$.

1. Introduction

A missile guidance system is a closed feedback loop which has at least one input which is the target acceleration (or target maneuverability) and one major output which is the missile - target relative separation called the miss - distance. When studying guided missile performance, a common goal is to find out major contributors to the total miss - distance, as well as to produce error budgets. It also may be desirable to examine performance parameters other than the miss - distance. Furthermore, the missile performance may need to be determined at various times through the homing period rather than exclusively at the final time. Due to the existence of nonlinear elements in the guidance loop, statistical inputs and disturbances, analysts usually resort to computational rather than analytical methods to determine the effects from such factors on the overall performance of the missile guidance system. Among different computational methods, Monte Carlo simulation techniques are the most commonly used. The technique is a numerical computation method that consists of repeated simulation trials plus ensemble averaging [1-2]. A major disadvantage of this technique is the large number of trials required to provide confidence in the accuracy of the results. Other procedures such as the linear adjoint method [3], the covariance analysis describing function technique (CADET) [4] and the statistical linearization adjoint method (SLAM) [5] have been more efficient and faster in solving certain problems.

In 1961, Peterson [6] illustrated how the linear adjoint method could be applied to perform guided missile analysis. Additionally, analytical expressions for miss - distance were derived by Howe [7] using the adjoint theory. In the past few decades, utilization of shaping filters [8-9] for target maneuvers has appended the use of the adjoint method in the missile guidance problem. It is applied to linear proportional guidance systems and gives trusted results [10].

The basic concept of linear adjoint method is that by reversing a system, i.e. its inputs become outputs (and its outputs become inputs), and applying a unit impulse as an input to the reversed system, we can accurately predict the contribution percentage of each input just by observing the output coming from the reversed impulse input. The output of the original system due to a white noise input can be found by squaring, integrating, and then taking the square root of the impulse response of the adjoint system in only one computer run [3]. This technique has the advantage of providing information concerning the characteristic behavior of the system at any time. Although this technique is very fast and very exact, up until now, it is only applicable to linear system models [11-12].

SLAM is another analysis method that uses statistical linearization [13] to deal with nonlinear systems. With this method, each nonlinear element is replaced by an equivalent gain that depends upon the assumed form of the input signal to the element. Thereafter, conventional probability analysis techniques are employed to get the statistical properties of the system inputs and disturbances. Finally, the linear adjoint method is applied to the statistically linearized system. CADET is a different technique for analyzing statistical behavior of nonlinear stochastic systems. It entails the use of the resulting linearized system model together with the conventional covariance techniques [14] to propagate statistics of the system state vector, recognizing that the describing function gains are functions of these statistics, and computes the root-mean-square miss - distance at the intercept time from elements of the system covariance matrix.

Since the previous two methods for nonlinear systems (SLAM and CADET) are based on statistical analysis of the system components, they are quite complicated and rely upon statistical data that usually is not readily available for flight hardware. Instead of studying input contributions, this paper expands the scope of the linear adjoint method by adopting it in studying the impacts of missile parameter variations on the performance of a nonlinear missile guidance system. As an example for such application of the linear adjoint method, a nonlinear missile guidance model is constructed from typical data and the effects from missile flight time variations on its miss-distance are studied.

2. Linear adjoint method

The linear adjoint method is used to simulate and analyze linear time-varying systems such as homing missile guidance loops. It is based on the system impulse response and gives a good inside look to the system performance and accurate knowledge of the contribution of each system input to this performance. This method can deal with probabilistic inputs as well as deterministic ones. As indicated earlier, this paper presents a study on using the linear adjoint method for evaluating the effect of parameter variations. Although a deterministic problem is considered in this paper, the same rules used can be applied to probabilistic ones with small changes in the algorithm. The rules for the adjoint method are as follows [3]:

(1) Modify system inputs:
For inputs of deterministic nature, transform each input to its impulse representation. For probabilistic inputs, outputs of shaping filters, driven by white noises, are squared and integrated to give the best representations of these input cases.

(2) Modify the time-varying coefficients:
For each time-varying parameter, replace t by t_f - t and vice versa. For gain tables, the entries are reversed in time, gains for t_f come first, etc., when used in the adjoint system.

(3) Modify signal flow direction:
The direction of signal flow in the system is reversed. The input ports are converted into output ports.

(4) Modify nodes and summing junctions:
Nodes are changed to summing junctions and summing junctions to nodes.

