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ANALYSIS OF OFDMA RESOURCE ALLOCATION WITH LIMITED FEEDBACK

FACULTY OF TECHNOLOGY,
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Abstract

Radio link adaptation, multiple antenna techniques, relaying methods and dynamic radio resource assignment are among the key methods used to improve the performance of wireless communication networks. Opportunistic resource block (RB) allocation in downlink orthogonal frequency division multiple access (OFDMA) with limited feedback is considered. The spectral efficiency analysis of multiuser OFDMA with imperfect feedback path, multiple antenna methods and relaying methods is a particular focus.

The analysis is derived for best-*M* feedback methods and for a RB-wise signal-to-noise ratio (SNR) quantization based feedback strategy. Practical resource fair round robin (RR) allocation is assumed at the RB assignment, i.e., each user gets the same portion of the available RBs. The fading of each RB is modelled to be independent and identically distributed (IID). This assumption enabled a communication theoretic approach for the performance evaluation of OFDMA systems The event probabilities related to the considered OFDMA systems are presented so that the feedback bit error probability (BEP) is a parameter in the expressions. The performance expressions are derived for the BEP in the case of binary phase-shift keying (BPSK) modulation and single antenna methods. Asymptotic BEP behavior is considered for the best-*M* feedback methods when the mean SNR tends to infinity. The system outage capacity and the average system spectral efficiency are investigated in the case of multiple antenna schemes. Antenna selection and space-time block coding (STBC) are considered in multiple antenna schemes when each RB is allocated exclusively to a single user. Simple OFDMA-spatial division multiple access (SDMA) schemes are also analyzed when zero forcing (ZF) detection is assumed at the receiver.

Relay enhanced dynamic OFDMA with single and multiple antennas at each end is considered for fixed infrastructure amplify-and-forward (AF) relaying methods. The average spectral efficiency has been derived for the best-*M* and RB-wise one bit feedback schemes, antenna selection and STBC methods.

The best choice for a combination of multiple antenna scheme and feedback strategy depends on several system parameters. The proposed analytical tools enable easy evaluation of the performance of the investigated schemes with different system parameters. The fundamental properties of the combinations of feedback and multiple antenna schemes are extensively studied through numerical examples. The results also demonstrate that the analytical results with idealized IID fading assumption are close to those obtained via simulations in a practical frequency selective channel when RBs are selected properly. Dynamic RB allocation is attractive for practical OFDMA systems since significant performance gain over random allocation can be achieved with a practical allocation principle, very low feedback overhead and an imperfect feedback channel.

Keywords: bit error probability, capacity, limited feedback, MIMO, OFDMA, resource allocation

To my family

Preface

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Oulu, September 3, 2009 Jouko Leinonen

Symbols and abbreviations

$(\cdot)!$	factorial of the argument
$\mathcal{A}$	set of the available RBs
C	average capacity
C_{ZYX}	average capacity for the case abbreviated by XYZ
C_n^{XYZ}	average capacity of the nth best RB for the case abbreviated by XYZ
$C_{P_{\mathrm{out}}}$	outage capacity for outage probability P_{out}
E_1	exponential integral
f_{XYZ}	PDF of SNR for the case abbreviated by XYZ
F_{XYZ}	CDF of SNR for the case abbreviated by XYZ
F_{M+}^{XYZ}	CDF of SNR of $N-M$ worst RBs for the case abbreviated by XYZ
$g(\gamma)$	bit error probability of uncoded BPSK in non-fading AWGN channel SNR value γ
$\mathcal{I}$	integral
k	user index
K	number of users
M	number of indicated RBs in the best-M method
$\mathcal{M}$	performance measure
$\mathcal{M}_{\mathrm{RB}}$	performance measure of a RB
$\mathcal{M}_n^{\mathrm{RB}}$	performance measure of the nth best RB
$\mathcal{M}_{\mathrm{Q}}$	performance measure for the quantization
$\mathcal{M}_l^{\mathrm{Q}}$	performance measure for the lth quantization region
$\mathcal{M}_{M+}^{\mathrm{XYZ}}$	cumulative performance of the $N-M$ worst RBs for the case abbreviated by XYZ
n	index of the nth best RB
N	number of RBs
N_e	number of bit errors in the received feedback word
N_t	number of transmit antennas
N_r	number of receive antennas
N_b	number of RBs in sub-block
p_{ASE}	probability of antenna selection error
p_b	feedback bit error probability
$p_e(N_e)$	probability of N_e bit errors
$p_{\mathrm{E}i}$	probability of the event Ei
$p_n(k, p_b)$	probability that the kth user gets the nth best RB when feedback BEP is p_b
p_l^{Q}	cumulative of SNR of lth quantization region
p_{γ_1}	probability that SNR exceeds the threshold γ_1

$p_{M+}(k)$	probability that the kth user gets the nth best RB and $M < n < N$.
p_w	error probability of best-M feedback word
$P_n(\bar{\gamma})$	BEP of the nth best RB with mean SNR $\bar{\gamma}$
P_{out}	outage probability
$P_{\text{Q}}(\bar{\gamma}, k)$	BEP for the kth user with $\bar{\gamma}$
$\mathcal{P}_m(x)$	the mth order Poisson distribution
$q(k, p_b)$	probability that the kth user gets a RB which is among the M best RBs when feedback BEP is p_b
$q_l(k, p_b)$	probability that the SNR of the allocated channel for the kth user belongs to the lth quantization region when feedback bit error probability is p_b
$q^{(R_i)}$	probability that the ith is applied
$q_l^{(R_i)}$	probability that the SNR of the allocated RB belongs to the lth quantization region when the ith is used
$q_l^{(R_i,\text{C})}$	probability that the SNR of allocated RB belongs to the lth quantization region when the ith is used and antenna selection is correct
$q_l^{(R_i,\text{E})}$	probability that the SNR of allocated RB belongs to the lth quantization region when the ith is used and antenna selection is erroneous
$\mathcal{R}$	Root of the polynomial equation
$\mathcal{R}_i$	Root of the polynomial equation in the case that the ith rate is used
w_i	correct feedback word indicating the ith best RB in the Obest-M scheme
$\hat{w}_i$	received feedback word w_i
$\mathcal{W}$	correct feedback word in the Obest-M scheme
$\hat{\mathcal{W}}$	received feedback word $\mathcal{W}$
γ	instantaneous SNR
$\bar{\gamma}$	mean SNR
γ_{AS}	SNR at the best antenna
γ_l	lth quantization boundary in SNR quantization
$\gamma_{\text{out}}(P_{\text{out}})$	SNR value which provides outage probability P_{out}
γ_{T}	threshold value
$\gamma_{\text{out},i}(P_{\text{out}})$	SNR value which provides outage probability P_{out} on the condition that the ith is used
$\Gamma(x)$	Gamma function
ν	indicator function for the lth quantization region
ξ	normalized threshold value in quantization
2G	second generation cellular systems

3G	third generation cellular systems
3GPP	Third Generation Partnership Project
AF	amplify-and-forward
AS	antenna selection
ASE	antenna selection error
AWGN	additive white Gaussian noise
BEP	bit error probability
BPSK	binary phase-shift keying
BS	base station
CDF	cumulative distribution function
CP	cyclic prefix
CQI	channel quality indicator
CSI	channel state information
DF	decode-and-forward
DZF	system with diversity transmission and zero forcing detection
EDGE	Enhanced Data Rates for GSM Evolution
EF	estimate-and-forward
ETSI	European Telecommunications Standards Institute
FDD	frequency division duplex
FFT	fast Fourier transform
GPRS	generalized packet radio service
GSM	Global System for Mobile communications
HIPERLAN	High Performance Radio LAN
HSPA	high-speed packet access
IEEE	Institute of Electrical and Electronics Engineers
IID	independent and identically distributed
IMT-A	international mobile telecommunications-advanced
IP	internet protocol
ISI	inter-symbol interference
ITU	International Telecommunication Union
LAN	local area network
LOS	line-of-sight
LTE	long-term evolution
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
Obest-M	ordered best-M
MRC	maximum ratio combining
OFDM	orthogonal frequency division multiplexing
OFDMA	orthogonal frequency division multiple access
PAPR	peak-to-average power ratio
QoS	quality of service
PDF	probability density function
QoS	quality of service

RA	random allocation
RB	resource block
RBAS	resource block-wise antenna selection
RR	Round Robin
RF	radio frequency
SBB-M	sub-block based best-M
SDMA	space division multiple access
SINR	signal-to-interference-plus-noise ratio
SIMO	single-input multiple-ouput
SISO	single-input single-ouput
SNR	signal-to-noise ratio
STBC	space-time block coding
TDD	time division duplex
TBAS	threshold based antenna selection
UMTS	universal mobile telecommunication system
VoIP	voice over internet protocol
WiMAX	worldwide interoperability for microwave access
WLAN	wireless local-area network
ZF	zero forcing

Contents

1 Introduction

Wireless communication networks have globalized rapidly in recent years. In addition to traditional voice services, the fast development of wireless communication has created the opportunity to provide new services through radio devices. The next generation networks should provide a large range of new data and multimedia services with low costs, high reliability and quality. However, the available radio spectrum is scarce, energy consumption is limited and the wireless channel is dispersive by nature. Consequently, system designers have faced challenging problems to meet the frequently increasing expectations of wireless communications. Several sophisticated features have been adopted for the existing networks, but still further development for future wireless systems is needed. This thesis concentrates on the analysis of some key methods developed to improve the spectral efficiency of wireless communications. The analysis of the resource allocation in multiuser orthogonal frequency division multiple access (OFDMA) with practical feedback information is of particular interest. The advanced features of the physical layer such as rate adaptation and multiple-input multiple-output (MIMO) techniques as well as relaying methods are included in the studied opportunistic OFDMA systems.

1.1 Background

The background of the technologies and prior work related to the research of thesis are described in this section. Section 1.1.1 presents the trend of wireless communication development. Section 1.1.2 presents some sophisticated wireless technologies which are commonly used to improve the performance of wireless systems and they are also included in the analyzed systems. Literature review of opportunistic multiuser systems with limited feedback is presented in Section 1.1.3. Section 1.1.4 includes an extensive review of literature related to resource allocation and feedback design in OFDMA communication. The most relevant prior work for the thesis, i.e., articles considering analytical performance evaluation on OFDMA with limited feedback, are also summarized in Section 1.1.4.

1.1.1 Wireless system development

The conventional fixed-mode systems, e.g., the second generation (2G) Global System for Mobile communications (GSM) standard [11], are designed to provide voice and low data rate services for all channel conditions. Such traditional design of wireless systems has focused on increasing reliability so that communication succeeds also in the worst channel conditions [12], [13]. The evolutions of the 2G systems, namely the Generalized Packet Radio Service (GPRS) and Enhanced Data Rates for GSM Evolution (EDGE), are able to provide higher data rates and new services such as web browsing and they are in active commercial use [14, 15]. A step towards high data rates is taken in the development of the third generation (3G) Universal Mobile Telecommunication System (UMTS) [16] and in its evolution High Speed Packet Access (HSPA) [17]. Several advanced features of wireless communications are already in commercial use in 3G systems including, e.g., link adaptation and dynamic radio resource management (RRM). In addition, wireless local area networks (WLAN), e.g., the family of 802.11 [18] standards developed by Institute of Electrical and Electronics Engineers (IEEE) and high performance radio LAN (HiperLAN) [19] developed by the European Telecommunications Standard Institute (ETSI), are widely used to provide wireless internet connections through communication techniques beyond 2G networks.

In addition to limited transmission power and channel dispersion, the major problem for commercial communication systems is still the fact that the same spectrum has to be shared with an increasing number of high data rate services. More efficient solutions for the physical, link and network layers are needed to fulfil the further increased requirements of the future wireless communication systems. Thus, the long term evolution (LTE) of the 3G [20, 21] standard and IEEE 802.16 worldwide interoperability for microwave access (WiMAX) standards [22, 23] are under development. The aim of evolved 3G systems is to improve the spectral efficiency and the flexible sharing of wide bandwidth, enabling services comparable with wireline technologies for numerous users. Link adaptation, orthogonal frequency division multiplexing (OFDM), and multiple antenna methods are among the key physical layer solutions in most of the evolved 3G systems like in 3G LTE and the WiMAX standards. In the link layer, the dynamic allocation of time-frequency-space resource blocks (RBs) is one of the most prominent technical solutions to efficiently guarantee the desired quality of service (QoS) of multiple users. Thus, adaptive multiuser MIMO-OFDMA transmission is a candidate also for the systems beyond 3G [24]. The requirements of wireless systems beyond 3G are included in the International Mobile Telecommunications-Advanced (IMT-A) concept created by the International Telecommunication Unit (ITU). The third generation partnership project (3GPP) is developing LTE-Advanced [25] to meet the requirements of the IMT-A.

Wireless relaying can be used to provide further diversity or extension for the coverage area [26, 27]. Relay nodes (RN)s can be employed to mitigate channel attenuation, dividing the long direct link into two short hops. Thus, the combination of adaptive multiuser MIMO-OFDMA methods and relaying tackles the most important problems of wireless communications. These techniques are also considered in this thesis and they are briefly described in the next section.

1.1.2 High data rate techniques

An efficient approach to mitigate the detrimental effects of a dispersive channel is to adapt the transmission parameters according to the instantaneous channel conditions [28, 29]. Channel capacity with channel side information has been presented for the different power and rate adaptation cases in [30]. Further spectral efficiency results in [31, 32] show the efficiency of power and rate adaptation also with a practical set of modulation alphabets. Adaptive modulation provides a significant performance enhancement also with fixed transmission power [32, 33]. The mechanism behind the adaptive modulation is simple. If the channel is in a deep fade, a low order modulation is employed or the transmission is even truncated in this channel for a while. In the case of good channel conditions, a high data rate can be achieved employing high order modulation. Similarly, code rate and transmission power can be optimized according to the instantaneous channel conditions and the required transmission rate and reliability. Adaptive modulation and coding is a crucial part of all evolved communication standards.

OFDM is an attractive method for wireless wide bandwidth systems since it provides several benefits such as inter-symbol interference (ISI) mitigation with simple receiver implementation and the possibility for frequency domain adaptation among others [34]. The bandwidth is divided into several overlapping and orthogonal flat fading narrow band sub-carriers [35, 36]. By using a cyclic prefix, ISI can be simply mitigated at the cost of small degradation of the system efficiency [36]. In a frequency selective channel, subcarriers experience different channel attenuation and the transmission parameters can be adjusted in the frequency domain resulting in high spectral efficiency [34, 37–39]. On the other hand, OFDM transmission leads to a high peak-to-average power ratio (PAPR) requiring an expensive power amplifier. The sensitivity to frequency synchronization errors is another drawback of the OFDM [36]. Thus, OFDM techniques are most feasible for downlink communications with the current cost of the radio frequency (RF) equipment.

Techniques with multiple antennas at the transmitter and receiver, which are commonly referred to as MIMO methods have been recently developed and studied extensively in order to improve the performance of the wireless communications [15, 40–44]. Multiple antenna methods improve performance without extra bandwidth making them very attractive for future wireless systems. However, increased costs and space requirements due to the added antennas, RF parts and multidimensional signal processing slow down the implementation of MIMO methods especially in small handsets [15]. Multiple antennas can be used to increase the spectral efficiency through multiplexing, to provide reliability through diversity or to cancel interference through smart antennas [15]. Remarkable MIMO channel capacity gain has been presented in [42–44]. If the transmitter has full MIMO channel knowledge, eigenbeamforming and waterfilling are efficient methods to increase spectral efficiency [15, 42]. Diversity transmission through space-time coding [45–47] can be employed as an alternative for data multiplexing to improve performance in a fading channel. Transmit diversity methods can be employed also in the case of a single antenna receiver or highly correlated channels. Multiple antenna transmission can also be designed to provide simultaneous diversity and multiplexing gain [46, 48]. In OFDM, multiple antenna transmission can be adjusted individually for the sub-bands[1] [49–51]. Channel aware adaptive MIMO-OFDM provides an efficient air interface and it has been selected for several standards like long term evolution (LTE) of the third generation (3G) standard [21] and the WiMAX standard [22].

In many practical systems, the channel knowledge is based on the feedback information from the receiver to the transmitter [31, 52]. Thus, the research on antenna selection [53–56] and beamforming [57, 58] based on the limited feedback information has received a lot of attention in the research community. Feedback design is a challenging problem especially in MIMO-OFDM systems due to the huge number of varying channels [59].

Wireless relaying has been known already for thirty years as a method to enhance the performance of the wireless communication [60]. However, the recent developments in radio technology have created opportunities to implement low cost terminals, enabling wireless relaying for future standards. Consequently, relaying methods have been extensively studied. Relay nodes can be used to extend the coverage area of a cell or to provide cooperative diversity through collaborating relay terminals [61–64]. Several variations of the amplify-and-forward (AF), decode-and-forward (DF) relaying protocols have been presented in [64]. The estimate and forward (EF) relay strategy has also been proposed [60]. Resource allocation in OFDMA relay links is an exiting research problem due to the number of different links between the source and the destination [65, 66]. Optimization of resource allocation in OFDMA based relay links is still under investigation and there are numerous open problems, several of those in the area of feedback design [52].

So called multiuser diversity provides a further degree-of-freedom that can be utilized in multiuser communication systems. When the number of users is large, there is a high probability that at least one of the users has a strong channel [67]. The basic principle to maximize multiuser diversity is to transmit to the users with the largest receive signal-to-noise ratio (SNR) at each channel resulting in maximum system throughout [13, 15, 67]. Dynamic channel assignment for multiple users can be performed in the time [67], frequency [68] and space [13, 15] domains for both the uplink and downlink directions. The allocation which maximizes the system throughput may lead to an unfair situation where users close to the base station occupy most of the channels whereas users at the cell edge are dropped at the scheduling. Thus, a practical approach for scheduler design is to maximize multiuser diversity subject to fairness and QoS constraints. Thus, joint link adaptation and dynamic subchannel assignment with different

[1] A sub-band may consist of several adjacent correlated sub-carriers.

QoS constraints have been shown to provide high spectral efficiency, see, e.g., [68–72]. On the other hand, multiple shared channels increase signaling overhead, which reduces the efficiency of dynamic OFDMA but it is still advantageous with a proper system design [73]. Another problem in multiuser systems is the feedback design [52, 74] which is considered in the next two sections.

1.1.3 Opportunistic multi-user communication with limited feedback

Channel knowledge is required to perform link adaptation and resource assignment. Full CSI has usually been justified for a time division duplex (TDD) technique based on channel reciprocity at the uplink and downlink [39]. However, the channel is not reciprocal in frequency division duplex (FDD) based communication, and also in practical TDD systems full CSI is difficult to obtain. Thus, the channel state has to be estimated at the receiver and some information about the channel conditions is conveyed to the transmitter via a feedback channel. The more feedback information is available at the adaptive transmission, the larger system capacity can be provided. However, it is desirable to minimize the feedback information per user in order to keep the overhead of feedback at an acceptable level. The feedback design is a crucial problem especially in multiuser MIMO-OFDMA systems due to the huge number of available channels.

Recent studies show that significant multiuser diversity gain can be achieved by utilizing only limited feedback information in resource management. Thorough presentations of limited feedback in wireless communications can be found in [52, 74]. Single channel multiuser systems with quantized feedback and a perfect feedback channel have been considered, e.g., in [75–79]. Interestingly, one bit feedback per user can provide nearly maximum sum-rate capacity with a properly defined threshold when the number of users is large [78, 79]. Multiuser diversity and threshold optimization with an imperfect estimator and a noisy feedback channel have been addressed in [80].

The feedback load of multiuser systems can be reduced by using selective multiuser diversity [81–83] where only the users enjoying a good channel state send a feedback message. Inspired by the selective multiuser diversity, the opportunistic feedback strategy has been studied to reduce the total feedback overhead of multiple users [84–86]. In the opportunistic feedback strategy, users send their feedback information in a random access manner via a common shared feedback channel if their received SNR values exceed a predefined threshold. The feedback words of different users may collide in the feedback channel due to the contention based feedback transmission, resulting in a loss of system performance. However, the bandwidth of the feedback channel is limited despite of the number of users. Opportunistic feedback provides a good tradeoff between the system performance and the feedback overhead [81].

In a multiuser system with limited feedback, MIMO methods are usually based on either opportunistic beamforming [58, 87, 88], antenna selection [89, 90] or a fixed set of the precoding vectors [52, 91]. Opportunistic beamforming can also be used for space-division multiple-access (SDMA) [92]. Precoder design for SDMA with limited feedback is a difficult problem, because users' signals cannot be perfectly orthogonalized due to the quantization errors [52]. In the case of a multiple antenna receiver, per antenna scheduling without precoding can provide multiuser diversity gain [91, 93, 94]. Opportunistic feedback for the per antenna scheduling without precoding has been shown to provide a relatively high sum rate with a significantly reduced feedback overhead [86].

The analytical performance evaluation of multiuser diversity in single carrier systems with outdated feedback information has been investigated in [95, 96]. The study of multiuser diversity with imperfect feedback has been analyzed in [97]. The closed form capacity results for scheduling with imperfect one bit feedback and with perfect antenna selection have been presented in [98]. A sum rate analysis with a one bit feedback scheme and a large number of users has been presented in [77, 78]

1.1.4 Frequency allocation in OFDMA systems

A major problem in many practical OFDMA systems arises from the fact that the total feedback load increases with the number of users and subchannels. Thus, frequency assignment with limited feedback has received remarkable attention in the research communities [69, 79, 99–106, 106–110, 110–114]. Properly designed low rate feedback schemes provide substantial performance improvement through dynamic resource allocation. For example, the practical system level study in [104] demonstrates 40 % allocation gain with a channel quality indicator (CQI) word size of only 30 bits per user in a 10 MHz LTE system.

Most limited feedback schemes for OFDMA frequency allocation fall into two categories. The feedback information of the first category is based on the quantized SNR that is conveyed from the receiver to the transmitter from each RB. Usually, RB-wise one bit quantization is applied to attain low feedback load [79, 100, 104]. A one bit feedback per time-frequency resource block (RB) can yield a high sum-rate provided that the threshold is properly selected and the number of users is large [79, 100, 101]. In addition, suboptimal sub-carrier-wise 1-bit quantization is shown to provide good performance when transmission power is minimized and channels are assigned subject to rate constraints [107]. A study of the optimization of channel quantization and resource allocation to minimize transmit power subject to rate requirements and limited quantization regions has been presented in [108, 109]. The joint space-frequency domain RB allocation with RB-wise one bit feedback information has been studied also through practical LTE system level simulations [93]. The results in [93] indicate that the performance loss by the use of RB-wise one bit feedback instead of full feedback per RB is only f 7 % to 10 %. The drawback to the RB-wise feedback method is the fact that the feedback overhead grows linearly with the increasing number of RBs. The feedback overhead can be reduced by using a group-wise one bit strategy, in which one bit indicates the quality of the group of channels [106, 110].

In the second category, the feedback overhead is reduced using a so called best-M feedback strategy, where only the indices of the M best channels are conveyed to the scheduler [69, 104][2]. The best-M methods have been considered also for practical networks such as LTE [115]. Moreover, the total feedback load can be controlled using an adaptive and selective feedback scheme which takes the number of users, fairness and the QoS into account [69]. A QoS aware best-M feedback method has been considered also for MIMO systems [116]. Toufik, Kim and Koutouris [99, 111] proposed different feedback schemes for the opportunistic MIMO-OFDMA systems, where the subcarriers are divided into groups and the best or a set of the best subcarriers and beams are sent to the scheduler. The drawback of the best-M feedback scheme is the fact that it is sensitive to feedback bit errors [104]. For that reason, Kovacs *et al.* [112] use a bit mask to indicate the M best channels resulting in the same overhead as in the RB-wise one bit feedback scheme.

In a practical frequency selective channel, the feedback load can be reduced by utilizing channel correlation in the design of the quantization [117], [118]. Feedback reduction in [117] is based on delta-modulation. In [118], users feed back the channel quality of only a sub-set of their strongest subcarriers, and channel correlation has been utilized to estimate the SNR for subcarriers around the indicated ones.

In order to reduce the total feedback load caused by multiple users, opportunistic feedback has been examined also for OFDMA systems in several studies [105, 106, 110, 113, 114]. The authors in [105, 113, 114] have studied weighted sum rate maximization for sequential and contention based feedback schemes.

Most of the research on relay enhanced OFDMA networks concentrates on the resource allocation problems with full CSI [65, 66]. Kim and Lee [119] show that the feedback information formed based on the second hop SNR is more advantageous than that of end-to-end SNR based feedback when fixed AF relaying and a single carrier system is considered. Siriwongparat *et al.* [120] address the outage probability of a cooperative protocol for multiuser OFDM networks with incremental relaying. Kaneko *et al.* [121, 122] have proposed resource a allocation method which reduces feedback overhead notably compared to full CSI. Closed form[3] capacity results for OFDMA with AF relaying and sub carrier pairing have been presented by Riihonen *et al.* [123]. The study in [123] assumes that the allocation is fixed at the second hop and channel knowledge is

[2]There is no unique term for this feedback scheme. For example, selective feedback [69] and max-n feedback [103] refers to the same method as best-M does. The best-M name is most common and it is adopted also for this thesis.
[3]Closed form solutions means that the capacity results are presented in terms of well known integrals, such as the exponential integral and gamma function.

available at the RN.

Analytical performance evaluation has been presented mostly for the SNR quantization based feedback schemes. The asymptotic sum rate capacity behavior of the one bit feedback scheme has been considered in [79, 124]. The sum rate capacity of the OFDMA system with practical frequency selective fading and the RB-wise one bit feedback scheme has been expressed in [100] for a large number of users. An asymptotic sum rate evaluation approach has been presented also in [106, 110] where the contention based and sequential feedback schemes are compared when the number of subchannels and the number of users tend to infinity with a fixed ratio. In addition, the characterizations of optimal group size and threshold for a sequential scheme have been investigated in [106, 110]. Closed-form expressions for the average throughput of multi-user OFDMA with uncoded adaptive modulation and imperfect feedback have been presented in [102]. The feedback scheme in [102] is based on RB-wise SNR quantization and the effects of practical aspects such as the CQI estimation error, outdated CQI and feedback errors are considered. Interesting theoretical throughput results for an SDMA-OFDMA system with RB-wise CQI have been addressed in [91], where the analysis takes proportional fair scheduling and practical modulation and coding into account. The analytical results in [91] are close to those obtained with the more practical system level simulations. Furthermore, the results in [91] indicate that SDMA-OFDMA provides significant throughput gain in a micro cell environment also without precoding. Jung *et al.* [103] have presented the system average capacity with maximum SNR allocation and the best-M feedback method. A closed form capacity formula for the case of best-1 feedback information, an SISO link and perfect feedback has been provided in [103]. Moreover, an upper bound and an approximate performance for the case of the best-M information with $M > 1$ have been presented in [103].

1.2 Motivation and scope of the research

So far, most of the studies considering OFDMA with limited feedback concentrate on the design of feedback strategies and subchannel allocation via computer simulations. Analytical performance evaluation has received less attention. Furthermore, most of the analytical studies address asymptotic sum rate analysis with a large number of users, SISO communications and perfect feedback. The theoretical analysis of multiuser OFDMA with limited feedback includes several open research issues including, e.g., closed form bit error probability (BEP) and spectral efficiency results when the numbers of users and channels are fixed, the characterization of the impact that an imperfect feedback path has on system performance, a practical subchannel allocation method, multiple antenna schemes and relaying techniques. This thesis addresses those issues. The aim is to contribute new useful analytical tools which enable easy performance evaluation of limited feedback based OFDMA systems of practical interest.

In practical communication systems, feedback bit errors impact crucially on the performance of the multiuser systems based on limited feedback. Thus, a particular focus in the thesis is to propose analytical expressions including feedback BEP among the parameters. In addition to the reliability of the feedback path, the performance of the dynamic multiuser communications based on a particular feedback scheme depends on several system parameters such as the number of sub-channels and the number of users. Theoretical results are helpful in order to study the fundamental properties of different feedback based MIMO-OFDMA schemes. In this thesis, the research problems are the following

- Derivation of the BEP and spectral efficiency performance measures for MIMO-OFDMA systems with limited feedback. Best-M feedback information or SNR quantization based feedback information is used at the RB allocation. MIMO methods include open loop STBC, transmit antenna selection as well as simple SDMA.
- Derivation of the probabilities related to the practical RB allocation method and the considered feedback methods so that the feedback BEP is a parameter in the proposed expressions.
- Extension of the investigated results for the fixed infra-structure AF relay enhanced OFDMA link.
- Examination of the fundamental properties of the considered feedback methods and multiple antenna methods through numerical examples based on the proposed analysis.

1.3 Outline of the thesis

Chapter 2 presents a system model for limited feedback based multiuser OFDMA communications. The assumptions for the analytical performance evaluation approach are described. The basic mechanism behind the best-M feedback schemes and the RB-wise SNR quantization based feedback information are explained. In addition, the considered multiple antenna schemes are presented.

Chapter 3 investigates the performance of OFDMA with the feedback information based on SNR quantization. Performance expressions are derived for the system outage capacity and average system spectral efficiency to measure the performance of the MIMO-OFDMA schemes. Antenna selection and STBC are considered in multiple antenna schemes when each RB is allocated exclusively to a single user. Further, simple OFDMA-SDMA schemes are analyzed when zero forcing (ZF) detection is assumed at the receiver. The BEP performance formulas are presented for the case of BPSK and an SISO link.

Chapter 4 considers the RB allocation in OFDMA according to the best-M feedback schemes. The same MIMO schemes as well as performance measures considered in Chapter 3 are analyzed also for the best-M feedback schemes. Furthermore, asymptotic BEP behavior when the mean SNR tends to infinity is considered for the best-M feedback methods.

Chapter 5 focuses on a relay enhanced OFDMA link. Spectral efficiency analysis has been derived for infrastructure based AF relaying when the best-M or RB-wise one bit feedback information for RB allocation and optimal rate adaptation are assumed. The analysis has been investigated for the SISO channel as well as for the case of multiple antenna nodes. AS and STBC are applied at the relay node (RN).

Chapter 6 presents numerical examples that compares combinations of feedback strategy and MIMO methods. Furthermore, analytical results with an idealized channel model are compared to those obtained via computer simulations with a practical frequency selective fading channel.

Chapter 7 concludes the thesis. The content of the thesis is summarized. Future research work related to the presented analysis and open research problems on OFDMA networks with limited feedback are proposed.

1.4 Author's contributions to the publications

The thesis is based in part on one journal paper [1], eight conference publications [2–5, 7–10], and one journal manuscript [6] to be submitted. The author has had the main responsibility in performing analysis, writing all the publications, generating numerical examples and running the simulations. Section II in [1] has been partly written by the second author and the asymptotic BEP analysis has also been contributed in cooperation with the second author. The analysis in Section III B in [6] was also developed in cooperation with the second author. Furthermore, the second author in [1–9] who is also the third author in [10] has provided ideas as well as guidance for the analysis and writing process. Sections I and II in [10] have been partly written by the second author. The last author has provided guidance, ideas and criticism during the writing process.

2 System model

The analysis considered in the thesis concentrates on the single cell multiuser system illustrated in Fig. 1. Downlink communication is considered so that the BS employs the adaptive multiuser transmission to users $1, 2, \ldots, K$ according to the CQI information received from the uplink feedback channel.

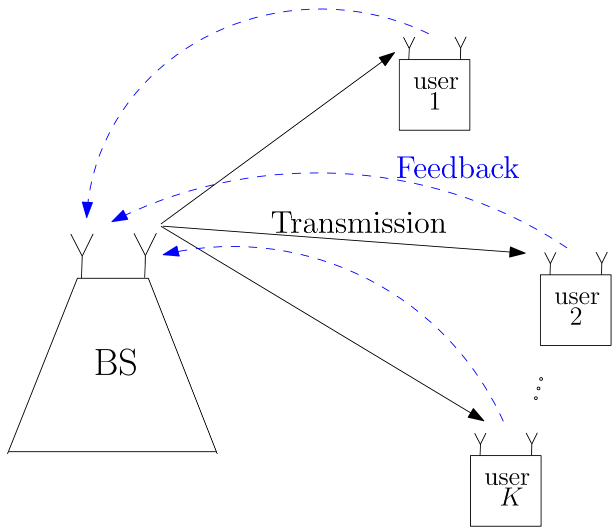


Fig 1. Downlink multiuser communications in a single cell environment. Solid arrows represent downlink transmission to the K users and dashed lines illustrate uplink feedback transmission.

A block diagram of the link from the BS to the kth mobile user terminal is shown in Fig. 2. Each mobile terminal estimates the channel and sends an instantaneous feedback information for resource block allocation and link adaptation via the feedback channel. Resource blocks are allocated at the base station based on the received feedback word. The RB consists of several adjacent sub-carriers and consecutive time domain symbols. The feedback form depends on the feedback strategy, rate adaptation, and multiple antenna scheme used. After channel assignment, rate adaptation, transmitter side OFDM processing and spatial processing are applied.

