ANALYSIS OF PARTIAL VOLUME EFFECTS IN DIFFUSION-TENSOR MRI

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Introduction

The organization of tissues in the brain is heterogeneous and complex. Consequently, partial volume effects may significantly influence the accuracy of DT-MRI measurements. A recent study showed that diffusion measurements in human brain were not adequately described by a single DT [1]. This is particularly true for most DT-MRI studies that use EPI techniques with relatively large voxels (~2-4 mm on a side). In this study, we present a two-DT compartment model to describe simple partial volume effects.

Properties of the diffusion tensor

In a DT-MRI experiment, the measured signal

$$S(b,\hat{\boldsymbol{g}}) = S_o \exp(-b\hat{\boldsymbol{g}}^T \underline{\boldsymbol{D}} \hat{\boldsymbol{g}}) \quad (1)$$

depends upon the diffusion-weighting, b, the encoding unit vector, $\hat{\boldsymbol{g}}$, and $\underline{\boldsymbol{D}}$. A minimum of six non-colinear measurements ($\hat{\boldsymbol{g}}$) are required to fully estimate the DT. Eq 1 demonstrates that the signal is exponentially weighted projections of the DT. The projected diffusion values in the measurement space are equivalent to

$$D_m(\hat{\boldsymbol{g}}) = \hat{\boldsymbol{g}}^T \underline{\boldsymbol{D}} \hat{\boldsymbol{g}} = \frac{\ln(S(b,\hat{\boldsymbol{g}})) - \ln S_o}{b}. \quad (2)$$

Examples of the measurement profiles are shown in Fig.1

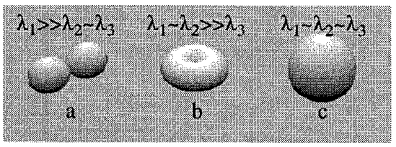

Fig 1. 3D $D_m(\hat{\boldsymbol{g}})$ profiles for (a) linear, (b) planar, and (c) spherical diffusion tensors.

Partial volume effects in DT-MRI

For a 2-DT compartment system, the signal will depend upon the exchange between compartments. Only two extreme cases -- rapid exchange (rx) and no exchange (nx) -- are considered. The behavior for intermediate exchange cases will be between the two extreme cases. For rx, the DT compartments are indistinguishable,

$$S_{rx}(b,\hat{\boldsymbol{g}}) = S_o \exp(-b\hat{\boldsymbol{g}}^T (f\underline{\boldsymbol{D}}_1 + (1-f)\underline{\boldsymbol{D}}_2)\hat{\boldsymbol{g}}), \quad (3)$$

where f and $(1 - f)$ are the relative proportions of the two tensor components $\underline{\boldsymbol{D}}_1$ and $\underline{\boldsymbol{D}}_2$. Conversely, the signal for no exchange between the compartments is

$$S_{nx}(b,\hat{\boldsymbol{g}}) = S_o(f\exp(-b\hat{\boldsymbol{g}}^T \underline{\boldsymbol{D}}_1 \hat{\boldsymbol{g}}) + (1-f)\exp(-b\hat{\boldsymbol{g}}^T \underline{\boldsymbol{D}}_2 \hat{\boldsymbol{g}})). \quad (4)$$

It is clear in this case that a single tensor model (Eq 1) will lead to erroneous calculations.

Materials and Methods

The 2-DT compartment model was simulated for several cases. We considered three DTs [λ_1, λ_2, λ_3] (10^{-3} mm^2/sec) that represent common tissues in the brain - white matter (WM) [1.4, 0.35, 0.35], gray matter (GM) [0.7, 0.7, 0.7], and CSF [2.0, 2.0, 2.0]. Three combinations were considered: (1) WM & WM, (2) WM & GM, and (3) WM & CSF. For the WM & WM case, the angle between the major eigenvectors of the DTs were varied between 0 and 90 degrees. f was assumed to be 0.5 in all cases. The simulation was performed for a range of b - between 100 and 3000 sec/mm^2. Two common DT encoding schemes were studied and compared for each of the cases: (i) orthogonal encoding (ORTH): $\hat{\boldsymbol{g}}$ = (1 0 0), (0 1 0), (0 0 1), 0.707(1 1 0), 0.707(1 0 1), 0.707(0 1 1), and (ii) oblique double gradient encoding (ODG) - see [3]. The orientation of the DTs were rotated with respect to the encoding axes to examine the rotational dependence. For each simulation, the trace, $\mathcal{T}$ (10^{-3} mm^2/sec), and fractional anisotropy, FA, [2] of the estimated single-compartment DT was calculated. Effects of measurement noise were not considered.

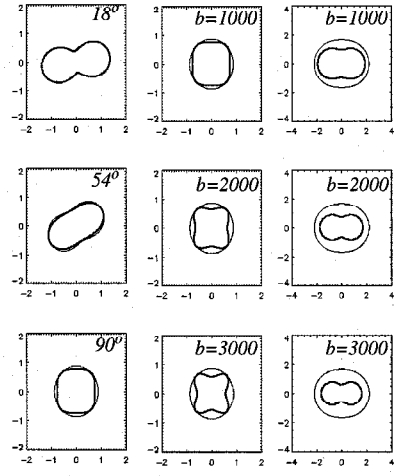

Fig 2. 2D measurement profiles for 2-DT compartments. The rx and nx (bold) profiles are both plotted. *Left column*: two WM compartments at different angles. b=1000. *Middle column*: two perpendicular WM components for different b-factors. *Right column*: WM and CSF compartments at different b-factors.

Results

$D_m(\hat{\boldsymbol{g}})$ profiles (Eq 2) for both rx (Eq 3) and nx (Eq 4) partial volume simulations are shown in Fig 2. The estimated $\mathcal{T}$ and FA values for ODG encoding and two perpendicular WM compartments are listed in Table 1. The rx values can be compared with the minimum and maximum values obtained for the nx case.

Table 1. tr and FA values for WM & WM with ODG encoding (Fig 2mid)

b (sec/mm^2)	$\mathcal{T}_{rx}$	$\mathcal{T}_{min}$	$\mathcal{T}_{max}$	FA_{rx}	FA_{min}	FA_{max}
500	2.1	2.03	2.06	0.41	0.32	0.44
1000	2.1	1.96	2.03	0.41	0.22	0.46
1500	2.1	1.90	2.00	0.41	0.13	0.49
2000	2.1	1.85	1.96	0.41	0.05	0.52

Note: For single WM compartment; tr = 2.1 & FA = 0.71

The results for the ORTH encoding were similar except that the FA range was less sensitive to b, while the $\mathcal{T}$ range was larger. The WM & CSF case showed a significant dependence for the nx case as the b-factor was increased (both encodings). The DT measurements appeared more isotropic for low b and more anisotropic for high b. The difference between the nx and rx cases was very small and relatively independent of b for the WM & GM case.

Discussion

Partial volume effects are significant in DT-MRI. For little or no exchange, the trace will tend to be underestimated (Table 1). The uncertainty of FA values also increased significantly with b. For very high b (=3000) and linear tensors [2.1, 0, 0], $\mathcal{T}_{min}$ decreased by approximately 75% (data not shown).

Underestimation of the trace could lead to misleading results when evaluating potential patients with stroke or ischemia. Uncertainties in the anisotropy measurements will influence the characterization of WM pathology and WM tracking. These results obviously warrant further investigation into the impact of partial voluming in DT-MRI.

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References

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