3. Nonlinear models of missile and guidance system

In this paper, we choose, as an example of the application of this method to nonlinear guidance systems, to study the effect of missile flight time variation on the miss - distance. For the purpose of this study, the nonlinear equations of motion of a missile in the pitch plane are developed and used along with a traditional proportional navigation algorithm under the following assumptions:

(1) The missile's out of plane motion and air density changes are insignificant.
(2) The missile is a symmetric rigid body.
(3) The earth is the inertial frame of reference, i.e. the atmosphere is fixed w.r.t. the earth and the coordinate system XeYeZe fixed to the earth.
(4) Atmospheric winds are assumed zero.
(5) The missile drag polar has a parabolic shape.

Based on [15-16], the state space form ($\dot{X} = f(X, u)$) of the missile's equations of motion, in the pitch plane w.r.t. an XYZ coordinate system fixed in the missile, is given as

$$\begin{bmatrix} \dot{\alpha} \\ \dot{V} \\ \dot{q} \\ \dot{\theta} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \frac{1}{mV}[mg\cos(\theta-\alpha) - L - T\sin(\alpha)] + q \\ \frac{1}{m}[T\cos(\alpha) - D - mg\sin(\theta-\alpha) - \dot{m}V] \\ \frac{1}{I_Y}M_Y + \frac{\dot{I}_Y}{I_Y}q \\ q \\ -f_f \end{bmatrix} \quad (1)$$

$$u(t) = \delta(t) \quad (2)$$

Where,

$$L = 0.5\rho S V^2 (C_{L0} + C_{L\alpha} + C_{L\alpha^2}\alpha^2 + C_{L\delta}\delta) \quad (3)$$

$$D = 0.5\rho S V^2 (C_{D0} + \frac{1}{\pi}C_L^2) \quad (4)$$

$$M_Y = 0.5\rho S V^2 l (C_{M\alpha}\alpha + C_{Mq}q + C_{M\delta}\delta) \quad (5)$$

Fig. 1 illustrates the relevant missile description.

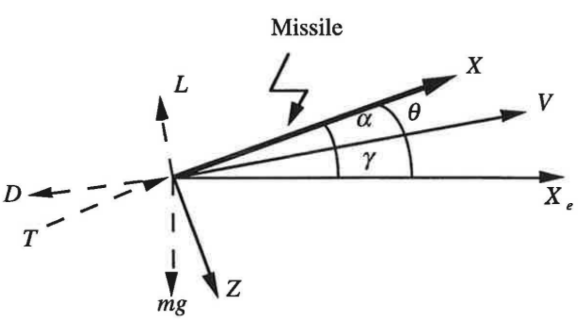

Fig. 1 Missile angle inclinations

From the missile - target engagement geometry, shown in Fig. 2, several relations can be drawn

(1) The target motion can be simulated by the following equations:

$$\dot{\beta} = \frac{N_T(t)}{V_T(t)} \quad (6)$$

$$X_{eT}(t) = -\int_{t_0}^{t} V_T(t)\cos(\beta)\,dt + X_{eT}(t_0) \quad (7)$$

$$Y_{eT}(t) = \int_{t_0}^{t} V_T(t)\sin(\beta)dt + Y_{eT}(t_0) \quad (8)$$

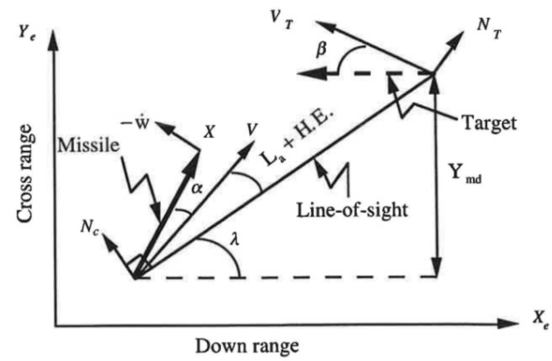

Fig. 2 Missile-target engagement geometry

(2) The engagement angles can be calculated through the following relations:

$$\gamma = \theta - \alpha = L_a + H.E. + \lambda \quad (9)$$

$$L_a + H.E. = \sin^{-1}\left[\frac{V_T}{V}\sin(\beta+\lambda)\right] \quad (10)$$

$$\lambda = \tan^{-1}\left[\frac{Y_{eT} - Y_{eM}}{X_{eT} - X_{eM}}\right] \quad (11)$$