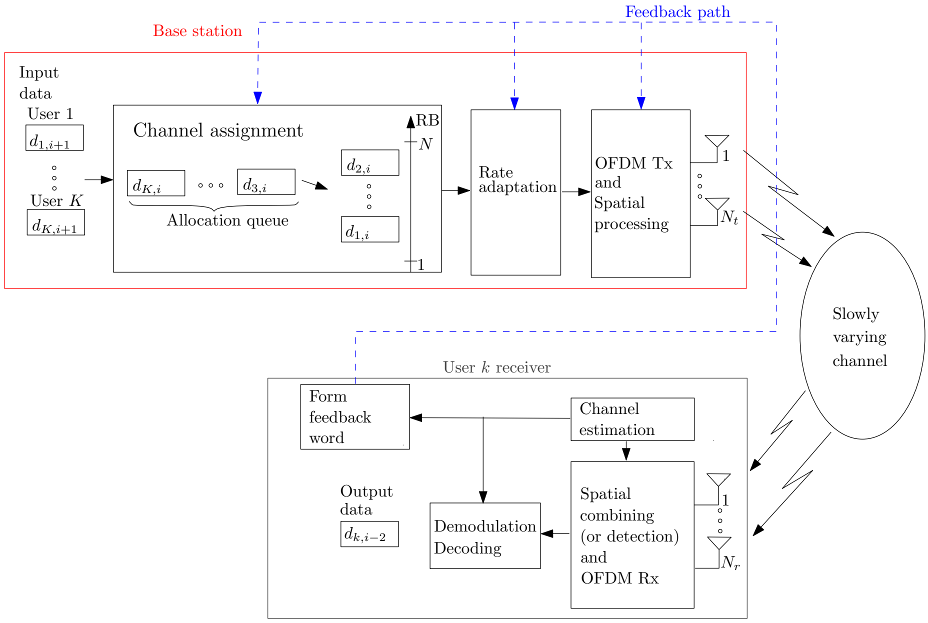


Fig 2. The block diagram of the link between the BS and the kth user receiver.

2.1 Assumptions

The aim is to analyze practical RB allocation and MIMO communications so that closed-form performance expressions are tractable. In order to achieve this goal the following assumptions regarding the general scheduling framework have been adopted:

1. Channel estimation is assumed to be perfect at the receiver.
2. The feedback word is uncoded and bit errors are uniformly distributed. Feedback BEP is p_b and it is known by the base station.
3. During an allocation time interval, an allocation queue of K users is formed in the transmitter and it grants a single RB to each of the K users from a pool of N resource blocks. RB consists of several adjacent sub-carriers and consecutive time domain symbols.
4. The number of admitted users for a resource block pool is controlled so that $K \leq N$.
5. Resource block assignment is fast when compared to channel coherence time, i.e., temporal feedback delay is neglected.
6. The resource blocks are of equal size and the channel is fully correlated inside a RB.

7. A Rayleigh fading channel is assumed so that the received SNR from each transmit antenna to each receive antenna admits independently and identically distributed (IID) exponential fading statistics for each RB. Thus, the probability density function (PDF) of the SNR is expressed as

$$f_{\mathrm{R}}(\gamma) = \frac{e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}}, \tag{1}$$

where γ is the SNR and $\bar{\gamma}$ is the average received SNR. The corresponding cumulative density function of the SNR is of the form

$$F_{\mathrm{R}}(\gamma) = 1 - e^{-\gamma/\bar{\gamma}}. \tag{2}$$

8. The mean SNR $\bar{\gamma}$ is known at the transmitter.

The third assumption means that the allocation is based on a practical round robin (RR) principle which is evidently not the best from the performance point of view but its clear benefit is the strict delay control and simplicity. There is no need for demanding real time computation. This RR allocation is also fair in the sense that each user gets a RB. Thus, system performance is not maximized as in greedy allocation. Fairness can be further improved by changing users' allocation queue position properly as the allocation queue is not formed according to the channel condition. Users may experience different mean SNR values. A good practical example service is the voice over internet protocol (VoIP). Furthermore, we note that the fifth assumption is valid for low-mobility users whose channel can be tracked by the base station by using feedback information.

The sixth assumption does not hold perfectly in practical frequency selective fading channels. However, the aim in system design is to select suitable RB size resulting in high correlation inside a RB because the same transmission mode is used for all symbols in the RB [38].

The seventh assumption is not fully true in any practical frequency selective channel. However, an idealized channel model enables an analytical performance evaluation approach. Thus, this assumption has been adopted in several studies [79, 102, 103, 105, 106, 113]. We also note that the performance of our idealized system provides a lower bound on the BEP and a higher bound on spectral efficiency for practical systems. Furthermore, simulation results in numerical examples show that the seventh assumption is approximately valid when the RBs are properly selected. Let us assume a wide bandwidth OFDMA system where it is possible to form pools of N uncorrelated time-frequency blocks provided that N is small compared to the total number of RBs. In such a system, groups of K users attached to each resource block pool may be formed. Finally, note that although RBs in pools are uncorrelated they may not admit IID statistics, since the mean received power may vary in frequency depending on the temporal channel profile. However, if users perform random hopping between pools, the impact of slightly different mean powers in different parts of the frequency band is diminishing.

2.2 Rate adaptation and performance measures

Rate adaptation depends on the selected performance measure. In the case of the BEP performance measure, uncoded BPSK is always used and rate adaptation is not needed.

The outage capacity is the performance measure when the transmitter does not know the instantaneous received SNR [15]. However, the mean SNR is known at the base station and the transmission rate is adapted according to the mean SNR and the outage probability constraint. Thus, the rate adaptation follows the slowly varying large scale fading and path loss which define the mean SNR. The outage capacity for the Rayleigh fading channel is defined as

$$C_{P_{\mathrm{out}}} = \log_2(1 + \gamma_{\mathrm{out}}(P_{\mathrm{out}})), \tag{3}$$

where $\gamma_{\mathrm{out}}(P_{\mathrm{out}}) = -\bar{\gamma} \ln(1 - P_{\mathrm{out}})$ provides the desired outage probability P_{out} [13]. The transmission rate can be chosen to be either constant irrespectively of the allocated RB or variable according to the available feedback information of the allocated RB. For example, assume now that the transmitter has one bit knowledge of the instantaneous SNR value so that it knows that the SNR is above or below a threshold γ_{T}. The higher rate is applied if $\gamma > \gamma_{\mathrm{T}}$ and the lower rate is applied otherwise. Now the outage capacity for the Rayleigh channel with one bit feedback information can be defined as

$$C_{P_{\mathrm{out}}} = \sum_{i=1}^{2} \log_2(1 + \gamma_{\mathrm{out},i}(P_{\mathrm{out}})), \tag{4}$$

where $\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = F_{\mathrm{R}}^{-1}(P_{\mathrm{out}}|i)$ is the SNR value that provides the outage probability P_{out} on the condition that the ith rate is applied. The outage capacity study focuses on "variable rate" transmission since it outperforms "constant rate transmission"[4] with the same feedback overhead.

The average capacity measures system spectral efficiency when instantaneous and optimal rate adaptation is applied [30]. In this case, the RB allocation is performed first and then the optimal transmission rate is used for the allocated channel. It is assumed that the channel is fixed and flat inside the time-frequency RB. Furthermore, a RB is assumed to be long in the time domain so that the channel capacity $\log_2(1 + \gamma)$ can be achieved during the transmission of the RB. The average capacity or the Shannon capacity without power adaptation, with transmitter and receiver side information, is of the form

$$C = \int_0^{\infty} f_{\mathrm{RB}}(\gamma) \log_2(1 + \gamma) d\gamma, \tag{5}$$

[4]Both the schemes vary the transmission rate according to the slow large scale fading. Variable rate transmission applies fast adaptation using the limited feedback information available at the transmitter.

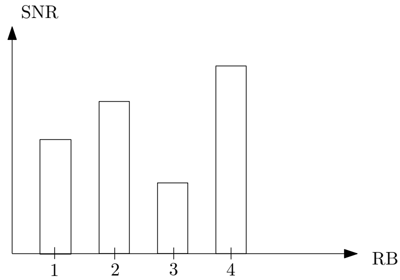
where $f_{\mathrm{RB}}(\gamma)$ is the PDF of the SNR of the RB [15, 30]. The capacity in (5) is the same as the channel capacity with receiver CSI [15, 125]. In the considered system, rate adaptation is needed because slow fading is assumed and the codeword cannot cover the fading. The feedback information for rate adaptation is assumed to be ideal and the further overhead is not taken into account. Optimal rate adaptation is widely used since it provides the achievable spectral efficiency of systems [79, 100, 101, 103, 126].

2.3 Feedback strategies

2.3.1 Best-M feedback method

Several variations of the so called best-M feedback algorithm have been considered. In all the best-M feedback methods, the basic principle is that the M best resource blocks are selected out of the total of N available resource blocks at the receiver. The information of only the M best RBs is only conveyed to the transmitter.

In the best-M scheme, the index which indicates the selected combination of the M best RBs is signaled back to the transmitter [104]. There are $\binom{N}{M} = N!/M!(N-M)!$ different possible feedback words in the best-M feedback scheme. An example case is shown in Fig. 3, where $N = 4$ and $M = 2$. All possible RB combinations and corresponding feedback words are shown in Fig. 3 (b). Thus, the feedback method requires $\lceil \log_2 \left(N!/(M!(N-M)!)\right) \rceil$ bits per user. In the allocation, a RB is randomly selected from the indicated pool if possible. If all the M best resource blocks have already been allocated to the previously scheduled users, a resource block is selected randomly from the available ones. Note that the instantaneous channel quality of the M best resource blocks is not assumed to be available at the transmitter.



(a) Channel condition

Index of RB combination	Index of RB combination in bits
12	000
13	001
14	010
23	100
24	101
34	110

Feedback word: 101

(b) Best-M feedback case

	RB index	RB index in bits
sub-band 1	1	0
	2	1
sub-band 2	3	0
	4	1

Feedback word: 11

(c) SBB-1 feedback case with two sub-blocks

RB index	RB index in bits
1	00
2	01
3	10
4	11

Feedback word: 1101

(d) Ordered best-M feedback case

Fig 3. Example of the best-M feedback schemes with $N = 2$ and $M = 2$. (a) Channel condition, i.e., the two best RBs are the second and the fourth RB. (b) Feedback word for the best-M scheme. (c) Feedback word for the SBB-M scheme. (d) Feedback word for the Obest-M scheme.

Robustness against feedback bit errors can be improved by using a modified best-M feedback strategy, namely, the sub-block based best-M (SBB-M) method. In the SBB-M feedback strategy, the available resource blocks are divided into sub-blocks. The number of the RBs in a sub-block is denoted by N_b. There are N/N_b sub-blocks provided that $N_b < N$ and N_b is the same for each sub-block[5]. The best-M feedback word is formed individually for each sub-block. In Fig. 3 (c), $N_b = 2$ and there are two sub-blocks. Thus, one bit is needed to inform the best RB of a sub-block in this case. Furthermore the feedback overhead is reduced compared to the best-M_1 word when the SBB-M_2 word indicates the M_1 channels ($M_2 < M_1$). Results for the SBB-M method can be easily derived from the results determined for the best-M method by the changing parameters.

Ordered best-M (OBest-M) means that each user individually conveys the indices of the M best resource blocks to the transmitter in a descending order, i.e., the index of the best resource block is signalled first. Fig. 3 (d) illustrates the example case of the Obest-2 feedback word. If a user holds the kth position in the allocation queue then it is possible that some or all of the indicated RBs are allocated to the previously scheduled $k - 1$ users. If some of the indicated

[5]This feedback strategy could also be formed so that N_b varies from one sub-block to another.

RBs are available, the best available one is selected. Otherwise, a RB is selected randomly from the available ones. The feedback load is $M \cdot \lceil \log_2(N) \rceil$ bits per user. The link performance of an individual user strongly depends on its position in the allocation queue, especially if M is much smaller than N. The Obest-M method is not usually used in OFDMA systems due to the higher feedback load than that of the conventional best-M method. Furthermore, the system analysis of Obest-M with imperfect feedback is straightforward but tedious. Therefore, the study focuses on the conventional best-M scheme and the SBB-M scheme.

2.3.2 SNR quantization

RB-wise quantization of the SNR means that the SNR of each RB is monitored at the receiver and a feedback word is selected according to the predefined quantization. Fig. 4 illustrates the example case of the one bit quantization. The resulting bits are conveyed to the transmitter. At the transmitter, a RB for the kth user in the allocation queue is selected randomly from the set of the best available resource blocks. The feedback overhead is $N \cdot \lceil \log_2(L+1) \rceil$ when the SNR is quantized into $L+1$ regions. The drawback of the RB-wise feedback method is that the feedback overhead is prohibitive when the number of resource blocks is large. Thus, one bit feedback per resource block is commonly applied in OFDMA systems [79, 93, 100, 101].

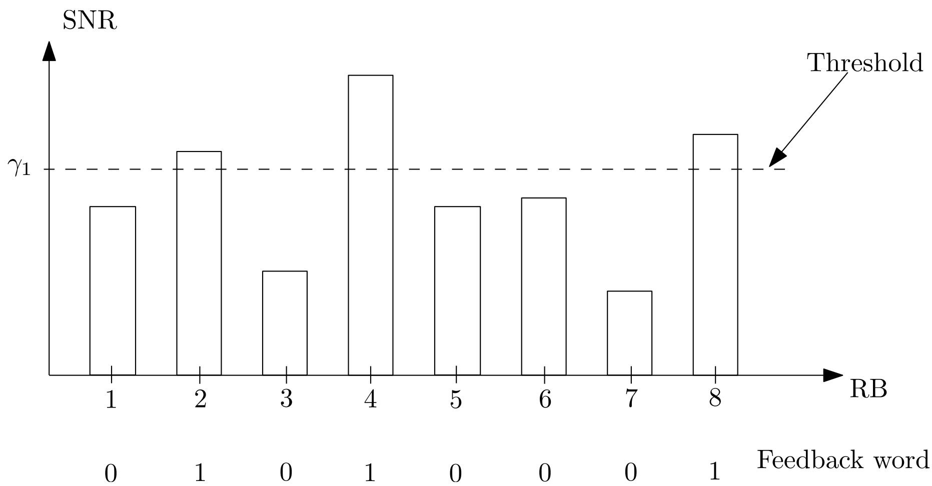


Fig 4. The example of the RB-wise one bit quantization.

2.4 Multiple antenna methods

In the considered single user MIMO methods, a time-frequency RB is allocated exclusively for a single user. STBC and antenna selection schemes are studied. For single user MIMO methods, the maximum number of the transmit and receive antennas are $N_t = 2$ and $N_r = 2$, respectively. Maximum ratio combining (MRC) is applied at the receiver when $N_r = 2$. However, several proposed expressions are valid for arbitrary antenna configurations, but in some cases the limitation for $N_t = N_r = 2$ enables closed form formulas.

2.4.1 Space-time block coding

Conventional STBC is an attractive open loop multiple antenna scheme which offers significant diversity gain without feedback information [45]. The received SNR obeys the Gamma distribution with the parameter $\mathcal{K} = N_t N_r$ when an MRC receiver is applied [127]. For full rate STBC and MRC, the probability density function (PDF) of the SNR is given by

$$f_{\mathrm{STBC}}(\gamma) = \frac{\gamma^{\mathcal{K}-1}}{\Gamma(\mathcal{K})} \left(\frac{N_t}{\bar{\gamma}}\right)^{\mathcal{K}} \exp\left(\frac{N_t \gamma}{\bar{\gamma}}\right). \tag{6}$$

where $\Gamma(\mathcal{K}) = (\mathcal{K}-1)!$ is the Gamma function [127]. The cumulative density function (CDF) of the SNR is expressed as

$$F_{\mathrm{STBC}}(\gamma) = 1 - \mathcal{P}_{\mathcal{K}}(N_t \gamma / \bar{\gamma}), \tag{7}$$

where

$$\mathcal{P}_m(x) = \sum_{i=0}^{m-1} (x^i / i!) e^{-x} \tag{8}$$

denotes the Poisson distribution.

2.4.2 Antenna selection

The performance and feedback overhead depends on the selected AS mechanism. The best performance is achieved when RB-wise AS (RBAS) is employed [128] at the cost of the high overhead. RB-wise antenna selection (RBAS) means that the index of the best antenna of each resource block is individually conveyed from the receiver to the transmitter. Since RBAS requires $N\lceil \log_2(N_t) \rceil$ feedback bits, the feedback overhead increases linearly with N. For a system with correct antenna selection and an MRC receiver, the SNR distribution of a RB is given by

$$f_{\mathrm{AS}}(\gamma) = N_t f_{\mathrm{MRC}}(\gamma) F_{\mathrm{MRC}}(\gamma)^{N_t - 1}, \tag{9}$$

where $f_{\mathrm{MRC}}(\gamma)$ and $F_{\mathrm{MRC}}(\gamma)$ are given by (6) and (7) with the parameters $N_t = 1$ and $\mathcal{K} = N_r$ [55]. The CDF of the SNR is expressed as

$$F_{\mathrm{AS}}(\gamma) = \left(1 - \mathcal{P}_{N_r}(\gamma/\bar{\gamma})\right)^{N_t}. \tag{10}$$

In the case of the single antenna receiver, the PDF of the SNR is reduced to $f_{\mathrm{AS}}(\gamma) = N_t f_{\mathrm{R}}(\gamma) F_{\mathrm{R}}(\gamma)^{N_t-1}$, where $f_{\mathrm{R}}(\gamma)$ and $F_{\mathrm{R}}(\gamma)$ are given in (1) and (2), respectively. The CDF for antenna selection with a single antenna receiver is written as $F_{\mathrm{AS}}(\gamma) = F_{\mathrm{R}}(\gamma)^{N_t}$.

An alternative low feedback AS scheme is to choose one antenna for all the RBs [128, 129]. Thus, a threshold based AS (TBAS) scheme is proposed to be used together with the RB-wise SNR quantization feedback strategy. In the TBAS scheme, the selected antenna operates on the whole frequency band. Pilot symbols are transmitted from each transmit antenna so that estimation of N_t channels related to each resource block is enabled. The index of an antenna, for which the number of detected SNR values that exceed the predefined threshold is the largest, is conveyed to the transmitter. Users' data packets are allocated to the channels of the selected antenna. TBAS requires only $\lceil \log_2(N_t) \rceil$ feedback bits irrespectively of the value of N. Thus, it is suitable also for large N_t and N. In the case of the single antenna receiver, the SNR of a RB is exponentially distributed. For the MRC receiver, the CDF of the SNR is given in (7) with $N_t = 1$.

2.4.3 Space-division multiple access

In the considered SDMA schemes, transmission for two users can be applied at the same time-frequency slot through different BS antennas. Zero forcing (ZF) detection is assumed at the receiver to distinguish spatial streams. In the simplest SDMA, the data of different users can be allocated to parallel spatial streams at the same time-frequency RB without any precoding. The performance of a single stream is comparable to the conventional spatial multiplexing systems. All detection methods developed for spatial multiplexing are applicable. In this paper, we analyze a ZF receiver for which the PDF of the post-detection signal-to-interference plus noise ratio (SINR) γ of the spatial stream is Gamma distributed as $\mathcal{G}(N_r - N_t + 1, \bar{\gamma}/N_r)$ [130]. The PDF of the Gamma distribution $\mathcal{G}(\alpha, \beta)$ is given as $f(\gamma; \alpha, \beta) = \gamma^{\alpha-1} e^{-\gamma/\beta} / (\Gamma(\alpha)\beta^{\alpha})$. For the 2×2 MIMO system, the PDF of the post-detection SINR of the spatial stream is expressed as

$$f_{\mathrm{ZF}}(\gamma) = \frac{2}{\bar{\gamma}} e^{-2\gamma/\bar{\gamma}}. \tag{11}$$

The corresponding CDF is given by

$$F_{\mathrm{ZF}}(\gamma) = 1 - e^{-2\gamma/\bar{\gamma}}. \tag{12}$$

SDMA with STBC means a combination of the simple diversity technique [45] and SDMA. The data symbols of a user are space-time block coded over two transmit antennas and transmitted at the selected frequency resource block in which the other group of two antennas can be used for another user's STBC transmission at the same time. It is shown in [131] that the SNR of the ZF detected spatial stream converges to Gamma distribution $\mathcal{G}(2N_r - 2, \bar{\gamma}/N_t)$ when N_r grows. It turns out that this distribution is very accurate also with $N_r = 2$ and $N_t = 4$. In this study, we consider the 4×2 MIMO system for the diversity-ZF (DZF) system. Thus, the PDF of the γ is expressed as

$$f_{\mathrm{DZF}}(\gamma) = \frac{16}{\bar{\gamma}^2}\gamma, e^{-4\gamma/\bar{\gamma}} \tag{13}$$

and the CDF is of the form

$$F_{\mathrm{DZF}}(\gamma) = 1 - \mathcal{P}_2\left(\frac{4\gamma}{\bar{\gamma}}\right). \tag{14}$$

3 Analysis for the SNR quantization

Since the feedback load of the SNR quantization increases rapidly with the number of quantization levels, one bit quantization has been commonly studied [79, 80, 101, 104, 124]. Furthermore, the probabilities related to the considered dynamic OFDMA system can be expressed in a tractable form for one bit quantization also with an imperfect feedback channel. Thus, the system probabilities and numerical examples are considered with RB-wise one bit quantization although most of the analytical performance expressions are presented for the RB-wise $\lceil \log_2(L+1) \rceil$ bit quantization.

Quantization of SNR is denoted by $Q = \{\xi_l\}_{l=0}^{L+1}$:

$$0 = \xi_0 < \xi_1 < \cdots < \xi_L < \xi_{L+1} = \infty, \quad \xi_l = \gamma_l / \bar{\gamma}, \tag{15}$$

where $\gamma_l = \xi_l \bar{\gamma}$ is the true SNR boundary point. Optimal quantization is system specific, since it varies with the mean SNR, the number of available RBs [79], the user position in the allocation queue, the feedback bit error probability and the selected optimization criterion [76]. For example, the quantization threshold for the system in [104] has been found through simulations and in [75], the thresholds are selected according to the performance of the available modulation methods and coding rates with a predefined error rate constraint. Optimization of the SNR regions for quantization is out of the scope of this thesis. The presented performance formulas are valid for an arbitrary one bit quantization and, besides, in several cases for the quantization given by (15).

Let $q_l(k, p_b)$ be the probability that the kth user occupies a shared channel whose received SNR is in the interval (γ_l, γ_{l+1}) when the feedback BEP is p_b. In the case of AS, the SNR of the best antenna of the allocated RB belongs to (γ_l, γ_{l+1}) with probability $q_l(k, p_b)$. The expected performance of the kth user in the allocation queue is denoted as

$$\mathcal{M}_Q(k) = \sum_{l=0}^{\lceil \log_2(L+1) \rceil} q_l(k, p_b) \mathcal{M}_l^Q, \tag{16}$$

where $\mathcal{M}_l^Q$ is the average performance on the condition that the SNR of the assigned RB belongs to (γ_l, γ_{l+1}). For the AS case, $\mathcal{M}_l^Q$ is the performance when the SNR of the best antenna of the allocated RB belongs to (γ_l, γ_{l+1}), although erroneous AS may provide a SNR out of this range.

According to (16), the PDF of the SNR with quantization Q can be expressed in the form [132]

$$f_{\mathrm{Q}}(\gamma, k) = \sum_{l=0}^{L} q_l(k, p_b) \frac{\nu_l(\gamma) f_{\mathrm{RB}}(\gamma)}{p_l^Q}, \tag{17}$$

where $f_{\mathrm{RB}}(\gamma)$ refers to the PDF of the SNR of a RB and

$$\nu_l(\gamma) = \begin{cases} 1, & \gamma \in (\gamma_l, \gamma_{l+1}), \\ 0, & \text{otherwise}, \end{cases} \quad p_l^Q = \int_{\gamma_l}^{\gamma_{l+1}} f_{\mathrm{RB}}(\gamma) d\gamma. \tag{18}$$

We note that the fraction term in (17) is the PDF of the SNR on the condition that the received SNR belongs to (γ_l, γ_{l+1}). The notation $f_{\mathrm{Q}}(\cdot, k)$ emphasizes that the PDF also depends on the user position in the allocation queue.

3.1 Probabilities of quantization levels

The system probabilities $q_l(k, p_b)$ are defined for one bit quantization per RB which is feasible for practical OFDMA systems. Furthermore, the event probabilities are valid for the RR allocation presented in Chapter 2.

Ideal feedback channel

The SNR of the channel allocated to the kth user is below γ_1 if all resource blocks for which $\gamma > \gamma_1$ have already been assigned to the previously scheduled users and there are $k-1$ "good" channels at maximum. The received SNR of a resource block is above γ_1 with a probability of $p_{\gamma_1} = P(\gamma > \gamma_1)$. The probability of the case that the user has u channels whose SNR exceeds γ_1 obeys the binomial law and it is expressed as

$$p_{\gamma_1}(u) = \binom{N}{u} p_{\gamma_1}^u (1 - p_{\gamma_1})^{N-u}. \tag{19}$$

If a user k has u, $u \leq k-1$, resource blocks with SNR values above γ_1 they are reserved with probability $\binom{N-u}{k-1-u} / \binom{N}{k-1}$. Thus, we notice that

$$\begin{aligned} q_0(k, 0) &= \sum_{u=0}^{k-1} \frac{\binom{N-u}{k-1-u}}{\binom{N}{k-1}} \binom{N}{u} p_{\gamma_1}^u (1 - p_{\gamma_1})^{N-u} \\ &= \sum_{u=0}^{k-1} \binom{k-1}{u} p_{\gamma_1}^u (1 - p_{\gamma_1})^{N-u}. \end{aligned} \tag{20}$$

The probability of the event that a user gets a channel which has a SNR above γ_1 can be simply calculated as $q_1(k, 0) = 1 - q_0(k, 0)$.

Imperfect feedback channel

The resource manager may select the wrong channel for the kth user due to feedback word errors in two cases. 1) Errors occur in the available channels with SNR below γ_1. 2) All feedback bit errors in the available resource blocks have occurred in all available channels whose SNR is above γ_1. These cases can become true for many possible allocations of the preceding users and error events.

The probability of N_e bit errors obeys the binomial distribution and it is given by

$$p_e(N_e) = \binom{N}{N_e} p_b^{N_e}(1-p_b)^{N-N_e}. \tag{21}$$

The event that j errors out of N_e occur in the resource blocks whose SNR is above γ_1 is denoted by E1. The probability of the event E1 is expressed as

$$p_{\mathrm{E1}}(j,u,N_e,k) = \binom{u}{j}\binom{N-u}{N_e-j}\Big/\binom{N}{N_e}, \tag{22}$$

where the numerator refers to the number of potential error combinations in event E1 and the denominator represents the number of possible error events of the N_e errors. The number of errors in the good resource blocks can be between $\min(j) = \max(0, N_e - N + u)$ and $\max(j) = \min(u, N_e)$. the number of bits that correctly indicate the channels with SNR values above γ_1 is $u - j$; $N_e - j$ errors occur in the channels with SNR below γ_1. The number of resource blocks whose SNR values are above γ_1 according to the received feedback message is denoted as $N_1 = u - j + N_e - j$. The event denoted by E2 is stated so that all resource blocks whose SNR values are above γ_1 according to the received feedback word have been allocated to the preceding $k-1$ users. The probability of the event E2 is expressed as

$$p_{\mathrm{E2}}(N_1,k) = \binom{N-N_1}{k-N_1-1}\Big/\binom{N}{k-1}. \tag{23}$$

Assume that u, N_e, j, and E2 has occurred when a resource block to the kth user is allocated. Thus, random allocation for available resource blocks is applied. A channel having a SNR value below γ_1 is selected with probability $(N-u-N_e+j)/(N-N_1)$ in this case. Assume now that the event E2 has not occurred and a resource block is randomly selected from the set of the free channels which are good according to the feedback word. In this case, the SNR of the allocated resource block is below γ_1 with probability $(N_e-j)/N_1$. Thus, RB allocation based on the erroneously received one bit feedback information yields

$$\begin{aligned} q_0(k,p_b) =& \sum_{u=0}^{N}\sum_{N_e=0}^{N}\sum_{j=j_s}^{j_e} p_{\gamma_1}(u)p_e(N_e)p_{\mathrm{E1}}(j,u,N_e,k) \\ &\cdot \left(p_{\mathrm{E2}}(N_1,k)\frac{N-u-N_e+j}{N-N_1} + (1-p_{\mathrm{E2}}(N_1,k))\frac{N_e-j}{N_1}\right) \end{aligned} \tag{24}$$

where $j_s = \max(0, N_e - N + u)$, $j_e = \min(u, N_e)$. The probability that a resource block whose SNR is above γ_1 is allocated to the user can be calculated as $q_1(k, p_b) = 1 - q_0(k, p_b)$.

3.2 BEP analysis in SISO systems

The average BEP performance is achieved by evaluating the integral

$$P_{\mathrm{Q}}(k) = \int_0^\infty f_{\mathrm{Q}}(\gamma, k) g(\gamma) d\gamma, \tag{25}$$

where $g(\gamma)$ is the BEP function of the applied modulation in a non-fading AWGN channel [132]. With the substitution of (17) into (25), the average performance of the kth user becomes [132]

$$P_{\mathrm{Q}}(k) = \sum_{l=0}^{L} q_l(k, p_b)/p_l^{\mathrm{Q}} \cdot (I_l - I_{l+1}), \tag{26}$$

where $I_l = \int_{\gamma_l}^\infty f(\gamma) g(\gamma) d\gamma$ and $I_{L+1} = 0$. In the case of the BEP performance measure and BPSK, we have $g(\gamma) = \Gamma\big(\frac{1}{2}, \gamma\big)/\sqrt{4\pi}$, where $\Gamma(\cdot, \cdot)$ is the complementary incomplete gamma function defined as $\Gamma(t, x) = \int_x^\infty s^{t-1} e^{-s} ds,\ x \geq 0$. The derivative of the function $g(\gamma)$ is given by

$$g'(\gamma) = -e^{-\gamma}/\sqrt{4\pi\gamma} \tag{27}$$

(see [133, Eqs. (6.5.17), (7.1.2) and (7.1.19)]). Applying integration by parts to I_l, using substitution $x = \gamma(1 + 1/\bar{\gamma})$ we obtain [132]

$$I_l = \frac{1}{\sqrt{4\pi}} \Big(e^{-\xi_l} \Gamma\big(\frac{1}{2}, \xi_l \bar{\gamma}\big) - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}} \Gamma\big(\frac{1}{2}, \xi_l (1 + \bar{\gamma})\big) \Big). \tag{28}$$

Now the BEP for the kth user is found for a multiple channel system with the resource-wise one bit feedback method by substituting (24), $q_1(k, p_b) = 1 - q_0(k, p_b)$, and (28) into (26).

3.3 Outage capacity analysis

The outage capacity can be determined for a constant rate transmission and for a variable rate transmission. In the constant rate transmission, the transmission rate is chosen according to the given outage probability, the mean SNR, the feedback bit error probability, and the user's position in the allocation queue. On the other hand, a higher system throughput is achieved if variable rate transmission is applied according to the limited knowledge about the allocated channel. Constant rate transmission is considered for the SISO system and variable rate transmission is analyzed also with multiple antenna methods.

3.3.1 Constant rate transmission

In the case of fixed rate transmission, the outage capacity with a given outage probability for a slowly Rayleigh fading channel is given by

$$C_{P_{\text{out}}}(k) = \log_2(1 + \gamma_{\text{out}}(P_{\text{out}}, k)), \tag{29}$$

where $\gamma_{\text{out}}(P_{\text{out}}, k)$ is the SNR value that provides the desired outage probability for the kth user. Thus, the value of $\gamma_{\text{out}}(P_{\text{out}}, k)$ has to be determined from the CDF of the SNR and then it is simply substituted to the mutual information formula in an AWGN channel [13].

The PDF of the SNR of the kth user is given in (17), and, consequently, the outage probability P_{out} with a given SNR γ_{out} and $\bar{\gamma}$ can be calculated as

$$P_{\text{out}}(\gamma_{\text{out}}, k) = \int_0^{\gamma_{\text{out}}} \sum_{l=0}^{L'} q_l(k, p_b) \cdot \frac{\nu_l(\gamma) f(\gamma)}{p_l^{\text{Q}}} d\gamma$$

where L' is chosen so that $\gamma_{L'} < \gamma_{\text{out}} < \gamma_{L'+1}$. For example, in the case of the SISO transmission, we have

$$P_{\text{out}}(\gamma_{\text{out}}, k) = \sum_{l=0}^{L'-1} q_l(k, p_b) + \frac{q_{L'}(k, p_b)}{p_{L'}^{\text{Q}}} \left(e^{-\gamma_{L'}/\bar{\gamma}} - e^{-\gamma_{\text{out}}/\bar{\gamma}} \right).$$

The SNR value $\gamma_{\text{out}}(P_{\text{out}}, k)$ that provides the desired outage probability P_{out} for user k can be derived as

$$\gamma_{\text{out}}(P_{\text{out}}, k) = -\bar{\gamma} \ln \left(e^{-\gamma_{L'}/\bar{\gamma}} - \frac{p_{L'}^{\text{Q}}}{q_{L'}(k, p_b)} \left(P_{\text{out}} - \sum_{l=0}^{L'-1} q_l(k, p_b) \right) \right). \tag{30}$$

The outage capacity for the desired outage probability is obtained by substituting (30) into (29). Note that the outage probability is assumed to be fixed regardless of the feedback bit error probability. The effect of feedback errors on throughput at constant outage probability is considered.

3.3.2 Variable rate transmission

Variable rate transmission utilizes the available feedback information. The transmission rate can be chosen from $L + 1$ alternatives according to the quality of the assigned channel when quantization in (15) is applied. The average outage capacity of the variable rate transmission for the kth user is expressed as

$$C_{P_{\text{out}}}(k) = \sum_{i=0}^{L} q^{(R_i)}(k, p_b) \log_2(1 + \gamma_{\text{out},i}(P_{\text{out}})), \tag{31}$$

where $q^{(R_i)}(k, p_b)$ is the probability that the ith rate is applied and the SNR value $\gamma_{\text{out},i}(P_{\text{out}})$ provides the desired outage probability for the ith transmission rate. The logarithm term in (31) expresses the outage capacity [13] on the condition that the ith rate R_i is used.

In the outage capacity analysis, the outage probability is fixed and $\gamma_{\text{out},i}(P_{\text{out}})$ is solved from the CDF of the SNR on the condition that the ith rate is applied at the base station. Due to the assumption that the base station knows p_b, the desired outage probability is achieved in spite of the imperfect feedback. We now assume that antenna selection is perfect and postpone the analysis of the selection errors to Section 3.3.3. Although the transmitter allocates the resource block whose SNR is in (γ_i, γ_{i+1}) according to the received feedback word, its real SNR can be outside this range due to the feedback errors. The probability that the SNR of the allocated resource block belongs to (γ_l, γ_{l+1}) is denoted by $q_l^{(R_i)}(p_b)$ when the ith rate is used.

Proposition 1 *Assuming perfect antenna selection and quantization in* (15)*, the SNR value that provides the outage capacity for the ith transmission rate with the desired outage probability P_{out} can be derived from*

$$\gamma_{out,i}(P_{out}) = F_{RB}^{-1}\left(F_{RB}(\gamma_{L'}) - A(p_b, P_{out})\right), \tag{32}$$

where L' is chosen so that $\gamma_{L'} < \gamma_{out,i}(P_{out}) < \gamma_{L'+1}$ and

$$A(p_b, P_{out}) = \frac{p_{L'}^{Q}}{q_{L'}^{(R_i)}(p_b)} \left(\sum_{l=0}^{L'-1} q_l^{(R_i)}(p_b) - P_{out} \right). \tag{33}$$

Proof. For a given SNR γ_o, the outage probability on the condition that the ith rate is used can be expressed as

$$\begin{aligned} P_{\text{out}}^{(R_i)}(\gamma_o) &= \int_0^{\gamma_o} \sum_{l=0}^{L'-1} q_l^{(R_i)}(p_b) \frac{\nu_l(\gamma) f_{\text{RB}}(\gamma)}{p_l^{\text{Q}}} d\gamma \\ &= \sum_{l=0}^{L'} q_l^{(R_i)}(p_b) + \frac{q_{L'}^{(R_i)}(p_b)}{p_{L'}^{\text{Q}}} \left(F_{\text{RB}}(\gamma_o) - F_{\text{RB}}(\gamma_{L'})\right), \end{aligned} \tag{34}$$

where L' is chosen so that $\gamma_{L'} < \gamma_o < \gamma_{L'+1}$, ν_l and p_l^{Q} are given in (18). We can obtain the results in (32) and (33) by fixing P_{out} and solving γ_o from (34). □

Now, the SNR value $\gamma_{\text{out},i}(P_{\text{out}})$ can be easily determined for different schemes based on $F_{\text{RB}}^{-1}(\gamma)$, (32) and (33). Then, after some manipulations we find that

$$\gamma_{\text{out},i}(P_{\text{out}}) = -\bar{\gamma}\ln\left(e^{-\gamma_{L'}/\bar{\gamma}} + A(p_b, P_{\text{out}})\right), \quad \text{for an SISO system} \tag{35}$$

$$\gamma_{\text{out},i}(P_{\text{out}}) = \bar{\gamma}\mathcal{P}_{N_r}^{-1}\left(\mathcal{P}_{N_r}\left(\frac{\gamma_{L'}}{\bar{\gamma}}\right) + A(p_b, P_{\text{out}})\right), \text{ for an SIMO system} \tag{36}$$

$$\gamma_{\text{out},i}(P_{\text{out}}) = \frac{\bar{\gamma}}{N_t}\mathcal{P}_{\mathcal{K}}^{-1}\left(\mathcal{P}_{\mathcal{K}}\left(\frac{N_t\gamma_{L'}}{\bar{\gamma}}\right) + A(p_b, P_{\text{out}})\right), \quad \text{for STBC} \tag{37}$$

$$\gamma_{\text{out},i}(P_{\text{out}}) = -\bar{\gamma}\ln\left(1 - \left(F_{\text{R}}(\gamma_{L'})^{N_t} + A(p_b, P_{\text{out}})\right)^{1/N_t}\right), \tag{38}$$

for RBAS, $N_r = 1$

$$\gamma_{\text{out},i}(P_{\text{out}}) = \bar{\gamma}\mathcal{P}_{N_r}^{-1}\left(1 - \left(F_{\text{MRC}}(\gamma_{L'})^{N_t} + A(p_b, P_{\text{out}})\right)^{1/N_t}\right), \tag{39}$$

for RBAS and MRC

$$\gamma_{\text{out},i}(P_{\text{out}}) = \frac{-\bar{\gamma}}{2}\ln\left(e^{-2\gamma_{L'}/\bar{\gamma}} + A(p_b, P_{\text{out}})\right), \tag{40}$$

for a 2 × 2 MIMO system with SDMA

$$\gamma_{\text{out},i}(P_{\text{out}}) = \frac{\bar{\gamma}}{4}\mathcal{P}_2^{-1}\left(\mathcal{P}_2\left(\frac{4\gamma_{L'}}{\bar{\gamma}}\right) + A(p_b, P_{\text{out}})\right), \tag{41}$$

for a 4 × 2 MIMO system with SDMA and STBC,

where $A(p_b, P_{\text{out}})$ is given in (33) and $\mathcal{P}^{-1}$ denotes the inverse value of the Poisson CDF. The inverse value of the Poisson CDF cannot be solved in a closed form. However, it exists because $\mathcal{P}_{\mathcal{K}}(x)$ is monotonically decreasing. It is also possible to generate a look-up-table to calculate $\mathcal{P}^{-1}(x)$ for $x > 0$.

Utilizing the event probabilities presented in Section 3.1, the probability $q^{(R_0)}(k, p_b)$ for the RB-wise one bit feedback strategy can be easily determined to be of the form

$$q^{(R_0)}(k, p_b) = \sum_{u=0}^{N}\sum_{N_e=0}^{N}\sum_{j=j_s}^{j_e} p_{\gamma_1}(u)p_e(N_e)p_{\text{E1}}(j, u, N_e)p_{\text{E2}}(N_1, k). \tag{42}$$

Thus, the higher rate is applied with probability $q^{(R_1)}(k, p_b) = 1 - q^{(R_0)}(k, p_b)$.

3.3.3 Antenna selection

Threshold based antenna selection

For the TBAS scheme, the SNR values $\gamma_{\text{out},i}(P_{\text{out}})$ are derived similarly as for the single antenna transmission, i.e., they are given in (35) and (36) for $N_r = 1$ and $N_r = 2$, respectively. The TBAS scheme, however, provides performance gains over single antenna transmission. Note that the probabilities $q_l^{(R_i)}(p_b)$

have to be calculated so that antenna selection is taken into account. Consider one bit quantization and $N_r = 1$ for an example case. A higher threshold value and, therefore, a higher transmission rate can be applied in the TBAS case than in the SISO case, if the users get the good channel with the same probability in both the cases. If the threshold value is the same in the SISO and TBAS schemes, the performance at the good channel is the same in both the schemes, but TBAS provides more good channels.

Imperfect antenna selection has to be taken into account when the probabilities $q_l^{(R_i)}(p_b)$ are defined. The probability of antenna selection error (ASE) is denoted as

$$p_{\mathrm{ASE}} = 1 - (1 - p_b)^{\lceil \log_2(N_t) \rceil}. \tag{43}$$

Now the probability $q_l^{(R_i)}(p_b)$ can be expressed as

$$q_l^{(R_i)}(p_b) = q_l^{(R_i,\mathrm{C})}(p_b)(1 - p_{\mathrm{ASE}}) + q_l^{(R_i,\mathrm{E})}(p_b)p_{\mathrm{ASE}}, \tag{44}$$

where $q_l^{(R_i,\mathrm{C})}(p_b)$ is the probability that the received SNR belongs to (γ_l, γ_{l+1}) when the rate R_i is applied and antenna selection is correct, and $q_l^{(R_i,\mathrm{E})}(p_b)$ is the corresponding probability when the antenna selection is erroneous due to the feedback error. Resource block allocation is random in the latter case, or $q_l^{(R_i,\mathrm{E})}(p_b) = q_l^{(R_j,\mathrm{E})}(p_b), \forall i, j$.

Let us consider the RB-wise one bit feedback scheme and the two transmit antenna case. In the case of correct antenna selection, we have $q_i^{(R_i,\mathrm{C})}(p_b) = 1 - p_b$ and $q_l^{(R_i,\mathrm{C})}(p_b) = p_b$ for $l \neq i$. Assume that an antenna is selected erroneously. The probability to get a RB whose SNR is below γ_1 is $(N - u_2)/N$ when u_2 is the number of good channels of the erroneously selected antenna. the number of resource blocks having SNR above γ_1 at the best antenna is denoted by $u_1 \geq u_2$. The probability $q_0^{(R_j,\mathrm{E})}(p_b)$ is given by

$$q_0^{(R_j,\mathrm{E})}(p_b) = \sum_{u_2=0}^{N} \frac{N - u_2}{N} \left(\sum_{u_1=u_2+1}^{N} 2p_{\gamma_1}(u_1)p_{\gamma_1}(u_2) + (p_{\gamma_1}(u_2))^2 \right), \tag{45}$$

where $p_{\gamma_1}(u)$ is the same as for the SISO scheme. The probability to allocate the channel whose SNR exceeds the threshold is $q_1^{(R_i,\mathrm{E})}(p_b) = 1 - q_0^{(R_j,\mathrm{E})}(p_b)$.

Resource-block-wise antenna selection

The results in (38) and (40) hold also when feedback for resource block allocation is imperfect if antenna selection is perfect. For simplicity, the outage probability with imperfect antenna selection is expressed for two transmit antennas although it could be generalized for the N_t antenna case. In order to the take antenna selection error into account, we have to determine the CDF of the SNR for the ASE case on the condition that the SNR of the best antenna belongs to (γ_l, γ_{l+1}). The SNR at the best antenna is denoted as γ_{AS}. The conditional CDF of the SNR is given as

$$\begin{aligned} F_{\text{ASE}l}(\gamma_o|\gamma_{\text{AS}}) &= \begin{cases} \frac{2F(\gamma_o)(F(\gamma_{l+1})-F(\gamma_l))}{F(\gamma_{l+1})^2-F(\gamma_l)^2} \\ \frac{2F(\gamma_o)(F(\gamma_{l+1})-F(\gamma_o))}{F(\gamma_{l+1})^2-F(\gamma_o)^2} \end{cases} \\ &= \begin{cases} \frac{2F(\gamma_o)}{F(\gamma_{l+1})+F(\gamma_l)} & \gamma_o < \gamma_l, \gamma_l < \gamma_{\text{AS}} < \gamma_{l+1} \\ \frac{2F(\gamma_o)}{F(\gamma_{l+1})+F(\gamma_o)} & \gamma_l < \gamma_o < \gamma_{\text{AS}}, \gamma_o < \gamma_{\text{AS}} < \gamma_{l+1}. \end{cases} \end{aligned} \tag{46}$$

In the upper case in (46), the numerator is the joint CDF for the case where $\gamma < \gamma_o$ in one antenna and $\gamma_l < \gamma < \gamma_{l+1}$ in the other antenna. The numerator in the lower case in (46) is the joint CDF for the case where in one antenna and $\gamma_o < \gamma < \gamma_{l+1}$ in the other antenna. The denominator in (46) is the probability that $\gamma_l < \gamma_{\text{AS}} < \gamma_{l+1}$ or $\gamma_o < \gamma_{\text{AS}} < \gamma_{l+1}$. Now we can obtain the outage probability of the SNR for quantization. For the SNR value γ_o, $\gamma_{L'} \leq \gamma_o \leq \gamma_{L'+1}$, the outage probability is given as

$$\begin{aligned} P_{\text{out}}^{(R_i)}(\gamma_o, k) = &\sum_{l=0}^{L'-1} q_l^{(R_i)}(p_b) \\ &+ q_{L'}^{(R_i)}(p_b)\Bigg(\frac{F_{\text{MRC}}(\gamma_o)^2 - F_{\text{MRC}}(\gamma_{L'})^2}{p_{L'}^{\text{Q}}}(1 - p_{\text{ASE}}) \\ &+ \frac{2F_{\text{MRC}}(\gamma_o)}{F_{\text{MRC}}(\gamma_{L'+1}) + F_{\text{MRC}}(\gamma_o)} p_{\text{ASE}}\Bigg) \\ &+ \sum_{l=L'+1}^{L} q_l^{(R_i)}(p_b) \frac{2F_{\text{MRC}}(\gamma_o)}{F_{\text{MRC}}(\gamma_{l+1}) + F_{\text{MRC}}(\gamma_l)} p_{\text{ASE}}, \end{aligned} \tag{47}$$

where the second term represents the CDF when antenna selection is perfect, the third term represents the CDF of the erroneously selected antenna on the condition that the SNR of the best antenna belongs to $(\gamma_o, \gamma_{L'+1})$, and the fourth term represents the CDF of the erroneously selected antenna on the condition that the SNR of the best antenna belongs to (γ_l, γ_{l+1}), $l > L'$. The probabilities $q_l^{(R_i)}(p_b)$ are again given as $q_i^{(R_i)}(p_b) = 1 - p_b$ and $q_l^{(R_i)}(p_b) = p_b$ for $l \neq i$ when RB-wise one bit feedback is applied. In order to determine the SNR value $\gamma_{\text{out},i}(P_{\text{out}})$ that provides the desired outage probability P_{out}, the value of $F_{\text{MRC}}(\gamma)$ has to be solved from (47). The (correct) root for the fourth order polynomial equation formed from (47) with a given P_{out} is denoted as $\mathcal{R}_i(P_{\text{out}}) = F_{\text{MRC}}(\gamma)$. Thus, the SNR value that provides the desired outage probability for the single antenna receiver is

$$\gamma_{\text{out},i}(P_{\text{out}}) = -\bar{\gamma} \ln\left(1 - \mathcal{R}_i(P_{\text{out}})\right), \tag{48}$$

and for the MRC receiver is

$$\gamma_{\text{out},i}(P_{\text{out}}) = \bar{\gamma}\mathcal{P}_{N_r}^{-1}\left(1 - \mathcal{R}_i(P_{\text{out}})\right). \tag{49}$$

3.4 Average capacity analysis

The average spectral efficiency of the system with optimal rate adaptation and RB-wise feedback information based on the quantization in (15) is derived in this section. The average capacity with optimal rate adaptation can be expressed as

$$C(k) = \sum_{l=0}^{L} \frac{q_l(k, p_b)}{p_l^{\text{Q}}}(\mathcal{I}_l^{\text{Q}} - \mathcal{I}_{l+1}^{\text{Q}}), \tag{50}$$

where $\mathcal{I}_l^{\text{Q}} = \int_{\gamma_l}^{\infty} f_{\text{RB}}(\gamma)\log_2(1+\gamma)d\gamma$ and p_l^{Q} is given in (18). Note that the formula in (50) is valid for AS schemes when the AS is correct. The capacity in (50) provides the theoretical upper bound of the achievable spectral efficiency of the system. Using integration by parts, $\mathcal{I}_l^{\text{Q}}$ can be expressed as

$$\mathcal{I}_l^{\text{Q}} = (1 - F_{\text{RB}}(\gamma_l))\log_2(1+\gamma_l) + \log_2(e)\underbrace{\int_{\gamma_l}^{\infty} \frac{1 - F_{\text{RB}}(\gamma)}{1+\gamma}d\gamma}_{\mathcal{I}_{\text{Q}}(\gamma_l)}. \tag{51}$$

Note that for $l = L+1$ the integral is $\mathcal{I}_{\text{Q}}(\gamma_{L+1}) = 0$ and $\mathcal{I}_0^{\text{Q}} = \mathcal{I}_{\text{Q}}(0)$. For the STBC and SDMA schemes, the capacity results are expressed for the arbitrary $\lceil \log_2(L+1) \rceil$ bit quantization in (15). In the case of the AS, one bit quantization is assumed for simplicity.

3.4.1 Space-time block coding

Proposition 2 *The integral $\mathcal{I}_Q(\gamma_l)$ in* (51) *for the STBC scheme can be presented as*

$$\mathcal{I}_Q(\gamma_l) = \mathcal{P}_{\mathcal{K}}\left(-\frac{N_t}{\bar{\gamma}}\right)E_1\left(\frac{N_t(1+\gamma_l)}{\bar{\gamma}}\right) + \sum_{m=1}^{\mathcal{K}-1}\frac{\mathcal{P}_m(N_t(1+\gamma_l)/\bar{\gamma})}{m}\mathcal{P}_{\mathcal{K}-m}\left(-\frac{N_t}{\bar{\gamma}}\right), \tag{52}$$

where $\mathcal{K} = N_r N_t$.

Proof. Substituting the CDF (7) into the integral $\mathcal{I}_{\text{Q}}(\gamma_1)$ in (51), we have $\mathcal{I}_{\text{Q}}(\gamma_1) = \int_{\gamma_1}^{\infty} \frac{\mathcal{P}_{\mathcal{K}}(N_t\gamma/\bar{\gamma})}{1+\gamma}d\gamma$. The result for $\mathcal{I}_{\text{Q}}(0)$ has been derived in [127]. The result in (52) is obtained by changing the parameters for the proof of the Shannon capacity of STBC in [127]. □

The integral $\mathcal{I}_{\mathrm{Q}}(\gamma_l)$ can be easily reduced from (52) for the SIMO and SISO schemes. For the SISO scheme we have

$$\mathcal{I}_{\mathrm{Q}}(\gamma_l) = e^{1/\bar{\gamma}} E_1((1+\gamma_l)/\bar{\gamma}), \tag{53}$$

where $E_1(a) = \int_a^\infty \frac{e^{-u}du}{u}$ is the exponential integral [133].

3.4.2 Antenna selection

Threshold based antenna selection

Similarly as for the outage capacity analysis in Section 3.3.3, the spectral efficiency expressions are the same as those for the SISO and SIMO cases when $N_r = 1$ and $N_r = 2$, respectively. Thus, the integral $\mathcal{I}_{\mathrm{Q}}(\gamma_l)$ is given in (53) for $N_r = 1$ and in (52) with parameters $N_t = 1$ and $\mathcal{K} = N_r = 2$ for $N_r = 2$.

The performance difference compared to single antenna transmission is due to the different event probabilities $q_l(k, p_b)$ and the quantization boundaries. Imperfect antenna selection has to be taken into account as presented in Section 3.3.3. For simplicity and clarity, RB-wise one bit feedback is considered for the TBAS scheme. The channel whose SNR is below the threshold is allocated with the probability

$$q_0(k, p_b) = q_0^{\mathrm{C}}(k, p_b)(1 - p_{\mathrm{ASE}}) + q_0^{\mathrm{E}}(p_b) p_{\mathrm{ASE}}, \tag{54}$$

where $q_0^{\mathrm{C}}(k, p_b)$ is given in (24) for the case of correct AS and $q_0^{\mathrm{E}}(p_b)$ is given in (45) for erroneous AS.

Resource block-wise antenna selection

For simplicity, one bit quantization is assumed in the RBAS scheme although the results could be extended for arbitrary quantization in (15). Thus, the capacity with optimal rate adaptation can be expressed as

$$\begin{aligned} C(k) =& q_1(k, p_b)((1 - p_{\mathrm{ASE}})C_{\mathrm{AS}1}(\gamma_1) + p_{\mathrm{ASE}} C_{\mathrm{ASE}1}(\gamma_1)) \\ &+ q_0(k, p_b)((1 - p_{\mathrm{ASE}})C_{\mathrm{AS}0}(\gamma_1) + p_{\mathrm{ASE}} C_{\mathrm{ASE}0}(\gamma_1)), \end{aligned} \tag{55}$$

where p_{ASE} is the probability of antenna selection error (ASE),

$$C_{\mathrm{AS}1}(\gamma_1) = \int_{\gamma_1}^{\infty} f_{\mathrm{AS}}(\gamma)/p_1^{\mathrm{Q}} \log_2(1+\gamma)d\gamma, \tag{56}$$

$$C_{\mathrm{ASE}1}(\gamma_1) = \int_{0}^{\infty} f_{\mathrm{ASE}1}(\gamma|\gamma_{\mathrm{AS}}) \log_2(1+\gamma)d\gamma, \quad \gamma_{\mathrm{AS}} > \gamma_1 \tag{57}$$

$$C_{\mathrm{AS}0}(\gamma_1) = \int_{0}^{\gamma_1} f_{\mathrm{AS}}(\gamma)/p_0^{\mathrm{Q}} \log_2(1+\gamma)d\gamma, \tag{58}$$

$$C_{\mathrm{ASE}0}(\gamma_1) = \int_{0}^{\gamma_1} f_{\mathrm{ASE}0}(\gamma|\gamma_{\mathrm{AS}}) \log_2(1+\gamma)d\gamma, \quad \gamma_{\mathrm{AS}} < \gamma_1 \tag{59}$$

where γ_{AS} is the SNR at the best antenna. The capacity $C_{\mathrm{AS}1}(\gamma_1)$ provides the average spectral efficiency for the channel whose SNR exceeds the threshold when the antenna selection is correct. The capacity in (58) corresponds to a channel whose SNR is below the threshold. For erroneous antenna selection, the notations $C_{\mathrm{ASE}1}(\gamma_1)$ and $C_{\mathrm{ASE}0}(\gamma_1)$ refer to spectral efficiencies when the SNR of the best antenna is above and below the threshold, respectively. For the MISO case, $C_{\mathrm{AS}1}(\gamma_1)$ and $C_{\mathrm{AS}0}(\gamma_1)$ can be found in [98]. In the following proposition, we provide the capacity $C_{\mathrm{AS}1}(\gamma_1)$ for the MRC receiver.

Proposition 3 *The capacity $C_{AS1}(\gamma_1)$ for 2×2 MIMO systems with the MRC receiver can be expressed as*

$$\begin{aligned} C_{AS1}(\gamma_1) =& \frac{\log_2(e)}{p_1^Q}\Big(p_1^Q \ln(1+\gamma_1) + 2\mathcal{I}_2(0, 1/\bar{\gamma}, \gamma_1) + 2\mathcal{I}_2(1, 1/\bar{\gamma}, \gamma_1) \\ &- \mathcal{I}_2(0, 2/\bar{\gamma}, \gamma_1) - 2/\bar{\gamma}\mathcal{I}_2(1, 2/\bar{\gamma}, \gamma_1) - 1/\bar{\gamma}^2\mathcal{I}_2(2, 2/\bar{\gamma}, \gamma_1)\Big), \end{aligned} \tag{60}$$

where $\mathcal{I}_2(a, b, c)$ is given by (169) *in Appendix 1.*

Proof. Applying integration by parts to (56) and the CDF formula in (10), the capacity can be expressed as

$$C_{\mathrm{AS}1}(\gamma_1) = \frac{\log_2(e)}{p_1^{\mathrm{Q}}}\left(p_1^{\mathrm{Q}} \ln(1+\gamma_1) + \int_{\gamma_1}^{\infty} \frac{2\mathcal{P}_2(\gamma/\bar{\gamma}) - \mathcal{P}_2^2(\gamma/\bar{\gamma})}{1+\gamma} d\gamma\right). \tag{61}$$

The result in (56) is obtained by substituting the integral $\mathcal{I}_1(i, \alpha, \gamma_1)$ given in (170) into (61). □

The capacity in (58) can be derived as $C_{\mathrm{AS}0}(\gamma_1) = C_{\mathrm{AS}1}(0) - C_{\mathrm{AS}1}(\gamma_1)$. Similarly, the capacity for system with antenna selection and any SNR quantization could be calculated so that the capacity is $C_{\mathrm{AS}l}^{\mathrm{Q}} = (C_{\mathrm{AS}l}(\gamma_l) - C_{\mathrm{AS}l+1}(\gamma_{l+1}))/p_l^{\mathrm{Q}}$ when the SNR of the allocated channel belongs to (γ_l, γ_{l+1}), and $C_{\mathrm{AS}l}(\gamma_l)$ is given by (60) with the substitutions $\gamma_1 = \gamma_l$ and $p_1^{\mathrm{Q}} = 1 - F_{\mathrm{AS}}(\gamma_l)$.

The capacity in the case of imperfect antenna selection on the condition that the SNR of the best channel is below or exceeds the threshold, i.e., $C_{\mathrm{ASE}1}(\gamma_1)$ or $C_{\mathrm{ASE}0}(\gamma_1)$, can be solved as detailed in the following proposition.

Proposition 4 *In the case of one bit quantization, antenna selection error, two transmit antennae, and MRC detection, the capacity can be expressed as*

$$\begin{aligned} C_{ASE1}(\gamma_1) &= \frac{2\log_2(e)}{2-\mathcal{P}_2(\gamma_1/\bar{\gamma})}\Big(-\mathcal{P}_2(\gamma_1/\bar{\gamma})\ln(1+\gamma_1)/2 + \mathcal{P}_2(-1/\bar{\gamma})E_1(1/\bar{\gamma}) \\ &\quad + 1 - \mathcal{P}_2(-1/\bar{\gamma})E_1((1+\gamma_1)/\bar{\gamma}) - e^{-\gamma_1/\bar{\gamma}}\Big) \\ &\quad + \log_2(e)\sum_{i=1}^{\infty}\frac{1}{2^i}\sum_{m=0}^{i}\binom{i}{m}\bar{\gamma}^{-m}\mathcal{I}_2(m, i/\bar{\gamma}, \gamma_1) \end{aligned} \tag{62}$$

for $\gamma_{AS} > \gamma_1$ *and for* $\gamma_{AS} < \gamma_1$ *as*

$$\begin{aligned} &C_{ASE0}(\gamma_1) = \log_2(1+\gamma_1)\left(1-\frac{2}{B}\right) \\ &- 2\log_2(e)\sum_{i=1}^{\infty}\left(\frac{1}{B^{i+1}}+\frac{1}{B^i}\right)\sum_{m=0}^{i}\binom{i}{m}\bar{\gamma}^{-m}\left(\mathcal{I}_2(m, i/\bar{\gamma}, 0) - \mathcal{I}_2(m, i/\bar{\gamma}, \gamma_1)\right), \end{aligned} \tag{63}$$

where $B = 1 - \mathcal{P}_2(\gamma_1/\bar{\gamma})$ *and* $\mathcal{I}_2(a, b, c)$ *is given by* (169) *in Appendix 1.*

Proof. In the case of one bit quantization, antenna selection error, two transmit antennae and MRC detection, the conditional CDF of the SNR can be expressed as

$$\begin{aligned} F_{\mathrm{ASE1}}(\gamma_*|\gamma_{\mathrm{AS}}) &= \begin{cases} \frac{2F_{\mathrm{MRC}}(\gamma_*)(1-F_{\mathrm{MRC}}(\gamma_1))}{1-F_{\mathrm{MRC}}(\gamma_1)^2} \\ \frac{2F_{\mathrm{MRC}}(\gamma_*)(1-F_{\mathrm{MRC}}(\gamma_*))}{1-F_{\mathrm{MRC}}(\gamma_*)^2}, \end{cases} \\ &= \begin{cases} \frac{2F_{\mathrm{MRC}}(\gamma_*)}{1+F_{\mathrm{MRC}}(\gamma_1)} & \gamma_* < \gamma_1, \quad \gamma_{\mathrm{AS}} > \gamma_1 \\ \frac{2F_{\mathrm{MRC}}(\gamma_*)}{1+F_{\mathrm{MRC}}(\gamma_*)} & \gamma_* > \gamma_1, \quad \gamma_{\mathrm{AS}} > \gamma_*, \end{cases} \end{aligned} \tag{64}$$

$$F_{\mathrm{ASE0}}(\gamma_*|\gamma_{\mathrm{AS}}) = \frac{2F_{\mathrm{MRC}}(\gamma_*)}{F_{\mathrm{MRC}}(\gamma_1)+F_{\mathrm{MRC}}(\gamma_*)} \quad \gamma_* < \gamma_1, \quad \gamma_* < \gamma_{\mathrm{AS}} < \gamma_1. \tag{65}$$

In the upper case in (64), the nominator is the joint CDF for the case where $\gamma < \gamma_*$ in one antenna and $\gamma > \gamma_1$ in the other antenna. The nominator in the lower case in (64) is the joint CDF for the case where $\gamma < \gamma_*$ in one antenna and $\gamma > \gamma_*$ in the other antenna. The denominator in (64) is the probability that $\gamma_{\mathrm{AS}} > \gamma_1$ or $\gamma_{\mathrm{AS}} > \gamma_*$. Similarly, the CDF formula in (65) can be determined for the case where $\gamma_{\mathrm{AS}} < \gamma_1$. The capacity $C_{\mathrm{ASE1}}(\gamma_1)$ has to be integrated in two parts from 0 to γ_1 and from γ_1 to ∞. By using integration by parts we have

$$
\begin{aligned}
&C_{\mathrm{ASE}1}(\gamma_1) = (F_{\mathrm{ASE}1}(\gamma_1|\gamma_{\mathrm{AS}}) - 1)\log_2(1+\gamma_1) \\
&+ \int_0^\infty \frac{1 - F_{\mathrm{ASE}1}(\gamma|\gamma_{\mathrm{AS}})}{1+\gamma} d\gamma \\
&= \frac{2\log_2(e)}{2 - \mathcal{P}_2(\gamma_1/\bar{\gamma})} \Big(- \mathcal{P}_2(\gamma_1/\bar{\gamma})\ln(1+\gamma_1)/2 + \mathcal{P}_2(-1/\bar{\gamma})E_1(1/\bar{\gamma}) + 1 \\
&- \mathcal{P}_2(-1/\bar{\gamma})E_1((1+\gamma_1)/\bar{\gamma}) - e^{-\gamma_1/\bar{\gamma}}\Big) + \log_2(e)\int_{\gamma_1}^\infty \frac{\mathcal{P}_2(\gamma/\bar{\gamma})}{(2-\mathcal{P}_2(\gamma/\bar{\gamma}))(1+\gamma)} d\gamma,
\end{aligned} \tag{66}
$$

which is valid for the MRC receiver with two receive antennas. No closed-form solution for the last integral in (66) exists but we can expand the term $1 - F_{\mathrm{ASE}1}(\gamma|\gamma_{\mathrm{AS}})$, for $\gamma > \gamma_1$, using a geometric series as follows

$$
\frac{\mathcal{P}_2(b\gamma)}{2 - \mathcal{P}_2(b\gamma)} = \frac{1}{2}\mathcal{P}_2(b\gamma)\sum_{i=0}^{\infty}\left(\frac{1}{2}\mathcal{P}_2(b\gamma)\right)^i = \sum_{i=1}^{\infty}\frac{1}{2^i}\mathcal{P}_2(b\gamma)^i \tag{67}
$$

provided that $\frac{1}{2}\mathcal{P}_2(b\gamma) < 1$. It is easy to see that this requirement is valid since $(\gamma > 0)$ $e^{b\gamma} = 1 + b\gamma + ... > 1 + b\gamma$ and thus $1 > (1+b\gamma)e^{-b\gamma} = \mathcal{P}_2(b\gamma)$. Actually, $\frac{1}{2}\mathcal{P}_2(b\gamma) < \frac{1}{2}$, and, hence, the series in (67) should converge rather rapidly. Combining the results in (66), (67), and (170) we obtain (62).