(3) The navigation algorithm used in the simulation is the true proportional guidance law in which the guidance commands are calculated as

$$N_c = N'Vc\dot{\lambda} \quad (12)$$

Acceleration commands perpendicular to the "instantaneous missile - target line of sight [3]" are related to the guidance commands by

$$\dot{w} = -\cos(\lambda)[\cos(\theta)]^{-1}N_c \quad (13)$$

These acceleration commands are translated into appropriate elevator deflections to increase or reduce the missile's normal acceleration $\dot{w}$ given by

$$\dot{w} = g\cos(\theta) - \frac{1}{m}\left[L\cos(\alpha) + D\sin(\alpha) - \frac{mwq}{\tan(\alpha)} + \dot{m}w\right] \quad (14)$$

(4) The closing velocity and the miss-distance are calculated as follows:

$$V_c = -\frac{d}{dt}\left(\sqrt{(X_{eT} - X_{eM})^2 + (Y_{eT} - Y_{eM})^2}\right) \quad (15)$$

$$\text{miss - distance} = Y_{md}(t_f) = \int_{t_0}^{t_f}\left[\int_{t_0}^{t}(N_T\cos(\beta) - N_c\cos(\theta-\alpha))dt\right]dt \quad (16)$$

(5) The miss-distance can also be calculated as

$$\text{miss-distance} = Y_{md}(t_f) = Y_{eT}(t_f) - Y_{eM}(t_f) = Y_{eT}(t_f) - \int_{t_0}^{t_f} V\sin(\theta - \alpha)dt \quad (17)$$

The relation between normal acceleration of the missile and elevator deflection (Equation 14) is empirically approximated to be $\dot{w} \approx -0.6\delta$, where $|\delta| \leq 0.35$ rad, in computer simulations. A block diagram for the guidance loop is shown in Fig. 3. The model's aerodynamic data are extracted from a HARM missile model distributed with the MISSILE DATCOM® software. For simulation validation of the nonlinear guidance loop, a hit case, is presented in Fig. 4. The parameter values of this case are given in Appendix A.

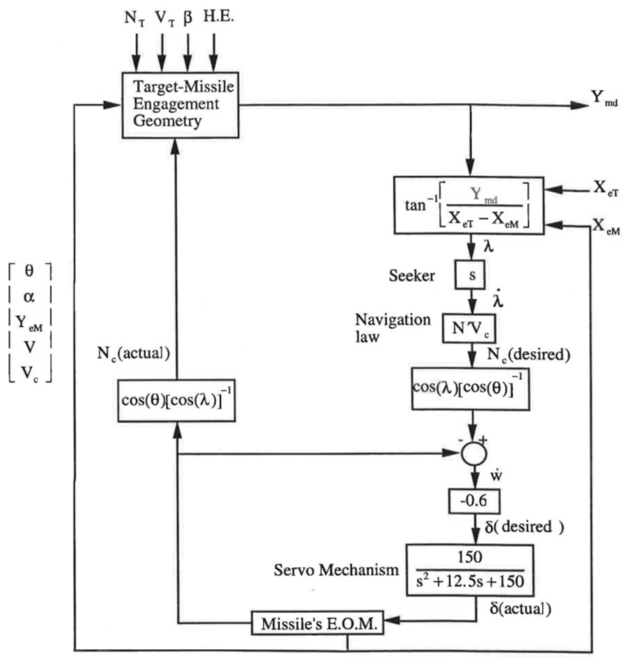

Fig. 3 Guidance loop simulation block diagram

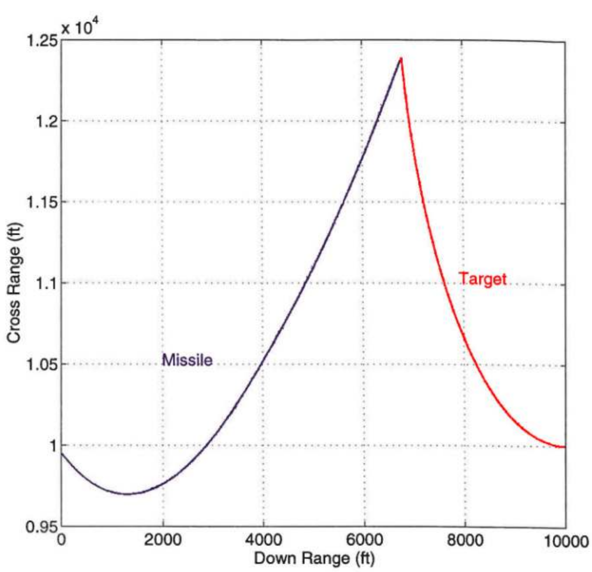

Fig. 4 An example of nonlinear simulation of missile-target hit engagement

4. Time-varying linearized model

For the purpose of using the linear adjoint method, the nonlinear guidance system model is linearized using small perturbation theory. This procedure gives the following time-varying linear missile system