By using integration by parts, the capacity for the case where the antenna selection is erroneous and the SNR at the best antenna is below the threshold can be derived as

$$
\begin{aligned}
C_{\mathrm{ASE}0}(\gamma_1) &= \log_2(1+\gamma_1) - \log_2(e)\int_0^{\gamma_1}\frac{F_{\mathrm{ASE}0}(\gamma|\gamma_{\mathrm{AS}} < \gamma_1)}{1+\gamma} d\gamma \\
&= \log_2(1+\gamma_1) - \frac{2}{B}\int_0^{\gamma_1}\frac{\log_2(e)}{(1-\mathcal{P}_2(\gamma/\bar{\gamma})/B)(1+\gamma)} d\gamma \\
&+ \frac{2}{B}\int_0^{\gamma_1}\frac{\log_2(e)\mathcal{P}_2(\gamma/\bar{\gamma})}{(1-\mathcal{P}_2(\gamma/\bar{\gamma})/B)(1+\gamma)} d\gamma,
\end{aligned} \tag{68}
$$

where $B = 1 - \mathcal{P}_2(\gamma_1/\bar{\gamma})$. The capacity expression in (63) is obtained by using the geometric series similarly as in (67) and combining the results in (68) and (170). $\square$

3.4.3 Space-division multiple access

Proposition 5 *The integral $\mathcal{I}_Q(\gamma_l)$ in (51) for SDMA without diversity transmission can be presented as*

$$
\mathcal{I}_Q(\gamma_l) = e^{2/\bar{\gamma}}E_1(2(1+\gamma_l)/\bar{\gamma}) \tag{69}
$$

and for the SDMA with STBC as

$$\mathcal{I}_Q(\gamma_l) = \mathcal{P}_2\left(-\frac{4}{\bar{\gamma}}\right) E_1\left(\frac{4(1+\gamma_l)}{\bar{\gamma}}\right) + e^{-4\gamma_l/\bar{\gamma}}. \tag{70}$$

Proof. Substituting (12) into the integral $\mathcal{I}_{\mathrm{Q}}(\gamma_l)$ in (51) and using $\mathcal{I}_3(2(1+\gamma_{\mathrm{T}})/\bar{\gamma}, \gamma_{\mathrm{T}})$ given by (171) in Appendix 1, the result in (69) is obtained. Setting $\mathcal{K} = 2$ and $N_t = 4$ in Proposition 2, the result for SDMA with STBC in (70) is achieved. □

3.5 Numerical examples

The RB-wise one bit feedback scheme is considered in the numerical examples. In this Section, the performance curves and marker points illustrate analytical performance results unless mentioned otherwise. Some simulation results are shown to validate the proposed analysis. Marker points are used to present possible simulation results and curves always present analytical results.

It is assumed that all SNR quantization regions have equal probabilities unless mentioned otherwise. This gives a natural starting point for a suboptimal and practical fixed quantization [132]. This quantization has also turned out to be a good choice for the resource-wise one bit feedback scheme in an unreliable feedback channel. Thus, the threshold value γ_1 can be easily determined for each scheme from

$$F_{\mathrm{RB}}(\gamma_1) = \frac{1}{2} \tag{71}$$

$$\gamma_1 = F_{\mathrm{RB}}^{-1}(1/2), \tag{72}$$

where $F_{\mathrm{RB}}(\gamma_1)$ depends on the considered multiple antenna scheme.

Section 3.5.1 investigates BEP performance in the SISO link. Section 3.5.2 considers the outage capacity with constant rate transmission in the SISO link and variable rate transmission for SISO and single user MIMO systems. Section 3.5.3 presents the system average capacity results with the optimal rate adaptation and single user MIMO schemes. Section 3.6 considers example results for the SDMA methods and also comparisons between the SDMA and AS schemes.

3.5.1 BEP performance

Fig. 5 illustrates BEP performance versus users' position in the allocation queue when the mean SNR is 10 dB, $N = 12$ and SISO transmission is applied. The BEP performance is shown with error free feedback channel and with feedback BEP $p_b = 0.05$. The reference results are a random allocation (RA) scheme without feedback information [12, Eq. (14-3-7)] and optimal RR allocation, where each user is assigned to the best available RB. We see that the simulation results match with the proposed analytical results. For the case where the channel has a SNR is above the threshold, the BEP is about $8 \cdot 10^{-6}$. On the other hand, the expected BEP is about 0.047 when a bad channel is allocated. Thus, the BEP performance of the uncoded transmission is sensitive to feedback bit errors and also to the user position in the queue when feedback is reliable. For example, the first user gets a good channel with a probability of 0.95 when $p_b = 0.05$, but the performance provided by the bad channel dominates the average BEP performance.

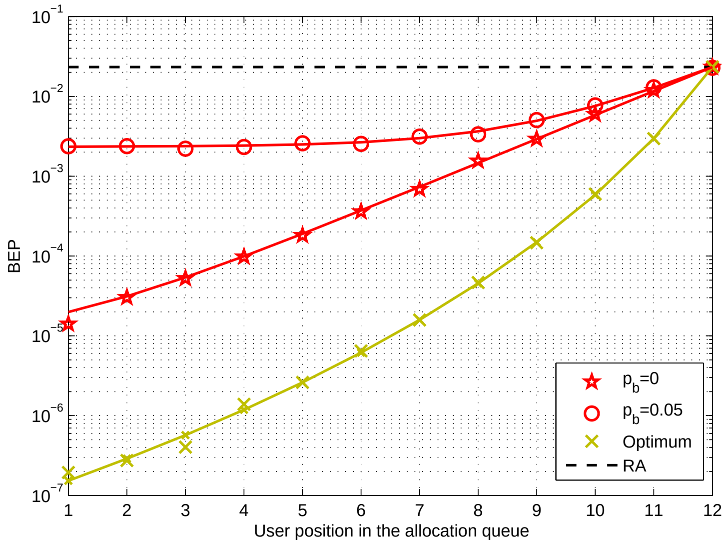


Fig 5. BEP vs. the user position in the allocation queue with SISO link, RB-wise one bit quantization, $N = 12$ and $\bar{\gamma} = 10$ dB. Marker points present simulation results.

3.5.2 Outage capacity

The outage capacity results in Figs. 6 and 7 show that the RB allocation based the RB-wise one bit information is robust against feedback errors. RR allocation based on RB-wise one bit feedback provides fair performance for the first users in the allocation queue.

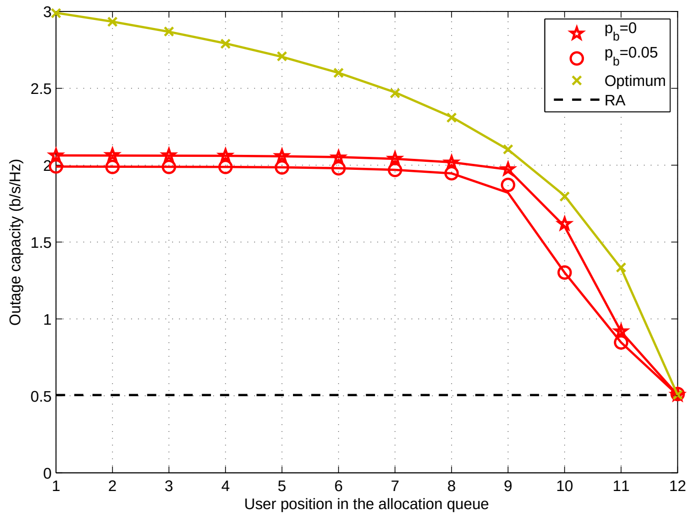


Fig 6. Outage capacity vs. the user position in the allocation queue with SISO link, RB-wise one bit quantization, constant rate transmission, $N = 12$ and $\bar{\gamma} = 6$ dB. Marker points present simulation results.

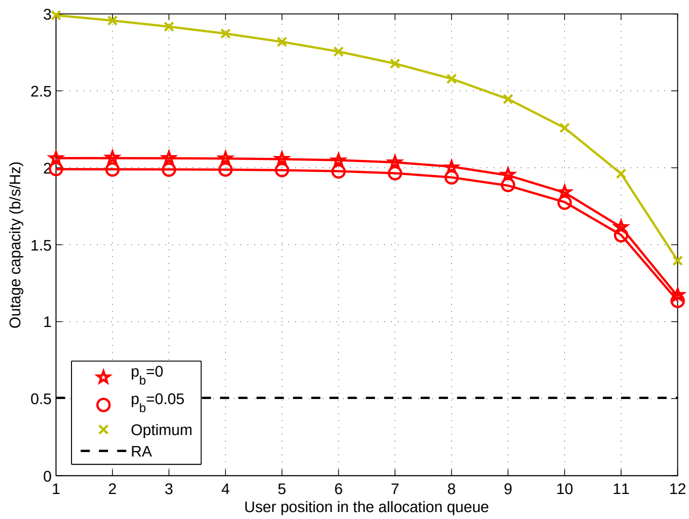


Fig 7. Outage capacity vs. the user position in the allocation queue with SISO link, RB-wise one bit quantization, variable rate transmission, $N = 12$ and $\bar{\gamma} = 6$ dB. Marker points present simulation results.

The value of the "maximum" provided outage capacity and also the number of users who achieve this maximum spectral efficiency depends on the threshold value. There is only one RB available for the last user in the allocation queue and its performance equals to that of the RA in the constant rate transmission. In variable rate transmission, the one bit information is utilized in rate adaptation and the higher rate is applied with a probability of 0.5 in the transmission to the last user. It can be seen that the variable rate transmission outperforms the constant rate for the latest users. Therefore, numerical examples focus on variable rate transmission for the rest of this section.

The outage capacity versus the user position in the allocation queue is illustrated for $N_t = 2$, $N_r = 1$ and $N_r = 2$ in Fig. 8 when the RB-wise one bit feedback method is used, $P_{\text{out}} = 0.1$, $\bar{\gamma} = 10$ dB, $N = 8$, and $p_b = 0.05$. The feedback overhead is shown in the legend. The threshold for resource block allocation and antenna selection in the TBAS scheme is set to be $\xi_l = 0.9$ and $\xi_l = 2$ for $N_r = 1$ and $N_r = 2$ when the expected number of channels having a SNR above the threshold is the same as in the other schemes. We can see again that the simulation results validate the proposed analytical results. The performance of the last user in the allocation queue corresponds to the performance of RA with rate adaptation based on the one bit quantization. The antenna selection schemes provide more allocation gain than the STBC scheme does when $p_b < 0.05$. On the other hand, the STBC scheme with the one bit feedback method provides fairer performance in the sense that it does not depend as much on the user position in the allocation queue.

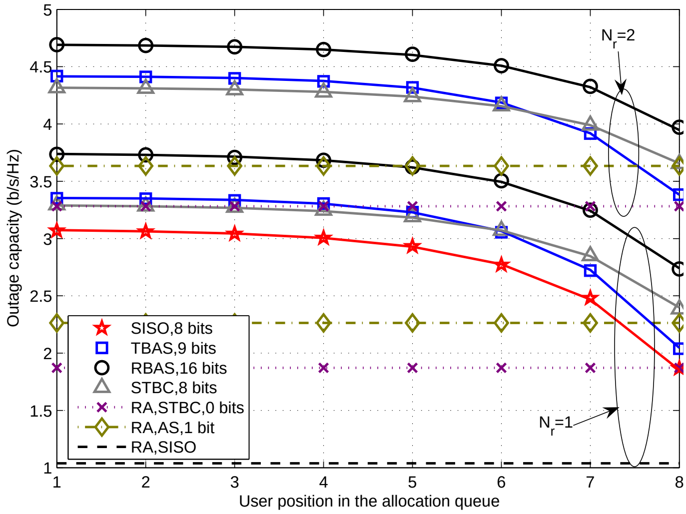


Fig 8. Outage capacity vs. the user position in the allocation queue with RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = 2$, $N = 8$, $\bar{\gamma} = 10$ dB, and $p_b = 0.05$. Marker points present simulation results.

The fairness can be further improved by giving any allocation queue position with equal probability for any user. The average outage capacity of the fair allocation becomes

$$C_{P_{\text{out}}} = \frac{1}{K}\sum_{k=1}^{K} C_{P_{\text{out}}}(k) \tag{73}$$

when users having the same $\bar{\gamma}$ have the same average performance. The average outage capacity of the fair allocation versus the feedback bit error probability is illustrated in Figs. 9 and 10 with $P_{\text{out}} = 0.1$ and $P_{\text{out}} = 0.05$, respectively, for one bit per resource block feedback strategy. The STBC scheme seems to be the most robust scheme against feedback bit errors as is understandable due to its open-loop nature. Resource block allocation based on the one bit quantization is robust against feedback bit errors provided that $p_b < P_{\text{out}}$. If $p_b > P_{\text{out}}$, the SNR value γ_o in (34) is below the quantization threshold γ_1. Thus, the sensitivity for feedback bit errors increases with decreasing outage probability. An antenna selection increases sensitivity to feedback errors. Antenna selection error leads to a random allocation for the "erroneous" antenna in the TBAS scheme. Thus, the performance of the TBAS method goes even below the SISO scheme when p_b increases.

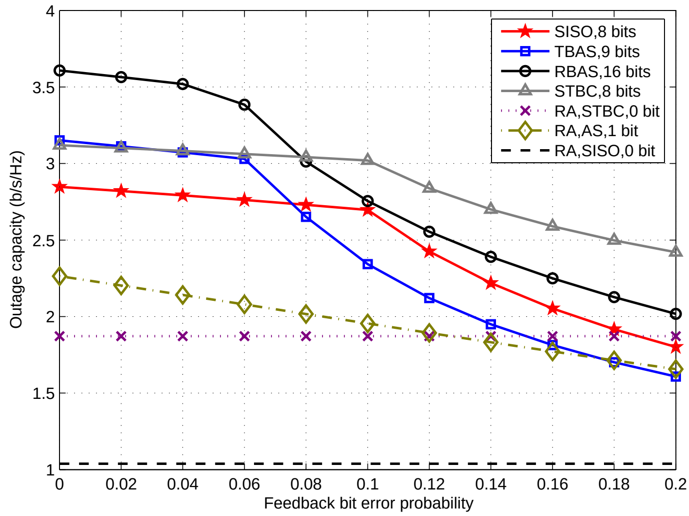


Fig 9. Outage capacity of fair allocation vs. feedback bit error probability with RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = 2$, $N_r = 1$, $N = K = 8$, and $\bar{\gamma} = 10$ dB.

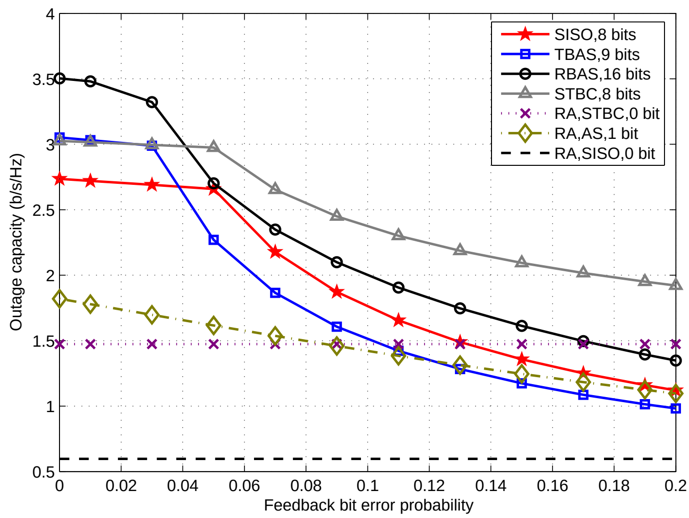


Fig 10. Outage capacity of fair allocation vs. feedback bit error probability with RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.05$, $N_t = 2$, $N_r = 1$, $N = K = 8$, and $\bar{\gamma} = 10$ dB.

The outage capacity results in Fig. 11 show that substantial SNR gain is achieved also with an imperfect feedback channel when the receiver is equipped with a single antenna. In the case of the SISO link, simple RR allocation provides about 7 dB of savings in the required SNR needed to achieve 2 b/s/Hz although the threshold is suboptimal. STBC and TBAS offer one dB of further improvement for the SNR, and RBAS a bit more than 2 dB gain when 2 b/s/Hz is desired and compared to the RB-wise one bit SISO scheme. The allocation gain is decreased when an antenna is added to the receiver and MRC is applied as can be seen from Fig. 12. Basically, the increased space diversity decreases multiuser diversity since the variability of the channel decreases [134]. However, the combination of STBC and the RB-wise one bit feedback scheme is attractive when slow link adaptation is applied due to the remarkable allocation gain and robustness against feedback bit errors.

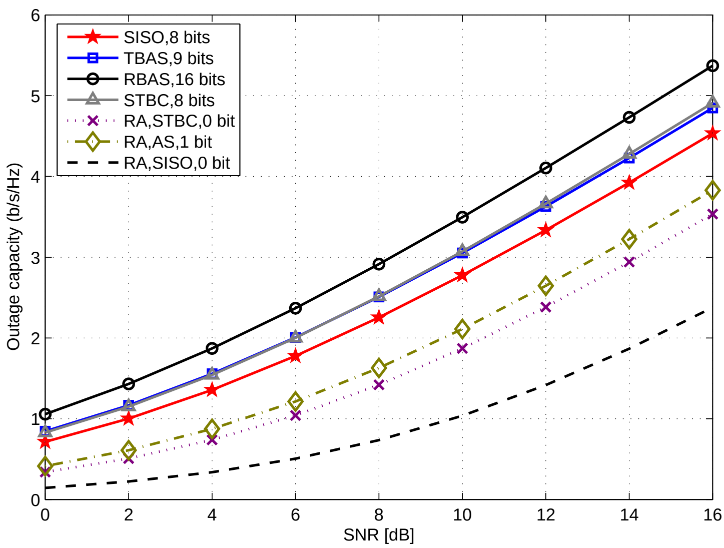


Fig 11. Outage capacity of fair allocation vs. SNR with RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = 2$, $N_r = 1$, $N = K = 8$, and $p_b = 0.05$ dB.

3.5.3 *Average spectral efficiency with optimal rate adaptation*

Fig. 13 illustrates the average capacity with optimal rate adaptation versus the user position in the allocation queue with the RB-wise one bit feedback method, $p_b = 0.1$ and $\bar{\gamma} = 10$ dB. In the fully loaded case $K = N$, the performance of the last user is the same as that of the RA. We can see that simple and fair allocation with a suboptimal threshold provides significant allocation gain compared to the RA. RBAS provides the best performance at the cost of the highest feedback overhead. The TBAS scheme provides a good tradeoff between performance and feedback overhead. On the other hand, antenna selection with random allocation provides also relatively good performance. It is interesting that the system capacity in the SISO channel has the same order as that of the STBC scheme when $N_r = 1$. If the outage capacity is considered, the STBC scheme clearly outperforms the SISO scheme as was seen in Figs. 8–12. Actually, OFDMA with STBC outperforms also the RBAS case when $p_b = 0.1$ and the outage capacity is considered. The average performance of the STBC scheme outperforms the SISO system in the case of random allocation. The resource block allocation based on one bit quantization with the STBC scheme provides good outage capacity performance due to the good MIMO diversity gain regardless of the feedback link quality. However, the fading of the SISO channel is more severe than that of the channel with STBC. Thus, utilizing multiuser diversity and optimal rate adaptation in the SISO-OFDMA system, the performance is close to that of the OFDMA with STBC. The results in [134] show that the maximum achievable throughput with scheduling and transmit diversity is definitely worse than that for the system with no diversity when the number of users and channels is large. Antenna selection further increases the number of channels, and hence, the achievable rates of good time-frequency resource blocks, and significant allocation gain is provided also in the presence of imperfect feedback.

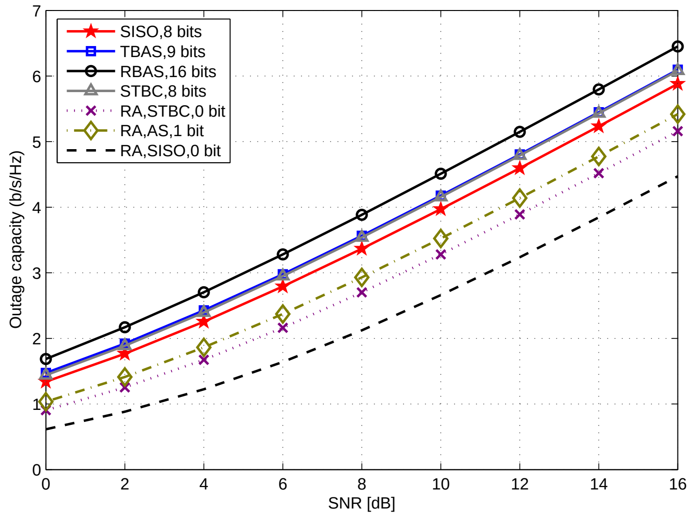


Fig 12. Outage capacity of fair allocation vs. SNR with RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = 2$, $N_r = 2$, $N = K = 8$, and $p_b = 0.05$ dB.

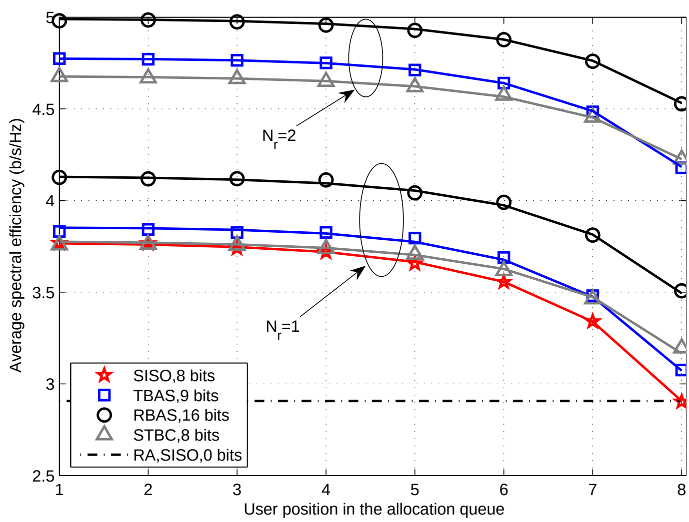


Fig 13. Average spectral efficiency vs. the user position in the allocation queue with RB-wise one bit quantization, optimal rate adaptation $N_t = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB, and $p_b = 0.1$. Marker points present simulation results.

3.6 Space division multiple access

The outage capacity results of the OFDMA-SDMA systems with the RB-wise one bit feedback strategy are illustrated in Fig. 14, where the number of time-frequency RBs is 8 and the total number of RBs is 16. For the same parameters, the average spectral efficiency with optimal rate adaptation is shown in Fig. 15. The spectral efficiency results for the SDMA case are shown for the used RB or equivalently per spatial stream. Since the spatial streams interfere with each other at the ZF detection, the spectral efficiency per RB is lower than that of the systems without spatial allocation. The spectral efficiency results of RB the allocation according to the RB-wise one bit information with optimal rate adaptation, with and without SDMA, are compared in Fig. 16. In order to obtain a simple and fair comparison, it is assumed that each user has the same mean SNR. The fully loaded case is assumed so that $K = N = 8$ and $K = N = 16$. The latter case is considered for the SDMA case to show that the performance is not improved significantly with N because threshold value is fixed. SDMA provides good spectral efficiency for moderate and high SNR values when ZF detection is able to distinguish the spatial streams. Surprisingly, SDMA with RA provides performance comparable to that achieved by the AS schemes when the SNR is high. However, the spectral efficiency per stream or per RB is half of the value shown in Fig. 16. Therefore, SDMA without precoding is suitable when the number of users is high and moderate data rate services are applied.

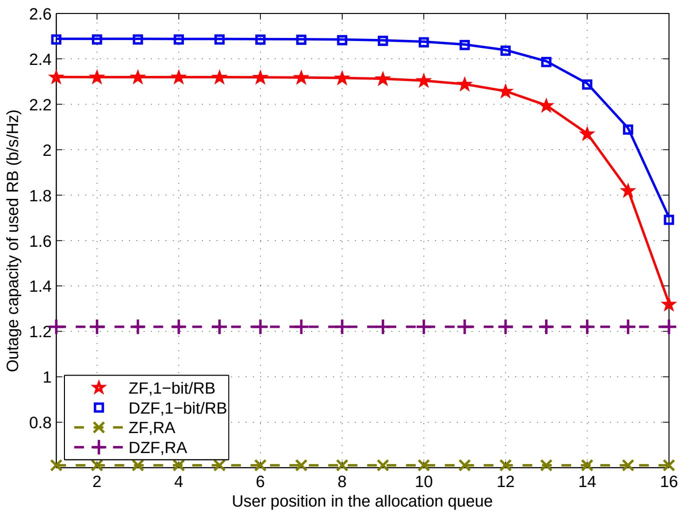


Fig 14. Outage capacity vs. the user position in the allocation with OFDMA-SDMA, RB-wise one bit quantization, variable rate transmission, $P_{\text{out}} = 0.1$, $N = 16$, $\bar{\gamma} = 10$ dB, and $p_b = 0$. Marker points present simulation results.

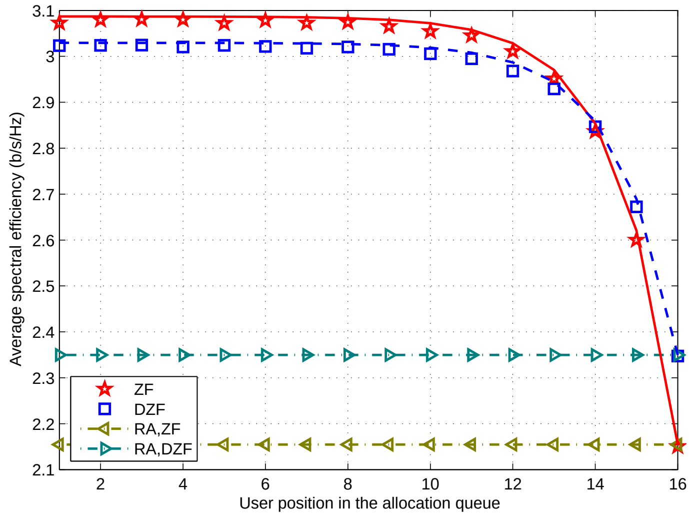


Fig 15. Average spectral efficiency vs. the user position in the allocation queue with OFDMA-SDMA, RB-wise one bit quantization, optimal rate adaptation, $N = 16$, $\bar{\gamma} = 10$ dB, and $p_b = 0$. Marker points present simulation results.

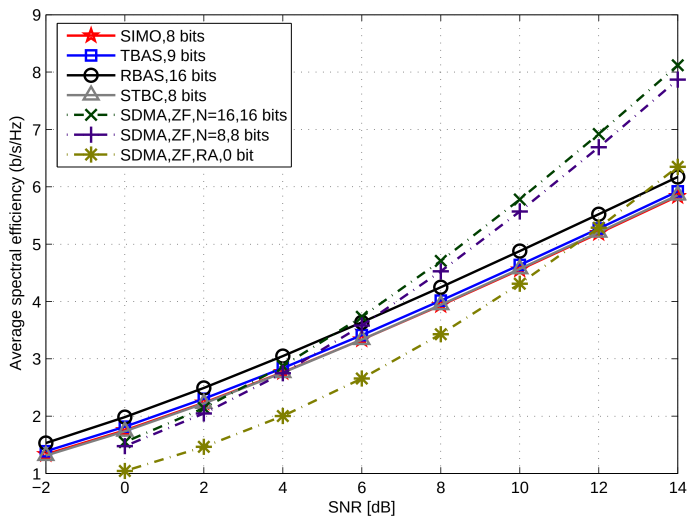


Fig 16. Spectral efficiency of the fair allocation vs. SNR with RB-wise one bit quantization, optimal rate adaptation, $N_t = 2$, $N_r = 2$, and $p_b = 0.05$ dB. In SDMA system, $N = K = 8$ and $N = K = 16$ are shown.

3.7 Summary

The performance of OFDMA systems with RR allocation based on the RB-wise SNR quantization was analyzed. Single antenna transmission and several multiple antenna schemes were considered with an imperfect feedback channel. BEP performance, outage capacity results and the system average capacity with optimal rate adaptation were considered. Numerical results based on the analysis were compared to those obtained by computer simulations to validate the correctness of the proposed analysis.

The fundamental properties of RB-wise one bit feedback information based allocation were studied through the numerical examples based on the proposed analysis. The RB-wise one bit feedback information provided significant performance improvement over random or fixed allocation although the allocation method and the threshold value were suboptimal. The combination of RB-wise one bit feedback and STBC was promising if the feedback channel was unreliable and outage capacity was considered. Otherwise, antenna selection was an attractive method to enhance multiuser diversity especially with the optimal rate adaptation. The results of SDMA with ZF detection indicated that spatial allocation without precoding is an effective method to obtain high system capacity for moderate and high SNR values.

4 Analysis of the best-M feedback method

This chapter considers feedback strategy based on the best-M information. The best-M method means that feedback information is formed only from the M best RBs out of N alternatives, see Section 2.3.1 for furhter details. Let $\mathcal{M}$ be a performance measure such as the BEP, the average spectral efficiency or the cumulative density function (CDF) of the SNR. On the condition that the kth position in the queue is admitted to the user, the performance of the RB allocation based on the best-M feedback scheme is given by

$$\mathcal{M}(k) = \frac{q(k,p_b)}{M}\sum_{n=1}^{M}\mathcal{M}_n^{\mathrm{RB}} + \frac{1-q(k,p_b)}{N-M}\sum_{n=M+1}^{N}\mathcal{M}_n^{\mathrm{RB}}, \tag{74}$$

where $q(k,p_b)$ refers to the probability that a RB among the M best blocks is assigned to the kth user when the feedback bit error probability is p_b, and $\mathcal{M}_n^{\mathrm{RB}}$ is the average performance of the nth best RB (out of N units)[6]. Note that $\mathcal{M}_n^{\mathrm{RB}}$ depends on the mean SNR and is equal for users having the same mean SNR, since the same fading statistics are assumed for each user.

In order to simplify the performance derivations, the result in (74) can be written in relation to the average performance of a RB over fading. The average performance of the RB, i.e., the average performance of the random allocation (RA) or fixed allocation, is denoted as $\mathcal{M}_{\mathrm{RB}}$.

Proposition 6 *The performance expression in* (74) *can be represented as*

$$\mathcal{M}(k) = \frac{Nq(k,p_b)-M}{M(N-M)}\sum_{n=1}^{M}\mathcal{M}_n^{RB} + \frac{(1-q(k,p_b))N}{N-M}\mathcal{M}_{RB}. \tag{75}$$

Proof. In order to simplify notations and capacity derivations, we use the result

$$\begin{aligned}\mathcal{M}_{\mathrm{RB}} =& \frac{1}{N}\sum_{n=1}^{N}\mathcal{M}_n^{\mathrm{RB}}\\ \Longleftrightarrow \sum_{n=M+1}^{N}&\mathcal{M}_n^{\mathrm{RB}} = N\mathcal{M}_{\mathrm{RB}} - \sum_{n=1}^{M}\mathcal{M}_n^{\mathrm{RB}} \triangleq \mathcal{M}_{M+}^{\mathrm{RB}}.\end{aligned} \tag{76}$$

Combining (74) and (76) we obtain (75). □

If the order information is available at the allocation, the performance of the allocation based on the ordered best-M principle is of the form

$$\mathcal{M}(k) = \sum_{n=1}^{N} p_n(k,p_b)\mathcal{M}_n^{\mathrm{RB}}, \tag{77}$$

[6] The Nth order statistics is considered, i.e., $\mathcal{M}_n^{\mathrm{RB}}$ is the nth best RB out of N alternatives, unless mentioned otherwise.

where $p_n(k, p_b)$ is the probability that the kth user gets the RB with the nth largest SNR. For $n > M$, the probability $p_n(k, p_b)$ turns out to be constant and it is denoted by $p_{M_+}(k, p_b)$ in this case. Thus, using the result in (76), we have

$$\begin{aligned}\mathcal{M}(k) &= \sum_{n=1}^{M} p_n(k, p_b)\mathcal{M}_n^{\mathrm{RB}} + p_{M_+}(k, p_b)\left(N\mathcal{M}_{\mathrm{RB}} - \sum_{n=1}^{M} \mathcal{M}_n^{\mathrm{RB}}\right) \\ &= \sum_{n=1}^{M} \left(p_n(k, p_b) - p_{M_+}(k, p_b)\right)\mathcal{M}_n^{\mathrm{RB}} + p_{M_+}(k, p_b)N\mathcal{M}_{\mathrm{RB}}.\end{aligned} \tag{78}$$

According to the order statistics, the PDF of the SNR of the nth best RB is expressed as

$$f_n^{\mathrm{RB}}(\gamma) = \frac{N!}{(N-n)!(n-1)!}F_{\mathrm{RB}}(\gamma)^{N-n}(1 - F_{\mathrm{RB}}(\gamma))^{n-1}f_{\mathrm{RB}}(\gamma), \tag{79}$$

where $F_{\mathrm{RB}}(\gamma)$ and $f_{\mathrm{RB}}(\gamma)$ are the CDF and PDF of the SNR of a RB, respectively [135]. The corresponding CDF is of the form [135]

$$F_n^{\mathrm{RB}}(\gamma) = \sum_{j=N-n+1}^{N} \binom{N}{j} F_{\mathrm{RB}}(\gamma)^j(1 - F_{\mathrm{RB}}(\gamma))^{N-j}. \tag{80}$$

4.1 Event probabilities

Consider first the best-1 feedback scheme and the perfect feedback channel[7]. Obviously, the first user always gets the indicated channel. The best RB of the kth user in the allocation queue has been assigned to one of the $k-1$ previous users with probability $(k-1)/n$. Thus, the best RB is assigned to the kth user with probability

$$q(k, 0) = p_1(k, 0) = \frac{N-k+1}{N}, \quad M = 1. \tag{81}$$

In the case of an imperfect feedback channel and the best-1 feedback scheme, the probability that the best RB is assigned for the kth user can be expressed as

$$\begin{aligned}q(k, p_b) = p_1(k, p_b) &= (1 - p_w)p_1(k, 0) + p_w(1 - p_1(k, 0))\frac{1}{N-1} \\ &= (1 - p_w)\frac{N-k+1}{N} + p_w\frac{k-1}{N(N-1)},\end{aligned} \tag{82}$$

where $p_w = 1 - (1 - p_b)^{\lceil \log_2(N) \rceil}$ is the error probability of the feedback word. From (82) we find that there are two alternative mechanisms through which the

[7] The ordered best-1 and the best-1 feedback schemes are exactly the same.

best RB can be obtained. In the first alternative, the feedback word is correct and the best RB is available for the kth user. In the second alternative, the feedback word is incorrect but the wrong feedback word refers to a RB that is already in use. Thus, the scheduler selects a RB randomly and there is the unlikely possibility that the best one is selected. The nth best RB is allocated with probability $p_{1_+}(k, p_b) = (1 - p_1(k, p_b))/(N - 1)$ for $n > 2$.

Consider next the best-M feedback scheme for $M > 2$ and an error free feedback word. If $k \leq M$, there is always an unoccupied RB that is within the set of the M best RBs and $q(k, 0) = 1$. Assume that $k > M$. Then

$$q(k,0) = 1 - \binom{k-1}{k-M-1} \Big/ \binom{N}{M}, \tag{83}$$

where in the second term, the numerator is the number of combinations that include none of the M best RBs and the denominator gives the total number of groups. For an erroneous feedback channel, the feedback word error probability is given as $p_w = 1 - (1 - p_b)^{\lceil \log_2(N!/M!(N-M)!) \rceil}$. A RB from the M best blocks is granted to a user if the feedback word is correct and such RBs are available. On the other hand, if the feedback word is incorrect, it recommends all possible sets of the M RBs except the correct one with equal probability. Thus, the erroneous set best-M feedback word leads to almost random allocation and a good RB is obtained with a probability of approximately M/N. Thus, we find that

$$q(k, p_b) \approx (1 - p_w) q(k, 0) + p_w M/N. \tag{84}$$

Even a one bit error in feedback causes random allocation. Furthermore, the feedback word error probability increases with N and M resulting in high sensitivity to feedback errors.

For the error free ordered best-M feedback case and $k \leq M$, the RB that is at least the kth best is always granted to the user. Hence, if $n > k$ and $k \leq M$, $p_n(k, 0) = 0$. Assume that $n \leq k$ and $2 \leq n \leq M$. The preceding users take $k - 1$ RBs from N possible ones before the kth user. The number of possible combinations of $k-1$ user allocations for N RBs is $\binom{N}{k-1} = N!/(k-1)!(N-k+1)!$ and we need to identify those combinations which contain the $n - 1$ best RBs from the kth user's perspective, and, on the other hand, do not contain the nth best RB. Thus, we can select $n - k$ RBs out of $N - n$ alternatives freely and the probability $p_n(k, 0)$ attains the form

$$p_n(k,0) = \begin{cases} (N-k+1)/N & n = 1, \\ \binom{N-n}{k-n} \Big/ \binom{N}{k-1} & n \leq k, \; 2 \leq n \leq M, \\ 0 & n > k, \; k \leq M. \end{cases} \tag{85}$$

If $n > M$, all the M best RBs have already been assigned to the preceding users and a RB for the kth user is allocated randomly from the set of unoccupied RBs.

The product of probabilities related to these consecutive events form $p_n(k,0)$ for $n > M$ and $k > M$. Thus, we have

$$p_n(k,0) = \left[\binom{N-M}{k-M-1} \Big/ \binom{N}{k-1} \right] \frac{1}{N-M}, \tag{86}$$

for $n > M$, $k > M$. Hence, $p_n(k,0) = p_{M_+}(k,0)$ does not depend on n when $n > M$. We further note that $p_{M_+}(k,0) = 0$ if $k \leq M$.

When $M > 1$, the exact probability $p_n(k,p_b)$ for the ordered best-M feedback schemes becomes cumbersome and we determine approximations only for the cases $M = 2$ and $M = 3$. The correct feedback word is denoted by $\mathcal{W} = [w_1, w_2, \ldots, w_M]$, where w_x is the index of the xth best RB. The received feedback word is denoted by $\hat{\mathcal{W}} = [\hat{w}_1, \hat{w}_2, \ldots, \hat{w}_M]$. Furthermore, the set of the indices of the available RBs is denoted by $\mathcal{A}$. We note that $p_{w_x} = 1-(1-p_b)^{\lceil \log_2(N) \rceil}$ for all x. Thus, we omit the index x from p_{w_x} for clarity. A couple of assumptions are used to simplify the determination of $p_n(k,p_b)$. It is assumed that $\hat{w}_i \neq \hat{w}_j$ when $i \neq j$. The other assumption is stated so that all available RBs are allocated with the same probability when $\hat{w}_x \notin \mathcal{A}$ for all $x = 1, \ldots, M$ and random allocation has been applied. The index of the selected RB by random allocation (RA) is denoted by w_{RA}.

The probabilities of all possible events where the nth best RB is allocated to the kth user have to be summed to obtain $p_n(k,p_b)$. Using the two described assumptions, the probability $p_1(k,p_b)$ for the ordered best-2 feedback scheme is approximated as

$$\begin{aligned} &p_1(k,p_b) \approx P(w_1 \in \mathcal{A} \cap \hat{w}_1 = w_1) \\ &+ P(w_1 \in \mathcal{A} \cap \hat{w}_1 \neq w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap \hat{w}_2 \neq w_2 \cap \hat{w}_2 = w_1) \\ &+ P(w_1 \in \mathcal{A} \cap \hat{w}_1 \neq w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap \hat{w}_2 = w_2 \cap w_2 \notin \mathcal{A} \cap w_{\mathrm{RA}} = w_1) \\ &+ P(w_1 \in \mathcal{A} \cap \hat{w}_1 \neq w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap \hat{w}_2 \neq w_2 \cap w_2 \notin \mathcal{A} \cap w_{\mathrm{RA}} = w_1). \end{aligned} \tag{87}$$

We note that $P(\hat{w}_1 \in \mathcal{A}) = p_1(k,0)$, $P(\hat{w}_1 \in \mathcal{A} \cap \hat{w}_2 \notin \mathcal{A}) = p_2(k,0)$, and $P(\hat{w}_2 = w_1 | \hat{w}_2 \neq w_2) = 1/(N-1)$. Hence, the probability (87) can be expressed as

$$p_1(k,p_b) \approx p_1(k,0)(1-p_w) + \frac{p_2(k,0)}{N-1} p_w^2 + p_{M_+}(k,0) p_w, \tag{88}$$

where the last term is the sum of the two last terms in (87). Similarly, the probability that the second best RB is allocated to the kth user is defined by the probabilities

$$
\begin{aligned}
&p_2(k,p_b) \approx P(\hat{w}_1 \neq w_1 \cap w_2 \in \mathcal{A} \cap \hat{w}_1 = w_2) \\
&+ P(\hat{w}_1 = w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap w_2 \in \mathcal{A} \cap \hat{w}_2 = w_2) \\
&+ P(\hat{w}_1 \neq w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap w_2 \in \mathcal{A} \cap \hat{w}_2 = w_2) \\
&+ P(\hat{w}_1 = w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap \hat{w}_2 \neq w_2 \cap \hat{w}_2 \notin \mathcal{A} \cap w_{\mathrm{RA}} = w_2) \\
&+ P(\hat{w}_1 \neq w_1 \cap \hat{w}_1 \notin \mathcal{A} \cap \hat{w}_2 \neq w_2 \cap \hat{w}_2 \notin \mathcal{A} \cap w_{\mathrm{RA}} = w_2), \\
&\approx \frac{p_1(k,0)}{N-1} p_w + p_2(k,0)(1-p_w) + p_{M_+}(k,0)p_w. \qquad (89)
\end{aligned}
$$

The probability $p_n(k,p_b)$ for $n > 2$ is simply calculated as $p_n(k,p_b) = (1 - (p_1(k,p_b) + p_2(k,p_b)))/(N-2)$.

The probability $p_n(k,p_b)$ for the ordered best-3 feedback scheme can be derived similarly as for the ordered best-2 feedback scheme. Thus, the probabilities $p_n(k,p_b)$ for the ordered best-3 feedback scheme are shown without details and they are given as

$$
\begin{aligned}
p_1(k,p_b) \approx& p_1(k,0)(1-p_w) + \frac{p_2(k,0)}{N-1} p_w^2 + \frac{p_3(k,0)}{N-1} p_w^2 + p_{M_+}(k,0)p_w, && (90)\\
p_2(k,p_b) \approx& \frac{p_1(k,0)}{N-1} p_w + p_2(k,0)(1-p_w) + \frac{p_3(k,0)}{N-1} p_w^2 + p_{M_+}(k,0)p_w, && (91)\\
p_3(k,p_b) \approx& \frac{p_1(k,0)}{N-1} p_w + \frac{p_2(k,0)}{N-1} p_w + p_3(k,0)(1-p_w) + p_{M_+}(k,0)p_w, && (92)\\
p_n(k,p_b) \approx& (1 - (p_1(k,p_b) + p_2(k,p_b) + p_3(k,p_b)))/(N-3), \quad n \geq 3, && (93)
\end{aligned}
$$

where the two approximations given above are applied.

4.2 BEP analysis in SISO systems

The average BEP of the nth best RB is denoted by P_n The average BEP performance of the RB having the nth highest SNR can be calculated as

$$
P_n = \int_0^\infty f_n^{\mathrm{R}}(\gamma) g(\gamma) d\gamma, \qquad (94)
$$

where $frnn(\gamma)$ is the PDF of the nth best RB in the case of a Rayleigh fading channel and $g(\gamma)$ is the BEP function of the applied modulation in a non-fading AWGN channel. Applying integration by parts to (94) we get

$$
P_n = -\int_0^\infty F_n^{\mathrm{R}}(\gamma) g'(\gamma) d\gamma, \qquad (95)
$$

where $F_n^{\mathrm{R}}(\gamma)$ is the CDF of the SNR of the nth largest SNR. Assuming IID fading over RBs, the CDF of the nth largest SNR out of N alternatives is achieved by substituting (2) into (80) and it is of the form

$$F_n^{\mathrm{R}}(\gamma) = \sum_{j=N-n+1}^{N} \binom{N}{j} \left(1 - e^{-\gamma/\bar{\gamma}}\right)^j e^{-(N-j)\gamma/\bar{\gamma}}. \tag{96}$$

Using the binomial theorem [133, Eq. (3.1.1)], (96) can be expressed in the form

$$F_n^{\mathrm{R}}(\gamma) = \sum_{j=N-n+1}^{N} \binom{N}{j} \sum_{l=0}^{j} \binom{j}{l} (-1)^l e^{-\gamma/\bar{\gamma}(N-j+l)}. \tag{97}$$

Substituting (27) and (97) to (95), using the substitutions $x = -\gamma/\bar{\gamma}(N-j+l+\bar{\gamma})$ and $\Gamma(1/2, 0) = \sqrt{\pi}$, we obtain

$$P_n = \frac{1}{2} \sum_{j=N-n+1}^{N} \binom{N}{j} \sum_{l=0}^{j} \binom{j}{l} (-1)^l \sqrt{\frac{\bar{\gamma}}{\bar{\gamma} + N - j + l}}. \tag{98}$$

In the case of Rayleigh fading and fixed allocation or RA, the BEP performance is given as [12, Eq. (14-3-7)]

$$P_{\mathrm{R}} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right). \tag{99}$$

Now we are able to calculate the BEP for a user allocated according to the best-M or the ordered best-M feedback method by substituting (98) and (99) into (75) or (78).

4.2.1 Asymptotic BEP analysis

The asymptotic BEP analysis is also derived for the BPSK. Let us assume that $M << N$. Thus, P_n tends to zero for $n < M$ relatively rapidly when the SNR tends to infinity. Using (75), the BEP of the best-M feedback case for the large SNR values can be approximated as

$$P(k) \approx (1 - q(k, p_b))N/(N - M)P_{\mathrm{R}}, \ \bar{\gamma} >> 1, \tag{100}$$

where P_{R} is the one given in (99). Thus, the asymptotic decay of the BEP of best-M allocation is the same as that for a fully random allocation. Yet, there is a gain $(1 - q(k, p_b))N/(N - M)$ in the BEP. In the allocation based on the ordered best-M feedback scheme, the slope of the mean BEP is also the same as that with fully random allocation. We can see from (78) that the BEP gain is approximately $p_{M+}(k, p_b)N$ when the SNR is large.

Consider now the asymptotic SNR gain for large SNR values between allocation according to the best-M feedback method and random allocation without feedback information. The SNR required to achieve the BEP value P_{R} in fully random allocation is denoted by γ_{R}. The corresponding SNR for the kth user allocated according to the best-M feedback scheme is denoted by γ_k. The BEP gain in dB is denoted as

$$\gamma_{\mathrm{G}} = 10 \log_{10} \frac{\gamma_{\mathrm{R}}}{\gamma_k}. \tag{101}$$

From (99) we can obtain the value of γ_{R} to be of the form

$$\gamma_{\mathrm{R}} = \frac{(1-2P_{\mathrm{R}})^2}{1-(1-2P_{\mathrm{R}})^2}. \tag{102}$$

It is easy to derive that for large SNR values, γ_k is approximately given by

$$\gamma_k = \frac{(1-2\alpha_k P_{\mathrm{R}})^2}{1-(1-2\alpha_k P_{\mathrm{R}})^2}, \tag{103}$$

where $\alpha_k = (N-M)/((1-q(k,p_b))N)$ for the set best-M scheme and $\alpha_k = 1/(Np_{M_+}(k,p_b))$ for the ordered best-M scheme. Substituting (102) and (103) into (101) and assuming that P_{R} goes to zero when $\bar{\gamma}$ goes to infinity, the asymptotic BEP gain with large SNR values can be expressed as

$$\gamma_{\mathrm{G}} = 10 \log_{10} \alpha_k. \tag{104}$$

If $p_{M_+}(k,p_b) = 0$ or $q(k,p_b) = 1$, the asymptotic BEP gain tends to infinity.

4.3 Outage capacity analysis

Constant rate transmission is considered in Section 4.3.1 for the Obest-M and best-M methods when single antenna transmission and reception are assumed. The outage capacity of variable rate transmission is straightforward but tedious to determine for the Obest-M scheme due to the cumbersome system probabilities as seen in Section 4.1. The analysis for variable rate transmission focuses on the best-M strategy without order information. Section 4.3.2 presents the outage capacity of variable rate transmission for the STBC and SDMA schemes. The outage capacity of variable rate transmission is investigated in Section 4.3.3 for AS.

4.3.1 Constant rate transmission

In the case of constant rate transmission, the outage capacity with a given outage probability for a slowly Rayleigh fading channel is given by

$$C_{P_{\mathrm{out}}}(k) = \log_2(1 + \gamma_{\mathrm{out}}(P_{\mathrm{out}}, k)), \tag{105}$$

where $\gamma_{\text{out}}(P_{\text{out}}, k)$ is the SNR value that provides the desired outage probability to the kth user. Thus, the value of $\gamma_{\text{out}}(P_{\text{out}}, k)$ has to be determined from the CDF of the SNR and then it is simply substituted in the mutual information formula for a non-fading AWGN channel [13].

The CDF for the best-M or the Obest-M scheme have been achieved by substituting $F_n^{\text{RB}}(\gamma)$ and $F_{\text{RB}}(\gamma)$ into (75) or (78), respectively. Thus, they are given as

$$F(\gamma) = \frac{Nq(k, p_b) - M}{M(N-M)} \sum_{n=1}^{M} F_n^{\text{RB}} + \frac{(1 - q(k, p_b))N}{N - M} F_{\text{RB}}, \tag{106}$$

for the best-M scheme and as

$$F(\gamma) = \sum_{n=1}^{M} \big(p_n(k, p_b) - p_{M_+}(k, p_b)\big) F_n^{\text{RB}} + p_{M_+}(k, p_b) N F_{\text{RB}} \tag{107}$$

for the Obest-M strategy. In order to determine the SNR value $\gamma_{\text{out}}(P_{\text{out}}, k)$, the Nth order polynomial equations are formed from (106) or (107) so that $F(\gamma) = P_{\text{out}}$, where P_{out} is the desired outage probability. The root for the polynomial equation is denoted as $\mathcal{R}(P_{\text{out}}) = F_{\text{RB}}(\gamma)$ and it can be determined through root finding. In the case of the SISO link, the SNR value $\gamma_{\text{out}}(P_{\text{out}}, k)$ for the desired outage probability is derived as

$$\gamma_{\text{out}}(P_{\text{out}}) = -\bar{\gamma} \ln\left(1 - \mathcal{R}(P_{\text{out}})\right). \tag{108}$$

4.3.2 Variable rate transmission

Two transmission rates can be instantaneously adapted based on the fast best-M feedback information. The higher rate R_1 is employed for channels indicated to be in the set of the M best channels. If all indicated channels have been occupied and the used channel is selected from the available RBs, the lower rate is used. Transmission rates depend on the $\bar{\gamma}$, P_{out}, p_b, M, and N. The outage capacity for the kth user using the best-M scheme, is expressed as

$$C_{P_{\text{out}}}(k) = \sum_{i=1}^{2} q^{(R_i)}(k) \log_2(1 + \gamma_{\text{out},i}(P_{\text{out}})), \tag{109}$$

where $q^{(R_i)}$ is the probability that the ith rate is applied and the SNR value $\gamma_{\text{out},i}(P_{\text{out}})$ provides the desired outage probability. The SNR values that provide the desired P_{out} are solved from the CDF of the received SNR on the condition that the ith rate is applied and they are then simply substituted to the mutual information formula for an AWGN channel [13].

If $k \leq M$, there is always a free RB that is within the group of the M best RBs and the higher rate is applied, i.e., $q^{(R_1)}(k) = 1$. The probability that a user gets a channel from the group of the M best RBs when $k > M$ is given as $q^{(R_1)}(k) = 1 - \binom{k-1}{k-M-1} \Big/ \binom{N}{M}$, see further details in Section 4.1. The lower rate is employed with probability $q^{(R_2)}(k) = 1 - q^{(R_1)}(k)$.

The notation (γ) is omitted in the CDF expressions for clarity in this section. In the case that the indicated channel is available, the antenna selection is perfect and $M = 1$, the CDF of the SNR with an imperfect feedback channel is of the form

$$F^{(R_1)} = (1-p_w)F_1^{\mathrm{RB}} + \frac{p_w}{N-1}\sum_{n=2}^{N} F_n^{\mathrm{RB}} = \left(1 - p_w - \frac{p_w}{N-1}\right) F_1^{\mathrm{RB}} + \frac{Np_w}{N-1} F_{\mathrm{RB}}, \tag{110}$$

where $p_w = 1 - (1 - p_b)^{\lceil \log_2(N) \rceil}$ is the word error probability of the best-1 feedback word and the result in (76) has been utilized, i.e.,

$$F_{\mathrm{RB}} = \frac{1}{N}\sum_{n=1}^{N} F_n^{\mathrm{RB}} \Longleftrightarrow \sum_{n=M+1}^{N} F_n^{\mathrm{RB}} = NF_{\mathrm{RB}} - \sum_{n=1}^{M} F_n^{\mathrm{RB}} \triangleq F_{M+}^{\mathrm{RB}}. \tag{111}$$

The CDF for the lower rate is given as

$$\begin{aligned} F^{(R_2)} =& \frac{1-p_w}{N-1} F_{2+}^{\mathrm{RB}} + \frac{p_w}{N-1} F_1^{\mathrm{RB}} + p_w \frac{N-2}{(N-1)^2} F_{2+}^{\mathrm{RB}} \\ =& \left(\frac{2p_w - 1}{N-1} - p_w \frac{N-2}{(N-1)^2}\right) F_1^{\mathrm{RB}} + \frac{N}{N-1}\,\frac{N-1-p_w}{N-1} F_{\mathrm{RB}}, \end{aligned} \tag{112}$$

where in the upper expression, the first term represents the case of correct feedback, the second term presents the case that the best RB is randomly allocated when the feedback word is erroneous, and the third term represents the case where one of the $N-1$ worst resources is randomly selected.

Consider next the best-M feedback scheme for $M > 1$. The feedback word error probability is given as $p_w = 1 - (1 - p_b)^{\lceil \log_2(N!/M!(N-M)!) \rceil}$. A RB from the M best blocks is granted to a user if the feedback word is correct and such RBs are available. On the other hand, if the feedback word is incorrect, it may recommend any of all the possible sets of the M RBs except the correct one with equal probability. Thus, the erroneous best-M feedback word leads to almost random allocation and the CDF of the SNR is given as

$$F^{(R_1)} \approx \frac{1-p_w}{M} \sum_{n=1}^{M} F_n^{\mathrm{RB}} + p_w F_{\mathrm{RB}}, \tag{113}$$

when the higher rate is applied. Even a one bit error in the feedback word causes approximately random allocation. Similarly in the case that $k > M$ and the indicated RBs have already been allocated to the preceding users in the allocation queue, the CDF of the SNR is approximated to be of the form

$$F^{(R_2)} \approx \frac{1-p_w}{N-M} F_{M+}^{\mathrm{RB}} + p_w F_{\mathrm{RB}} = -\frac{1-p_w}{N-M} \sum_{n=1}^{M} F_n^{\mathrm{RB}} + \left(\frac{N - M p_w}{N-M} \right) F_{\mathrm{RB}}. \tag{114}$$

In order to determine the SNR value $\gamma_{\mathrm{out},i}(P_{\mathrm{out}})$ corresponding to the outage capacity value in (109), the value of F_{RB} has to be solved based on (110)–(114) with a given $P_{\mathrm{out}} = F^{(R_i)}$. The (correct) root for the Nth order polynomial equation formed from (110)–(114) with a given P_{out} is denoted as $\mathcal{R}_i(P_{\mathrm{out}}) = F_{\mathrm{RB}}(\gamma)$. Numerical root finding is needed to find the solution for $\mathcal{R}_i(P_{\mathrm{out}})$. Now the desired SNR value can be generically derived as $\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = F_{\mathrm{RB}}^{-1}(\mathcal{R}_i(P_{\mathrm{out}}))$. Thus, for different transmission schemes, we have

$$\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = -\bar{\gamma} \ln\left(1 - \mathcal{R}_i(P_{\mathrm{out}})\right), \quad \text{for an SISO link} \tag{115}$$

$$\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = \bar{\gamma} \mathcal{P}_{N_r}^{-1}\left(1 - \mathcal{R}_i(P_{\mathrm{out}})\right), \quad \text{for an SIMO link with MRC} \tag{116}$$

$$\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = \frac{\bar{\gamma}}{N_t} \mathcal{P}_{\mathcal{K}}^{-1}\left(1 - \mathcal{R}_i(P_{\mathrm{out}})\right), \quad \text{for the STBC scheme} \tag{117}$$

$$\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = \frac{-\bar{\gamma}}{2} \ln\left(1 - \mathcal{R}_i(P_{\mathrm{out}})\right), \tag{118}$$

for a 2-by-2 MIMO link with SDMA w/o precoding

$$\gamma_{\mathrm{out},i}(P_{\mathrm{out}}) = \frac{-\bar{\gamma}}{4} \mathcal{P}_2^{-1}\left(1 - \mathcal{R}_i(P_{\mathrm{out}})\right) \tag{119}$$

for a 4-by-2 MIMO link with SDMA and STBC.

4.3.3 Antenna selection

The CDFs of the SNR are given by (110)–(114) when antenna selection is perfect. However, the CDFs are more complicated with imperfect antenna selection and only the case of $N_t = 2$ is considered for $M > 1$.

In the case of the best-1 feedback method and the MRC receiver, the CDF of the SNR for the higher rate is approximated as

$$\begin{aligned} F^{(R_1)} \approx &(1-p_w)(1-p_{\mathrm{ASE}}) F_1^{\mathrm{AS}} + p_w (1-p_{\mathrm{ASE}}) F_{1+}^{\mathrm{AS}} \\ &+ (1-p_w) p_{\mathrm{ASE}} F_{1+}^{\mathrm{MRC}} + p_w p_{\mathrm{ASE}} F_{\mathrm{ASE}}, \end{aligned} \tag{120}$$

where $F_{1+}^{\mathrm{AS}} = \sum_{n=2}^{N} F_n^{\mathrm{AS}}/(N-1)$, $F_{1+}^{\mathrm{MRC}} = \sum_{n=2}^{NN_t} F_n^{\mathrm{MRC}}/(NN_t - 1)$ and $F_{\mathrm{ASE}} = \sum_{n=2}^{N_t} F_n^{\mathrm{MRC}}/(N_t - 1)$. The first term in (120) represents the case of the correct best-1 feedback word and antenna selection and the second term is the CDF when the best-1 feedback word is erroneous and antenna selection is correct. The third term in (120) refers to the CDF when the best-1 word is correct and antenna selection is erroneous since the allocated RB can be any of the available

NN_t channels except the best one. If the best-1 feedback word and antenna selection are erroneous the CDF is well approximated by the F_{ASE} that is found in the last term in (120). In the case where the best RB is allocated to previously scheduled users according to the received feedback, the CDF of the random allocation is approximated as

$$\begin{aligned} F^{(R_2)} \approx &(1-p_w)(1-p_{\text{ASE}})F_{1+}^{\text{AS}} \\ &+ p_w(1-p_{\text{ASE}})\left(\frac{1}{N-1}F_1^{\text{AS}} + \frac{N-2}{(N-1)^2}F_{1+}^{\text{AS}}\right) + p_{\text{ASE}}F_{\text{ASE}}, \end{aligned} \tag{121}$$

where the first term is the CDF for the correct feedback word, the second and third terms form the CDF for the case with an erroneous best-1 feedback word and correct antenna selection and the last term refers to the CDF for the case with an antenna selection error.

For $M > 1$ and R_1, the CDF can be determined straightforwardly by (120) and it can be written as

$$\begin{aligned} F^{(R_1)} \approx &(1-p_w)(1-p_{\text{ASE}})\sum_{n=1}^{M} F_n^{\text{AS}} + p_w(1-p_{\text{ASE}})F_{n+}^{\text{AS}} \\ &+ (1-p_w)p_{\text{ASE}}\frac{1}{M}\sum_{l=1}^{M} F_{l+}^{\text{MRC}} + p_w p_{\text{ASE}}F_{\text{ASE}}, \end{aligned} \tag{122}$$

when the indicated channel is allocated and the higher rate is employed. The third term in (122) represents the CDF when the best-M word is correct and antenna selection is erroneous. In this event, the complete form would include terms from F_{1+}^{MRC} to $F_{(2M-1)+}^{\text{MRC}}$ with a certain probability. The used approximation simplifies the CDF and causes only negligible error for the performance. In the case that the lower rate has to be used, the CDF of the SNR is well approximated by

$$F^{(R_2)} \approx (1-p_w)(1-p_{\text{ASE}})F_{M+}^{\text{AS}} + p_w(1-p_{\text{ASE}})F_{\text{AS}} + p_{\text{ASE}}F_{\text{ASE}}. \tag{123}$$

We can now form the NN_t order polynomial equation and solve F_{MRC} from (120)–(123) with a given P_{out}. The root of the polynomial equation for the ith rate is again denoted as $\mathcal{R}_i$. The SNR value $\gamma_{\text{out},i}(P_{\text{out}})$ corresponding to the outage capacity value in (109) is given in (115) and (116) for $N_r = 1$ and $N_r = 2$, respectively.

4.4 Average capacity analysis

For simplicity, the spectral efficiency analysis focuses on the best-M feedback strategy although the capacity expressions could be used for the Obest-M scheme as well. According to (75), the average capacity for the kth user is of the form

$$C(k) = \frac{Nq(k,p_b)-M}{M(N-M)}\sum_{n=1}^{M} C_n^{\text{RB}} + \frac{N(1-q(k,p_b))}{N-M}C_{\text{RB}}, \tag{124}$$

where

$$C_n^{\mathrm{RB}} = \int_0^\infty f_n^{\mathrm{RB}}(\gamma)\log_2(1+\gamma)d\gamma \tag{125}$$

is the capacity of the nth best RB and

$$C_{\mathrm{RB}} = \int_0^\infty f_{\mathrm{RB}}(\gamma)\log_2(1+\gamma)d\gamma \tag{126}$$

is the average capacity of a RB over fading. The results in (124)–(126) are valid for the RBAS scheme only with perfect antenna selection.

4.4.1 Space-time block coding

Proposition 7 *The average capacity of the nth best RB can be expressed as*

$$\begin{aligned} C_n^{STBC} =& \frac{N!\log_2(e)}{(N-n)!(n-1)!}\frac{1}{\Gamma(\mathcal{K})}\sum_{j=0}^{N-n}\binom{N-n}{j}(-1)^j \\ &\cdot \sum_{l=0}^{(j+n-1)(\mathcal{K}-1)} \beta_{l,j}\alpha^{l+\mathcal{K}}\mathcal{I}_4(l+\mathcal{K},(j+n)\alpha), \end{aligned} \tag{127}$$

where $\mathcal{K} = N_t N_r$, $\alpha = N_t/\bar{\gamma}$, the coefficient $\beta_{l,j}$ can be derived as

$$\beta_{l,j} = \sum_{r=l-\mathcal{K}+1}^{l} \frac{\beta_{r,j-1}}{(l-r)!} I_{[0,(j-1)(\mathcal{K}-1)]}, \tag{128}$$

$\beta_{0,0} = \beta_{0,n} = 1$, $\beta_{l,1} = 1/l!$, $\beta_{1,j} = j$, $I_{[a,b]} = 1$ when $a \le r, \le b$ and $I_{[a,b]} = 0$ otherwise, and

$$\mathcal{I}_4(a,b) = \int_0^\infty \gamma^{a-1}e^{-b\gamma}\ln(1+\gamma)d\gamma = (a-1)!e^b\sum_{s=0}^{a}\frac{\Gamma(a-s,b)}{b^s}. \tag{129}$$

Proof. The PDF of the SNR of the nth best RB for STBC transmission is attained by substituting (6) into (79). Then we have

$$\begin{aligned} f_n^{\mathrm{STBC}}(\gamma) =& \frac{N!}{(N-n)!(n-1)!}(1-\mathcal{P}_{\mathcal{K}}(\alpha\gamma))^{N-n}\mathcal{P}_{\mathcal{K}}(\alpha\gamma)^{n-1}\frac{\gamma^{\mathcal{K}}}{\Gamma(\mathcal{K})}\alpha^{\mathcal{K}}e^{-\alpha\gamma} \\ =& \frac{N!}{(N-n)!(n-1)!}\sum_{j=0}^{N-n}\binom{N-n}{j}(-1)^j\left(\sum_{l=0}^{\mathcal{K}-1}(\alpha\gamma)^l\frac{1}{l!}e^{-\alpha\gamma}\right)^{j+n-1}\frac{\gamma^{\mathcal{K}}}{\Gamma(\mathcal{K})}\alpha^{\mathcal{K}}e^{-\alpha\gamma} \\ =& \frac{N!}{(N-n)!(n-1)!}\sum_{j=0}^{N-n}\binom{N-n}{j}\frac{(-1)^j}{\Gamma(\mathcal{K})}\sum_{l=0}^{(j+n-1)(\mathcal{K}-1)}\beta_{l,j}\alpha^{l+\mathcal{K}}\gamma^{l+\mathcal{K}-1}e^{-(j+n)\alpha\gamma} \end{aligned} \tag{130}$$

after using the binomial expansion, the multinomial theorem, and $\alpha = N_t/\bar{\gamma}$. The coefficients $\beta_{l,j}$ can be derived as presented in (128), see [136], [137] for further details. The average capacity of the nth best RB is computed by substituting (130) into (124). We also need to derive the integral $\mathcal{I}_4(l+\mathcal{K},(j+n)\alpha) = \int_0^\infty \gamma^{l+\mathcal{K}-1} e^{-(j+n)\alpha\gamma} \ln(1+\gamma)d\gamma$, which is derived in [126] and is given in (129). $\square$

The average capacity (126) has been derived in [127]. Moreover, it can be obtained also from the result derived in Section 3.4.1, i.e., $C_{\mathrm{STBC}} = \log_2(e)\mathcal{I}_{\mathrm{Q}}(0)$, where $\mathcal{I}_{\mathrm{Q}}(0)$ is given in. Now, the average capacity for the system with the best-M feedback method and the STBC scheme is obtained by substituting (127) and C_{STBC} into (124).

The capacity for SISO, MISO or SIMO systems can be easily determined from (127). The capacity for the best RB in the Rayleigh channel, i.e., C_1^{R}, has also been derived in [103].

4.4.2 Antenna selection

The exact PDF of the SNR for a system with the RBAS scheme and the best-M feedback method is tedious to determine. Furthermore, the derivation of the capacity of the nth best RB for AS with the MRC receiver and $N_r > 2$ leads to complex expressions. In order to obtain a tractable solution, $N_r = N_t = 2$ is assumed and an approximate solution is provided in the following proposition.