$$\dot{X}(t) = A(t)X(t) + B(t)u(t) \quad (18)$$

$$A_{ij}(t) = \left.\frac{\partial f_i(X,u)}{\partial X_j}\right|_{(X,u)} \quad (19)$$

$$B_i(t) = \left.\frac{\partial f_i(X,u)}{\partial u}\right|_{(X,u)} \quad (20)$$

$$u(t) = \delta(t),\ i, j = 1,..5 \quad (21)$$

For the target, the motion equations are introduced into the linear model by taking their Laplace transformation to get

$$Y_{eT}(s) = \frac{N_T}{s^2 + \left(\frac{N_T}{V_T}\right)^2} \quad (22)$$

$$X_{eT}(s) = \frac{-V_T s}{s^2 + \left(\frac{N_T}{V_T}\right)^2} \quad (23)$$

For the line-of-sight angle calculation, Zarchan [3] suggests the following linear relation:

$$\lambda = \frac{Y_{eT} - Y_{eM}}{V_c(t_f - t)} \quad (24)$$

The linearized guidance loop is shown in Fig. 5.

To study the effect of missile flight time on the miss-distance, multiple linear and nonlinear simulations are run and miss-distance is calculated at the end of each run. If the parameter variation studied had a probabilistic nature, Monte Carlo techniques would have been used to determine the number of required simulations according to the statistical properties of the variation being analyzed. In this example, the target is assumed to start maneuvering at engagement time t = 0 with the initial missile and target data given in Appendix B. The missile flight time is varied from 0.1s to 10 s, in 0.1 s increment, and Fig. 6 shows the engagement miss-distance results of 100 simulation runs for both nonlinear and linearized missile guidance models. The same procedure used can also be applied to other guidance loop parameters of interest. As a stopping condition for computer simulation, missile to target distance less than 50 feet is considered a hit and the end of a simulation run. The nonlinear simulations suggest that a minimum flight time of 4.08 seconds is required for the missile to hit the target, while the linear simulation requires 4.18 seconds to reach the minimum miss - distance. The approximate nature of linearized model is responsible for its failure to achieve a perfect (zero miss – distance) interception.

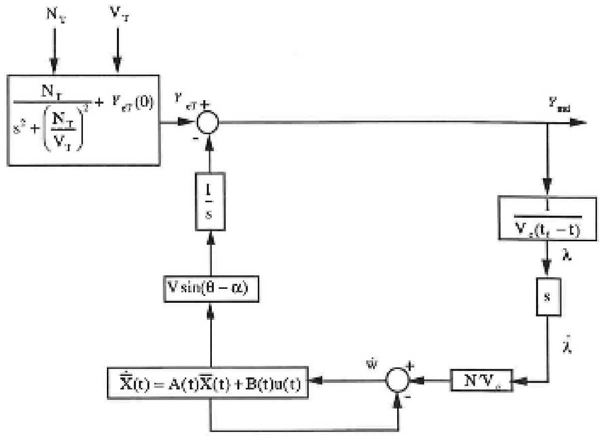

Fig. 5 Linear guidance loop

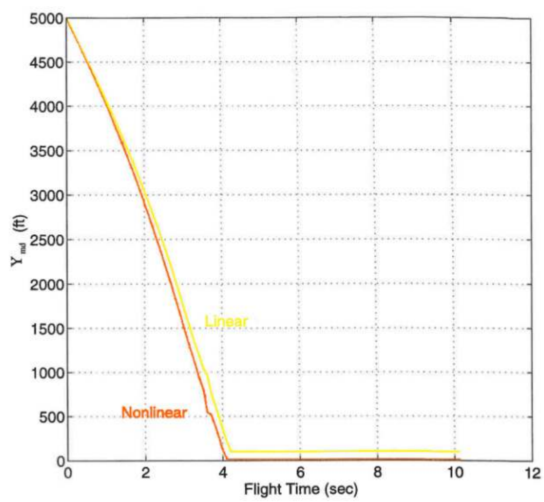

Fig. 6 Linear and nonlinear simulations of miss - distances vs. missile flight time

5. Linear adjoint model

To apply the linear adjoint method, the adjoint system is constructed according to the guiding rules described earlier in this paper. Fig. 7 shows the resulting systems.

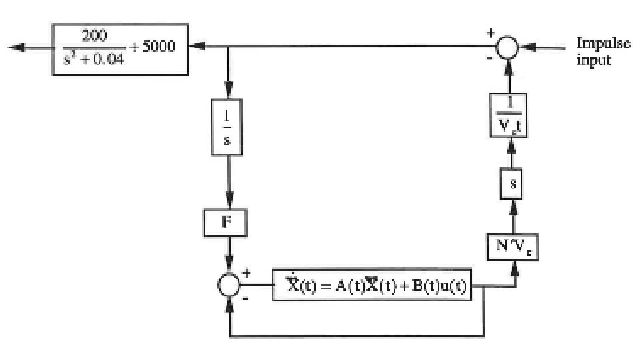

Fig. 7 Linear adjoint loop

To ultilize the adjoint system, two problems had to be overcome:

(1) Simulation of the time-varying missile linear system whose input is the lateral acceleration $\dot{w}$ and the output is the angle deflection of the elevatorδ.

(2) Development of the relationship between the rate of change of the missile's cross range Vsin(θ-α) and the lateral acceleration, marked as block F in Fig. 7.

For the first problem, which is the major hurdle, the nonlinear simulation is run with an impulse input for the target maneuverability, assuming a missile flight time of 10 s. The impulse input is chosen since, in the adjoint system, this input here will be the output of the adjoint simulation. The linearized coefficients are calculated for each time step of the nonlinear simulation. Those coefficients are then time-reversed and fed to the linear system during the adjoint simulation. For the second problem, from the above nonlinear simulation results, a first order numerical polynomial fit is used to approximate the relation between the lateral acceleration and the rate of change of the missile's cross range as follows

$$\dot{w} = 0.016[V\sin(\theta - \alpha)] + 23.2 \tag{25}$$

The linear adjoint simulation is then run and the results are shown in Fig. 8.

From Fig. 8, we can draw the following remarks:

(1) The linear and nonlinear simulation results take 100 runs each to get the miss - distance for different missile flight times, while it takes the linear adjoint method only one computer run.

(2) The adjoint simulation run is not stopped when the minimum value of miss - distance happens and it lasts through the entire flight period of 10 s. The point at which the linear adjoint system predicts minimum miss - distance (at about 4.7 seconds) gives the minimum missile flight time to achieve a hit.

(3) The linear adjoint simulation accurately approximates the results of the linear and nonlinear simulations.

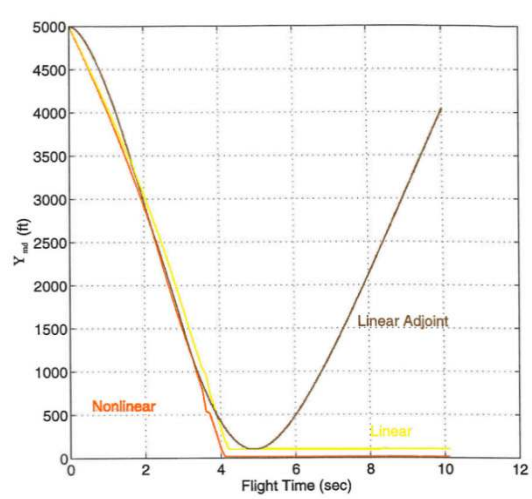

Fig. 8 Linear adjoint simulation result compared to the results of linear and nonlinear simulations

6. Conclusion

In this paper, a linear adjoint method is developed for a nonlinear time-varying missile guidance system to identify the contribution of system parameter variations to the system's miss - distance. A new technique that involves the calculation of the missile's linearized coefficients from the impulse simulation of the nonlinear system and then employs their time-reversed values in the adjoint system gives good estimation of the effect of missile flight time on the miss-distance. Other parameter variations can be studied using the same technique.

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APPENDIX

A. Missile-target engagement parameter values for Fig. 4:

$N_T = 300$ ft/s^2
$V_T = 1{,}000$ ft/s
$X_{eT}(0) = 10{,}000$ ft
$Y_{eT}(0) = 10{,}000$ ft
$X_{eM}(0) = 0$ ft
$Y_{eM}(0) = 10{,}000$ ft
$\theta(0) = -10°$
$\alpha(0) = 10°$
$V = 1{,}200$ ft/s

B. Missile-target engagement parameter values for Fig. 6 and Fig. 8:

$N_T = 200$ ft/s^2
$V_T = 1{,}000$ ft/s
$X_{eT}(0) = 10{,}000$ ft
$Y_{eT}(0) = 5{,}000$ ft
$X_{eM}(0) = 0$ ft
$Y_{eM}(0) = 0$ ft
$\theta(0) = 48°$
$V = 1{,}200$ ft/s