Proposition 8 *The average capacity for the kth user in OFDMA systems with $N_t = N_r = 2$, RBAS, the MRC receiver, and the best-M feedback strategy is of the form*

$$\begin{aligned} C(k) \approx{}& (1-p_{ASE})\left(\frac{Nq(k,p_b)-M}{M(N-M)}\sum_{n=1}^{M} C_n^{AS} + \frac{N(1-q(k,p_b))}{N-M}C_{AS}\right) \\ &+ p_{ASE}\left(\frac{q(k,p_b)}{M}\sum_{l=1}^{M} C_{l+}^{MRC} + (1-q(k,p_b))C_{ASE}\right), \end{aligned} \tag{131}$$

where

$$\begin{aligned} C_n^{AS}(\gamma) ={}& \frac{N!\log_2(e)}{(N-n)!(n-1)!}\sum_{l=0}^{2(N-n)+1}\binom{2(N-n)+1}{l}(-1)^l \\ &\cdot \sum_{m=0}^{n-1}\binom{n-1}{m}2^{n-m}(-1)^m \sum_{t=0}^{l+n-1+m}\binom{l+n-1+m}{s}\frac{\mathcal{I}_4(s+2,(l+n+m)/\bar{\gamma})}{\bar{\gamma}^{s+2}}, \end{aligned} \tag{132}$$

$$C_{ASE} = \log_2(e)(\mathcal{I}_2(0,2/\bar{\gamma},0) + 2/\bar{\gamma}\mathcal{I}_2(1,2/\bar{\gamma},0) + 1/\bar{\gamma}^2\mathcal{I}_2(2,2/\bar{\gamma},0)). \tag{133}$$

The Integrals $\mathcal{I}_4(a,b)$ and $\mathcal{I}_2(c,\alpha,x)$ are given in (129) *and* (169)*, respectively. Furthermore,*

$$C_{l+}^{MRC} = \frac{1}{NN_t - l} \sum_{n=l+1}^{NN_t} C_n^{MRC} = \frac{NN_t}{NN_t - 1} C_{MRC} - \frac{1}{NN_t} \sum_{n=1}^{l} C_n^{MRC}, \quad (134)$$

where the capacity of the nth best RB is denoted as C_n^{MRC} in (134) *when the MRC is applied.*

Proof. In the case of correct antenna selection, the capacity is exactly given by the first two terms in (131). If the correctly indicated RB is allocated but the antenna selection is erroneous, the complete capacity formula includes the capacities from C_{1+}^{MRC} to $C_{(2M-1)+}^{\mathrm{MRC}}$ with certain probabilities. For example, if the best RB is allocated and the antenna selection is erroneous, the capacity is exactly given by $C_{1+}^{\mathrm{MRC}} = \frac{1}{NN_t - 1} \sum_{n=2}^{NN_t} C_n^{\mathrm{MRC}}$, because the allocated channel can be any out of the NN_t channels except the best one. In the case that the correctly indicated RB is allocated and the antenna selection is erroneous, the capacity is well approximated as $\sum_{l=1}^{M} C_{l+}^{\mathrm{MRC}}/M$. This approximation provides performance very close to the correct one and simplify the calculus. The performance of the resource allocation based on the best-M method is almost random when the best-M feedback word is erroneous. Consequently, in case the feedback word and the antenna selection are erroneous, the capacity is given by the last term in (131), i.e., the average capacity of an erroneously selected antenna.

Now we prove the result in (132) which is the capacity of the nth best RB in the case of antenna selection. Substituting (9) and (10) into (79), applying binomial expansion, and after some manipulations, the PDF of the SNR of the nth best RB can be written as

$$f_n^{\mathrm{AS}}(\gamma) = \frac{N!}{(N-n)!(n-1)!} \sum_{l=0}^{2(N-n)+1} \binom{2(N-n)+1}{l} (-1)^l \cdot \sum_{m=0}^{n-1} \binom{n-1}{m} 2^{n-m} (-1)^m \sum_{t=0}^{l+n-1+m} \binom{l+n-1+m}{s} \frac{\gamma^{s+1} e^{(l+n+m)\gamma/\bar{\gamma}}}{\bar{\gamma}^{s+2}}. \quad (135)$$

Substituting (135) into (125) and applying (129) we obtain (132).

The CDF of the SNR for the ASE case is given as $F_{\mathrm{ASE}}(\gamma) = 2F_{\mathrm{MRC}}(\gamma) - F_{\mathrm{MRC}}(\gamma)^2$ for $N_t = 2$. Applying integration by parts into $C_{\mathrm{ASE}} = \int_0^\infty f_{\mathrm{ASE}}(\gamma) \log_2(1+\gamma) d\gamma$, we have

$$C_{\mathrm{ASE}} = \log_2(e) \int_0^\infty \frac{\mathcal{P}_2^2(\gamma/\bar{\gamma})}{1+\gamma} d\gamma. \quad (136)$$

The result in (133) is obtained from (136) by using (170) in Appendix 1. □
The capacity C_n^{MRC} in (134) is given by (127) with parameters $\mathcal{K} = N_r$, $N = NN_t$

and $\alpha = 1/N_t$. The average capacity for the MRC is derived as $C_{\text{MRC}} = \mathcal{I}_{\text{Q}}(0)$, where $\mathcal{I}_{\text{Q}}(x)$ is presented in (52) with the parameters $\mathcal{K} = N_r$ and $N_t = 1$. For antenna selection, we have $C_{\text{AS}} = C_{\text{AS1}}(0)$, where $C_{\text{AS1}}(x)$ is given in (60).

4.4.3 Space-division multiple access

Proposition 9 *In the case of SDMA without diversity transmission and a* 2×2 *MIMO channel, the capacity for the* n*th best RB* (125) *is expressed as*

$$C_n^{ZF}(\gamma) = \frac{N! \log_2(e)}{(N-n)!(n-1)!} \sum_{j=0}^{N-n} \binom{N-n}{j} \frac{(-1)^j}{j+n} e^{2(j+n)/\bar{\gamma}} E_1\left(\frac{2(j+n)}{\bar{\gamma}}\right), \tag{137}$$

Proof. For the SDMA without diversity transmission, the PDF of the nth best RB is obtained by substituting (11) into (79), and after using the binomial expansion we have

$$f_n^{\text{ZF}}(\gamma) = \frac{2N!}{(N-n)!(n-1)!\bar{\gamma}} \sum_{j=0}^{N-n} (-1)^j e^{-2(j+n)\gamma/\bar{\gamma}}. \tag{138}$$

The capacity in (137) is obtained by substituting (138) into (125) and using $\mathcal{I}_3(2(j+n)/\bar{\gamma}, 0)$ given by (171) in Appendix 1. □
The average capacity of the ZF detection can be computed by substituting (11) into (126), and we obtain $C_{\text{ZF}} = \log_2(e) e^{2/\bar{\gamma}} E_1(2/\bar{\gamma})$. The average capacity for the OFDMA-SDMA scheme is obtained by combining (124), (137), and C_{ZF}. Similarly for the SDMA with STBC, we state the following proposition.

Proposition 10 *In the case of the SDMA with STBC and a* 4×2 *MIMO channel, the capacity for the* n*th best RB can presented as*

$$\begin{aligned} C_n^{DZF}(\gamma) = &\frac{N! \log_2(e)}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} (-1)^i \binom{N-n}{i} \sum_{j=0}^{i} \binom{i}{j} \\ &\sum_{l=0}^{n-1} \binom{n-1}{l} \alpha^{j+l+2} \mathcal{I}_4(j+l+2, \alpha(i+n)), \end{aligned} \tag{139}$$

where $\alpha = 4/\bar{\gamma}$ *and* $\mathcal{I}_4(a,b)$ *is given in* (129).

Proof. In the case of SDMA with the STBC transmission, the PDF of the SNR of the nth best RB is found by substituting (13) into (79). After binomial expansion and some manipulations, the PDF becomes

$$
\begin{aligned}
f_n^{\mathrm{DZF}}(\gamma) = & \frac{N!}{(N-n)!(n-1)!} \sum_{i=0}^{N-n} (-1)^i \binom{N-n}{i} \sum_{j=0}^{i} \binom{i}{j} \\
& \sum_{l=0}^{n-1} \binom{n-1}{l} \alpha^{j+l+2} \gamma^{j+l+1} e^{-\alpha(i+n)\gamma},
\end{aligned}
\tag{140}
$$

where $\alpha = 4/\bar{\gamma}$. The capacity for the nth best RB can be achieved by substituting (140) into (125) and using (129). □

For an OFDMA-SDMA system with STBC transmission, the average capacities in (124) are provided by (139) and $C_{\mathrm{DZF}} = \log_2(e)\mathcal{I}_{\mathrm{Q}}(0)$, where $\mathcal{I}_{\mathrm{Q}}(a)$ is given in (70). The average system capacity for OFDMA-SDMA with STBC is obtained by substituting (139) and C_{DZF} into (124).

4.5 Numerical examples

Similarly to the numerical results in Chapter 3, the analytical results are presented for the best-M feedback strategies as detailed in Section 3.5[8]. Section 4.5.1 considers the BEP and outage capacity performance of the best-M, Obest-M, and SBB-1 feedback strategies in the SISO link. Section 4.5.2 investigates the asymptotic BEP performance when the SNR tends to infinity. The combinations of multiple antenna methods and the best-M feedback method and the SBB-1 method are studied through outage capacity and the average capacity results in Section 4.5.3.

4.5.1 SISO communications

The average BEP performance versus user position in the allocation queue is illustrated in Fig. 17 when $\bar{\gamma} = 10$ dB, $N = 12$ and the feedback word is error free. The corresponding outage capacity results with $P_{\mathrm{out}} = 0.1$ and $\bar{\gamma} = 6$ dB are shown in Fig. 18 for constant rate transmission and in Fig. 19 for variable rate transmission. The feedback overhead of each scheme is illustrated in Table 1. The reference results are a RA without feedback information and the optimum RR allocation where the user is always assigned to the best available channel. As expected, the performance of the resource assignment according to the ordered best-M feedback method provides optimum performance for the M first users. The performance degradation due to the missing order information in the best-M feedback scheme covers the first few users. The higher M is the more severe performance degradation is faced by the first users.

[8]The curves illustrate analytical results in this section, but Fig. 22 differs from this rule in that some curves present simulation results

Table 1. The lengths of the different feedback words in bits.

N	Best-1	OBest-2	OBest-3	Best-2	Best-3	SBB-1
12	4	8	12	7	9	6

The performance of the M first users using the best-M feedback scheme is constant, which can be predicted based on (74). In Figs 17–19, the SBB-1 method has three sub-blocks and $N_b = 4$. Thus, the first users obtain the average performance of the best RB out of four candidate RBs in the SBB-1 method. The SBB-1 feedback method degrades the performance of the first users further. The BEP of the ordered best-M feedback strategy based system deviates rapidly from the optimum one with increasing k. If the indicated RBs are not available, the expected performance with the best-M schemes is worse than that of fully random allocation. Thus, for users $k > M$, the BEP performance is dominated by random allocation of the available RBs if $M \ll N$.

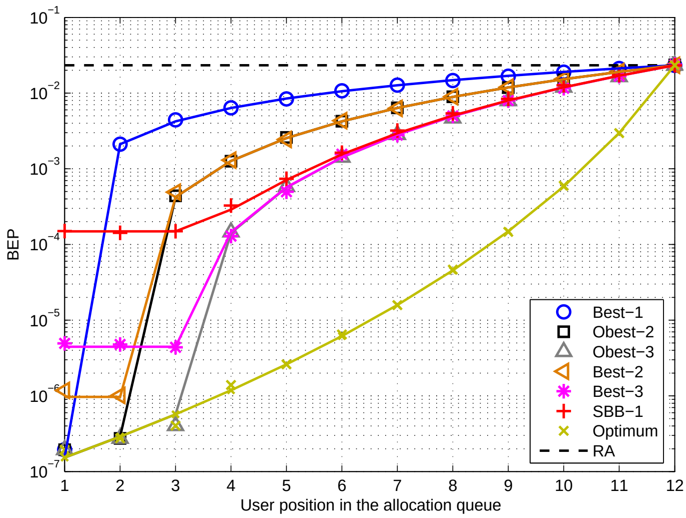


Fig 17. BEP vs. user position in the allocation with 12 RBs, $\bar{\gamma} = 10$ dB, and $p_b = 0$. Marker points present simulation results.

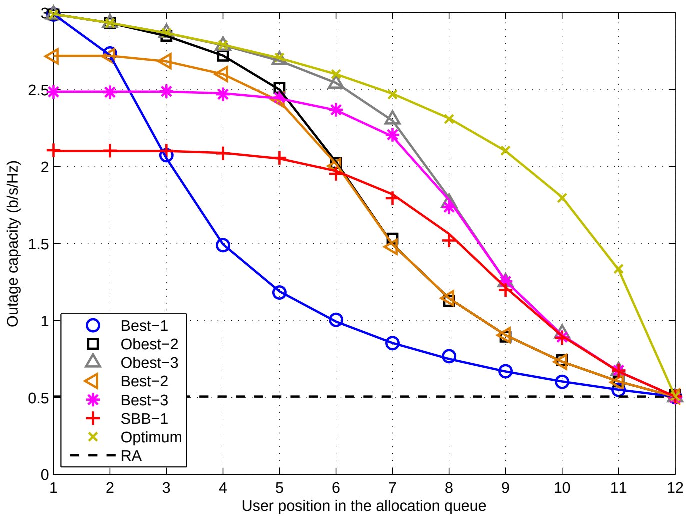


Fig 18. Outage capacity vs. user position in the allocation with constant rate transmission, $P_{\text{out}} = 0.1$, 12 RBs, $\bar{\gamma} = 6$ dB, and $p_b = 0$. Marker points present simulation results.

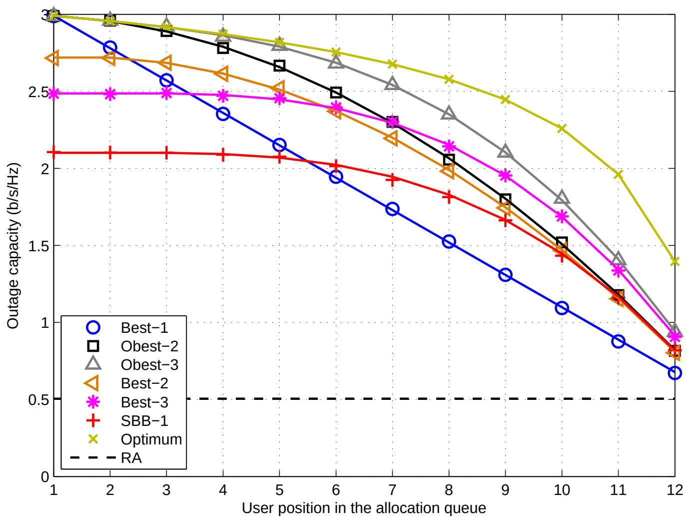


Fig 19. Outage capacity vs. user position in the allocation with variable rate transmission, $P_{\text{out}} = 0.1$, 12 RBs, $\bar{\gamma} = 6$ dB, and $p_b = 0$. Marker points present simulation results.

The good performance for the first few users provided by the best-M feedback strategies is unfortunately lost when more practical erroneous feedback is received at the base station. This can be seen from the BEP and outage capacity performance results shown in Figs. 20–22 where $p_b = 0.05$ and the other parameters the same as in Figs. 17–19. The SBB-1 feedback method seems to be remarkably more robust against feedback bit errors than that of the Obest-M and the conventional best-M schemes. The ordered best-1 feedback word indicates the wrong RB even in the case of a one bit error. The ordered best-2 and best-3 methods are also relatively sensitive to one bit errors. The set best-M feedback scheme is the most sensitive to feedback errors, because it leads easily to random allocation as explained in Section 4.1. If $M > 1$ and a one bit error occurs, the best-M feedback method leads to random allocation. The SBB-1 is not so sensitive to feedback errors because the feedback word related to the randomly selected sub-block can be correct when errors occur at the other sub-blocks. The error probability of the feedback word of a sub-block is also smaller than that of the Obest-M or best-M feedback words. Thus, the SBB-1 feedback strategy is an attractive one due to its good tradeoff between feedback overhead, robustness and allocation gain.

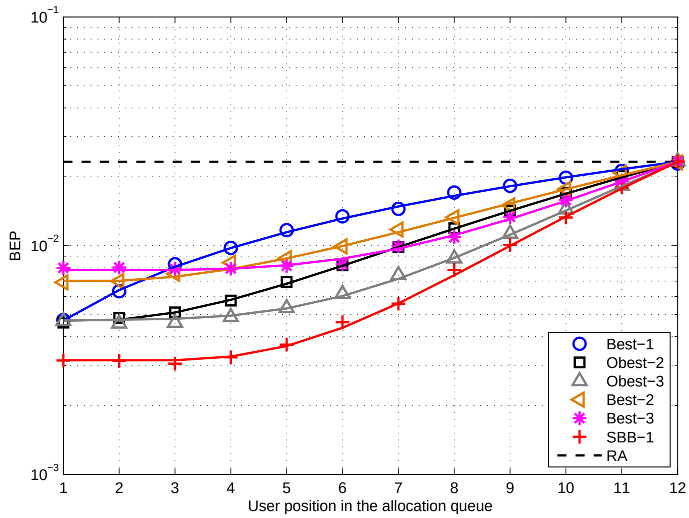


Fig 20. BEP vs. user position in the allocation with 12 RBs, $\bar{\gamma} = 10$ dB, and $p_b = 0.05$. Marker points present simulation results.

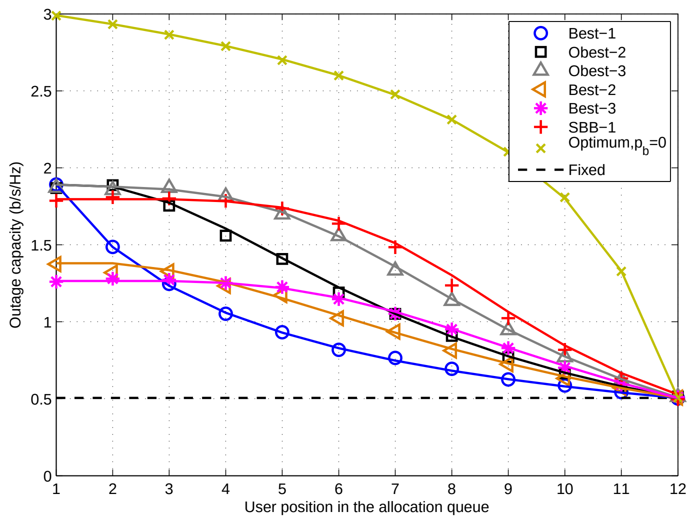


Fig 21. Outage capacity vs. user position in the allocation with constant rate transmission, $P_{\text{out}} = 0.1$, 12 RBs, $\bar{\gamma} = 6$ dB, and $p_b = 0.05$. Marker points present simulation results.

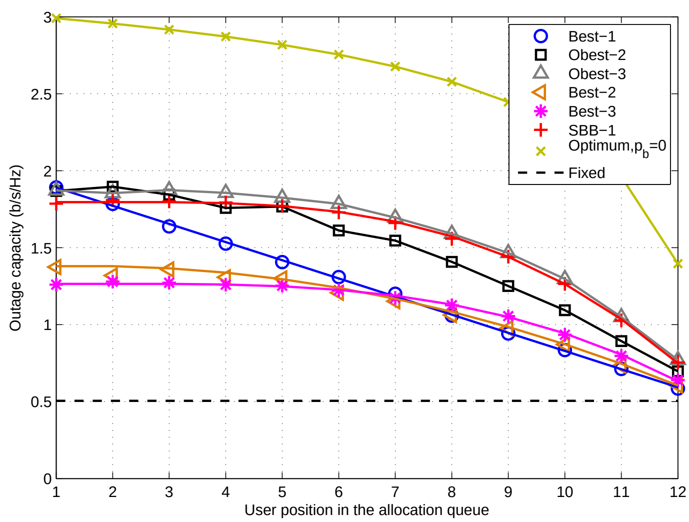


Fig 22. Outage capacity vs. user position in the allocation with variable rate transmission, $P_{\text{out}} = 0.1$, 12 RBs, $\bar{\gamma} = 6$ dB, and $p_b = 0.05$. Marker points present simulation results. The results for the Obest-M scheme have been simulated.

4.5.2 Asymptotic BEP analysis

The BEP performances of the allocation with the best-1 and best-2 feedback schemes are illustrated for the first and fourth users in Fig. 23. The full diversity of RB selection among N alternatives is available only for the first user using the ordered best-M feedback scheme when the feedback is error free. For $k \leq M$ and perfect feedback, the BEP gain between the best-M methods and random allocation approaches infinity as SNR goes to infinity. In the case where $k > M$ or the feedback is erroneous, the asymptotic slope of the BEP versus the SNR is the same as in random allocation regardless of the feedback schemes. The larger p_b or k is the smaller SNR value is needed to achieve an asymptotic BEP slope. A relatively low SNR is enough to achieve an asymptotic BEP slope when $p_b = 0.05$ as can be seen from Fig. 23. For example, for users $k = 1,\ 4$, the SNR gain provided by the best-1 feedback method for the BEP value 10^{-3} is exactly 7.95 dB, 2.84 dB, and according to (104) 7.89 dB, 2.83 dB.

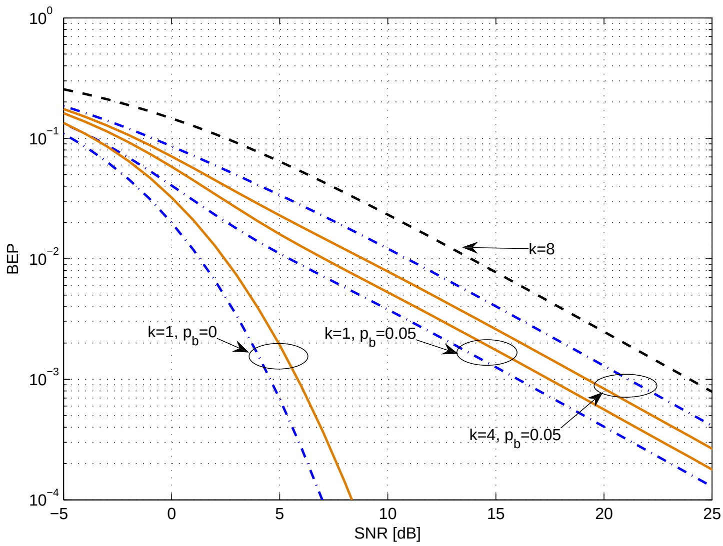


Fig 23. BEP vs. SNR. Dash dotted curves illustrate the best-1 scheme and solid curves illustrate the set best-2 scheme.

4.5.3 Multiple antenna methods

Fig. 24 illustrates the outage capacity versus user position in the allocation queue with the best-M feedback scheme, $P_{\text{out}} = 0.1$, $N_t = N_r = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB, and $p_b = 0.1$. We can see again that the simulation results validate the proposed analysis. The approximations in (120)–(123) also provide accurate results for the RBAS scheme. The larger the M is the less of on effect the allocation queue position has on the performance. The RBAS scheme outperforms the STBC scheme although $p_b = 0.1$. Note that in the case of the RB-wise one bit feedback method the STBC clearly outperforms the RBAS when $p_b = 0.1$, as can be seen in Figs. 9 and 10. The best-M feedback word is more likely to be erroneous than the feedback word dedicated to antenna selection. Thus, errors in the best-M word dominantly deteriorate the performance in the presence of feedback errors. The impact that the feedback bit error probability has on the performance has been shown in Fig. 25, which illustrates the outage capacity of fair allocation versus feedback bit error probability with $P_{\text{out}} = 0.1$, $\bar{\gamma} = 10$ dB. In the SBB-1 AS method the number of sub-block is two, $N_b = 2$ and the best antenna is indicated only for the indicated channels. Thus, two bits are needed to indicate the best RB of a sub-block and one bit per sub-block is needed for antenna selection. The combination of AS and the SBB-1 method is promising from the feedback overhead and robustness point of view.

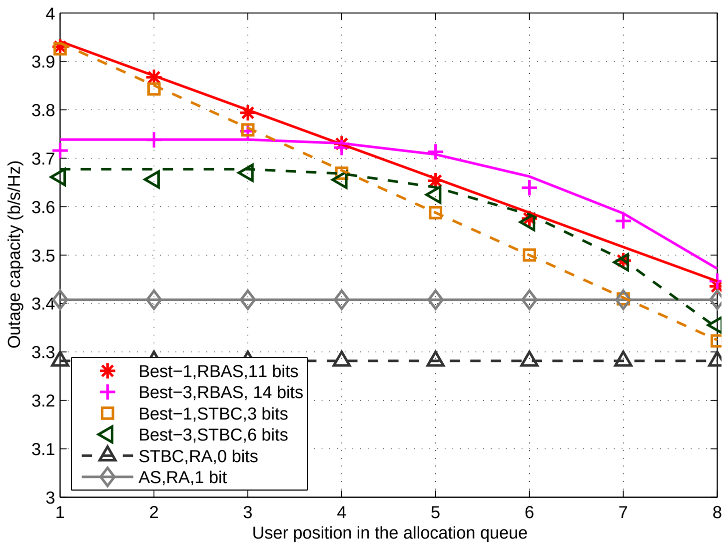


Fig 24. Outage capacity vs. user position in the allocation queue with variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = N_r = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB, and $p_b = 0.1$. Marker points present simulation results.

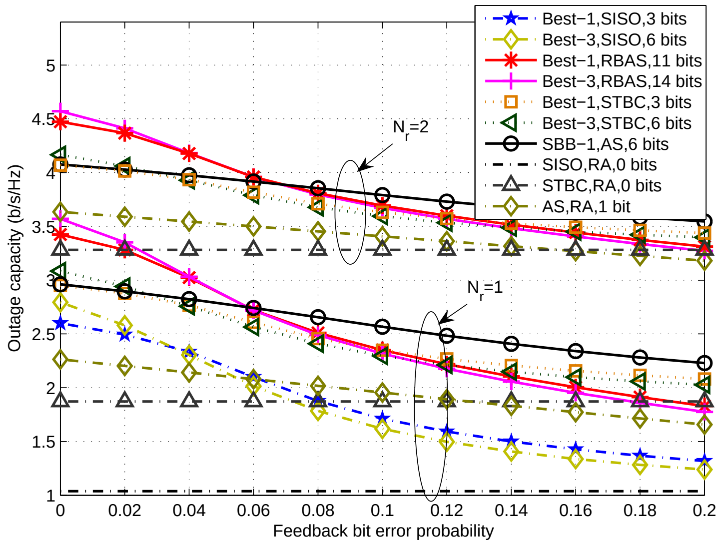


Fig 25. Outage capacity of fair allocation vs. feedback bit error probability with variable rate transmission, $P_{\text{out}} = 0.1$**,** $N_t = 2$**,** $N = K = 8$**,** $\bar{\gamma} = 10$ **dB.**

The average capacity versus user position in the allocation queue with the best-M feedback method, $p_b = 0.1$ and $\bar{\gamma} = 10$ dB is illustrated in Fig. 26. We can see that the analytical results again match the simulation results. STBC provides a small improvement against single antenna transmission when the best-M feedback method is applied and $N_r = 1$. The best-1 feedback method provides good performance when optimal rate adaptation is used and only a single RB per user is allocated. Note, that the best-M method with $M > 1$ could obtain significant performance enhancement if the instantaneous channel knowledge for the M best channels was available. On the other hand, the feedback overhead of such a system is high and it is sensitive to feedback errors when M is high [104].

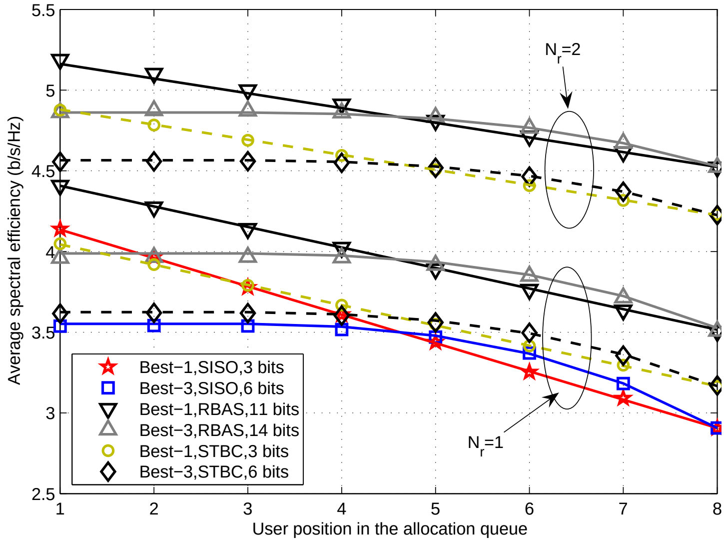


Fig 26. Average spectral efficiency vs. user position in the allocation queue with optimal rate adaptation, $N_t = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB, and $p_b = 0.1$. Marker points present simulation results.

The RBAS outperforms the STBC at the cost of feedback overhead. On the other hand, we have realized that STBC does not provide notable gain over SIMO transmission just as the combination of the best-M method and AS information only for the M RBs does not provide an attractive tradeoff between performance and feedback overhead. A promising method is to combine the AS method and the SBB-M feedback strategy. The performance of the SBB-M and other schemes are compared in Fig. 27, which illustrates the SNR that is needed to achieve 4 b/s/Hz in fair allocation for a given feedback BEP. In the SBB-1 method with AS, there are four sub-blocks and $N_b = 2$. Thus, one bit is needed to find the best RB of each sub-block and additional four bits are needed to find the best antennas of the indicated RBs. In the combination of RBAS and SBB-1, the RBs are divided into two sub-blocks with $N_b = 4$.

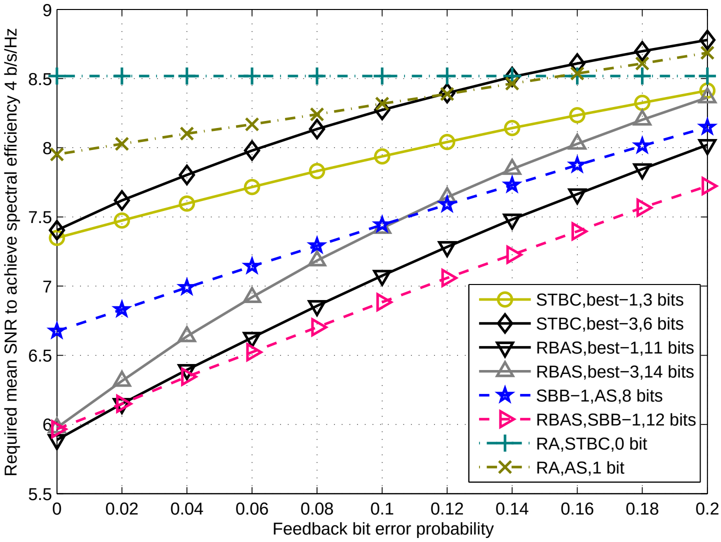


Fig 27. Required SNR needed to support 4 b/s/Hz vs. feedback BEP with fair allocation, optimal rate adaptation, $N = 8$, $K = 6$.

Fig. 28 illustrates the performance of fair OFDMA-SDMA allocation. Note that the outage capacity performance is shown in b/s/Hz per RB. The spectral efficiency per RB or per user in this case is lower than that in the schemes with no SDMA. On the other hand, SDMA enables two times more users compared to the single user MIMO schemes and the spectral efficiency in b/s/Hz is double compared to the results in Fig. 28. The best-3 case provides even worse performance than that of the best-1 scheme when $p_b > 0.03$. The error probability of the best-M feedback word increases with N and M. Thus, the SBB-M method is more suitable for a large N. In Fig. 28, the sub-block includes four space-time-frequency RBs and, consequently, there are three sub-blocks in the SBB-1 method. The index of the best RB of each sub-block is fed back to the base station in the SBB-1 method. We can see that the proposed SBB-1 is significantly more robust against feedback bit errors than the conventional best-M scheme. Furthermore, the SBB-1 method reduces feedback overhead when compared to the best-3 method. STBC transmission outperforms single antenna transmission when outage capacity is used to measure the performance of the OFDMA-SDMA system. However, the results in Fig. 29 illustrate that SDMA without STBC provides more multiuser diversity when optimal rate adaptation is applied. It is assumed that each user has the same mean SNR value and the average capacity over time-frequency RBs is shown in Fig. 29.

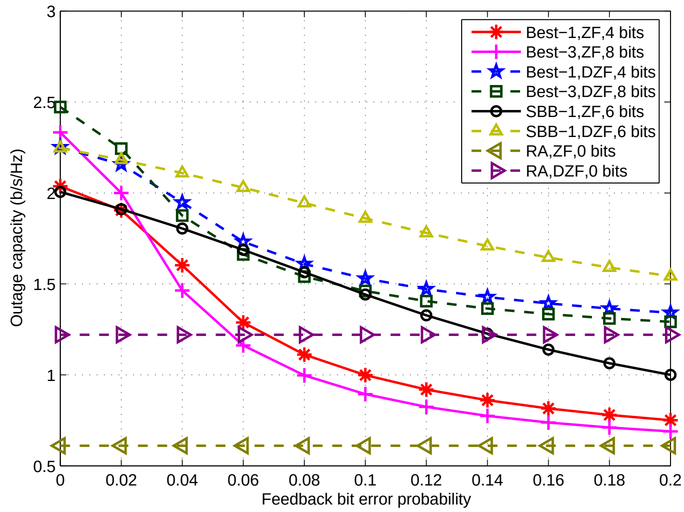


Fig 28. Outage capacity per RB of fair OFDMA-SDMA allocation vs. feedback bit error probability with variable rate transmission, $P_{\text{out}} = 0.1$, $N_t = 4$ for DZF, $N_t = 2$ for ZF, $N_r = 2$, $N = K = 12$, and $\bar{\gamma} = 10$ dB.

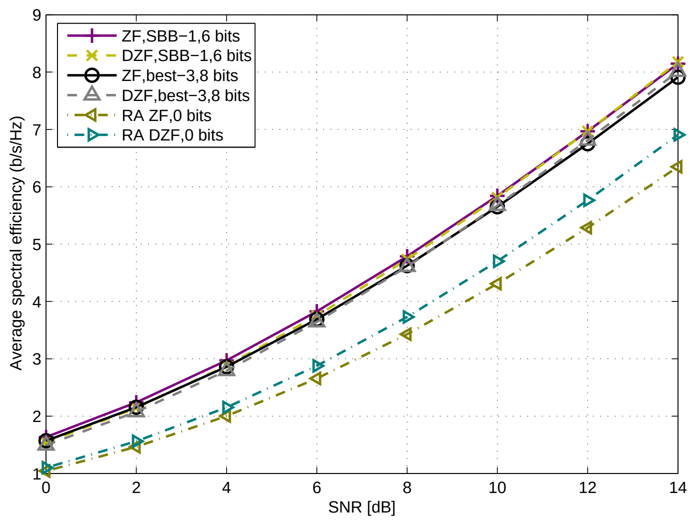


Fig 29. Average system spectral efficiency of fair OFDMA-SDMA allocation vs. SNR with optimal rate adaptation, $N_t = 4$ for DZF, $N_t = 2$ for ZF, $N_r = 2$, $N = K = 12$, $p_b = 0.05$.

Assume now that each user has the same mean SNR value. SDMA without STBC and AS are compared in Fig. 30. The number of RBs and users are different in the considered schemes to obtain the same feedback overhead and the number of time-frequency RBs is also the same in both schemes. For the SDMA system, $N_b = 4$ and the number of sub-blocks is four whereas in the AS system, the parameters are the same as in Fig. 27. SDMA provides a higher sum rate than that of the AS scheme. On the other hand, AS outperforms SDMA from the user specific throughput point of view.

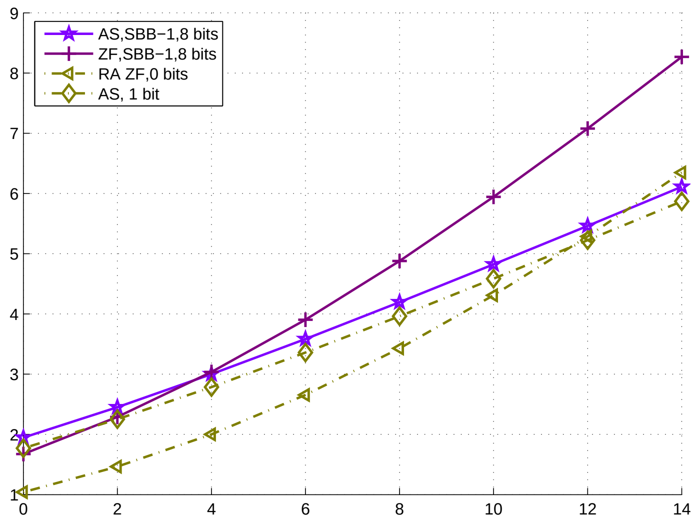


Fig 30. Average system spectral efficiency of fair allocation vs. SNR with optimal rate adaptation, $N_t = N_r = 2$ and $p_b = 0.05$. For AS $N = K = 8$ and for SDMA $N = K = 16$.

4.6 Summary

The analysis of RB allocation in an OFDMA system based on the best-M feedback strategies was studied. The BEP and outage capacity performances were investigated for the conventional best-M scheme, the Obest-M method as well as for the SBB-1 method when an SISO link was assumed. The analysis of the Obest-M feedback strategy turned out to be tedious with an imperfect feedback link and furthermore it causes the highest feedback overhead of the considered best-M based methods. Thus, the spectral efficiency study focused on the conventional best-M scheme and the SBB-1 method in terms of the outage capacity and the system average capacity with optimal rate adaptation. Antenna selection and STBC multiple antenna techniques were considered when a time-frequency RB was allocated exclusively to a single user. In addition, the combination of SDMA and OFDMA was analyzed with the assumption of two spatial RBs per time-frequency RB.

The best-M feedback strategies provides substantial allocation gain provided that the feedback channel is reliable. On the other hand, RB allocation based on the conventional best-M feedback information is sensitive to feedback bit errors. The modified best-M method, namely, the SBB-1 feedback strategy, is relatively robust against feedback errors and it also reduces the feedback overhead of the best-M method. The combination of the SBB-1 method and AS or SDMA turned out to provide promising spectral efficiency gain also with an imperfect feedback channel when compared to random allocation.

5 Analysis of relay enhanced link

5.1 System model

Two-hop downlink OFDMA communication from a BS to mobile user equipments (UE)s through a fixed infrastructure-based AF RN is considered. The study considers a single-cell multiuser OFDMA system, where K UEs share N available RBs and $K \leq N$. Fig. 31 illustrates the studied downlink relaying system. The RN employs the AF protocol and operates in half-duplex mode such that orthogonal (in time or in frequency) channels are allocated for reception and transmission in the RN. The study focuses on the situation where UEs are assumed to be far from the BS so that the path loss from the BS is too high to communicate directly with BS. Cooperative diversity through the BS and RN is not feasible. All UEs are connected to the RN and do not receive the direct transmission from the BS. In this system the RN is used to extend the coverage area similarly as also proposed for LTE-A networks [138, 139].

It is assumed that the source–relay (SR) channel is static and flat over the employed band of N subchannels. This is a realistic model, because both the BS and the RN are fixed infrastructure-based nodes and it is possible to achieve a line-of-sight (LOS) connection, where beamforming and power control can be used. The instantaneous signal-to-noise ratio (SNR) of the SR channel is denoted by γ_{SR}, which is equal to the average SNR $\bar{\gamma}_{\mathrm{SR}}$.

In the relay–destination (RD) link, the received SNR admits IID exponential fading statistics with the average $\bar{\gamma}_{\mathrm{RD}}$ in each transmit-receive antenna pair in each RB. The same assumptions as those given in Section 2.1 are valid for the RD link. Each UE estimates the RD channel and sends instantaneous feedback information to the RN via a feedback channel. The RN allocates RBs to the UEs according to the feedback information. Note that the RB allocation could also be performed at the base station since the allocation in the first hop does not have an impact on the performance in this system.

The end-to-end (E2E) SNR is given by [123, 140]

$$\gamma = \frac{\bar{\gamma}_{\mathrm{SR}}\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1}, \tag{141}$$

where γ_{RD} is the received SNR of the RD link. Now, we have probability

$$\begin{aligned} P(\gamma < x) &= P\left(\frac{\bar{\gamma}_{\mathrm{SR}}\gamma_{\mathrm{RD}}}{\bar{\gamma}_{\mathrm{SR}} + \gamma_{\mathrm{RD}} + 1} < x\right) \\ &= P\left(\gamma_{\mathrm{RD}} < \frac{(\bar{\gamma}_{\mathrm{SR}} + 1)x}{\bar{\gamma}_{\mathrm{SR}} - x}\right) \\ &= F_{\mathrm{RD}}\left(\frac{(\bar{\gamma}_{\mathrm{SR}} + 1)x}{\bar{\gamma}_{\mathrm{SR}} - x}\right). \end{aligned} \tag{142}$$

Thus, for full rate STBC transmission and MRC, the CDF of the E2E SNR $F_{\mathrm{RB}}(\gamma)$ can be expressed as

$$F_{\mathrm{STBC}}(\gamma) = 1 - \mathcal{P}_{\mathcal{K}}\left(\frac{N_t(\gamma_{\mathrm{SR}}+1)\gamma}{(\gamma_{\mathrm{SR}}-\gamma)\bar{\gamma}_{\mathrm{RD}}}\right), \tag{143}$$

where $\mathcal{P}_m(x)$ is given in (8). For a system with antenna selection at the RN and MRC at the UE, the CDF of the SNR is presented as

$$F_{\mathrm{AS}}(\gamma) = \left(1 - \mathcal{P}_{N_r}\left(\frac{(\gamma_{\mathrm{SR}}+1)\gamma}{(\gamma_{\mathrm{SR}}-\gamma)\bar{\gamma}_{\mathrm{RD}}}\right)\right)^{N_t}. \tag{144}$$

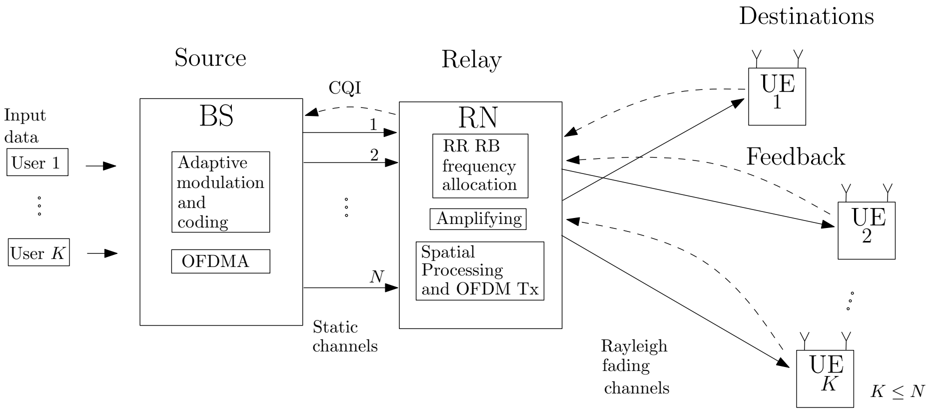


Fig 31. Downlink multiuser communications through fixed-infrastructure AF relay node.

5.2 Resource block wise one bit feedback scheme

In this section, the capacity derivation follows straightforwardly the same principles as presented in Section 3.4, but now the PDF and CDF of the E2E SNR have to be used. Only one bit quantization is considered for the relay enhanced link. The average capacity of the kth user in the allocation queue is expressed as

$$\begin{aligned} C(k) =& \frac{q_1(k,p_b)}{2p_1^Q} \underbrace{\int_{\gamma_{\mathrm{T}}}^{\bar{\gamma}_{\mathrm{SR}}} f_{\mathrm{RB}}(\gamma)\log_2(1+\gamma)d\gamma}_{C_1(\gamma_{\mathrm{T}})} \\ &+ \frac{q_0(k,p_b)}{2p_0^Q} \underbrace{\int_{0}^{\gamma_{\mathrm{T}}} f_{\mathrm{RB}}(\gamma)\log_2(1+\gamma)d\gamma}_{C_0(\gamma_{\mathrm{T}})}, \end{aligned} \tag{145}$$

where the factor $\frac{1}{2}$ comes from the half duplex relaying assumption. The expression in (145) is valid for the STBC scheme and for antenna selection with perfect antenna selection.

Space-Time Block Coding

Applying integration by parts to $C_1(\gamma_{\mathrm{T}})$ in (145), it becomes $C_{\mathrm{STBC}1}(\gamma_{\mathrm{T}}) = \log_2(e)[p_1^Q \ln(1+\gamma_{\mathrm{T}}) + \int_{\gamma_{\mathrm{T}}}^{\bar{\gamma}_{\mathrm{SR}}} (1 - F_{\mathrm{STBC}}(\gamma))/(1+\gamma)d\gamma]$. Similarly as in Appendix 2, we apply the substitution $t = \frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}$, $\alpha = N_t/\bar{\gamma}_{\mathrm{RD}}$ and partial fraction decomposition, resulting in the expression

$$\begin{aligned} C_{\mathrm{STBC}1}(\gamma_{\mathrm{T}}) &= \log_2(e)\Bigg[p_1^Q \ln(1+\gamma_{\mathrm{T}}) \\ &+ \int_{\gamma_{\mathrm{T}}'}^{\infty} \sum_{i=1}^{\mathcal{K}-1} \frac{t^i e^{-t}}{i!} \left(\frac{1}{\alpha+t} - \frac{1}{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)+t}\right) dt\Bigg] \\ &= \log_2(e)\left[p_1^Q \ln(1+\gamma_{\mathrm{T}}) + \sum_{i=1}^{\mathcal{K}-1} \left(\mathcal{I}_6(\gamma_{\mathrm{T}}', i, 1, \alpha) - \mathcal{I}_6(\gamma_{\mathrm{T}}', i, 1, \alpha(\bar{\gamma}_{\mathrm{SR}}+1))\right)\right], \end{aligned} \tag{146}$$

where $\gamma_{\mathrm{T}}' = \frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{\gamma_{\mathrm{SR}}-\gamma_{\mathrm{T}}}$, $\alpha = N_t/\bar{\gamma}_{\mathrm{RD}}$ and $\mathcal{I}_6$ is given by (174). The capacity $C_0(\gamma_{\mathrm{T}})$ can be derived using (146) so that $C_{\mathrm{STBC}0}(\gamma_{\mathrm{T}}) = C_{\mathrm{STBC}1}(0) - C_{\mathrm{STBC}1}(\gamma_{\mathrm{T}})$.

Antenna Selection

The capacity with optimal rate adaptation can be expressed as

$$\begin{aligned} C(k) &= \frac{q_1(k,p_b)}{2}((1-p_{\mathrm{ASE}})C_{\mathrm{AS}1}(\gamma_1) + p_{\mathrm{ASE}}C_{\mathrm{ASE}1}(\gamma_1)) \\ &+ \frac{q_0(k,p_b)}{2}((1-p_{\mathrm{ASE}})C_{\mathrm{AS}0}(\gamma_1) + p_{\mathrm{ASE}}C_{\mathrm{ASE}0}(\gamma_1)) \end{aligned} \tag{147}$$

where p_{ASE} is the probability of antenna selection error (ASE),

$$C_{\mathrm{AS}1}(\gamma_{\mathrm{T}}) = \int_{\gamma_{\mathrm{T}}}^{\infty} f_{\mathrm{AS}}(\gamma)/p_1^Q \log_2(1+\gamma)d\gamma, \tag{148}$$

$$C_{\mathrm{ASE}1}(\gamma_{\mathrm{T}}) = \int_0^{\infty} f_{\mathrm{ASE}1}(\gamma|\gamma_{\mathrm{AS}} > \gamma_{\mathrm{T}}) \log_2(1+\gamma)d\gamma, \tag{149}$$

$$C_{\mathrm{AS0}}(\gamma_{\mathrm{T}}) = \int_0^{\gamma_{\mathrm{T}}} f_{\mathrm{AS}}(\gamma)/p_0^{\mathrm{Q}} \log_2(1+\gamma)d\gamma, \tag{150}$$

$$C_{\mathrm{ASE0}}(\gamma_{\mathrm{T}}) = \int_0^{\gamma_{\mathrm{T}}} f_{\mathrm{ASE0}}(\gamma|\gamma_{\mathrm{AS}} < \gamma_{\mathrm{T}}) \log_2(1+\gamma)d\gamma, \tag{151}$$

and γ_{AS} is the SNR at the best antenna.

The capacity $C_{\mathrm{AS1}}(\gamma_{\mathrm{T}})$ provides the average spectral efficiency for the channel whose SNR exceeds the threshold when the antenna selection is correct. The closed form capacity can be derived by applying integration by parts to (148) and substituting (144) to the resulting integrals. Thus, the capacity is now of the form

$$\begin{aligned} C_{\mathrm{AS1}}(\gamma_{\mathrm{T}}) =& \log_2(e)\Bigg[p_1^Q \ln(1+\gamma_{\mathrm{T}}) + \\ & \int_{\gamma_{\mathrm{T}}}^{\infty} \frac{2\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right) - \left(\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)\right)^2}{1+\gamma} d\gamma\Bigg] \\ =& \log_2(e)\left[p_1^Q \ln(1+\gamma_{\mathrm{T}}) + 2\mathcal{I}_5(\gamma_{\mathrm{T}},1) - \mathcal{I}_5(\gamma_{\mathrm{T}},2)\right], \end{aligned} \tag{152}$$

where the integral $\mathcal{I}_5$ is derived in the Appendix 2.

The capacity in (150) corresponds to a best antenna whose SNR is below the threshold. It can be easily derived as $C_{\mathrm{AS0}}(\gamma_{\mathrm{T}}) = (C_{\mathrm{AS1}}(0) - p_1^Q C_{\mathrm{AS1}}(\gamma_{\mathrm{T}}))/p_0^Q$.

For erroneous antenna selection, the capacity is given by $C_{\mathrm{ASE1}}(\gamma_{\mathrm{T}})$ in the case where $\gamma_{\mathrm{AS}} > \gamma_{\mathrm{T}}$. This integral is also derived using integration by parts. Thus, we need to know the CDF of the SNR for the ASE case which is determined in (64) to be of the form

$$F_{\mathrm{ASE1}}(\gamma_*|\gamma_{\mathrm{AS}}) = \begin{cases} \frac{2F_{\mathrm{MRC}}(\gamma_*)}{1+F_{\mathrm{MRC}}(\gamma_1)} & \gamma_* < \gamma_{\mathrm{T}}, \gamma_{\mathrm{AS}} > \gamma_{\mathrm{T}} \\ \frac{2F_{\mathrm{MRC}}(\gamma_*)}{1+F_{\mathrm{MRC}}(\gamma_*)} & \gamma_* > \gamma_{\mathrm{T}}, \gamma_{\mathrm{AS}} > \gamma_*, \end{cases} \tag{153}$$

where $F_{\mathrm{MRC}}(\gamma_*) = 1 - \mathcal{P}_{N_r}\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)$ is now the E2E CDF of the system with single antenna AF relaying and the MRC receiver. The integral has to be integrated in two parts so that $C_{\mathrm{ASE1}}(\gamma_{\mathrm{T}}) = \int_0^{\gamma_{\mathrm{T}}} \frac{1-F_{\mathrm{ASE1}}(\gamma_*|\gamma_{\mathrm{AS}})}{1+\gamma} d\gamma + \int_{\gamma_{\mathrm{T}}}^{\infty} \frac{1-F_{\mathrm{ASE1}}(\gamma_*|\gamma_{\mathrm{AS}})}{1+\gamma} d\gamma = \log_2(e)(\mathcal{I}_{\mathrm{P0}}(\gamma_{\mathrm{T}}) + \mathcal{I}_{\mathrm{P1}}(\gamma_{\mathrm{T}}))$. The closed form solution can be easily found for the integral $\mathcal{I}_{\mathrm{P0}}(\gamma_{\mathrm{T}})$ and it can be expressed as

$$\begin{aligned} \mathcal{I}_{\mathrm{P0}}(\gamma_{\mathrm{T}}) =& \int_0^{\gamma_{\mathrm{T}}} \frac{-\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{\gamma_{\mathrm{SR}}-\gamma_{\mathrm{T}}}\right) + 2\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)}{\left(2-\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{\gamma_{\mathrm{SR}}-\gamma_{\mathrm{T}}}\right)\right)(1+\gamma)} d\gamma \\ =& -\frac{-\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{\gamma_{\mathrm{SR}}-\gamma_{\mathrm{T}}}\right)\ln(1+\gamma_{\mathrm{T}}) + 2\mathcal{I}_5(\gamma_{\mathrm{T}},1)}{2-\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{\gamma_{\mathrm{SR}}-\gamma_{\mathrm{T}}}\right)}, \end{aligned} \tag{154}$$

where $\mathcal{I}_5$ is derived in Appendix 2. The closed form solution for the integral $\mathcal{I}_{\text{P1}}(\gamma_{\text{T}}))$ does not exist, but we can expand the term $1 - F_{\text{ASE1}}(\gamma|\gamma_{\text{AS}})$, $\gamma > \gamma_{\text{T}}$, using a geometric series as follows $\frac{\mathcal{P}_2(b)}{2-\mathcal{P}_2(b)} = \frac{1}{2}\mathcal{P}_2(b)\sum_{i=0}^{\infty}\left(\frac{1}{2}\mathcal{P}_2(b)\right)^i = \sum_{i=1}^{\infty}\frac{1}{2^i}\mathcal{P}_2(b)^i$ because $\frac{1}{2}\mathcal{P}_2(b) < 1$. Thus, now we have

$$\begin{aligned}\mathcal{I}_{\text{P1}}(\gamma_{\text{T}}) &= \sum_{i=0}^{\infty}\frac{1}{2^i}\int_{\gamma_{\text{T}}}^{\bar{\gamma}_{\text{SR}}}\frac{\left(\mathcal{P}_2\left(\frac{\alpha(\gamma_{\text{SR}}+1)\gamma}{\gamma_{\text{SR}}-\gamma}\right)\right)^i}{1+\gamma}d\gamma,\\ &= \sum_{i=0}^{\infty}\frac{1}{2^i}\mathcal{I}_5(\gamma_{\text{T}}, i),\end{aligned} \tag{155}$$

where $\mathcal{I}_5$ is derived in the Appendix 2.

The capacity in (151) refers to the case where the antenna selection is erroneous and $\gamma_{\text{AS}} < \gamma_{\text{T}}$. The CDF for this case is given by

$$F_{\text{ASE0}}(\gamma_*|\gamma_{\text{AS}}) = \frac{2F_{\text{MRC}}(\gamma_*)}{F_{\text{MRC}}(\gamma_{\text{T}}) + F_{\text{MRC}}(\gamma_*)}, \tag{156}$$

where $\gamma_* < \gamma_{\text{T}}$ and $\gamma_* < \gamma_{\text{AS}} < \gamma_{\text{T}}$. Applying integration by parts and geometric series similarly as in (155), after some derivations, the capacity (151) can be rewritten as

$$\begin{aligned}C_{\text{ASE0}}(\gamma_{\text{T}}) =& \log(1+\gamma_{\text{T}})\left(1 - \frac{2}{B}\right)\\ &- 2\sum_{i=1}^{\infty}\left(\frac{1}{B^{i+1}} - \frac{1}{B^i}\right)\underbrace{\int_0^{\gamma_{\text{T}}}\frac{\left(\mathcal{P}_2\left(\frac{\alpha(\gamma_{\text{SR}}+1)\gamma}{\gamma_{\text{SR}}-\gamma}\right)\right)^i}{1+\gamma}d\gamma}_{\mathcal{I}_5(0,i)-\mathcal{I}_5(\gamma_{\text{T}},i)}\end{aligned} \tag{157}$$

where $B = 2 - \mathcal{P}_2\left(\frac{\alpha(\gamma_{\text{SR}}+1)\gamma}{\gamma_{\text{SR}}-\gamma}\right)$ and $\mathcal{I}_5$ is derived in the Appendix 2.

5.3 Best-M feedback scheme

Space-Time Block Coding

In the case of half duplex AF relaying, the expected capacity for the kth user can be derived as

$$C(k) = \frac{Nq(k,p_b) - M}{2M(N-M)}\sum_{n=1}^{M}C_n^{\text{STBC}} + \frac{N(1-q(k,p_b))}{2(N-M)}C_{\text{STBC}}, \tag{158}$$

where $C_n^{\text{STBC}} = \int_0^{\infty} f_n^{\text{STBC}}(\gamma)\log_2(1+\gamma)d\gamma$ refers to the capacity of the nth best RB and $C_{\text{STBC}} = \int_0^{\infty} f_{\text{STBC}}(\gamma)\log_2(1+\gamma)d\gamma$ is the capacity of a RB over fading.

Applying integration by parts to C_n^{STBC}, the capacity of the nth best channel can be derived as

$$C_n^{\mathrm{STBC}} = \log_2(1+\bar{\gamma}_{\mathrm{SR}}) - \int_0^{\bar{\gamma}_{\mathrm{SR}}} \frac{\log_2(e) F_n^{\mathrm{STBC}}(\gamma)}{1+\gamma} d\gamma. \tag{159}$$

Now we need to define the CDF of the nth best RB for the system with STBC transmission at the RN and MRC at the UE. Substituting (143) into (80), this CDF can be expressed

$$\begin{aligned} F_n^{\mathrm{STBC}}(\gamma) = & \sum_{j=N-n+1}^{N} \binom{N}{j} \left(1 - \mathcal{P}_{\mathcal{K}}\left(\frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)\right)^j \\ & \left(\mathcal{P}_{\mathcal{K}}\left(\frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{(\bar{\gamma}_{\mathrm{SR}}-\gamma)}\right)\right)^{N-j} \\ = & \sum_{j=N-n+1}^{N} \binom{N}{j} \sum_{l=0}^{j} \binom{j}{l} (-1)^l e^{-(l+N-j)\frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{(\bar{\gamma}_{\mathrm{SR}}-\gamma)}} \\ & \sum_{r=0}^{(\mathcal{K}-1)l+N-j} \beta_{r,l+N-j} \left(\frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{(\bar{\gamma}_{\mathrm{SR}}-\gamma)}\right)^r, \end{aligned} \tag{160}$$

where $\alpha = N_t/\bar{\gamma}_{\mathrm{RD}}$ and the coefficient $\beta_{l,j}$ is given in 128. The second expression in (160) is obtained using binomial expansion. Substituting (160) into (159) and using the substitution $t = \frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{(\bar{\gamma}_{\mathrm{SR}}-\gamma)}$ similarly as in the Appendix 2, after some derivations we end up with the result

$$\begin{aligned} C_n^{\mathrm{STBC}}(\gamma) = & \sum_{j=N-n+1}^{N} \binom{N}{j} \sum_{l=0}^{j} \binom{j}{l} (-1)^l \\ & \sum_{r=0}^{(\mathcal{K}-1)l+N-j} \beta_{r,l+N-j} \big(\mathcal{I}_6(0,r,l+N-j,\alpha) - \mathcal{I}_6(0,r,l+N-j,\alpha(\bar{\gamma}_{\mathrm{SR}}+1))\big), \end{aligned} \tag{161}$$

where $\mathcal{I}_6$ is given in (174).

Antenna Selection

In the case of perfect antenna selection, the closed form solution can be derived. When the antenna selection is erroneous, the exact PDF or CDF of the SNR for the system with the best-M feedback method is tedious to determine. Based on the capacity formula in (131), the capacity for the antenna selection is well approximated by

$$
\begin{aligned}
C(k) \approx{}& (1-p_{\mathrm{ASE}})\left(\frac{Nq(k,p_b)-M}{2M(N-M)}\sum_{n=1}^{M} C_n^{\mathrm{AS}} + \frac{N(1-q(k,p_b))}{2(N-M)}C_{\mathrm{AS}}\right) \\
&+ p_{\mathrm{ASE}}\left(\frac{q(k,p_b)}{2M}\sum_{l=1}^{M} C_{l+}^{\mathrm{MRC}} + (1-q(k,p_b))C_{\mathrm{ASE}}\right),
\end{aligned}
\tag{162}
$$

where p_{ASE} is the probability of antenna selection error given in (43). Furthermore, $C_{l+}^{\mathrm{MRC}} = \frac{1}{NN_t-l}\sum_{n=l+1}^{NN_t} C_n^{\mathrm{MRC}} = \frac{NN_t}{NN_t-1}C_{\mathrm{MRC}} - \frac{1}{NN_t}\sum_{n=1}^{l} C_n^{\mathrm{MRC}}$, where the capacity of the nth best RB is denoted as C_n^{MRC} when the MRC is applied.

The capacity C_n^{AS} refers to the nth best RB with correct antenna selection. It can be derived as

$$
C_n^{\mathrm{AS}} = \log_2(1+\bar{\gamma}_{\mathrm{SR}}) - \int_0^{\bar{\gamma}_{\mathrm{SR}}} \log_2(e)\frac{F_n^{\mathrm{AS}}(\gamma)}{1+\gamma}d\gamma, \tag{163}
$$

where we have applied integration by parts and $F_n^{\mathrm{AS}}(\gamma)$ refers to the CDF of the nth best channel of the best antenna. For the RN with $N_t = 2$ and the MRC receiver with $N_r = 2$, the CDF $F_n^{\mathrm{AS}}(\gamma)$ can be expressed as

$$
\begin{aligned}
F_n^{\mathrm{AS}}(\gamma) ={}& \sum_{j=N-n+1}^{N}\binom{N}{j}\left(1-\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)\right)^{2j} \\
&\left(2\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right) - \left(\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)\right)^2\right)^{N-j} \\
={}& \sum_{j=N-n+1}^{N}\binom{N}{j}\sum_{l=0}^{2j}\binom{2j}{l}(-1)^l\sum_{m=0}^{N-j}\binom{N-j}{m} \\
&2^{N-j-m}(-1)^m\left(\mathcal{P}_2\left(\frac{\alpha(\gamma_{\mathrm{SR}}+1)\gamma}{\gamma_{\mathrm{SR}}-\gamma}\right)\right)^{l+m+N-j}
\end{aligned}
\tag{164}
$$

With the aid of the results in Appendix 2, the capacity in (163) can be rewritten as

$$
\begin{aligned}
C_n^{\mathrm{AS}} ={}& \log_2(1+\bar{\gamma}_{\mathrm{SR}}) - \log_2(e)\sum_{j=N-n+1}^{N}\binom{N}{j}\sum_{l=0}^{2j}\binom{2j}{l} \\
&(-1)^l\sum_{m=0}^{N-j}\binom{N-j}{m}2^{N-j-m}(-1)^m\mathcal{I}_5(0,l+m+N-j)
\end{aligned}
\tag{165}
$$

The average capacity of AS over fading, i.e., C_{AS} in (162) is given by $C_{\mathrm{AS}} = C_{\mathrm{AS1}}(0) = 2\mathcal{I}_5(0,1) - \mathcal{I}_5(0,2)$, where C_{AS1} is presented in (152). The capacity C_n^{MRC} is given by (161) with the parameters $\mathcal{K} = N_r$, $N = NN_t$ and $\alpha = 1/N_t$. The average capacity for the MRC is derived as $C_{\mathrm{MRC}} = C_1(0)$, where $C_1(0)$ is presented in (146) with the parameters $\mathcal{K} = N_r$ and $\alpha = 1/N_t$.

The term C_{ASE} in (162) refers to the case where both the best-M feedback word and the antenna selection are erroneous. The CDF of the SNR for the ASE case is given as $F_{\mathrm{ASE}}(\gamma) = 2F_{\mathrm{MRC}}(\gamma) - F_{\mathrm{MRC}}(\gamma)^2$ for $N_t = 2$. Applying integration by parts into $C_{\mathrm{ASE}} = \int_0^{\bar{\gamma}_{\mathrm{SR}}} f_{\mathrm{ASE}}(\gamma)\log_2(1+\gamma)d\gamma$, we have

$$C_{\mathrm{ASE}} = \log_2(e)\underbrace{\int_0^{\infty}\frac{\left(\mathcal{P}_2\left(\frac{(\gamma_{\mathrm{SR}}+1)\gamma}{(\gamma_{\mathrm{SR}}-\gamma)\bar{\gamma}_{\mathrm{RD}}}\right)\right)^2}{1+\gamma}d\gamma}_{\mathcal{I}_5(0,2)} \tag{166}$$

where $\mathcal{I}_5$ is derived in the appendix 2 with $\alpha = 1/\bar{\gamma}_{\mathrm{RD}}$.

5.4 Numerical examples

In numerical examples, it is assumed that $N_t = N_r = 2$ and there are eight RBs and users, i.e, $N = K = 8$. The SNR at the first hop is $\bar{\gamma}_{\mathrm{SR}} = 20$ dB for each RB. The threshold for one bit quantization was set so that $P(\gamma > \gamma_{\mathrm{T}}) = 0.3125$. This threshold was determined numerically to provide the best average performance over allocation queue positions when $p_b = 0.05$ and $\gamma_{\mathrm{RD}} = 15$ dB. Fig. 32 illustrates the performance of the expected capacity versus user position in the allocation queue when the feedback BEP is 0.05 and $\bar{\gamma}_{\mathrm{RD}} = 10$ dB. The performance of the last user is the same as that in random allocation (RA) whereas the first users gets a good channel with a high probability. However, the capacity variation between users is smaller than in the system in [123] with optimal pairing where the worst SR channel is paired with the worst RD channel, resulting in performance for the last user that is remarkably lower than that in RA. As already demonstrated in Sections 3.5.3 and 4.5.3, AS outperforms STBC also with an imperfect feedback link. On the other hand, AS requires feedback information. If the index of the best antenna is requested separately for the assigned channel, one bit of information is enough for AS (with two transmit antennas). In the case of the fast AS, N bits of RB-wise feedback information for AS have been sent together with the feedback information dedicated for RB allocation.

Relatively high SNR values $\bar{\gamma}_{\mathrm{RD}}$ are needed to provide high data rate communications through the half duplex relay protocol. Fig. 33 illustrates the $\bar{\gamma}_{\mathrm{RD}}$ values that are needed to provide 2 b/s/Hz E2E spectral efficiency in queue position fair allocation, in which each user gets each queue position with equal probability. The performance differences between the feedback schemes are relatively small in the case of AS. Otherwise, RB allocation with different feedback schemes behaves similarly as the one presented in Sections 3.5 and

4.5. AS with RA provides performance close to that of the STBC scheme. All the considered feedback schemes provide substantial performance enhancement also with imperfect feedback. The tradeoff between feedback overhead, system performance, and robustness against feedback errors is promising in the SBB-1 method with $N_b = 4$ and 2 sub-blocks. In a partially loaded case, i.e., when $K < N$, the allocation gain would be larger.

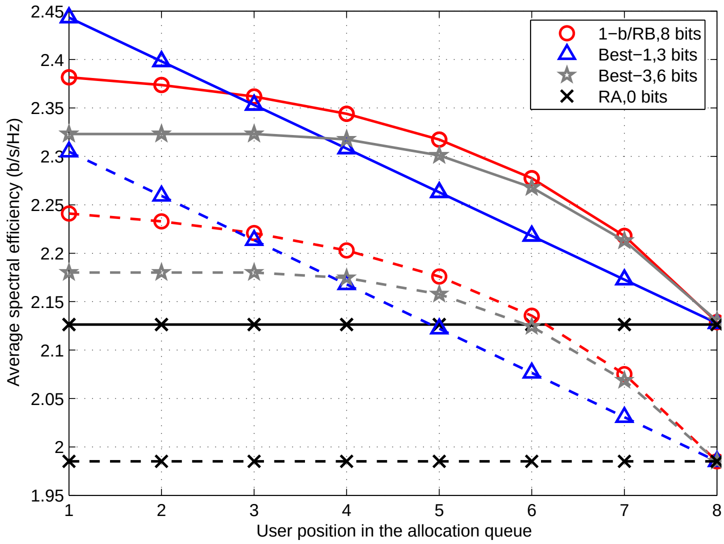


Fig 32. Average spectral efficiency vs. user position in the allocation queue with $N_t = N_r = 2$, $\bar{\gamma}_{\text{SR}} = 20$dB, $\bar{\gamma}_{\text{RD}} = 10$dB, $N = K = 8$ and $p_b = 0.05$. Solid curves illustrate the performance of the AS scheme and dashed curves present the performance of the STBC scheme.

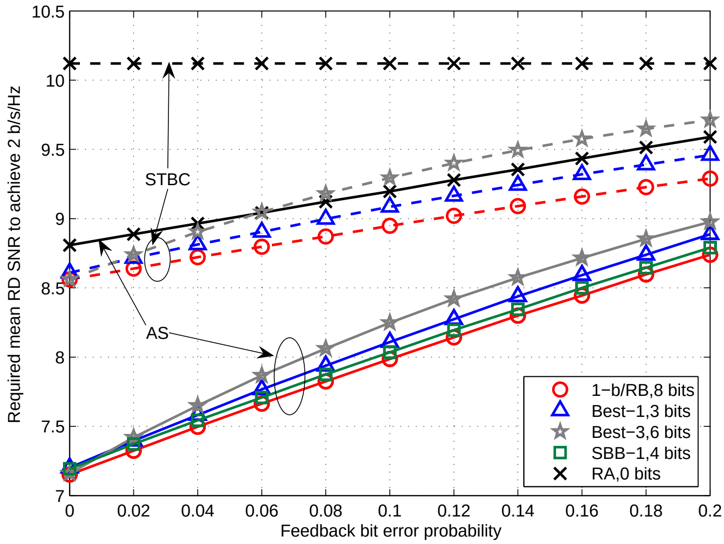


Fig 33. Required $\bar{\gamma}_{\text{RD}}$ to achieve spectral efficiency of 2 b/s/Hz vs. feedback BEP when $N_t = N_r = 2$, $\bar{\gamma}_{\text{SR}} = 20$dB and $N = K = 8$. Allocation queue position fair channel assignment is applied. Solid curves illustrate the performance of the AS scheme and dashed curves present the performance of the STBC scheme.

5.5 Summary

System average capacity results were investigated for a system with dynamic OFDMA and a wireless AF relay node. The presented analysis was straightforward extension of the analysis presented for single-hop systems previously. Multiple antennae were assumed for each end. RBs were allocated according to the limited feedback information which was based on the best-M information or RB-wise one bit quantization. In particular, STBC and AS multi-antenna schemes were studied. Optimal rate adaptation was assumed for the assigned channels. The results indicated that simple and practical round robin allocation provides significant performance gain over random allocation with very limited feedback information. In particular, the SBB-1 feedback method provided substantial performance gain with low feedback overhead in the case where a single RB per user was allocated. The RB-wise one bit feedback strategy achieved the best performance as well as robustness against feedback bit errors.

6 Comparisons

The basic properties of RB allocation based on the best-M feedback strategies and RB-wise one bit quantization were studied in Sections 3.5 and 4.5, respectively, when IID fading for each RB was assumed. The purpose of this chapter is two-fold. The first aim is to compare the best-M feedback strategies and the RB-wise one bit feedback schemes through further numerical examples, since the best choice of the combination of feedback scheme for RB allocation and MIMO scheme depends on the number of users, the number of RBs and feedback channel reliability, and performance measure. The combinations of feedback strategies and multiple antenna schemes are compared in Section 6.1.

The seventh assumption in Section 2.1 enabled the proposed analytical approach. Furthermore, it was explained that the IID channel condition can be approximately achieved when feedback is only formed from few uncorrelated RBs. The second purpose is to show the validity of this RB selection method. Thus, the analytical results with an idealized channel model are compared to those obtained from the simulation results with practical frequency selective fading in Section 6.2.

6.1 Best-M versus RB-wise one bit SNR quantization

Fig. 34 illustrates the average outage capacity of fair allocation versus N when $K = N$, $\bar{\gamma} = 10$ dB, $p_b = 0$, and $p_b = 0.05$. Table 2 illustrates the feedback word length for each feedback method. The feedback schemes are not fair in the sense of feedback overhead. The performance of the allocation based on the 1-bit/RB feedback method improves with increasing N, while the performance of both best-M schemes slightly degrades with imperfect feedback. The feedback word error probability of the best-M scheme increases with N and M. However, the best-2 and best-3 feedback methods provide significant allocation gain with a low feedback overhead. On the other hand, some redundancy could be inserted for the set best-M feedback to improve performance so that the overhead would be the same as that for the 1-bit/RB feedback method.

Table 2. The lengths of different feedback words in bits.

N	Best-1	OBest-2	OBest-3	Best-2	Best-3	1-bit/RB
6	3	6	9	4	5	6
8	3	6	9	5	6	8
12	4	8	12	7	9	12
16	4	8	12	7	10	16
24	5	10	15	9	11	24

Antenna selection is an attractive method to enhance spectral efficiency when optimal rate adaptation is applied. The SNR gain of fair allocation with different feedback methods and antenna selection schemes is illustrated in Fig. 35, which depicts the SNR which is required to achieve spectral efficiency of 4 b/s/Hz with a given p_b. Note that the partially loaded case is considered in Fig. 35 so that $N = 8$ and $K = 6$. In the TBAS scheme, the feedback word emphasizes resource block allocation. Only one bit is used for the antenna selection and about 1 dB SNR of gain can be achieved against random allocation. The SBB-1 method provides more performance gain than the TBAS scheme does. In the SBB-1 method, four bits are used for frequency allocation and four bits are used for antenna selection. More precisely, $N = 8$ resource blocks are divided into four sub-blocks. A sub-block includes two resource blocks, and one bit is needed to indicate the best one. In addition, one bit is used to refer to the best antenna of the indicated frequency resource block for each sub-block. The performance can be further improved by using RBAS at the cost of the increased feedback overhead. In the RBAS schemes, the antenna selection is emphasized in the feedback word. Resource block allocation on top of the antenna selection enables to finding spectrally efficient channels. The three bit best-1 feedback word significantly enhances the performance when RBAS is applied. If the best channel is allocated and the antenna selection is correct, the peak rate out of the NN_t potential channels is obtained. The SBB-1 feedback principle is the most promising one when RBAS is applied. The combination of the RBAS and SBB-1 is based on the two sub-blocks in Fig. 35. Thus, four feedback bits are needed to indicate the best resource blocks of the sub-blocks. On the other hand, if the rate adaptation follows only the large scale fading and outage capacity constraint, the RB-wise one bit scheme with STBC provides attractive outage capacity as illustrated in Fig. 36. The RB-wise one bit scheme with an optimized threshold value would provide even more performance gain over random or fixed allocation.

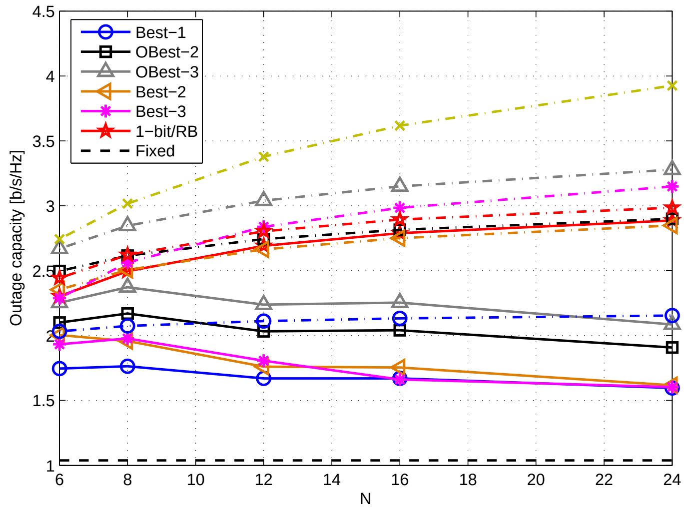


Fig 34. Average outage capacity of fair allocation vs. the number of RBs with constant rate transmission, $K = N$, $P_{\text{out}} = 0.1$, and $\bar{\gamma} = 10$ dB. Solid curves represent performance with $p_b = 0.05$ and dash dotted curves with $p_b = 0$. Cross marked dash dotted curve shows the performance of the optimal allocation with $p_b = 0$.

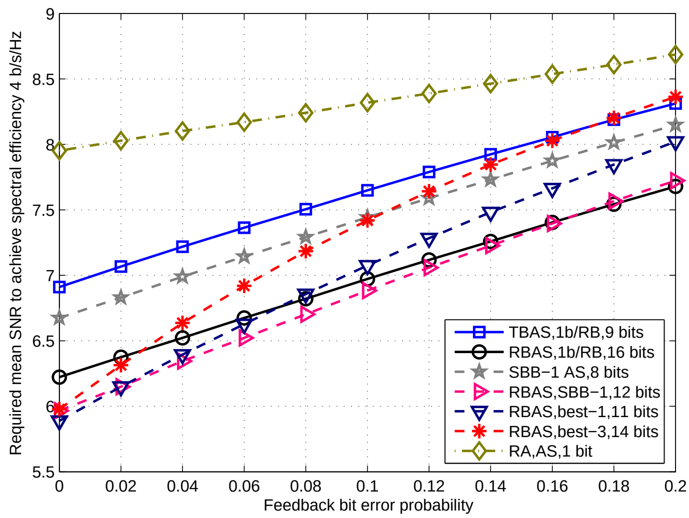


Fig 35. Required mean SNR to support 4 b/s/Hz/user vs. feedback bit error probability with fair RB allocation, $N = 8$, $K = 6$, $N_t = N_r = 2$.

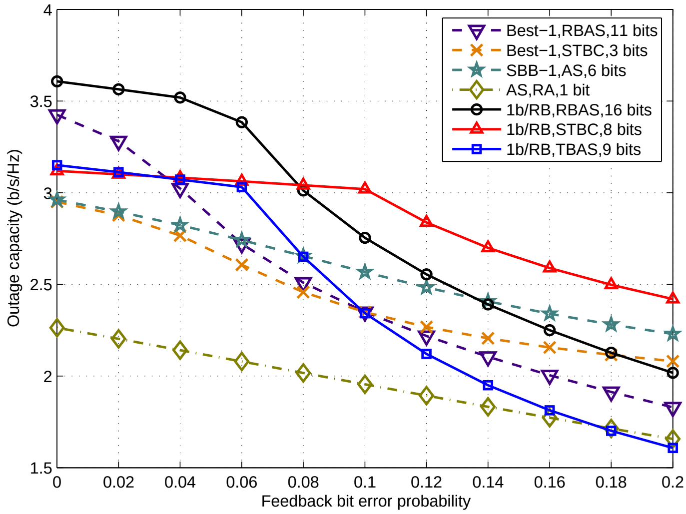


Fig 36. Outage capacity vs. feedback BEP in fair OFDMA with variable rate transmission $N = K = 8$, $P_{\text{out}} = 0.1$, $N_t = 2$, $N_r = 1$.

The results for the SDMA systems are basically similar as those for the MIMO schemes with a single user per RB. The RB-wise one bit feedback scheme is advantageous when outage capacity is examined whereas the SBB-1 feedback method is attractive when optimal rate adaptation is applied at the BS. The SNR gain of the fair OFDMA-SDMA RB allocation is illustrated in Fig. 37 with $N = K = 16$. Resource block allocation based on the best-1 feedback method improves the average capacity significantly, since only a single resource block is allocated for each user. If more resource blocks are allocated per user, the best-1 feedback method would probably not be suitable. In the SBB-1 feedback method, the space-time-frequency resource blocks are divided into four sub-blocks, i.e, it recommends four channels out of 16 alternatives. The tradeoff between the average capacity and the feedback overhead provided by the SBB-1 feedback strategy is advantageous.

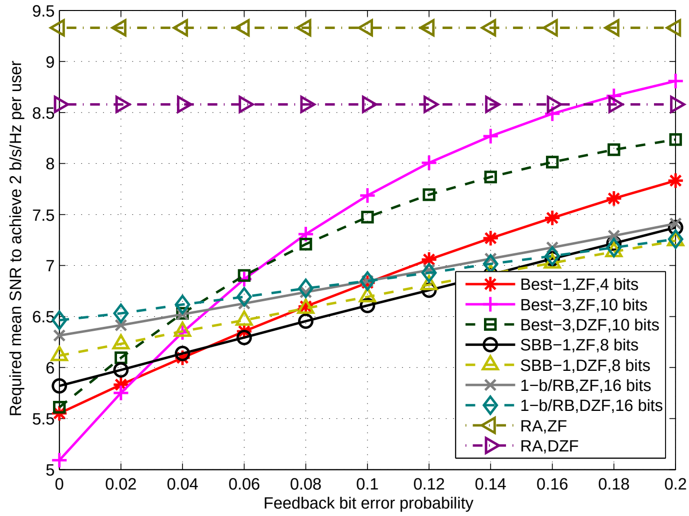


Fig 37. Required mean SNR to support 2 b/s/Hz/user vs. feedback bit error probability with fair OFDMA-SDMA, $N_t = 4$ **for DZF,** $N_t = 2$ **for ZF,** $N_r = 2$**,** $N = K = 16$**.**

6.2 Analysis versus practical fading channel

In order to show that analytical results provide suitable approximations of the performance in a practical environment, spectral efficiency results are simulated with the RB selection described in Section 2.1. The size of FFT is 2048 and 1728 subcarriers are used. A RB consists of 12 adjacent subcarriers. Users form feedback information from 8 selected RBs so that there are 18 RBs between the used RBs.

Figs. 38 and 39 illustrate the capacity versus user position in the allocation queue with $p_b = 0.05$, $N_t = N_r = 2$, $N = K = 8$ and $\bar{\gamma} = 10$ dB. A single sample SNR is taken from each RB. An exponentially decaying channel with 20 MHz bandwidth and a 0.5 microsecond root mean square value of delay are used in the simulations. We can see that the analytical results are close to those obtained in simulations.

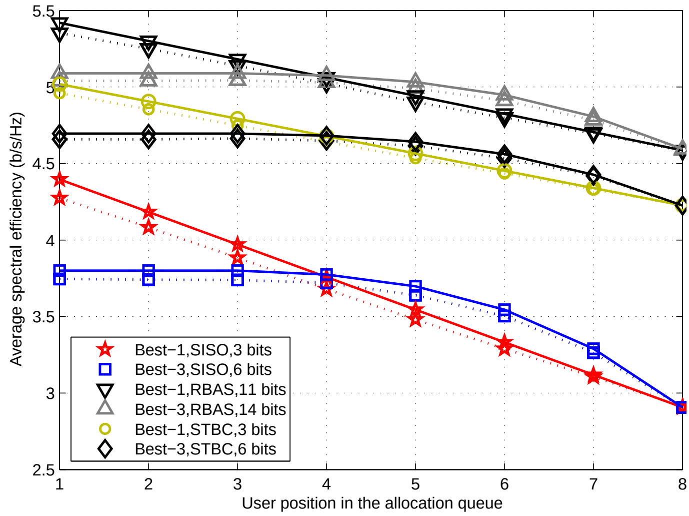


Fig 38. Average spectral efficiency vs. user position in the allocation queue with best-M feedback, $p_b = 0.05$, $N_t = N_r = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB. Solid curves present analytical results and dotted curves illustrate simulation results.

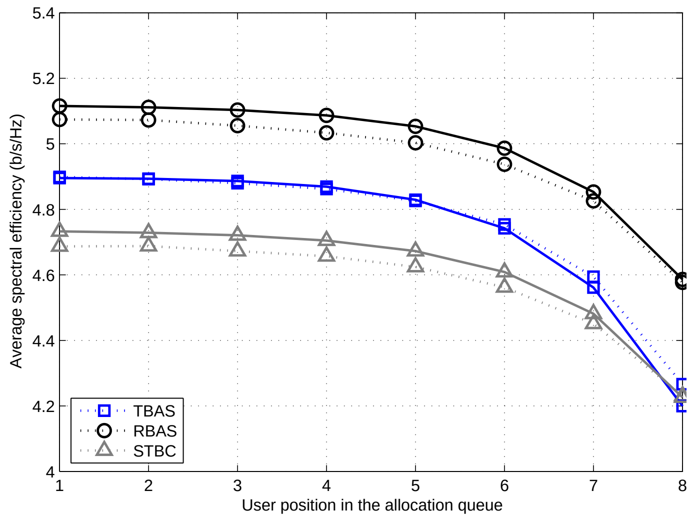


Fig 39. Average spectral efficiency vs. user position in the allocation queue with RB-wise one bit feedback, $p_b = 0.05$, $N_t = N_r = 2$, $N = K = 8$, $\bar{\gamma} = 10$ dB. Solid curves present analytical results and dotted curves illustrate simulation results.

Further example results are simulated using an extended typical urban (ETU) channel, which represents high frequency selectivity [141]. Analytical and simulated outage capacity results are collected into Table 3 when $\bar{\gamma} = 6$ dB and $P_{\mathrm{out}} = 0.1$. The performance was evaluated based on the average SNR over subcarriers at the RB. Note that in the ETU channel the subcarriers in the RB are not fully correlated. Thus, averaging removes the effect of the peak and the most faded subcarriers. Consequently, the performances of the last users are better in the simulations than in analytical results. If a RB consists of one subcarrier, the performance of the last user would be the same as that in random allocation.

Table 3. Outage capacity (b/s/Hz) of the kth user, $P_{\mathrm{out}} = 0.1$ $\bar{\gamma} = 6$ dB.

method / user	Best-1	Best-2	Best-3	1-b/RB	optimum
analysis / $k = 1$	2.70	2.70	2.70	2.06	2.70
simulation / $k = 1$	2.49	2.49	2.49	2.03	2.49
analysis / $k = 4$	1.04	2.07	2.30	2.02	2.31
simulation / $k = 4$	1.21	1.97	2.17	2.00	2.19
analysis / $k = 6$	0.68	0.96	1.40	1.60	1.80
simulation / $k = 6$	0.88	1.12	1.46	1.56	1.78
analysis / $k = 8$	0.51	0.51	0.51	0.51	0.51
simulation / $k = 8$	0.73	0.74	0.75	0.75	0.75

7 Conclusion

7.1 Summary

The analysis of a MIMO-OFDMA system with limited feedback was considered in this thesis. The study focused particularly on MIMO-OFDMA resource block (RB) allocation based on the best-M feedback strategies and on RB-wise quantized SNR information. The first chapter included background for the system model and a literature review of the prior work related to the topic in consideration. The system model was presented in Chapter 2. The mobile terminals were assumed to send feedback information only from a few uncorrelated resource blocks in order to achieve a reasonable feedback overhead also with a large number of resource blocks in the frequency domain. Consequently, the fading of the resource blocks was modelled to be IID. This assumption enabled a communication theoretic approach for the performance evaluation of OFDMA systems. The allocation was based on the practical round robin principle, which is suitable for delay critical communications such as VoIP or video streaming.

Chapter 3 addressed the analysis of the OFDMA system with the feedback information based on SNR quantization. The analysis concentrated on RB-wise one bit quantization which generates feasible feedback overhead for practical systems. However, many performance expression were also valid for more general quantizations with an arbitrary number of bits. The BEP performance was analyzed for a SISO link when uncoded BPSK was assumed. The analytical expressions for outage capacity and system average capacity with optimal rate adaptation were derived for several MIMO methods. AS and STBC were used at the transmitter and MRC was applied at the receiver when each RB was allocated exclusively to a single user. Simple OFDMA-SDMA schemes were also analyzed when zero forcing (ZF) detection was assumed at the receiver. The RB-wise one bit feedback method provided significant allocation gain with regard to the random or fixed allocation and robustness against feedback bit errors with suboptimal quantization and simple RR RB allocation. The combination of STBC and RB-wise one bit feedback turned out to be promising when the feedback channel was erroneous and the outage capacity was used to measure the performance. The combination of RBAS and RB-wise one bit feedback provided good average capacity with optimal rate adaptation at the cost of increased feedback overhead. TBAS yielded AS gain over single antenna transmission with low feedback overhead, provided that the feedback channel was reliable. SDMA-OFDMA without diversity transmission and with optimal rate adaptation provided the highest system spectral efficiency for moderate and high SNR values. On the other hand, the spectral efficiency per user was lower in SDMA than that provided by AS.

The performance of OFDMA RB allocation with the best-M feedback strategies was characterized in Chapter 4. The BEP performance and the outage capacity provided by three different best-M methods were investigated when a SISO link was assumed. The outage capacity and the average capacity with optimal rate adaptation were investigated for the same MIMO methods as those considered for the RB-wise SNR quantization scheme. The best-M feedback strategies provided substantial allocation gain provided that the feedback channel was reliable. On the other hand, RB allocation based on the conventional best-M feedback information was sensitive to feedback bit errors. The modified best-M method, namely, the SBB-1 feedback strategy, was relatively robust against feedback errors and it also reduced the feedback overhead of the best-M method. The combination of the SBB-1 method and AS or SDMA turned out to provide promising spectral efficiency gain with an imperfect feedback channel too when compared to random allocation.

Chapter 5 investigated the average capacity of a system with dynamic OFDMA and a wireless infrastructure AF relay node. Multiple antennas were assumed for each end. Frequency were allocated according to the limited feedback information which was based on the best-M information or RB-wise one bit quantization. STBC and AS multi-antenna schemes were studied. Optimal rate adaptation was assumed for the assigned channels. The results indicated that simple and practical RR allocation provides significant performance gain with very limited feedback information. The SBB-1 feedback method provided substantial performance improvement over random allocation with low feedback overhead in the case where a single RB per user was allocated. The RB-wise one bit feedback strategy achieved the best performance as well as robustness against feedback bit errors.

The results demonstrated that the best feedback method for the resource block allocation and multiple antenna transmission is not always the same, but it depends on the employed performance measure and several system parameters. Thus, the introduced analytical expressions offer useful tools to compare the feedback reduction schemes with desired parameters. One of the purposes of Chapter 6 was to compare the combinations of feedback and multiple antenna schemes. The results indicated that RB-wise SNR based quantization is more robust against feedback bit errors than the best-M feedback methods are. On the other hand, the SBB-1 method turned out to provide good robustness. When the outage capacity was considered with an unreliable feedback channel, the STBC or SDMA and RB-wise one bit feedback scheme was a good choice, whereas in the case of optimal rate adaptation, the SBB-1 method with AS or SDMA provided an attractive tradeoff between feedback overhead and performance.

The analytical expressions were derived based on the assumption of IID fading. The other purpose of Chapter 6 was to show that the analytical results with an idealized channel model provide a suitable performance benchmark when the RBs are selected properly. In the simulations, the receivers conveyed feedback information to the transmitter only from a few RBs which were as far from each in the frequency domain as possible. The results in Section 6.2 illustrated that the analytical results are close to those obtained via computer simulations in a practical frequency selective fading channel.

7.2 Future work

There are several open research issues related to RB allocation with limited feedback. In this thesis, RB allocation was based on the RR allocation in which a single RB was assigned per user. In addition, the number of users was limited to be equal or less than the number of RBs. The results could be different when several RBs per user are assigned. Based on the results proposed in this thesis, the analysis could be straightforwardly extended for RR allocation assigning several RBs per user. In the best-M feedback schemes, the parameter M should be equal or larger than the number of the allocated channels to obtain good performance. Similarly, the number of the indicated channels or sub-blocks should be large enough in SBB-M methods.

The average capacity results were presented for optimal rate adaptation, which is not the most suitable performance measure for practical systems. There is a separate feedback word for the modulation and code rates in practical systems. For example, the same transmission rate could be used for each RBs above the threshold in the RB-wise one bit quantization case [104]. Only a single transmission rate is needed to be indicated from the receiver to the transmitter. The analytical expressions could be straightforwardly extended to cover such practical rate adaptation. In the best-M feedback methods, several feedback methods for the instantaneous transmission rate could be studied. If the transmission rate was indicated for all the M best channels, the performance would be improved at the cost of increased feedback overhead

In this thesis, the threshold value in the RB-wise one bit feedback scheme was suboptimal. One could study threshold optimization in the RB-wise one feedback scheme. It would be interesting to see the impact that the number of required channels per user has on the threshold value. Practical rate adaptation migth also need threshold optimization [104].

The used RR allocation is simple and practical, but the performance of more sophisticated RB allocation methods could also be worthy of further study. The allocation gain could also be studied in the case where the number of users is larger than the number of RBs and only the preferable users according to the received feedback information are allocated. In such allocation the fairness issue is important to consider using, e.g., proportional fair allocation.

Yet, another important future research topic is to extend the results for transmit beamforming with limited feedback. In an OFDMA system the feedback design for beamforming is a difficult problem due to the increasing feedback overhead. Beamforming based on a similar principle as TBAS could be interesting to analyze.

Relay based OFDMA networks also includes several open research problems. The presented results could be extended for the case of the practical rate adaptation. The feedback methods and analytical results could be extended for a selection between the RN and BS when UEs receive a strong signal from both nodes. It would be interesting to compare RB-wise node selection and node selection where the UE is connected only to a single node. Furthermore, the optimization of the RB allocation and feedback design for multiuser relay networks includes open problems.

Since the most suitable combination of feedback method and multiple antenna scheme depends on several parameters, the design of adaptive feedback through system level simulations could be an important research issue to find a practical feedback protocol which provides good performance for several services with different QoS. The feedback form could be adapted according to user location, i.e., average performance and QoS. Adaptive and opportunistic feedback protocols could be combined with the RB selection method considered in the thesis. Moreover, the interference limited multicell environment could have on impact on the feedback design.

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Appendix 1 Useful integrals

In this Appendix, the closed-form expressions of useful integrals are derived in terms of the exponential integral $E_1(a) = \int_a^\infty \frac{e^{-u}du}{u}$ and the complementary incomplete gamma function defined as $\Gamma(t,x) = \int_x^\infty s^{t-1}e^{-s}ds,\ x \geq 0$ [133].

An integral needed to derive capacity results in single hop systems is expressed as

$$\mathcal{I}_1(i, \gamma_{\mathrm{T}}, \alpha) = \int_{\gamma_{\mathrm{T}}}^\infty \frac{\mathcal{P}_2^i(\alpha\gamma)}{1+\gamma} d\gamma, \tag{167}$$

when the rate adaptation is based on the quantized SNR information and the MRC receiver with $N_r = 2$ is applied. The capacity for the STBC scheme which is $\mathcal{I}_1(1, 0, N_t/\bar{\gamma})$ can be found in [127]. The function $\mathcal{P}_2(\alpha x)$ can be expressed as $(1+\alpha\gamma)^i e^{-i\alpha\gamma} = \sum_{l=0}^i \binom{i}{l}(\alpha\gamma)^i e^{-i\alpha x}$ after the binomial expansion. It remains to integrate the formula

$$\begin{aligned}
\mathcal{I}_2(i, \theta, \gamma_{\mathrm{T}}) &= \int_{\gamma_{\mathrm{T}}}^\infty \frac{\gamma^i e^{-\theta\gamma}}{1+\gamma} d\gamma \\
&= \int_{1+\gamma_{\mathrm{T}}}^\infty \frac{(t-1)^i e^{-\theta(t-1)}}{t} dt \\
&= e^\theta \sum_{m=0}^i \binom{i}{m} (-1)^{i-m} \int_{1+\gamma_{\mathrm{T}}}^\infty t^{m-1} e^{-\theta t} dt,
\end{aligned} \tag{168}$$

where we have substituted $t = 1+\gamma$ for the first expression and then the binomial expansion. Using substitution $s = \theta t$ we obtain

$$\mathcal{I}_2(i, \theta, \gamma_{\mathrm{T}}) = e^\theta (-1)^i E_1(\theta(1+\gamma_{\mathrm{T}})) + e^\theta \sum_{m=1}^i \binom{i}{m} (-1)^{i-m} \theta^{-m} \Gamma(m, \theta(1+\gamma_{\mathrm{T}})). \tag{169}$$

The capacity in (167) can be rewritten as

$$\mathcal{I}_1(i, \gamma_{\mathrm{T}}, \alpha) = \sum_{l=0}^i \binom{i}{l} \alpha^i \mathcal{I}_2(i, i\alpha, \gamma_{\mathrm{T}}). \tag{170}$$

In several capacity derivations, we need the integral expressed as

$$\mathcal{I}_3(\alpha, a) = \int_a^\infty \frac{e^{-\alpha\gamma} d\gamma}{1+\gamma} = e^\alpha \int_{\alpha(1+a)}^\infty \frac{e^{-\gamma} d\gamma}{\gamma} = e^\alpha E_1(\alpha(1+a)). \tag{171}$$

Appendix 2 Integrals for the analysis of relay enhanced link

Integrals needed in the capacity analysis of a relay enhanced link is derived in this appendix. We need to derive to integral expressed as

$$\mathcal{I}_5(\gamma_{\mathrm{T}}, i) = \int_{\gamma_{\mathrm{T}}}^{\bar{\gamma}_{\mathrm{SR}}} \frac{\left(\mathcal{P}_2\left(\frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{\bar{\gamma}_{\mathrm{SR}}-\gamma}\right)\right)^i}{1+\gamma} ds. \tag{172}$$

Applying binomial expansion, the substitution $t = \frac{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)\gamma}{(\bar{\gamma}_{\mathrm{SR}}-\gamma)}$ and partial fraction decomposition to (172), after some calculus we have

$$\begin{aligned}\mathcal{I}_5 &= \int_{\gamma'_{\mathrm{T}}}^{\infty} \sum_{j=0}^{i} \binom{i}{j} t^j e^{-it} \left(\frac{1}{\alpha+t} - \frac{1}{\alpha(\bar{\gamma}_{\mathrm{SR}}+1)+t}\right) dt \\ &= \sum_{j=0}^{i} \binom{i}{j} \Big(\mathcal{I}_6(\gamma'_{\mathrm{T}}, j, i, \alpha) - \mathcal{I}_6(\gamma'_{\mathrm{T}}, j, i, \alpha(\bar{\gamma}_{\mathrm{SR}}+1))\Big), \end{aligned} \tag{173}$$

where $\gamma'_{\mathrm{T}} = \frac{(\bar{\gamma}_{\mathrm{SR}}+1)\gamma_{\mathrm{T}}}{(\bar{\gamma}_{\mathrm{SR}}-\gamma_{\mathrm{T}})}$ and $\mathcal{I}_6(\gamma'_{\mathrm{T}}, j, i, \alpha) = \int_{\gamma'_{\mathrm{T}}}^{\infty} t^j e^{-it}/(\alpha+t)dt$. The second required integral $\mathcal{I}_6$ can be expressed as

$$\begin{aligned}\mathcal{I}_6(\gamma'_{\mathrm{T}}, j, i, \alpha) &= e^{i\alpha}(-\alpha)^j E_1(i(\gamma'_{\mathrm{T}}+\alpha)) \\ &+ e^{i\alpha} \sum_{m=1}^{i} \binom{i}{m} i^{-m}(-\alpha)^{i-m}\Gamma(m, i(\gamma'_{\mathrm{T}}+\alpha)), \end{aligned} \tag{174}$$

after the substitution $\gamma = \alpha + t$, binomial expansion and the substitution $x = \alpha\gamma$.

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